# Science in Court Society

Giovan Battista Benedetti's
*Diversarum speculationum mathematicarum et physicarum liber*
(Turin, 1585)

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Pietro Daniel Omodeo and Jürgen Renn

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# Introduction

Giovanni Battista Benedetti is today a lesser known figure in the history of early modern science. This relative oblivion is in striking contrast to the fame he enjoyed during his lifetime as a prominent mathematician and mathematical philosopher of Venetian origin and reputable courtier to the Savoy dukes in Turin. Among his admirers, the astronomer Johannes Kepler regarded him as one of the few Italians to significantly contribute to the advancement of mathematics in his time: "The Italians are asleep with the sole exceptions of Commandino and Giovanni Battista Benedetti. And in fact, Clavius is German."[1] For his part, the mathematician of the *Collegio Romano*, Christopher Clavius, extolled Benedetti's scientific merits in the 1589 edition of his reputed commentary of Euclid, *Euclidis Elementorum Lib[ri] XV*. In the dedicatory letter to Carlo Emanuele I of Savoy (*Serenissimo Principi ac Domino D. Carolo Emmanueli Sabaudiae Duci*), he praised "his court mathematician" Benedetti as "very expert in mathematics" (*scientissimus rerum Mathematicarum*).[2] In contrast to the eulogies of the Imperial mathematician Kepler and the most prominent astronomers of the Jesuit order, the Urbino school gathered around the mathematical purist Federico Commandino was rather reluctant to acknowledge his achievements, probably due to rivalry, reciprocal misunderstandings, and different philosophical and cultural choices. Among Commandino's pupils, Guidobaldo Del Monte severely criticized Benedetti's approach to mechanics and his claim to originality, as documented by his manuscript annotations on mathematical issues.[3] Another member of the Urbino school, Bernardino Baldi, gave an extremely negative and reductive portrait of Benedetti in his collection of short biographies of mathematicians from all epochs, *Cronica de matematici.*

> The Venetian Giovanni Battista Benedetti occupied himself with mathematics, a field in which he served the dukes of Savoy. He wrote a book on gnomonics, which dealt with many proofs belonging to this discipline. However, he has been criticized by the most exquisite scholars for not having respected the method and the purity of explanation which mathematics requires and which was respected by the Greek masters and by their followers. He also wrote some light things of no great import.[4]

Such a harsh judgment can only be explained on the basis of a profound enmity held by Commandino's followers against Benedetti. This deserves attention since it also influenced the reception of his work. Therefore, we will offer a reconstruction of Benedetti's

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[1] Kepler 1937–2001, 390: "Itali somniant (preter unum Commandinum et Joh[annem] Baptistam Benedictum, Clavius enim Germanus est)."

[2] Clavius 1589, ff. *4r–*5r. The list of Benedetti's admirers also includes the Pisa philosopher Jacopo Mazzoni, the Venetian intellectual leader Paolo Sarpi, and the French scholar Marin Mersenne. See Cappelletti 1966, 262.

[3] See Renn and Damerow 2012 and Renn and Omodeo 2013.

[4] Baldi 1707, 140: "GIO[VANNI] BATTISTA Benedetti veneziano attese alle matematiche, nelle quali servì i Duchi di Savoia. Scrisse un libro di gnomonica, il quale toccò molte cose appartenenti alle dimostrazioni della detta disciplina, se non che viene ripreso da più esquisiti di non haver'osservato quel metodo, e quella purità dell'insegnare, che ricercano le matematiche, et è stato osservato dagl'ottimi Greci, e dagl'imitatori loro. Scrisse anco alcune altre cose leggiere, e di nessun momento." Here and in the subsequent pages, Italian and Latin grammar (e.g., capitalization and punctuation) has been modernized.

cultural environment and the scientific culture of Renaissance Italy in order to understand his work, its roots, and legacy.

Although Benedetti was recognized by his contemporaries, in many respects the theoretical and historical relevance of his work remains obscure. The obscuration of Benedetti is also the result of the prominence of the Urbino school and their bias towards mathematical purism. Moreover, the hagiographic historiography of science sought out the precursors to Galileo and therefore almost naturally adopted the criticism of Benedetti by Galileo's patron Guidobaldo del Monte. Intrinsic factors also played a role in the eclipse of Benedetti's fame, principal among which is the relative rarity of his major work, *Diversarum speculationum mathematicarum et physicarum liber* (Book Including Various Mathematical and Physical Speculations, 1585), not to mention that of most of his earlier publications. Benedetti's fragmentary style is a special difficulty faced by the reader, a defect that was overemphasized by Baldi and denounced by early modern scholars such as Claude-François Milliet Dechales. Dechales dealt with Benedetti's geometry in his *Cursus seu mundus mathematicus* (1690), observing that "the end of his work [the *Diversae speculationes*] comprises many miscellaneous geometrical remarks, some of which are good, in particular with regard to their special concern, but [they are] disordered."[5]

Following historiographic commonplaces and their nineteenth-century crystallization, recent historians of science have tended to neglect Benedetti's work due to the lack of extensive translations of his writings into modern languages, with the exception of the excerpts included in Drake and Drabkin, *Mechanics in Sixteenth-Century Italy* (1969). This anthology isolated passages that were exclusively devoted to what the editors saw, in hindsight, as the most relevant contributions for the progress of mechanics. Due to its selective nature, this publication did little justice to the complexity and richness of Benedetti's stature in the history of science. In fact, isolating certain results elides recognition of the multilayered architecture of a book such as the *Diversae speculationes*, which is characteristic of Renaissance science. Benedetti's showcase of mathematical erudition and scholarship is thus omitted and obscured. Drake and Drabkin's selection could only yield a reductive and rather misleading image of Benedetti as a scientist and thinker. As we will argue, entire paragraphs or chapters expunged from Drake and Drabkin's translation, for instance those concerning philosophy and cosmology, are relevant for an understanding of the author's general conception of mechanics and physics. From a historical viewpoint, it is hardly possible to trace disciplinary boundaries in the Renaissance that fit those established today. Mechanics was an emerging discipline at the crossroads of mathematics, engineering, and natural philosophy. Hence, a reassessment of Benedetti's work in its entirety is necessary not only to understand his personality but also to grasp the scientific culture of his age as the result of interdisciplinary controversies.

This open access edition makes the *Diversarum speculationum mathematicarum et physicarum liber* accessible to a large scholarly readership. Benedetti's volume is a major contribution to Renaissance science, especially due to its insights into mechanics, the mathematization of (or geometrical approach to) natural investigation, and the connection of celestial and terrestrial dynamics in a post-Copernican perspective. The first edition of this work was an elegant folio, which included heterogeneous writings on technical and philosophical issues as well as on mathematics and physics. Benedetti presented them as short treatises (*tractatus*) or letters (*epistolae*) addressed to gentlemen, courtiers, scholars, engineers, and practitioners of different arts. The volume was printed by Niccolò Bevilac-

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[5]Ventrice 1985, 188: "in fine sui operis multa habet miscellanea geometrica, quorum nonnulla ad sectiones praesertim pertinentia bona sunt, sed inordinata."

qua's heir (*apud haeredem Nicolai Bevilacquae*), who was the owner of the main printing house in Turin, which was directly supported by the dukes of Savoy. The *Diversae speculationes* appeared in a series of prestigious volumes aimed at celebrating the magnificence of the court and the capital, including the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), on Turin, *Inclytorum Saxoniae Sabaudiaeque principum arbor gentilitia* (1581), on the genealogy of the ruling family, and *Sindon evangelica* (1581), on Christ's shroud, which had been recently transferred from Chambéry to Turin. The *Diversae speculationes* exhibits the same celebratory intention. The volume aimed to make the quality of the court mathematician's research and skills publicly appreciable. It also bore witness to the intensity of the cultural debates going on in Turin, and connected this city with other centers of learning, especially Venice.[6] Its miscellaneous and epistolary form was suitable for displaying the variety of the author's interests and for praising his patrons, friends, and colleagues by naming them as dedicatees or addressees of the different sections and letters.

The structure of the *Diversae speculationes*—its occasional and fragmentary character, its celebratory purpose, and the epistolary display of a network of personal connections—mirrors the socio-cultural embedment of Benedetti's work. We regard it as exemplary of "science in court society." As Norbert Elias has pointed out, the *höfische Gesellschaft*, or court society, is a particular social configuration (*gesellschaftliche Figuration*) characteristic of the transitional phase to an industrial and capitalist Europe, which we conventionally refer to as the early modern period.[7] During the Renaissance and the Ancien Régime, the court was (or became) a hegemonic center from which powerful elites mediated between the ruler and the socio-political environment. Benedetti's activities as a court mathematician exemplify such a function. In his role he was expected to interact with the upper classes and respect aristocratic etiquette, and to act as the Savoy "expert" on a wide range of pedagogical and technical issues linked to his profile as a mathematician and mathematical philosopher. He was required to teach geometry to the offspring of the ducal family, to supervise engineering and architecture projects, to produce instruments or machines for practical purposes, warfare, and recreation (such as fountains, sundials, or nautical instruments). He had to adhere to shared court values, norms, and behaviors, primarily those linked to honor and prestige. These courtly principles are reflected in the epistemic values permeating his scientific production, for instance in the value of scientific disinterestedness that marks his theoretical approach to practical as well as to speculative problems. In a hierarchical and aristocratic society, his theoretical attitude marked at once the continuity and the distance between his role as a court mathematician and those involved in practical activities. Moreover, the primacy of courtly interests over those of science as a purely scholarly endeavor (as it was pursued at universities and academies) is evident from Benedetti's networking strategies, which were aimed at not so much exchange with other scholars as at giving advice to a wide range of people, beginning with the ruling elites of the country. In other words, he was not primarily concerned with establishing a réseau, as was typical for the Republic of Letters. As we will show, he did not regard himself and his activity as part of a learned network but rather as the center of courtly interaction. This center-periphery structuring of his network mirrors—in two senses—the "knowledge economy" his work is embedded in. Sociologically, the central-

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[6]Cecchini and Roero 2004.

[7]As Norbert Elias put it (Elias [1969] 2002, 73): "Durch das Bemühen um die Struktur der höfischen Gesellschaft und damit um das Verständnis einer der letzten großen nicht-bürgerlichen Figurationen des Abendlandes eröffnet man sich also mittelbar zugleich einen Zugang zum erweiterten Verständnis der eigenen berufsbürgerlich-städtischen-industriellen Gesellschaft."

izing character of court society is reproduced in scientific policies through Benedetti's function as a technocrat; epistemologically, the fluid style and fragmentary form of the *Diversae speculationes* is an expression of the expert-advice character of his scientific work. Thus, in order to properly understand his work, we deem it necessary not only to investigate the technical and theoretical dimensions of Benedetti's science, but also to analyze evidence concerning the ties between these dimensions and the social and cultural environment.

Among the studies on Benedetti, Giovanni Bordiga's monograph *Giovanni Battista Benedetti: filosofo e matematico veneziano del secolo XVI* (1926) still stands out as one of the most important references as far as prosopographical information is concerned.[8] The excellent research accomplished by Carlo Maccagni and the proceedings of the conference on Benedetti held in Venice in 1985 at the *Istituto Veneto di Scienze, Lettere ed Arti* investigated many aspects of Benedetti's contribution.[9] In spite of the accuracy of these relatively recent Italian studies, Alexandre Koyré's evaluation of Benedetti's role in the first stages of modern science, in the *Études galiléennes* (1939), has had a more direct influence on his international reception. Koyré emphasized the link between the incipient mathematical science of motion and heliocentrism in Benedetti's speculations. On account of this, Paul Lawrence Rose regarded him as a herald of the "Italian Renaissance of mathematics".[10] Koyré's grand narrative of the Scientific Revolution, which he conceived as a development with a "prologue" (Copernicus) and "epilogue" (Newton) in the heavens, included Benedetti as a precursor to Galileo because of the interconnection of mathematical and physical themes in the former's work. Koyré's main thesis was that classical physics (the mathematical science of nature of Galileo, Descartes, and Newton) emerged as a direct consequence of Copernicus's geokinetic system, which undermined the traditional (Aristotelian and Ptolemaic) worldview. Although Benedetti's relevance as a source for Galileo cannot be denied, scholars now view Koyré's narrative as dubious due to its abstract treatment of the history of science, conceived of as an internal development of ideas. In the years of the Cold War (or shortly before it), this viewpoint embodied the ideological reaction to Marxist or materialist-oriented accounts, which stressed the technological, empirical, and social roots of modern mechanics, as was the case with Leonardo Olschki and Edgar Zilsel.[11] Bendetti's approach to mechanics and post-Copernican astronomy therefore appears to be an appropriate case study for reconsidering this general historical *problematique*, beginning with a reassessment of the relation between mechanics and astronomy in early modernity. This implies a reconsideration of the basic questions of the historiography of science and of historical epistemology, such as the role of material and intellectual factors in the so-called Scientific Revolution.

In this edition of the *Diversae speculationes*, we aim to present Benedetti's achievement in its rich complexity. Benedetti is emblematic both of his time and of the non-linearity of the historical process of Renaissance science with its multicentric institutions and scientific networks. We will show that the apparently fragmentary nature of his work is expressive of the peculiar character of science in court society and, in spite of this form, it conceals a fundamental unity of his conception of nature and method, both of which rest on geometry. To be sure, Benedetti regarded mechanics as a model, but he enlarged his

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[8] Bordiga 1985.

[9] Bordiga 1926, repr. Bordiga 1985, Maccagni 1967b, Maccagni 1967a, Maccagni 1983, and Istituto Veneto di Scienze 1987.

[10] Rose 1975, 154–156.

[11] The cultural-political intentions of Koyré's approach emerge most vividly from his 1943 article on Galileo's Platonism, see Koyré 1943. Lefèvre stresses it in Lefèvre 2001.

perspective to include the most varied fields of investigation in order to concretely demonstrate the fruitfulness of his approach to universal knowledge about astronomy, physics, meteorology, and even literature and ethics.

**Overview and Acknowledgments**

Our first chapter is devoted to Benedetti's biography and comprises an overview of his publications. We consider the various dimensions of his identity, such as his nobility as both a Venetian patrician and a Savoy aristocrat in the retinue of Emanuele Filiberto and Carlo Emanuele at the Turin court. In particular, we discuss his role as a courtier, the tasks he was entrusted with, and the relations he established in this context. We also deal with his extra-academic education and his attitude towards mathematics, which he initially saw as an intellectual instrument to be used against the "bookish" culture of the universities and the "idle" rhetoric of the humanists. Later he softened the polemical tone that characterizes his early writings. Moreover, since he saw himself as a philosopher, more precisely as a court philosopher to the Dukes of Savoy, a discussion of Benedetti's mathematics cannot be separated from his philosophical project. Benedetti claimed for himself the right to discuss in mathematical terms issues of natural philosophy that traditionally belonged to the rather qualitative and conceptual approach of the peripatetics. The *Diversae speculationes* is an altogether magisterial example of this merging of philosophical and mathematical perspectives.

Chapter 2 is a reconstruction of the cultural life of Renaissance Turin, the town in which Benedetti spent his mature years and where he composed his major work. His achievement was embedded in the cultural ferment of the new capital of Savoy, a place of ambitious town planning and civil reforms. It was a time in which the arts, literature, and philosophy received a new impetus. Editorial projects were launched; the university was reopened and illustrious scholars were attracted there. The dukes' religious politics was informed by a sense of pragmatism, which is mirrored in the fluctuating relations between the ruling family, the Jesuits, and Rome. Benedetti's secular attitude towards science and philosophy mirrors the cultural politics of his patrons. In addition we discuss his involvement in various scientific debates divided into courtly conversations, academic controversies, and controversies going beyond the settings of the court and the university. Among such extra-academic public controversies, the most important was Benedetti's public defense of the reliability of astronomical calculation against a polemist, Benedetto Altavilla, who indirectly attacked his and others' astrological practice. Newly discovered documents show that Benedetti's successor as court mathematician, Bartolomeo Cristini, continued that polemic after Benedetti's death. Cristini discredited Benedetti's use of astronomical tables to cast horoscopes, in order to ingratiate himself with the dukes and successfully start a career at court. We trust that this chapter offers new insights into the scientific culture of the Renaissance by bringing Turin into focus, a cultural centre that has so far escaped in-depth consideration by historians of early modern science.

In chapter 3 we offer an overview of the structure of the *Diversae speculationes*. We introduce Benedetti's mathematical sections in general terms, focusing on his geometrical demonstrations for the solutions of problems of arithmetic—which were the result of his private teaching of mathematics to the Savoy prince—his sketchy annotations on the theory of proportions based Book 5 of Euclid's *Elements*, and his considerations on linear perspective aimed at supporting the work of painters and architects. The sections on physics, mechanics, and natural philosophy are not discussed in this chapter as they re-

ceive special treatment in other chapters. The last part of the *Diversae speculationes* was a miscellanea of scientific letters. We discuss them vis-à-vis their significance as a mirror of Benedetti's social capital. His epistles were mostly directed to aristocrats, beginning with his patrons, other courtiers, and diplomats, especially those from Venice. He also corresponded with professors, artists, engineers, and practitioners, some of whom lived north of the Alps. His network was markedly a center-periphery one, in which the court expert shared his views on the most varied topics with others seeking his advice or opinion. Thus, it was not a scholarly network implemented for the sake of exchange and the advancement of knowledge. Rather than a networking activity establishing a Republic of Letters, Benedetti's correspondence reflected court-society centralism.

Chapter 4 addresses Benedetti's epistemology on the basis of passages regarding the certitude of mathematics and his effective use of mathematics in physics. His role as an early champion of what would later become known as "physico-mathematics" is understood here against the background of the philosophy of mathematics in the Renaissance. Moreover, we deem the modal epistemology underlying his science of particular interest: his treatment of nature in mathematical terms did not imply the necessary or deterministic nature of physical processes. Rather, he embraced an ontology and an epistemology of contingency that constituted a bridge between medieval scholastic views on nature and the mathematical physics of the time of Galileo and Descartes. We dedicate an excursus to the vision of nature as the realm of contingency in the period extending from the medieval science of weights to seventeenth century mechanics, and ascribe to Benedetti a central position in this intellectual process.

Chapter 5 deals with the field in which Benedetti has received the most credit from historians: mechanics. Actually, Benedetti himself emphasized the importance of his contribution to mechanics as what would secure his fame in posterity. We summarize his theories on equilibrium and his critical reworking of earlier theories such as those developed by Jordanus Nemorarius and Niccolò Tartaglia. We consider Guidobaldo Del Monte's negative reaction to Benedetti's mechanics in detail, as well as the weaknesses and strengths of both authors. We regard this pluralism of clashing and integrating views as revealing the complex paths of discovery undertaken by students of mechanics in a period of the utmost relevance to its modern systematization. Moreover, the subterranean conflict of views and approaches between Benedetti and Del Monte affected Galileo's work. His mechanics drew from both authors, although he did not acknowledge Benedetti explicitly due to circumstances and opportunity.

Chapter 6 summarizes Benedetti's astronomical work. Although he did not see himself as an astronomer, his contribution is quite interesting. He should be acknowledged for his effort to develop a new mathematical physics in accordance with post-Copernican astronomy. His discussion of astronomical theory against the background of a general philosophical reform was strikingly innovative. His specific polemics on the reliability of astronomical calculation also receive close treatment here. Furthermore, in an appendix Günther Oestmann offers an assessment of Benedetti's astrological calculations on the basis of so-far neglected manuscript sources containing two of his horoscopes.

In chapter 7 we deal with Benedetti's natural philosophy as he presented it in Book 4 of the *Diversae speculationes*. Although he entitled it "Disputations on Some Opinions Held by Aristotle" (*Disputationes de quibusdam placitis Aristotelis*), it was a polemic directed "against" fundamental Aristotelian theses on motion, time, space, matter, and cosmology. This is the section in which Benedetti's commitment to "the system of Aristarchus and Copernicus" most clearly emerges. It is also a fundamental section on the existence of the physical void as the necessary presupposition of any local displacement and on free

fall through different media. We see this book of the *Diversae speculationes* as a major contribution to the Renaissance debate on the foundations of physics, going far beyond the treatment of mechanics and cosmology *strictu sensu*. Hence, we take into consideration Benedetti's definition of space as an "inter-bodily gap" (*intervallum corporeum*), his defense of the possibility of actual infinity in nature against Aristotle's veto, his understanding of time as an absolute frame complementary to space and its place in the philosophical debates of the Renaissance, the revision of the concepts of natural and violent motion, and finally, the surprising conclusion of the "Disputations on Some Opinions Held by Aristotle" with a Copernican note.

This volume is a continuation of an Edition Open Access project aimed at the publication and scholarly reassessment of the fundamental sources of Renaissance mechanics. This project began with Jürgen Renn and Peter Damerow's *Guidobaldo del Monte's Mechanicorum Liber* in 2010. Elio Nenci's open-access publication of Bernardino Baldi's *In mechanica Aristotelis problemata exercitationes* appeared in 2011 and, in 2013, Matteo Valleriani's *Metallurgy, Ballistics and Epistemic Instruments*, including a transcription and an English translation of Nicolò Tartaglia's *Nova scientia*. Ideas that were crucial for the writing of this introduction to Benedetti's *Diversae speculationes* are derived from another volume by Renn and Damerow, *The Equilibrium Controversy: Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Backgrounds* (2012).

# Chapter 1
# Prosopography

## 1.1  The Life and Career of a Renaissance Man

Giovanni Battista de Benedetti came from a patrician family of Venice. Although the title of nobility may appear superfluous to the historian of science, it was not so for him and his contemporaries. Benedetti often noted this in his publications, adding to his name the honorific "Patritius Venetus." Evidence for Benedetti's noble origins can be found in a document dated January 14, 1570. This is a patent through which Duke Emanuele Filiberto of Savoy conferred upon Giovanni Battista the privileges of Imperial nobility in addition to his previous titles:

> We make, create, and constitute the aforementioned Giovanni Battista Benedetti as a true noble of the Holy Roman Empire and of our Empire forever, alongside all his legitimate and natural sons and daughters (those who are already born and those that will be born). We will call and fully declare them such [nobles of the Holy Roman Empire]—although he and his predecessors are noble and were born from an ancient and noble progeny, as we are very well informed.[1]

In those years, the establishment of the Savoy court in Turin brought about a general transformation of the urban *patriziato* into an aristocratic class gravitating around the dukes.[2] This trend was parallel to the more general political-social shift from the civil humanism of the medieval municipalities toward the courtly culture of centralized territorial States.

On the occasion of the conferral of the patent on Benedetti, the cross of Savoy was added to his heraldic design along with the motto "sic vita veritas."[3] This motto, which indicated a conduct of a life dedicated to the search for truth, was the acknowledgment of his mathematical and philosophical excellence. In the preamble to the duke's patent of nobility, it was precisely Benedetti's devotion to the mathematical disciplines, the *humanae litterae*, and the philosophy that was extolled as an example to be imitated and a reason for the conferral of aristocratic privileges on him and his heirs.[4] In this case, scientific distinction led to higher social recognition and even served as a legitimation for it.

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[1] Bordiga 1985, 752: "Habbiamo creato, fatto et costituito, facciamo creamo et costituiamo il detto Giovan Battista de Benedetti con tutti i suoi figliuoli maschi e femine legittimi, et naturali, nati et che nasceranno, et saranno procreati di legittimo matrimonio, con tutti loro posteri et heredi et successori in perpetuo veri nobili del Sacro Romano Imperio et nostri, et per tali li chiamiamo et dicchiariamo per dabondante (ancora ch'egli insieme coi suoi predecessori siano nobili e nati di antica prole nobili come siamo benissimo informati)."

[2] Stumpo 1998, 138.

[3] Bordiga 1985, 601.

[4] Bordiga 1985, 752: "Emanuele Filiberto per gratia di Dio Duca di Savoia Principe di Piemonte etc. Essendoche le attioni che tendono alla Virtù, come che da quella prendano accrescimento et perfettione, sono ammirate et havute in pregio: così gl'huomini che in quelle di continuo si essercitano vengono da ogniuno istimati et tenuti in particolare consideratione, la onde havendomi sempre fatto conto delle persone che dirizzassero ogni loro pensiero al bene operare, et quanto più si potrà, cercassero col mezo delle scienze, et arti liberali sicure et vere guide alla virtù di venire alla cognitione di esso doppo l'haver noi ricercato che

During the Renaissance, nobility was more important than professional appurtenances or academic titles. For instance, the celebrated Danish astronomer Tycho Brahe, himself an appreciative reader of Benedetti, held aristocratic lineage in higher esteem than any status linked to university professorship, including the position of imperial mathematician—an appointment which, by contrast, raised the status of his fellow countryman and opponent Nicolaus Reimarus Ursus, who was of low extraction.[5] Accordingly, Brahe always emphasized Benedetti's lineage when citing his work, for instance his letter on the superlunary location of the supernova of 1577. The capitalization as well as the reverence in this passage from the *Astronomiae instauratae progymnasmata* (posthumous, 1602) is telling:

> The small star of Cassiopeia would not shine as brightly as this nova over the whole surface of the Earth because of the dry fumes placed in-between, if they had been only under that one, and did not affect in the same manner the other stars next to it and augmented that unusual light. But the most excellent philosopher GIOVANNI BATTISTA BENEDETTI, THE VENETIAN PATRICIAN, eminently and skillfully demonstrated this with geometric arguments, in [his] outstanding work concerning mathematical and physical speculations (around the end of his letters). Writing to Annibale Raimondo […] he clearly showed the absurdity which necessarily follows from his false assumption [i.e., the sublunary position of the nova].[6]
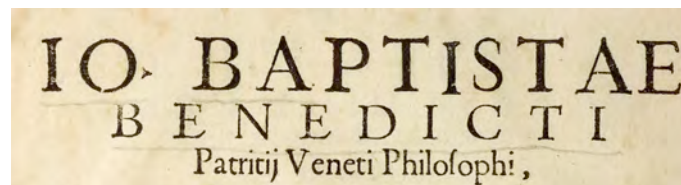


Figure 1.1: An example of the titles Benedetti added to his name in his publications. In the title page of *De gnomonum umbrarumque solarium usu* (1574), he called himself "Venetian Patrician, Philosopher." (Max Planck Institute for the History of Science, Library)

The prominence accorded to lineage is evident from Brahe's self-representation in the portrait at the beginning of his *Epistolarum astronomicarum libri* (1596), a collection of

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in questo ne sotisfacesse, massime nelle discipline matematiche. Al fine ci è pervenuto nelle mani il nobile messer Giovanni Battista de Benedetti venetiano, nostro mattematico il quale havendo consumato la maggior parte dell'età sua nelle bone lettere et studij di filosofia, et fatto professione delle dette mattematiche, et così divinamente et per eccellenza riuscito che si può dire in quelle (tra gl'altri) essere singolare cosa che si porge tal contento, et la sua servitù a noi molto grata tale soddisfattione che lo giudichiamo degno che partecipi de gl'honori dovuti alle sue virtù acciò che gl'accresca l'animo di perseverare et altri siano invitati a seguitare li suoi vestigij."

[5]This is why Brahe was not and could not desire to be imperial mathematician to Rudolph II, as has often been wrongly thought. See Voelkel 1999.

[6]Brahe 1916, 250: "Accedit et hoc, quod Stellula illa Cassiopeae in toto Orbe Terrarum ob siccas illas fumositates interpositas non tam splendide apparuisset atque haec Nova, si sub hac sola constitissent, et non reliquas illi vicinas pari modo attingissent, lumineque insueto auxissent. Hoc vero ultimum egregie et solerter ex excellentissimo Philosopho IOHANNE BAPTISTA BENEDICTO PATRICIO VENETO in praeclaro illo Opere quod de speculationibus Mathematicis et Physicis inscripsit, circa finem inter Epistolas eius evidenter et dilucide, Geometricis rationibus demonstratur. Ubi ad hunc ipsum Annibalem Raimundum scribens, absurdum, quod ex eius falsa assumptione necessario sequitur, dilucide ostendit."

epistles that arguably took Benedetti's collection in the *Diversae speculationes* as a model. Brahe's image is encircled by the heraldic designs of the family and makes the signs of his nobility very visible. In the same epistolary, Brahe's letters directed to aristocrats appear more prominently than those addressing "simple" professors or practitioners. He attached greater importance to his correspondence with the patron of sciences, Landgrave William IV of Hesse-Kassel, than to exchanges with the latter's court mathematician Christoph Rothmann.[7] Similarly, in the *Diversae speculationes*, Benedetti published with pride his letters to dukes or to illustrious aristocrats.

Apart from his nobility, we do not know much about Benedetti's origins. According to a horoscope that he cast for himself (Figure 1.2), and was printed by the Neapolitan astrologer Luca Gaurico in *Tractatus astrologicus* (*Astrological Treatise*, 1552),[8] Benedetti's father was a learned *Hispanus*, or Spaniard. Based on this thin evidence, his biographer, Giovanni Bordiga, speculated that his family could have been merchants trading with Spain.[9] Other archival documents caused him to speculate about Benedetti's marriage, around 1585, and about the existence of a daughter called Lodovica from an earlier relationship or marriage. She married a certain Domenico Pipino of Racconigi. Benedetti built a sundial for this son-in-law (*magnificus Dominus Dominicus Pipinus generus meus*), as indicated in *De gnomonum… usu* (1574). Lodovica died young, long before her father, in 1580.[10]

For the greater part of his life Benedetti was a courtier. For several years he served duke Ottavio Farnese of Parma, whom he joined in 1558 as "*lettore di filosofia e mathematica*."[11] Later, from 1567 up to his death on January 20, 1590, Benedetti served the Dukes of Savoy Emanuele Filiberto and Carlo Emanuele I. His duties were typical for a Renaissance court mathematician and are akin to those of Leonardo da Vinci in Milan, Guidobaldo del Monte in Urbino, Galileo in Florence, and Kepler in Prague, to mention only a few well-known names.[12] Benedetti was required to advise his patrons on issues of mathematical expertise. His fields of competence included engineering and architecture.[13] In Parma and Turin he built sundials (such as the modern one in Figure 1.3). He was also responsible for the construction of a fountain in the ducal park (*Parco di Viboccone*, later *Parco Regio*), which was destroyed by the French army during the siege of 1706.[14] Moreover, he was consulted on astronomy and music, both traditionally considered mathematical disciplines. In Parma he carried out astronomical observations, which he also reported on in the *Diversae speculationes*. In two letters to the Parma choirmaster de Rore, Benedetti explained musical consonance and dissonance of two tones by the ratio of oscillations of waves of air generated by the strings of musical instruments.[15] He claimed that the frequency of two strings of equal tension must have an inverse ratio to the lengths of the strings, and thus proposed to describe the degree of consonance or dissonance of two tones mathematically. In Turin he wrote a proposal for the calendar reform in 1578, *De temporum emendatione*, later reprinted in the *Diversae speculationes* as the

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[7]See Mosley 2007.

[8]Gaurico 1552, f. 76*r*.

[9]Bordiga 1985, 588.

[10]Bordiga 1985, 604–605.

[11]Bordiga 1985, 593–595.

[12]For the broad European context of patronage and the arts in the Early Modern Period, see Bedini 1999, Moran 1981, and Moran 1991.

[13]See Roero 1997 and Mamino 1989.

[14]Maccagni 1967a, 353–354.

[15]Benedetti 1585, 277–278.
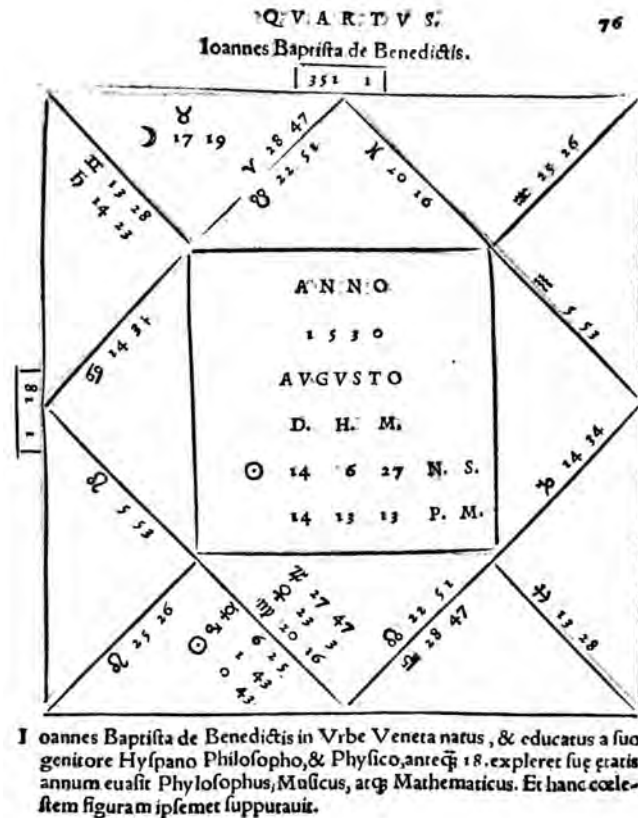
Figure 1.2: Benedetti's own horoscope, in Luca Gaurico, *Tractatus astrologicus* (1552), f. 76r.
(Bayerische Staatsbibliothek)

first of his epistles (to Duke Emanuele Filiberto).[16] This proposal was also sent to Rome and was meant as an aid to Clavius's efforts to correct the calendar.[17] At the same time, he taught mathematics to Emanuele Filiberto and his son Carlo Emanuele I.

Courtly life included participation in literary culture. Baldassar Castiglione, in his idealization of the court of Urbino in *Il libro del Cortegiano* [*The Book of the Courtier*] (1528), launched the model of a courtier with a refined literary education.[18] Following such cultural dispositions, a courtier versed in mathematics could advocate the usefulness of his expertise for the interpretation and assessment of "scientific" questions raised by classical sources, even poems. This attitude explains the inclusion of a letter on Ovid in the collection of epistles in the *Diversae speculationes*.[19] It was addressed to a certain Pancrazio Mellano, perhaps a courtier, asking Benedetti's opinion about the astronomical references in Book 2 of the *Metamorphoses*, in which Ovid tells the myth of Phaeton. According to the myth, Phaeton rode his father Apollo's chariot one day but he was unable to control the horses and keep the sun on its regular path. Finally, he was thrown out of the chariot, took a bad fall, and died. In the poem Ovid described the solar path in some detail but, according to Benedetti, he mixed up daily rotation and annual motion along the ecliptic: "Ovid unduly passes from the daily motion to the annual" (*Quod Ovidius transcurrit*

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[16]Benedetti 1585, 205–210.

[17]Benedetti's advice on the calendar reform is preserved in the Biblioteca Apostolica Vaticana under the signature cod. Vat. lat. 5645, 148r–150r. See Ziggelaar 1983, 211–214.

[18]Baldassar Castiglione, *Il libro del Cortegiano*, ed. Walter Barberis (Torino: Einaudi, 2017).

[19]Benedetti 1585, 417–418.

Figure 1.3: A modern sundial on the Church of San Lorenzo in Turin reminiscent of those designed by Benedetti. (Own photography)

*a motu diurno, ad motum annuum praeter rem*). To make his point clear, Benedetti listed the passages dealing with one or other of the two motions ascribed to the sun in ancient astronomy.[20]

As an exponent of the Turin elite, he was himself devoted to poems. For instance, the Milanese painter and writer Giovanni Paolo Lomazzo, who was linked to Savoy's court, celebrated Benedetti in verse as a philosopher, mathematician, and astrologer. In the first lines of a poem dedicated to him, Lomazzo declared himself delighted that Benedetti appreciated his paintings and cast his birth horoscope. Lomazzo's poem paints a vivid picture:

> Prudence and knowledge descend
> From Philosophy into [human] intellects;
> Which are perfect as far as their disposition is concerned,
> As each one receives its part of justice and reason.
> To Benedetti, he so wise
> And precious in the world,
> Belongs so much of this [philosophy]
> That it would be vain to try to equal him:
> So sublime does his value shine.
> All the more am I delighted that he appreciated

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[20]Omodeo 2012b.

> My painting so much so that he considered
> The time and the point in which I was born in the world.
> Oh splendor of our time, the sound [of your voice] silenced
> Every scholar of your art, who had to direct his judgment elsewhere,
> As it was overshadowed by yours, which is so deep.[21]

Benedetti received no formal or academic education. Like other Renaissance self-taught men (e.g., Niccolò Tartaglia and Tommaso Campanella), Benedetti was even proud of being removed from the academic habitus and training centers. This is particularly evident from the anti-academic tone of some of his polemics. In the preface to his first scientific treatise, *Resolutio omnium Euclids problematum* (1553) (On the solution to geometrical problems using a compass with a fixed opening), the twenty-three year old Benedetti emphasized the fact that he had not had a "common" (*quod vulgus solet*) education at some gymnasium or school. He boldly wrote to his patron, the Dominican abbot and diplomat Gabriel Guzman, that:

> Until now I have advanced without any mentor or teacher (under the guidance of God). I have never frequented any gymnasium or school. I have not learned what the vulgar (I mean this word without arrogance) use to estimate erudition, [such as limiting it] to the time spent at school, thus setting an end to learning when the seven years [of regular studies] are ended. As long as I live, I will continue [learning].[22]

It is possible that Benedetti was educated privately by his father, depicted in Gaurico's *Tractatus astrologicus* as "*Hyspanus, Philosophus, et Physicus*" (see Figure 1.2). For his part, Benedetti acknowledged only one teacher, namely the reputed mathematician and scientist-engineer Niccolò Tartaglia (ca. 1500–1557), for introducing him to the first four books of Euclid's *Elements*, probably between 1546 and 1548. In the *Diversae speculationes mathematicae et physicae*, Benedetti mentioned Tartaglia again as one of the very few authors of mathematical works whom he deemed worth reading.[23] However, in the

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[21]Lomazzo 2006, 177–178, III, 19, "Del Sig. Gio. Battista Benedetti Matematico":

> "De la Filosofia nasce e discende
> La prudenza e 'l saper de gli intelletti;
> Co' quali essendo nel dispor perfetti,
> A ognuno suo diritto e sua ragion si rende
> Di questa sì gran parte se ne prende
> Il saggio e raro al mondo Benedetti,
> Che d'agguagliarlo in vano è chi s'affetti:
> Tanto sublime suo valor s'estende.
> Però tanto godo io che sì gli piacque
> La mia pittura, e perciò egli volse
> L'ora et il punto nel qual nacqui al mondo.
> Splendor di questa etade al tuo suon tacque
> Ogn'un de l'arte tua, e altrove volse
> Il suo dir vinto dal tuo sì profondo."

[22]Benedetti 1553, f. 5*r*: "[…] huc usque progressus sum (Deo duce) sine monitore praeceptoreque ullo, nullum gymnasium unquam, nullamque scholam frequentavi, neque hoc studui, quod vulgus solet (sed absit verbo arrogantia) pro tempore in scholis transacto, eruditionem estimare, ac septennario finito finem studiis imponere, sed dum vivo, illa prosequi."

[23]One reads in the preface *ad lectorem* of the *Diversae speculationes* the following declaration: "In his autem meditandis, ex arithmeticis authoribus quos inspexi praecipuus fuit Nicolaus Tartalea, quippe quem

*Resolutio omnium Euclidis problematum*, he was quick to add that he had learned the rest of the *Elements* by himself:

> As it is honest and right to attribute to everybody his own merit, [I should acknowledge that] Niccolò Tartaglia taught only the first four of Euclid's books to me. I studied the rest alone with effort and diligence. In fact, for the one who wants to know, nothing is [too] difficult.[24]

Bordiga described such self-celebration as a sign of Benedetti's "pride in the assumed independence of his own thinking" (*orgoglio di creduta indipendenza del proprio pensiero*).[25] This is the same pride that would later lead to animosity with other prominent mathematicians such as Del Monte.

Moreover, in the preface to the *Resolutio*, Benedetti contrasted the simplicity of mathematics with the vanity of rhetoric. He went so far as to accuse learned and eloquent doctors of corrupting the sciences.

> Furthermore, mathematics does not require much [stylistic] splendor. If some language expert tried to improve its elegance, this would have no value, because a change of the mathematical language and of the scientific terminology could easily confuse the sense [of the reasoning] and render everything obscure. Therefore, I will follow the scholarly tradition and use plain words in my demonstrations, as I disapprove of deceptive elegance. In this respect, I follow the steps of the ancients who taught the sciences and the subjects themselves using plain words. Petty teachers (indeed, charlatans and babblers) corrupted this manner of teaching. Although they do not understand the subject, their babbling obtains the highest praise by the vulgar who regard them as learned scholars. This should not be surprising, considering that the most perfect and distinguished expertise in the sciences is attained by very few—despite the fact that many people write a great deal in all kind of sciences and arts, babbling a lot and capturing the attention of the uneducated with illusions and bombastic words.[26]

The same tone characterized Benedetti's next publication. Its title was intentionally polemical: *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (1554). In fact, this booklet put forward a novel theory of motion. He

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fere omnia ab aliis scripta collegisse constat, nec alios ex praecipuis quos legere potui omittendos duxi, inter quos sunt Hieronymus Cardanus, Michael Stifelius, Gemma Frisus, Ioanna Novimagus, Cuthbertus Tonstallus, caeterique huiusmodi."

[24]Benedetti 1553, f. 5*v*: "Caeterum quia cuiusque quod suum est reddi debet, nam et pium et iustum est, Nicolaus Tartalea, mihi quatuor primos libros solos Euclidis legit, reliqua omnia, privato et labore et studio investigavi, volenti namque scire, nihil est difficile."

[25]Bordiga 1985, 588 (4).

[26]Benedetti 1553, f. 5*v*: "Adde quod Mathematicae disciplinae, neque tantum requirunt splendorem, neque si quis peritus linguarum contendat ad elegantiam rem reducere, egregium quid effecerit, quia mutato usu Mathematicae loquendi, ipsiusque scientiae terminis, sensum facile perturbaverit, et ex nihilo nihil apprehensum obtinuerit. Quare morem scholarum sequutus, obstentatione elegantiae explosa, verbis nudis in demonstrationibus usus sum, hac in parte veterum vestigia sequutus, qui nudis verbis scientias resque ipsas docebant, quem modum docendi, nobis devastarunt scioli vel potius circulatores, garruli, rebus ipsoque iudicio destituti, garrulitate siquidem apud vulgus, laudem summam consequuntur, et pro doctis circunferuntur, nec mirum, cum scientiarum perfecta exquisitaque perita, paucissimis detur, non obstante quod multi permulta de omnis generis et scientiis et artibus scribant, permultaque garriant, fucis suis, et ampullis imperitorum oculos perstringentes […]."

argued that bodies of the same material fall through a given medium with the same speed, and not with speeds proportional to their weights, as Aristotle held. This is the reason for Benedetti's declaration of war "against Aristotle and all philosophers" in the title. Benedetti employed the Archimedean concept of buoyancy to account for the dependence of the motion of fall on their specific rather than absolute weight. As we shall see, these ideas played an important role in the *Diversae speculationes*. The use of Archimedean notions to improve on Aristotle's physics was probably stimulated by Tartaglia's Italian translation (1543) of Book 1 of Archimedes's treatise on bodies in water.[27] Benedetti's challenge to Aristotle must have raised considerable discussion, as is shown by the fact that, in his *Demonstratio*, he discussed Aristotle's views and responded to his critics at length. In the second edition of the *Demonstratio* (13 February 1554 *more veneto,* in fact, 1555), he showed that the resistance encountered by a falling body in a medium depends not on its volume, but on its surface area. Benedetti moreover explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. He had already outlined his theory of fall in the dedicatory letter of the *Resolutio*, explaining this anticipation as a means of avoiding plagiarism.[28] Still, in spite of his efforts to secure priority for his ideas by repeated publication, they were plagiarized by the Flemish polymath Jean Taisner in 1562 and spread through Europe with no clear acknowledgement of their origin.[29] This prompted Benedetti to express his indignation and rage at Taisner in the dedicatory letter of his *De gnomonum… usu* (1574).[30]

As was to be expected by his irreverent tone, some of the first reactions to Benedetti's early writings were rather critical. As he reports in the preface to the second edition of the *Demonstratio* (1555), some Roman scholars objected that his treatment of motion was in disagreement with Aristotle (*illam [meam propositionem] neutiquam esse iuxta mentem Aristotelis*). Benedetti was informed about their disapproval by a Dominican friend, Petrus Arches, an expert of Hebrew and Greek letters cultivated in philosophy and mathematics.[31] Benedetti replied that those scholars worshipped Aristotle like a pagan god (*veluti coeleste quoddam numen*) and did not admit that their *auctor* could make mistakes. He claimed that he had not misunderstood Aristotle; rather, that he simply disagreed with him.

> I remember that he [the very educated Doctor Peter Arches]—after many different conversations on various subjects—told me that many in Rome considered that proposition of mine (which I sent to you, Reverend Mr. Guzman, among other ones) and they mostly reacted with surprise for I did not specify that it was by no means in accordance with Aristotle's mind. Such was the reaction of those who considered my demonstration very attentively.
>
> They could not concede that Aristotle was mistaken in any way, because they do not regard him as a human being. Rather, they confer upon him the celestial condition of a pagan divinity. And they see even slight disagreement as a sin. Therefore [they believe that] I committed (and still commit) heresy if, according to their judgment, I do not follow the pure and authentic mind of Aristotle's doctrine in any manner.
>
> Thus, in order to escape the allegation of such an error or [the rumor] that I am dissimulating and hiding something, especially as far as this issue is con-

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[27] Archimedes 1543.

[28] Benedetti 1553, f. 10*v*. See Maccagni 1967a, 338–340 and Maccagni 1967b, 14–15.

[29] Taisner 1562, see the discussion in Maccagni 1967a, 344–455, n. 13.

[30] Benedetti 1574, f. 4*v*.

[31] Maccagni 1967b, 20–21, and 20, n. 14.

cerned, I decided to publish this new booklet in which I present my opinion more clearly. In this manner, everybody should become aware that I correctly understood Aristotle and that I disagree with him on a particular issue with considered reason. This is an unpleasant task for me. In fact, it is only unwillingly that I dissent with such a great man. I know nobody who could rival his excellence in all kind of doctrines. Nevertheless, his teaching is to take as true that which is supported by stronger reasons. He himself followed this precept, as he stated in the *Ethics*: "Plato is my friend, Socrates is my friend, but truth is even more friend to me."[32]

It is evident from these passages that Benedetti regarded mathematics as a support for conclusive rational argumentation in the treatment of natural issues. Therefore, as a *mathematicus* he claimed for himself the right to be called a *philosophus*. Already in the short biographical indication accompanying his birth horoscope, he was said to be a "*Phylosophus, Musicus, atque Mathematicus*" (see Figure 1.2). In his publications, Benedetti often stressed his quality as "*philosophus*" or "*filosofo.*" Galileo would later add the title of "philosopher" to that of "court mathematician" in Medici's Florence.[33] However, in Benedetti's case, it is evident that adding the title of "*philosophus*" was not part of a strategy aimed at social advance but rather mirrored his cultural and philosophical commitment to a mathematical philosophy of nature with all its consequences, among them that Aristotelian physics was open to critique by means of mathematical reasoning.

Thus, Benedetti not only dealt with fields of mathematical inquiry that traditionally belonged to the domain of mathematics (such as mechanics, optics, mathematical astronomy, and musical theory), but also addressed issues considered beyond the limitations of mathematics, especially terrestrial and celestial physics. The title of the *Diversae speculationes mathematicae et physicae* is itself provocative, as it brings together mathematics and natural philosophy (or *physica*), considered to be separate fields, one dealing with the *quia* (the "phainomena") and the other with the *propter quid* (the "causes"). In this respect, Benedetti's methodology is very close to that of Nicolaus Copernicus, whose heliocentric system he admired. In Book 1 of *De revolutionibus orbium coelestium* (1543) and in the *Narratio prima* (1540), Copernicus and his pupil Georg Joachim Rheticus (1514–1574) reversed the Peripatetic hierarchization of physics over mathematics, urging a reform of natural philosophy and celestial and terrestrial physics in order to bring them into accord with the geokinetic and heliostatic innovations in mathematical astronomy. Beyond astronomy, the issue of the status of mathematics and its role in natural investigations was

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[32]Maccagni 1967b, 20–21: "Memini eum [eruditissimum Doctorem Petrum Arches], post varia et diversa colloquia utro citroque inter nos habita, mihi retulisse quamplurimos Romae, conspecta mea illa propositione quae ultra reliquas tuae R[everende] D[omine] [Guzman]a me mittebatur, valde mirari solitos me addidisse illam neutiquam esse iuxta mentem Aristotelis, idque ab eis dictum ubi meam demonstrationem attentius considerarunt.

Ne vero Aristotelem ullo modo errasse concederent, cum illum non infra humanae conditionis terminum habeant, sed potius veluti coeleste quoddam numen sibi proponant, censeantque nefas esse si vel latum quidem unguem ab eo quis dissentiat, in hac potius haeresi fuisse, ac etiamnum esse, ut me germanum et genuinum sensum Aristotelicae opinionis nequaquam ex authoris mente assecutum existiment.

Ego vero ne mihi diutius talis impingatur error, neve quid maxime super hac re sentiam, aut dissimulem, aut reticeam, statui, hoc novo libello edito, meam sententiam clarius aperire, ut omnes intelligant me et Aristotelem ipsum antea recte intellexisse, et non temere hoc in loco ab eo discrepare, quod sane quanquam invitus facio (nec tamen libenter a tanto viro diversum sentio, quippe qui norim quam ille praeclarus extiterit in omni doctrinarum genere), docet tamen maiorem ratione veritatis habere, quo ipsemet facendum censuit, quam inquit in *Ethicis*: 'Amicus Plato, amicus Socrates, at magis amica veritas.'"
[33]Biagioli 1989, 49–50.

heatedly debated by philosophers and mathematicians during the Renaissance.[34] One ancient predecessor to praise mathematical physics was the Hellenistic "prince of astronomy and geography," Claudius Ptolemy. In the beginning of the *Almagest*, he pointed out the superiority of mathematics over theology and physics, and even argued for a possible extension of the method of mathematical astronomy to include the treatment of local motion in general, as well as theology and ethics.

> Only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we are able to the whole of it, but especially to the theory concerning divine and heavenly things. For this alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy, physics and theology] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre.[35]

Even after Copernicus, Ptolemy's methodological insights maintained their full importance and could guide scholars who intended to expand the realm of the application of mathematics far beyond the limits established by traditional philosophy. In the *Diversae speculationes*, Benedetti deepened the discussion of issues of natural philosophy such as the concepts of space, time, and motion, claiming for a mathematician a better and clearer insight into foundational problems of physics.

Astrology was another area of expertise for Benedetti. During the Renaissance, astronomy and astrology were never separated. Benedetti was expected to cast horoscopes and give astrological advice to his patrons, just as Brahe astrologically advised the King of Denmark, Kepler the Emperor, and Galileo the grand dukes of Tuscany.[36]

In Venice Benedetti frequented celebrated exponents of the astrological culture of the time, among them Annibale Raimondo of Verona and Francesco Giuntini. Raimondo reported about a meeting they had in the residence of the senator and poet Domenico Venier. On that occasion he and Benedetti tested Giuntini's astrological preparation:

> We gathered at Mr. Domenico Venier's place; his magnificence [came] first, followed by the most excellent Mr. Giovanni Battista Benedetti, many other

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[34]De Pace 1993.

[35]Ptolemy 1984, 35–37.

[36]A very informed case study on astrology at Italian Renaissance courts is Azzolini 2013.

gentlemen, myself (Annibale Raimondo), and finally the ex-reverend father Pacifico of Florence (now, as an ex-friar, known as 'excellent Mr. Francesco Giuntini'). As soon as the latter arrived, he was given the simple astrological chart of the revolution of the magnificent Venier, without any written indication around or below. The good father took countless and endless texts and aphorisms out of his scapular. He related them to the revolution as good as a physician might give prescriptions to sick people by saying 'God might help you.' Since the most excellent Mr. Benedetti and myself laughed uncontainably—thereby making the father believe that he could not have better done—the good father, who was already trotting, was spurred by our laugher to gallop so quickly that it became extremely difficult to bring him back to silence and prevent him from telling more stupidities.[37]

An astrological report by Benedetti, cast for Carlo Emanuele I (Turin, 19 October 1589), is still extant and preserved in the Civic Library of Turin (Coss. 4, ff. 1*r*-2*v*). It contains a day-by-day personalized astrological forecast for the month of November 1589. The days are qualified with adjectives such as "*buono*" (good), "*mediocre*," or "*cattivo*" (bad), but some are treated more specifically (the 9th of November is indicated as apt to "*negotii ingeniosi*," ingenious endeavors, whereas the 10th of November as "*buono in cose femminili ma nel resto cattivo*," that is, bad except for women's affairs). Benedetti signed this astrological letter as "*Matematico e Astrologiaro*."[38] This signature shows that his "professional" profile could vary depending on circumstances, since it depended in part on the kind of advice requested from him.

In the concluding letter of the *Diversae speculationes*, Benedetti envisaged a reform of astrology. He directed this letter to a German correspondent whose name he awkwardly Latinized as *Volfardus Aisestain*.

As for the question whether or not I regard as true all that is written in the books of judicial astrology, I respond that I do not. I even believe that much is wrong [...]. But you will be informed about all this in a special tract of mine, about which I told you on another occasion. In it, you will find many things I have proven through the evidence of many observations. I intend to publish that tract along with some other speculations of mine, if only I will have enough time to do that, before I meet the body of the adverse Mars as indicated by my horoscope. This is going to happen in 1592.[39]

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[37]Raimondo 1574: "Ritrovandosi nella camera del Clariss. M. Dominico Veniero prima la sua Mag.[,] lo eccellentissimo M. Gio. Battista Benedetti, molt'altri gentilhuomini, et Annibale Raimondo, che son quel io, vi sopraggiunse al'hora il Reverendo Padre Frate Pacifico Fiorentino de gli bene inculati, adesso per essersi sfratato lo Eccellente M. Francesco Giuntini, alquale, subito giunto, fu dato in mano la figura simplice del cielo della Revolutione del detto Mag. Veniero, senz'altra scrittura intorno, né appresso, il buono padre alhora mise mano al suo scapolario et cavò fuori testi, et afforismi senza fine, et senza fondo, allegandoli tanto a proposito della Revolutione, quanto facea quel buon medico le ricette che 'l dava ai suoi infermi, quando le dicea Dio te la mandi buona, et perché lo Eccell. M. Gio. Battista Benedetti et io se smassellavamo dalla risa, ben però in modo di maravigliarsi, come non fusse possibile a dir meglio di quello che dicea sua paternità, il buon padre per il nostro ridere sì come prima andava trottando, si misse a correr' de modo che fu gran fatica a poterlo tenere et farlo tacere che'l non dicesse più minchionerie." Cf. Corradeschi 2009, 111, n. 46. On Raimondo and Giuntini, see Ventrice 1989, 140–145.

[38]Roero 1997, 57–58.

[39]Benedetti 1585, 425–426: "Circa vero id de quo me interrogas, scilicet, utrum putem omnia vera esse, ea quae scripta reperiuntur in libris Astrologiae iudiciariae, respondeo quod non, imo puto plurima falsa esse [...]. Sed diffusius haec omnia videbis in meo illo particulari tractatu, de quo tibi alias dixi, in quo multa

This passage concludes his major work. In it, Benedetti predicted, using astrological means, his own death for the year 1592, but he actually died in January 1590.[40] This fact aroused some doubts about his proficiency as an astrologer, especially from his successor as court mathematician, Bartolomeo Cristini.[41]

To sum up, Benedetti's persona and work had various facets, his interests ranging from mathematics to cosmology and from natural philosophy to literature. In a certain sense, he can be seen as a Renaissance polymath. However, his profile can be better encompassed by the title of "*mathematicus*," as long as we do not take it too restrictively. A Renaissance mathematician like Benedetti was an engineer and a technical inventor, as well as a theoretician and a natural philosopher; someone with teaching and civil duties who served as a counsellor, also for astrological matters. Being a court mathematician implied benefiting from high recognition and visibility in society. Thus, this professional and intellectual appurtenance had nothing to do with the rather low acknowledgment that mathematicians received at universities, where physicians, lawyers, and theologians were higher placed and received better salaries.[42] The cultural environment of Turin, with which Benedetti interacted in the most important years of his career, shall be addressed in the next section.

## 1.2 Benedetti's Works and Publications

Benedetti published his first work at the age of 23, the *Resolutio omnium Euclidis problematum* (Resolution to All of Euclid's Problems, Venice 1553), which offered the solution to "all" geometrical problems using a compass with a fixed opening. The work reacted to a challenge that emerged from a controversy between Niccolò Tartaglia and Lodovico Ferrari in the years 1546–1548 and inserted Benedetti into the scientific debates of his time. One year earlier the astrologer Luca Gaurico had already paid homage to him, including in his *Tractatus astrologicus* a horoscope of the promising mathematician cast by Gaurico himself.

In 1554 Benedetti published a *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (Demonstration Concerning the Proportions of Local Motions against Aristotle and All Philosopers), which is not as famous for its polemical verve as for the presentation of an innovative theory of fall. As we have discussed in the preceding section, in this treatise Benedetti developed a theory of the motion of fall, first proposed in the dedicatory letter of the *Resolutio* of 1553. Benedetti maintained that bodies of the same material fall through a given medium with the same speed and not with speeds in proportion to their weights, as Aristotle and his followers claimed. Benedetti tried to overcome the fallacies of the Aristotelian theory of fall by employing the Archimedean concept of buoyancy, assuming that the motion of fall depends on their specific rather than absolute weight. As we have also discussed above, in the second edition of the *Demonstratio*, published in Venice in 1555,[43] Benedetti argued that the resistance incurred by a

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videbis, quae omnia ab experientia, ex multis a me observatis, comprobata sunt, quem quidem tractatum cum quibusdam aliis meis speculationes in lucem producere cupio, si fieri poterit, antequam ad directionem mei Horoscopi cum corpore Martis Anaeretae perveniam, quae quidem directo circa annum millesimum quingentesimum nonagesimum secundum eveniet."

[40] Benedetti was not the first mathematician who tried to forecast his own death. Among his predecessors are famous the cases of Johannes Stöffler and Girolamo Cardano. Cf. Omodeo 2014b, 3–4.

[41] Vernazza 1783, 16–18.

[42] On the lower status of mathematicians, see Henry 2011.

[43] Benedetti [1554] 1555, see Benedetti 1985.

falling body in a medium depends not on its volume, but on its surface area. This is also the view that he presented in the *Diversae speculationum mathematicarum et physicarum liber*, published in Turin in 1585. He explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. Such examples show how he dealt with new challenging problems, which were difficult and sometimes impossible to solve using the mainstream theories of his time, by bringing forth and promoting new ideas.

After the *Resolutio omnium Euclidis problematum* and the *Demonstratio proportionum motuum localium*, composed when Benedetti was still in Venice, the next extant works stem from the time when he had already settled in Turin. First, he composed two works on gnomonics, one in Italian and one in Latin. The former is a manuscript preserved in the Civic Library of Carignano (Turin, Italy), entitled *La generale et necessaria instruttione per l'intelligentia et compositione d'ogni sorte [di] Horologij Solari*, which was presumably written between 1567 and 1573. The latter was printed under the title *De gnomonum umbrarumque solarium usu liber* (1574). Here Benedetti dealt at length with the construction of sundials with faces of varying inclinations and also with cylindrical and conical surfaces. At ff. 107*r-v* one finds a discussion of a sundial that perhaps can still be seen today on a wall of the Royal Palace in Turin.[44]

In 1574 Benedetti also wrote about a trigonometrical measuring instrument of his own invention, *Descrittione, uso, et ragioni del Trigonolometro*. It was never printed and is preserved in manuscript form in the Civic Library of Carignano along with the Italian work on sundials, *Intelligentia et compositione d'ogni sorte [di] Horologij Solari*.[45] His next scientific treatise, *De temporum emendatione opinio* (1578), proposed correcting and reforming the calendar. In 1578 the duke initiated a public disputation at the University of Turin where Benedetti argued with Antonio Berga about whether there was more water or more land on the earth, following an argument by Alessandro Piccolomini. The views which Benedetti brought forth against his opponent were published in Turin in 1579 under the title *Consideratione… d'intorno al discorso della grandezza terra et dell'acqua del eccellent[e] sig[nor] Antonio Berga*. This polemic was renowned, as can be seen in the Italian translation and commentary of Sacrobosco's *Sphere* by the theologian, astronomer, and astrologer Francesco Giuntini in Lyon: "The excellent philosopher, Mr. Giovanni Battista Benedetti, mathematician to the serene duke of Savoy, resolved this question very aptly, arguing against the philosopher Berga, a famous reader at the University of Turin. The latter argues against Mr. Piccolomini that there is more water than earth. Benedetti defends the opposite view, which corresponds to truth: that there is less water than earth."[46]

Next came Benedetti's defense of the reliability of the mathematical computations underlying astrological predictions in the context of a heated polemic on this issue that burst out in Turin 1580–1581. Benedetti first communicated his views in Italian, in epistolary form: *Lettera per modo di discorso… all'illustre sig. Bernardo Trotto. Intorno ad alcune nuove riprensioni, et emendationi contra alli calculatori delle effemeridi* (Letter

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[44]Roero 1997, 47.

[45]Clara Silvia Roero published Benedetti's letter to Carlo Emanuele I (Turin, 19 October 1589), the index of the manuscript on gnomonics, as well as an excerpt from the manuscript on the mathematical instrument *trigoniometro* as appendices II and III of Roero 1997.

[46]Giuntini 1582, 95–96: "La qual questione ha resoluta molto dottamente lo eccellente filosofo, il signor Giovambattista Benedetti mathematico del serenissimo signor Duca di Savoia, contra il filosofo Berga, famoso lettore nella università di Turino: il quale contra l'opinione del signor Piccolomini defende che l'acqua è maggiore della terra: e il Benedetti defende il contrario in favore della verità: cioè che l'acqua è minore della terra."

in the Form of a Discourse… Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators) (1581). Benedetti later included a Latin translation of this letter in the *Diversae speculationes* (1585).[47] His commitment to astrological practice is testified to by an astrological report he wrote for Carlo Emanuele I, a handwritten letter (Turin, 19 October 1589) preserved in the Civic Library of Turin (Coss. 4, ff. 1*r*-2*v*).[48]

Finally, Benedetti had his major work, *Diversarum speculationum mathematicarum, et physicarum liber*, printed in 1585. It was issued again under slightly different titles in Venice in 1586 (*Speculationum mathematicarum et physicarum tractatus*) and, posthumously, in 1599 (*Speculationum liber*).

Two of Benedetti's manuscripts, preserved in the Biblioteca Nazionale Universitaria of Turin until 1904, are irreparably lost due to a fire that burst out in that year, destroying many valuable manuscripts. The first one was a collection of his letters, *Lettere di Giovanni Battista Benedetti, Veneziano, matematico del Duca Emanuele Filiberto e Carlo Emanuele I, in risposta ai quesiti fattigli dal Duca e da altri personaggi intorno alla matematica, fisica, musica e filosofia*.[49] The second one held similar content and was entitled *Lettere di Giovanni Battista Benedetti in risposta a quesiti di fisica e matematica* (Letters by Giovanni Battista Benedetti answering questions on physics and mathematics).[50]

Reprints of Benedetti's works are rather scarce. Excerpts on mechanics from Benedetti's work were included by Stillman Drake and Israel Edward Drabkin in their *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo and Galileo* (Madison, Wisc.-Milwaukee-London, 1969). Carlo Maccagni's *Le speculazioni giovanili "de motu" di Giovanni Battista Benedetti* (Pisa, 1967) includes excerpts from the dedicatory letter of the *Resolutio omnium Euclidis problematum* and the text of the two editions of the *Demonstratio proportionum motum localium contra Aristotilem et omnes philosophos*.

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[47]Benedetti 1585, 228–248, "Defensio ephemeridum."
[48]See Roero 1997, Appendix I.
[49]Peyron 1904, 73–74, Codex 83, N. II. 50.
[50]Peyron 1904, 95, Codex 94, N. III. 27.

# Chapter 2
# The Subalpine Environment

Benedetti's life, work, and reception are indissolubly linked to Turin and the Duchy of Savoy. As one reads in the *Diversae speculationes*, he chose to live in this town until the end of his life. There he benefited from the patronage of Duke Emanuele Filiberto (Figure 2.1) and, as a consequence, from a prominent social position and recognition. In the dedicatory epistle of his major work, addressed to Filiberto's successor, Carlo Emanuele I, Benedetti extolled the merits of the deceased duke, who had invited him to Turin almost two decades earlier:

> Nineteen years have passed since I was sent for by a letter of the most serene [Emanuele Filiberto] father of Your Highness [Carlo Emanuele I] and I moved from the town Parma to this municipality. Upon my arrival, he received me so humanely, and later I met with so much generosity as a reward for my services, that I began to desire vehemently that I could spend the rest of my life under his authority.[1]

As one reads, Benedetti and Emanuele Filiberto were so close that the patron even wanted his court mathematician to accompany him during his periods of residence in the countryside. On such occasions they often discussed scientific matters:

> His benevolence toward me, as well as my respect toward him, consolidated through the time we spent together, and our familiarity [grew] to the point that the duke wanted me to accompany him when he resided in the countryside. [He] often [even invited me] to stay with him overnight. In that time he discussed mathematics with me. He used my work in order to learn those sciences, asking questions on arithmetic, geometry, optics, music and astronomy [*astrologia*].[2]

Emanuele Filiberto's passion for mathematics was well known in his day. The Venetian ambassador to Turin, Giovanni Correr, reported on this singular aspect of his personality in 1566:

> That Duke is no man of letters but he loves the virtuosi. Hence, he has many of them by him; he likes to listen to their reasoning and he asks them questions. However, there is no subject that delights him more than mathematics, as

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[1] Benedetti 1585, f. A2*r*: "Agitur nonus decimus annus ex quo litteris Serenissimi patris tuae Celsitudinis, accersitus ex urbe Parmensi in hanc me civitatem contuli. Is advenientem tam humane excepit, tanta deinde liberalitate fuit complexus ego vicissim ei deserviendi, tam vehementi cupiditate fui accensus, ut sub eius ditione quod superesset vitae agere constituerem."

[2] Benedetti 1585, f. A2*r*: "Cuius in me banignitas, mea in illum observantia mirum in modum mutuo usu, et consuetudine est adaucta, ut idem Dux me secum dum rusticaretur esse vellet, saepe etiam secum pernoctare; quo quidem tempore de Mathematicis scientiis mecum agebat, in quibus perdiscendis mea opera utebatur, quaestiones, Arithmeticam, Geometriam, Opticen, Musicam, aut Astrologiam spectantes proponens."

this science is not only apt but also necessary to the profession of military commander.[3]



Figure 2.1: Portrait of Emanuele Filiberto from Tonso, *De vita Emmanuelis Philiberti* (1596). (Biblioteca Nazionale Universitaria di Torino)

The duke's passion for science and his special relation with his court mathematician is further confirmed by the Venetian ambassador Giovanni Francesco Morosini, who mentioned Benedetti in a speech delivered to the Senators of Venice in 1570:

> The duke of Savoy has a wonderful mind apt to every kind of science. However, he did not learn the sciences [*le lettere*] with the diligence that is necessary to become an expert, as his passion has always been the profession of

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[3]Firpo 1983, 123: "Non è quel duca litterato, ma ama li virtuosi, et però ne tiene alquanti appresso di sé, sente piacere a udirli ragionare, egli stesso li fa de quesiti, ma nessun ragionamento più li diletta, che quello delle matematiche, come scientia, che non solo è conveniente ma ancora è necessaria alla professione del capitano."

war […]. But since mathematics is very useful and [even] necessary to pro-
fessional warfare, His Excellency [Emanuele Filiberto] learns [mathematics]
with much pleasure and knows more of it than the average man. He is aware
that to receive substantial knowledge in any science one has to be in contact
with it and learn it continuously; therefore a certain Mr. Giovanni Battista
Benedetti of Venice imparts to him a lesson either on Euclid or on another
writer of those sciences every day. In my opinion, as well as according to
many other gentlemen, he is the most excellent scholar in this discipline in
our times. The duke likes him very much. In fact, not only has [Benedetti]
mastered this science, but he is also able to transmit it very well to others in
his lessons.[4]

However, Benedetti's activities in Turin cannot be fully grasped if we limit our considera-
tion to his relationship with the dukes. Rather, we should consider the wider political and
cultural environment in which this relationship was established.

## 2.1   Turin's Economy and Politics between Italy and Europe

From the point of view of economic exchanges as well as of the European balance of
power, Turin was located in a delicate and strategic position. It was in fact an obligatory
station on the commercial road connecting Italy and France through the *Val di Susa*. For
many centuries it had served as a transit point for merchants from Liguria, Lombardy,
and Piedmont on their way to Lyon and the French and Flemish markets, and vice versa.
Merchants were not the only visitors, as scholars from France, Flanders, and the British
Islands began their *iter Italicum* from Turin. Turin was also the first station in Italy of
Erasmus of Rotterdam, a key figure of the European Renaissance. On that occasion, on
September 4, 1506, he received an "Italian" degree in theology from the University of
Turin.

Its intermediate position between Italy and France made the town relevant not only
from the point of view of economics and culture but also for military reasons. When
Francis I of France and Charles V of Spain fought over Italian and European supremacy,
Turin acquired fundamental strategic importance. The French army conquered it in 1536,
together with most of Savoy and Piedmont, at the expense of Charles II of Savoy, brother
in law to Charles V of Spain. The King of France made Turin the most important center in
the region and a bulwark that was fundamental for consolidating his position on the Italian
peninsula. Some of the political and administrative reforms promulgated by the new ruler
were long-lasting. The most important of them were the creation of a parliament and of a
*Camera dei Conti* for the supervision of public finances.[5]

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[4]Firpo 1983, 211: "Ha il signor duca di Savoja un bellissimo ingegno capace d'ogni scienza: ma non ha
atteso alle lettere con quella diligenza, che si converria a chi ne volesse sapere, essendo la sua principal
professione il mestiero della guerra […]. E perché la scienza delle matematiche è molto utile e necessaria a
chi vuole fare questa professione de l'arme, però se ne diletta assai Sua Eccellenza [Emanuele Filiberto] e di
quella sa assai più che mediocremente. Con tutto questo sapendo che l'uomo tanto sa di ogni scienza quanto
continua in vederla e studiarla, però usa di udire ogni giorno una lezione o d'Euclide o d'altro scrittore di
quelle scienze da un messer Giovan Battista Benedetti veneto; uomo, per opinione non solamente mia, ma
di molti valentuomini ancora, il maggiore che oggi faccia professione, e di grandissimo gusto del Signor
Duca; perché oltre a possedere lui quella scienza eccellentissima sa anco così bene insegnarla ad altri che
con molta facilità ne fa restar capacissimo chi lo ascolta."
[5]Merlin 1998, 16.

Emanuele Filiberto, known as "testa di ferro" for his energy and capacity in military affairs, retook Turin on the battlefields. He conducted the campaign against the French as a captain in the service of the Habsburgs. In 1553 he was the supreme commander of Charles V's imperial army in Flanders and was nominated governor of the Netherlands by Philip II in 1556. His victory in the battle of Saint-Quentin led to the Peace of Cateau-Cambrésis (1559), according to which the Savoy and Piedmontese territories had to be restored. The French agreed to give them back to the dukes of Savoy with the significant exception of five fortified towns, occupied by their troops.

Turin was one of them. Therefore, it took some years before it was eventually returned to Emanuele Filiberto in 1562. In 1563 the duke entered the town and choose it as the new capital of his duchy instead of Chambéry. In this manner, he conferred an Italian identity to his duchy. This transfer set in motion political, social, and economic transformations, which were still in progress when Benedetti arrived in Turin in 1567. Moreover, the Piedmontese territories were politically fragmented. Apart from the centers under French control (Chieri, Pinerolo, Chivasso, and Villanova d'Asti), the region included the *Marchesati* of Monferrato and of Saluzzo. Moreover, the county of Tenda, connecting Piedmont with the Savoy possession of Nice, was an imperial fief. As for Geneva, a former possession of Savoy, it had become the "Jerusalem" of the Calvinists and would never be regained.

Within this difficult territorial and political constellation it was imperative that Emanuele Filiberto reestablish his authority after years of wars and foreign domination. In the European context, this meant striking a balance between the interests of Spain and France, who both wanted to annex the territories of the duchy either as a part of France or as a continuation of the Milanese territories. Piedmont was already split into a faction favorable to the French and one favorable to the Spaniards during the years of the war, and this division would also continue during the reigns of Emanuele Filiberto and Carlo Emanuele I.[6]

International diplomacy was comprised of marriage politics. Emanuele Filiberto received a French spouse, Margret of Valoys, daughter of Francis I of France and sister of King Henry II. This meant a strong political and cultural link to Paris. Margret was well known for her patronage of literati and artists, among them the poets of the Pléiade, Pierre de Ronsard, and Joachim Du Bellay. However, her son Carlo Emanuele I married a Habsburg, the daughter of Philip II of Spain, *infanta* Catherine Michelle, who arrived in Turin in 1585. This liaison was strongly encouraged by the pro-Spain party. Its leader was Andrea Provana of Leyní (1511–1592), with whom Benedetti was well acquainted. Four of the letters included in the epistolary of the *Diversae speculationes* are addressed to him. Benedetti judged the importance of his correspondence with this exponent of the Savoy aristocracy to be second only to those with Emanuele Filiberto (first epistle of his collection) and Carlo Emanuele I (second epistle). We can assume, taking his origins as a guide, that Benedetti supported Provana's pro-Spain party.

In his relations to other Italian States the duke also followed a politics of balance. He was particularly keen on having good relations with Venice, which he visited in 1566 and 1574. On the latter occasion he was even endowed with the title of *patrizio* of the town. In turn, a Venetian embassy was established in Turin. The Savoy relation with Rome was also cordial. The papacy regarded Turin as a bulwark to stop the dissemination of reformed ideas in Italy, especially from the Swiss cantons. For his part, Emanuele Filiberto saw "heresy" as a danger to the unity of his state and his authority. Therefore, on

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[6]Merlin 1998, 33 and Merlin and Stango 1998, 266–267.

Figure 2.2: Portrait of Carlo Emanuele I by Francesco Maria Ferrero di Labriano, *Augustae Regiaeque Sabaudae Domus Arbor Gentilitia* (Turin, 1702), p. 174. (Biblioteca Nazionale Universitaria di Torino)

matters of faith, the Roman interests and his own converged. Against the background of the confessional tensions of those years, his support for the Jesuits is comprehensible. Yet he was no fanatic of orthodoxy. He was influenced by the Imperial policy of mediation, as is shown by his ratification of a compromise with the Valdesans in 1561, in which he accorded to them religious freedom in their valleys.

The ties with Rome and Venice were reinforced through Savoy support for expeditions against the Turks. In 1565 Andrea Provana was sent with three galleys to Malta, as the court historian Pingone recounted in his history of Turin, *Augusta Taurinorum*:

> When Malta was besieged by the Turks, in June 1565, duke Emanuele [Filiberto] sent Andrea Provana of Leinì with four well-equipped triremes to bring supplies to the isle together with triremes from the Pope, Spain, and other [states]. First, Provana [*Leniacus*] arrived and assessed the difficulties. Then, he conveyed others [to the battle] and broke the siege with divine favor. The holy and vigorous order of the knights of Jerusalem was liberated under the superior command of the French Jean of Valetta. Public demonstrations of

immense joy and pious celebrations of thanks to God for the victory were displayed in Turin.[7]

In 1571 Provana was enlisted to defend Cyprus and contributed to the "holy" victory in the battle of Lepanto.

> In 1571, when duke Emanuele [Filiberto] ruled over Turin and a confederation was established between Pope Pius V, the king of Spain and the Venetian Republic, he was asked to command the fleet with everybody's agreement. But he had to renounce the offer owing to the present danger to his country engendered by local conflicts. [In his place] John of Austria, offspring of emperor Charles V, of great spirit and promising youth, was made commander. Chief Andrea Provana of Leyní joined this expedition with three triremes. It was fought near Nauplia with the support of the Greeks. The Christians had hardly two hundred triremes and the Ottomans more than three hundred. The battle [*Mars*] was undecided for a long time but finally victory was given to the Christians, with the favor of God or even as a miracle. Provana, who fought bravely in the commanding trireme, was hit by a gun bullet and could hardly escape under the protection of a galley. One of the [Savoy] triremes, named Margara, was scattered and sunk into the depth; [another one], Pedemontana, was saved many times from the enemy. That victory was celebrated in Turin with thanks given to God and holy days set aside for the people.[8]

On these occasions Benedetti served as an advisor to Provana. Three of the four epistles of the *Diversae speculationes* addressed to him deal with mathematical issues related to navigation. As one reads, Benedetti undertook to give Provana suggestions concerning navigation and the employment of navigational instruments.[9] The first epistle is entitled *Per eundem parallelum absque correctione semper navigari non posse ubi notantur Petri Nonii lapsus in correctione erroris navis et alii Petri Medinae errores* (That one cannot always navigate along the same parallel without correction, where an error by Petrus Nonius concerning the correction of the deviation of a ship and other [errors] by Petrus Medina are considered). The second and the third letter deal with a navigation instrument invented by Benedetti based upon the design of Gerardus Mercator (Figure 2.3). They are a description of the instrument accompanied by technical drawings and an explanation of its use. They

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[7]Pingone 1577, 85: "Anno Christi 1565 mense Iunio, Dux Emanuel, obsessa a Turcis Melita, Andream Provanam Leniacum cum triremibus quatuor instructissimis mittit, qui una cum Pontificiis, Hispanis et aliis triremibus suppetias insulae afferret. Prior Leniacus applicuit, difficultates exploravit, alios postea advexit, et soluta tandem faventibus superis obsidione, Hierosolymitanorum militum sacer, et strenuus ordo liberatus, Ioanne Valleta Gallo summum magisterium gubernante. Quam ob victoriam Taurini immensae laetitiae publica significatio reddita, et devotae superis gratiarum actiones."

[8]Pingone 1577, 88: "Anno Christi 1571 Emanuel Dux Taurini agens, confoederatione inita in Turcam Cypri vastatorem, inter Pium quintum Pontificem, Hispaniarum Regem, et Venetam Rempublicam, qui classi praeesset ab omnibus exposcitur: sed ob imminentiam a vicinis discordiis patriae discriminis, excusatus habetur. Ioannes vero Austriacus Caroli quinti Caesaris soboles, magni animi, et expectationis iuvenis praeficitur. At Dux Andream Provanam Leniacum tribus cum triremibus in eam expeditionem adiungit. Apud Naupactum Achaicum concursum, et decertatum. Christianorum vix ducentum triremes: Turcarum vero plusquam trecentum: Mars diu anceps, tandem Deo maximo favente, et quodam potius miraculo ad Christianos inclinavit victoria. Leniacus ex triremi Praetoriam fortiter dimicans sclopeto ictus in capite vix galeae praesidio evasit: triremium una Margaris nomine dissipata, mersaque penitus, Pedemontana semel atque iterum ab hostibus recepta. Ob eam victoriam, Taurini supplicationes superis, feriae mortalibus indictae.." See also Tonso 1596, 142, 161 and 177–179.

[9]Benedetti 1585, 214–216.

are entitled *De armilla nautica* (On the armillary nautical sphere) and *De usu armillae nauticae* (The utilization of the armillary nautical sphere), respectively.[10] As one reads, the letters follow private discussions with Provana on the difficulties linked to navigation using nautical maps.
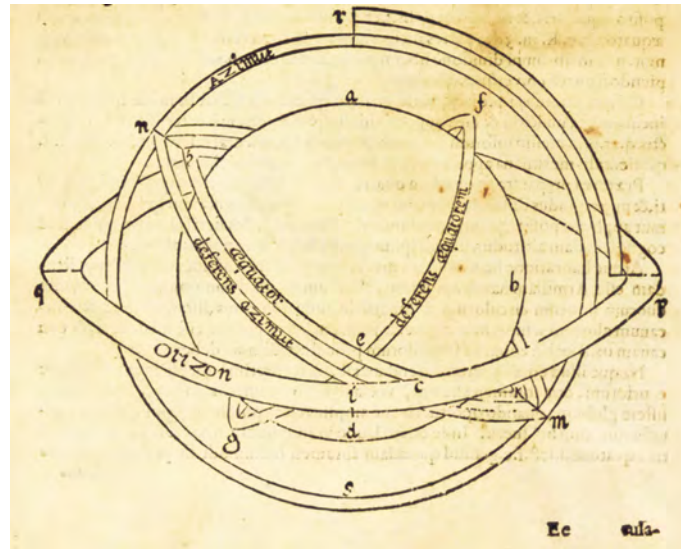


Figure 2.3: An armillary nautical sphere invented by Benedetti for Andrea Provana for navigation purposes, presumably in the Savoy military expeditions against the Turks. (Max Planck Institute for the History of Science, Library)

## 2.2   Civil Reforms and Military Policy

Emanuele Filiberto and his son were very different rulers. While the court of the former has been depicted as "*funzionale, ristretta e popolata di homines novi*" (functional, small and composed of *homines novi*) the latter's court was "*fastosa, aristocratica, centro propulsore di una politica culturale oramai intensamente barocca*" (pompous, aristocratic, irradiating center of a deeply baroque cultural politics).[11] Their common efforts were directed towards the consolidation of their state. Emanuele Filiberto implemented profound administrative, financial, and military reforms. He issued tax reforms and imposed the use of the vulgar tongue in official documents. As to his military policy, Emanuele Filiberto introduced the obligatory conscription of all men aged between 18 and 50 years. Thanks to this reform, which followed the Swiss example and Machiavelli's theory, Emanuele Filiberto trained his subjects to defend their territories in case of invasion, disposed of many thousands of soldiers, and limited the use of mercenary troops.[12] Part of his defensive strategy was the erection of new fortifications in Nice, Bourg-en-Bresse, Saint-Julien, and Montmélian (see Figure 2.4). The construction of the *cittadella* of Turin was particularly important and rapid. It was achieved in only two years, between 1564 and 1566, and was celebrated by the official town historian Emanuele Filiberto Pingone in the following terms:

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[10] Benedetti 1585, 217–219 and 219–220.
[11] Ricuperati 1998, XXII.
[12] See Stumpo 1993, 561.

> In that year [1564], the duke began building a fortification, which is commonly called the citadel, in the most sacred part of the town on the ruins of the temple of the divine Savior.[13]

> On March 15, 1566, the citadel of Turin was finished after a few months of work. It was admirable with its five bulwarks, serving all military purposes and built according to the art of architecture. He [the duke] let it be blessed with religious and pious blessings (Archbishop Della Rovere was in charge of the rite). Soon he organized the defenses, entrusting them to Giuseppe Caresana of Vercelli, a subject of his [*benemeritus*] and a man very expert in the military art.[14]

Francesco Horologi was responsible for the construction of the citadel, designed by engineer Francesco Paciotto, whom Emanuele Filiberto recruited in Flanders. Its pentagonal structure, responding to recent developments in warfare, was the model for later fortifications, such as the citadels created by the same Paciotto in Antwerp (1567) and Parma (1591).

As often occurred during the Renaissance, the military-political function of the citadel had two sides. On the one hand, it served to defend the town from possible assaults from outside. On the other, it affirmed the supremacy of the dukes over the new capital and had the function of dissuading the subjects from claiming too much autonomy.[15] As Martha Pollak remarked, "Paciotto proposed a five-sided fortress, with three bastions oriented towards the countryside, defending the approach to the city from the west, and two bastions facing the city, ready to bring it under control in case of riotous uprisings against the duke."[16] The new urban arrangement transformed Turin along with its political balance of power. During these changes the relations between the dukes and the local *patriziato* were often strained. In fact, all decisions had to be negotiated between civic administrators, state functionaries, and the court. Whereas the town council was eager to keep its medieval privileges, the dukes made the opposite effort of centralizing power in order to grasp control firmly in their hands.

A thorough knowledge of the surrounding territory through cartography, alongside fortification and military reforms, was also seen as an important element of defense. The intensity of mapping efforts in the Savoy in the second half of the sixteenth century is a noteworthy example. A large number of maps of great quality were made, for instance Forlani's *Savoia* (1552), Boileau de Boullion's work on the road from Lyon to Turin (1556), Nicolais's maps *Boulonnais* (1558), *Berry* (1566) and *Bourbonnais* (1569)[17] and, above all, Giacomo Gastaldi's *Pedemontanae vicinorumque regionum… descriptio* (1574). Many of the maps made in those years are still preserved in Turin, in the *Biblioteca Reale*, the *Biblioteca Nazionale Universitaria*, the *Archivio di Stato*, and in the wide collection of the *Archivio Storico della Città*.[18] Benedetti shared this interest in

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[13]Pingone 1577, 85: "Eo anno [1564] Dux in aeditiore parte civitatis, in ipsis templi Divi Solutoris ruinis Acropolis aedificare coepit, Cittadellam vulgo dicunt."

[14]Pingone 1577, 86: "Anno Christi 1566 idibus Martiis, absoluta paucis mensibus Taurinensi acropoli, quinis propugnaculis admiranda, servata omni rei militaris, et architectonicae artis ratione eam religiosa ac pia benedictione communiri curat, Archiepiscopo Rovereo sacris praeeunte: mox praesidiis firmat, eique praeficit Iosephum Caresanam Vercellensem de se benemeritum, ac rei militaris peritissimum."

[15]Merlin and Stango 1998, 118–119.

[16]Pollak 1991b, 16.

[17]See Broc and Greppi 1989, 113.

[18]The Archivio di Stato preserves Carracha's maps of Turin: *Augusta Taurinorum* (1577) and *Turino* (ca. 1580)—see Archivio Storico della Città di Torino 1982.

Figure 2.4: Fortification projects in a drawing by Benedetti's follower as court mathematician, Bartolomeo Cristini. (Biblioteca Nazionale Universitaria di Torino)

geography and topography. This especially emerges from some of his epistles, for instance those to the architect Gabriele Busca on topography and measuring instruments, to the imperial land surveyor (*agrimensor*) Anselm Rosenburg (presumably of the Bohemian aristocrat family Rožmberk) on measuring techniques, and to the Turin physician and natural philosopher Giovanni Battista Femello concerning cartographic errors about the position of islands, in particular Iceland.[19]

## 2.3   Engineering and Architecture

Countless engineers worked in Turin under Emanuele Filiberto and Carlo Emanuele I to implement the defenses and the hydraulic system, among them Francesco Paciotto, Ferrante Vitelli, Ascanio Vitozzi, and Vitozzo Vitozzi.

The leading Italian architect of that time, Andrea Palladio, visited Turin between May and June 1568. He might have been the architect behind the park of Viboccone (later known as *Regio Parco*) between the rivers Dora, Po, and Stura. Benedetti is said to have constructed his fountain there. It is also likely that Palladio gave Emanuele Filiberto advice on the organization of his popular militia according to the classical Roman model.[20] Later, he dedicated to the duke of Savoy the third of his four books on architecture, *Quattro libri dell'architettura* (Venice, 1570). This section deals with public constructions, streets, bridges, squares, basilicas, and gymnasia. In the letter to the reader, the author stressed the prestige deriving from public buildings, and the fruitful collaboration between Renaissance princes and architects: "[Public buildings] are bigger and more embellished with rare ornaments than the private ones; and they aim to be used by everybody. Therefore, through them, princes can display the greatness of their generosity to the world and

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[19]Benedetti 1585, 271–274, 405–408 and 267.
[20]Tessari 1993.

architects have the occasion to show how capable they are through beautiful and wonderful inventions."[21]  The dedication to Emanuele Filiberto by Palladio was motivated by his "heroic spirit"[22] as well as by his interest in and deep understanding of architecture:

> As your Highness is familiar with the most noble arts and sciences related to these issues [concerning architecture], you will have much pleasure and relief by considering the subtle and beautiful inventions of humankind as well as the true science of this art, which you understand very well and which has been brought to the most rare and almost absolute perfection. This is witnessed by the illustrious and royal buildings that have been constructed in many parts of your large and most happy state.[23]

Urban and military developments were accompanied by a flourishing literature on war and defense theory. Emanuele Filiberto was a great supporter and collector of such writings:[24] among other examples, Benedetti's correspondent Busca authored the treatise *Della espugnazione et difesa delle fortezze* (On the conquest and defense of fortresses, Turin, 1585), which followed the *Istruttione de' bombardieri* (Education of the bombardiers, Carmagnola, 1584). He would later publish the tract *Architettura militare* (1601) in Milan. Another acquaintance of Benedetti's, Giacomo Soldati composed *Discorso intorno al fortificare la città di Torino* (Discourse on fortifying the town of Turin).[25]

In this context of military reforms and architectural changes aimed at transforming Turin into the capital of an absolutist state, the skilled engineers implementing the dukes' vision gained high social recognition. An example is the career of the mathematician Cristini. In 1569 he was courtly librarian and later "calculatore" or "controlore delle fabriche," that is, supervisor of architectural projects. In this capacity, he become closely connected to the celebrated military and civil architect Ascanio Vitozzi. On December 13, 1582, he became "re d'arme dell'ordine dell'Annunziata," a honorific and administrative title that implied responsibility for the organization of courtly tournaments, feasts, and balls.[26] He entered the court as Benedetti's successor in the position of ducal mathematician. Cristini's career shows the enhanced status of Renaissance scientist-engineers bringing together mathematical and natural expertise, technical skills, administrative duties, and courtly honors. Benedetti addressed one of the letters of the *Diversae speculationes* to him (the one dealing with geometrical problems encountered in Ptolemy's *Geography*) calling him "Bartholomeo Christino Serenissimi Sabaudiae Ducis apparitor." The term *apparitor* can mean either "servant" or "functionary." An unknown hand corrected this title in a copy preserved in the Royal Library of Turin, substituting it for the more

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[21]Palladio 1570, III, 5: "Ne' quali [edificii publichi], perché di maggior grandezza si fanno, e con più rari ornamenti, che i privati, e servono a uso, e commodo di ciascun; hanno i Principi molto ampio campo di far conoscere al mondo la grandezza dell'animo loro; e gli Architetti bellissima occasione di dimostrar quanto essi vagliano nelle belle, et meravigliose invenzioni."

[22]Palladio 1570, III, 3: "Principe, il qual solo a tempi nostri con la Prudenza, e co'l valore s'assimiglia a quelli antichi Romani Heroi, le virtuosissime operazioni de' quali si leggono con maraviglia nell'historie, et parte si veggono nell'antiche ruine."

[23]Palladio 1570, III, 3: "Delle qual cose [concernenti l'architettura] essendo l'A[ltezza] V[ostra] dotata delle più nobili arti, e scientie; piglierà non poca contentezza, e consolazione considerando le sottili, e belle invenzioni degli huomini, e la vera scienza di quest'arte, da lei molto bene intesa, e ridotta a rara, e perfetta perfezione; come dimostrano gli illustri, e reali edifici fatti fare, e che tuttavia si fanno in diversi luoghi dell'amplissimo, e felicissimo suo stato."

[24]Pollak 1991a, 18–26.

[25]See Viglino Davico 2005, Pollak 1991a and Signorelli 1969–1970.

[26]Vernazza 1783, 8 and 11.

emphatic "P[rim]o Feciali," that is to say, "First Herald."[27] This was in fact the most appropriate title for the "Roy d'armes."[28] It is possible that this correction was inserted by Benedetti himself.
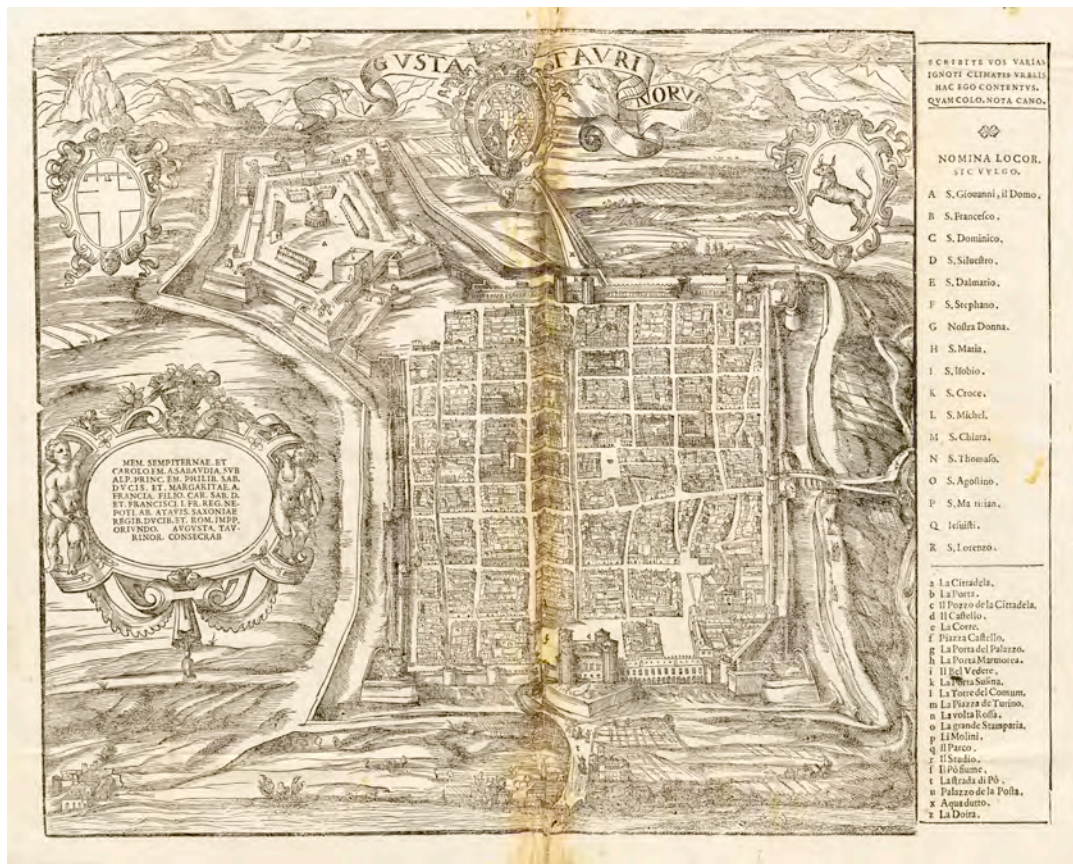


Figure 2.5: Map of Turin in Benedetti's times, from Pingone's *Augusta Taurinorum* (1577). (Staatsbibliothek zu Berlin)

Benedetti interacted with architects and engineers, as can be seen in his correspondence. Four of the scientific letters included in the *Diversae speculationes* are addressed to the architect Busca. Their topics, however, are not strictly related to building or engineering. In fact, their topics range from meteorology to instruments, topography, and natural philosophy.[29] Benedetti was closely related to the ducal "architect and cosmographer" (*Architetto e Cosmografo*) Soldati, who had worked as a hydraulic engineer and architect in Milan and in Lombardy, and joined the Turin court in 1576. Benedetti held him in great esteem, judging by the dedication to him in one of the most technical parts of the *Diversae speculationes*, that is, the entire second book, which is entitled "Explanation on Operations of Perspective" (*De rationibus operationum perspectivae*).[30] Besides, Benedetti's interest in engineering and measuring instruments emerges from many writings, especially from his work on gnomonics (1574) and from a manuscript analysing a measuring instrument that

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[27]The letter is to be found in Benedetti 1585, 330–331. The collocation of the volume in the *Biblioteca Reale di Torino* is G 43 8.

[28]Vernazza 1783, 37, n. 31.

[29]Benedetti 1585, 271–277.

[30]Benedetti 1585, 119–140. See Mamino 1989.

is preserved in the *Biblioteca Civica di Carignano*, entitled *Descrittione, uso, et ragioni del Trigonolometro* (1578).[31]

## 2.4   Intellectual Ferment: Arts, Literature, and Philosophy

Renaissance Turin was as appealing to mathematicians as to all other types of intellectuals, including artists, literati, and scholars in general. One could mention the names of two reputed theorists of art who were closely linked with the court: Federico Zuccari and Lomazzo. The former wrote *L'idea de' pittori, scultori, et architetti* (The idea of the painters, sculptors and architects, 1607) and the latter *Trattato dell'arte della pittura* (Treatise on the art of painting, 1584), dedicated to Carlo Emanuele I. Lomazzo also dedicated to the duke of Savoy his collection of poems, *Rime* (1587), including one for Benedetti. Among the artists appointed by the duke, the Flemish Jan Kraeck or "Carracha," who resided in Chambéry, not only painted; he also worked as a cartographer and made a large map of Turin in 1572 (385x397 cm) as part of a wider cartographic program.

Illustrious scholars came to Emanuele Filiberto's court or to the reopened university, first in Mondovì and later in Turin. Among them, the man of letters from Ferrara, Giovanni Battista Giraldi Cinzio, was appointed to teach humanities. His collection of *novelle*, the *Ecatommiti* (Mondovì, 1565), ended with a long celebratory poem mentioning the most visible intellectuals of the Peninsula. Many of them were linked to the duke of Savoy, either as professors or as courtiers.[32] For instance, Francesco Ottonaio of Florence, who taught mathematics at the university in Benedetti's years and exchanged views with him, is extolled for his expertise in astronomy, meteorology, and astrology:

> My Ottonaio moves his feet towards you along with the others.
> He received the gift of scrutinizing the heavens,
> of knowing the reasons for warmth and coldness,
> why the days are short or long,
> and what layer veils the Sun making it dark,
> the manner in which the year becomes adorned of beautiful flowers again,
> what nativity is a sign of honor and merit
> or of shame and disgrace,
> and what is the star presiding over
> a man's state from his birth
> until his vital light is extinguished
> one circle after the other.[33]

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[31] Mamino 1989, 432–433 and Roero 1997.

[32] Villari 1988, 93–95 and 107–110. See Doglio 1998, 599ff.

[33] Villari 1988, 93–94:

> "Move insieme con lor verso te il piede
> il mio Ottonaio, a cui scorrere il cielo, per grazia, diede.
> Del caldo la cagion saper, del gelo,
> e perché breve sia, sia lungo il giorno,
> e quale offoschi il sole oscuro il velo;
> come ritorni di bei fiori adorno
> l'anno e chi debba aver dal nascimento
> onore e pregio, e qual ingiuria e scorno;
> e da che stella prender de' argomento
> de lo stato suo l'uom, poi ch'egli è nato,
> insin che il suo vital lume sia spento

Giraldi Cinzio did not mention Benedetti as he had not yet come to Savoy, but he included the Turin physician and professor of medicine Antonio Berga, with whom Benedetti would later enter a controversy over the proportion of water and earth in the terraqueous globe:

> With his gentle and beautiful works
> he tries to subtract his name from the oblivion,
> defeating the stealing forces of greedy time.
> I refer to my gentle Antonio Berga,
> who shows the way to those who wish to learn
> by writing his papers for the common good.[34]

Two famous authors who visited Turin between 1576 and 1579 are the poet Torquato Tasso and the philosopher Giordano Bruno. Both arrived in the town as fugitives and both enjoyed their stay. Tasso interacted with the cultivated elites. The Turin philosopher and physician Agostino Bucci appears as *persona dialogans* in three dialogues of his (*Il Forno ovvero della nobiltà*, *Della dignità*, and *Della precedenza*). His connection with the court is further confirmed by the dedication of the 1581 edition of the *Gerusalemme liberata* to the prince of Savoy.[35]

As for Bruno, he did not establish lasting contacts in town. He visited Turin in 1576 (or at the beginning of 1577) for the first time after abandoning the Dominican cloister of Naples, where he was accused of heresy. However, as he stated later in his Inquisition trial, "non trovando trattenimento a mia satisfattione, venni a Venezia per il Po [as I did not find sufficient means, I came to Venice along the Po]."[36] He visited Turin again in 1578 and went to Chambéry, where he spent the winter of that year as a guest of the Dominicans. On that occasion he possibly carried a booklet, now lost, entitled *De' segni de' tempi*, that he had printed in Venice and that probably dealt with the comet of that year.[37] We will refer to his possible involvement in some polemics on the comet of 1577–1578 later. It is hard to say whether Bruno and Benedetti ever met or were informed of each other's views. In spite of the fact that they belonged to very distant milieus, there is some affinity between their outlooks. Both shared an aversion to Aristotle, the project of reforming natural philosophy, the support for the Copernican system, and other cosmological views.

The case of the philosophical poet Pandolfo Sfondrati also deserves our attention. He was active in Turin as a poet at the same time as Benedetti and authored poems that were often inserted in the first pages of books printed by the Bevilacqua printing house. Pandolfo made himself visible in Turin with celebratory poems that were included in important local publications, in particular in the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), *Inclytae Saxoniae Sabaudiaeque principum arbor gentilitia*

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di cerchi in cerchio."

[34]Villari 1988, 95:

> "E quel che, con gentil opre, e leggiadre,
> tenta che il nome suo da l'oblio s'erga,
> vinte del tempo avar le forze ladre,
> i' dico il mio gentile Antonio Berga,
> che addita, a chi imparar cerca, la strada,
> mentre, ad util comun, le carte verga."

[35]Doglio 1998, 621 and 625.
[36]Firpo 1993, 159. See Ricci 2000.
[37]Ernst 1992.

(1581) and *Sindon evangelica* (1581).[38] Hence, Sfondrati frequented the same courtly and cultural environment as Benedetti. It is likely that they discussed natural issues together, especially when considering that Sfondrati composed philosophically minded poems such as the Copernican *Inferiora regi dum syderis omnia motu*, which opens the *Animadversiones in Ephemeridas* by Benedetto Altavilla (Turin, 1580). There is also evidence that Benedetti was familiar with the Sfondratis, in particular with Paolo Sfondrati, who was a senator of Milan and an ambassador of Filippo II in Turin.[39] Moreover, Pandolfo Sfondrati authored the atomistic poem *Democriti prohibent nosci corpuscula formas* and a treatise on the tides, which he explained in mechanical terms as the result of the interaction of water particles warmed up by solar rays.[40]

## 2.5 Religious Policy

### 2.5.1 Pragmatic Counter-Reformation

The relics were moved from the old to the new capital: Christ's shroud traversed the Alps together with the court. Religion was an essential stabilizing factor. According to the report of the political thinker Giovanni Botero, Emanuele Filiberto declared that piety was essential to guarantee his authority in the state: "Those people who are zealous in their devotion [to religion] are more moderate: in consequence, they obey their Prince better than licentious people."[41] His religious politics were characterized by pragmatism. He undertook measures against the dissemination of the Reformation in his country and repressed the reformed communities only insofar as they jeopardized the integrity of the state or its relations with Rome or with other Catholic countries. The persecution of the Valdesans in the Alpine valleys, between 1559 and 1560, was part of a wider endeavor aimed at establishing a Catholic league that could favor his conquest of Geneva. However, when the prospect of an anti-Protestant confederation vanished, Emanuele Filiberto interrupted the aggression. The resistance of the Valdesans had been strong and persistent. It was a destabilizing factor for the duchy. A compromise was reached on June 5, 1561, when Emanuele Filiberto promulgated an edict, known as the Edict of Cavour, conceding to the Valdesans freedom of worship in their mountains. In exchange, he imposed the construction of new fortifications and strict military control of the Alpine passes.[42]

The other pole of Savoy religious politics was external. It concerned the regulation of jurisdictional issues with the Roman Church. A reason for friction was the taxation of the ecclesiastics, which Rome was reluctant to grant. The economic stake was high, since the estates belonging to the Church amounted to a third of the land; in some areas, it even reached values comprised between 40 and 70 percent. The ancient privileges of the dukes to select the candidates for the principal ecclesiastical positions had to be negotiated anew. It was only in 1573, under Gregory XIII, that these were confirmed. Finally, the introduction and the reinforcement of the Inquisition in the duchy fostered confessional uniformity but interfered with secular jurisdiction.[43]

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[38] See Griseri 1998.

[39] In a letter to Benedetti, Francesco Patrizi asked him to give his regards to Baron Sfondrati. See Patrizi 1975, 42–43.

[40] See Omodeo 2008b and Omodeo 2012a.

[41] "La gente infervorata di devotione è molto più regolata: e per consequenza più ubidiente al Suo Prencipe, che la dissoluta." Botero 1608, 241.

[42] De Simone 1958.

[43] See Merlin 1995, 238–267, chap. IX, "Tra Controriforma e Ragion di Stato."

The new Inquisition, established in the wake of the Council of Trent in order to counter the Reformation and reaffirm Catholic hegemony in Italy, had a centralized structure with ramifications for the entire Peninsula. The Holy Office lay at its heart. The various Italian states accepted it as a preventative measure against public disorder, especially against confessional conflicts such as the civil wars affecting France and other European areas. The Inquisition was a repressive control system binding Rome and the local powers. It was a compromise in which, in Adriano Prosperi's words, "l'aiuto era offerto e richiesto in nome della conservazione del potere, quello politico dei principi e quello della corporazione ecclesiastica" (The aid was offered and requested in order to preserve the power—the political one of the princes as well as that of the ecclesiastical corporation.)[44] Nonetheless, the jurisdiction problem remained acute: what was the legitimacy of a foreign tribunal—the Roman one—trying and condemning the subjects of other countries? In Piedmont, the interests of Turin and Rome were guaranteed through the mediation of the Papal nuncio, who was obliged to inform both the duke and the Holy Office in Rome about Inquisition trials.[45] For their part, these authorities could intervene in the trials and had the right to give their consent concerning the opportunity to carry them out. However, the opposition to the establishment of the Inquisition was strong, particularly in the French-speaking areas. Relying on its Gallican tradition, the local church in Savoy did not accept a form of direct control from Rome.[46]

The relations between Emanuele Filiberto and the Papacy were not always calm, and became strained after the Cavour edict of tolerance in 1561. Its promulgation provoked the protests of Rome and the commission of the intransigent Cardinal Inquisitor, Michele Ghisleri, to the diocese of Alexandria as Bishop of Mondovì, with the aim of reinforcing religious control. The relations between this champion of orthodoxy and the pragmatic politics of the Savoy dukes were tense, including after Ghisleri was elected pontiff as Pius V in 1566.

### 2.5.2 Jesuit Colleges in Piedmont

In 1561 Emanuele Filiberto supported the foundation of the first Jesuit college in his territories, in Mondovì, where the university was located at that time. A second college was established in Chambéry (1564), which was the former capital of the duchy. Among the Jesuit teachers, Antonio Possevino is one of the most renowned. The future compiler of the *Bibliotheca selecta* (1593) sojourned in Piedmont between 1560 and 1562.[47] From 1564 to 1567 the future cardinal and inquisitor Robert Bellarmine was also there but could not be appointed professor of philosophy in the Jesuit college of Turin, opened in 1567, due to the intransigent opposition of the university against the attempts to transfer this chair from the university to the Jesuit institution.[48] Both in Mondovì and in Turin the Jesuits took over the so-called "public schools," which essentially taught the rudiments of grammar.[49]

In those years, the chair of letters belonged to the Ferrara humanist Giovanni Battista Giraldi Cinzio, whose religious tendencies could be called Erasmian.[50] At the moment of the establishment of the Jesuit college in Turin, he sided with the humanistic legacy

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[44] Prosperi 1996, 57–58.

[45] See Prosperi 1996, III, "Inquisizione romana e stati italiani" and Black 2013, 30.

[46] Prosperi 1996, 103–105.

[47] Longo 1998, 475 and Scaduto 1959, 52.

[48] Grendler 2002, 42.

[49] Vallauri 1846, 19.

[50] For instance, Giraldi Cinzio defended the famous commentator on Aristotle's *Poetics*, Ludovico Castelvetro, who was excommunicated in 1560 as "eretico fuggitivo e impenitente" for his alleged bias towards Melanchthon. On this occasion Giraldi Cinzio argued that violence and coercion could only produce the

against their pedagogy. He paid the price of their expansion, as his chair was suppressed and transferred to the Jesuit college.[51] From 1567 to 1574, the Jesuits received 200 *scudi* per year to teach Greek and Latin grammar, humanities, and rhetoric to the youth (half of Giraldi's salary). Thus, the humanist had to abandon Turin for Pavia in 1569. In a letter to the Florentine philologist Pietro Vettori (Pavia, March 20, 1569), he lamented his expulsion, although he expressed his gratitude to the dukes for the donation of 500 *scudi* for his leave.[52] He particularly protested that his teaching as a learned humanist was being substituted by the teaching of elementary grammar.

The opening of the Turin college set off enduring hostilities between the Jesuits, on the one side, and the university and the municipality on the other. According to Grendler's reconstruction, between 1570 and 1572 the duke and the rector, Achille Gagliardi, made an agreement according to which nine chairs would be given to Jesuit professors. However, the project did not succeed owing to obstruction on the part of the town and the university.[53] In these struggles, the Archbishop Gerolamo della Rovere was among the most strenuous opponents of the Jesuits. The position of Emanuele Filiberto fluctuated. Initially, he supported the "reverend fathers" but later distanced himself from their educational projects. In 1575 he even reintroduced the teaching of humanities in the university. There is indirect evidence that Benedetti sided with the humanists in this anti-Jesuit controversy. In 1583 Francesco Patrizi, who belonged to the same Ferrara cultural environment as Giraldi Cinzio, asked him to support the candidacy of his friend Giovanni Giacomo Orgiazzo for the position of professor of humanities in 1583.[54]

Apart from the political interests at stake (the privileges of the town and of the university), the professors' resistance concerned the contents of the teaching, as one reads in a document from 1593, "Raggioni perché non sia bene che gli Rev[erendi] Padri Gesuiti leggano la filosofia tutta, et la logica nel loro Comento, et si lasci a leggerli nello Studio et pubbliche scuole, come sempre insino a qui si è fatto" (Reasons why it is not good that the Jesuit Reverend Fathers teach all philosophy and logic in their commentary and are allowed to teach at the university and in public schools, as has been the case until now).[55] According to the academics, philosophy should be imparted to students as the fundamental tenet of the study of medicine. Therefore, the focus should be set on Aristotle's natural philosophy and not on metaphysics and logic, as was the case with the Jesuits. Metaphysics, as one reads in the document, is the "last" and not the "first" part of philosophy. By contrast, the Jesuits began their teaching with the most abstract issues, e.g., the statute of ideas and universals, and divine ineffability ("*utrum Deus sit in praedicamento*" or "*utrum Deus sit infinitus*").

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opposite effects than those wished for by the defenders of orthodoxy. See Cinzio 1996, Letter n. 101, 371, n. 3.

[51] Vallauri 1846, 19 and Grendler 2002, 42–43.

[52] Cinzio 1996, Letter n. 127, 425: "Sed Taurino iam menses quatuor absum, Ticinique publice profiteor. Nam, praeter iacturam valetudinis, quam ibi quotidie faciebam, me ad abeundum urgentem, natio illa haec nostra studia nihil quidem facit. Hinc Princeps ille, qui oratoriam ac poeticam facultatem profiteretur, in Academia sua habere constituit neminem, quod satis esse censuerit Iesuitas nescio quos, suo in collegio, hoc muneris cum puerilis ac infantibus obire; qui, cum Deuspaterio quodam, barbaro plane auctore, mollia ingenia, obscurissima, ne dicam foedissima, imbuunt barbarie. Me tamen abeuntem, praeter annuam quadrigentorum aureorum nummum stipe, quam liberaliter exsolvit, centum etiam scutatis aureis donavit."

[53] Grendler 2002, 42–44.

[54] Patrizi to Benedetti (Ferrara, 21 March 1583), Patrizi 1975, 39.

[55] Archivio di Stato di Torino, Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701), Fascicolo 7/2. The document is included as an appendix to Omodeo 2014d.

### 2.5.3 Benedetti and the Counter-Reformation

What can be said about Benedetti's attitude toward the culture of the Counter-Reformation emerging after the Council of Trent? We can assume that his scorn for Aristotelian philosophy was not only a dispute with the university professors of his day, but also with the theology-oriented Aristotelianism propagated through the cultural apparatus of the post-Tridentine Church. We have no evidence that Benedetti frequented exponents of the Jesuit order, in spite of their presence in Turin. Rather, we have evidence of his connections with scholars who were not in the mainstream of the official Catholic culture of those years.

Among others, he corresponded with Francesco Patrizi of Cherso, whose Platonism was regarded with suspicion and even censured in Rome. The closeness between them is witnessed by the fact that Benedetti acted as a mediator between the philosopher and the duke of Savoy on at least one occasion. When Patrizi issued his *Della nuova geometria libri XV* (Fifteen books on the new geometry, Ferrara, 1587) with a dedication to Carlo Emanuele I, Benedetti passed on a copy to his patron. Patrizi expressed his gratitude with the following words (Ferrara, April 6, 1587):

> Very Magnificent and Excellent Signore,
>
> I rejoice with your Lordship that you recovered from sickness quicker than believed. And I am very thankful to you for presenting my book to the very serene Prince and promising to inform me about his remarks after he has read it. If by chance the book will be forgotten, due to his many duties [negozii], I hope at least that you will remember me. If his High Serenity will give some sign that he appreciated it [my book], I will be very glad and I will be particularly grateful to your Lordship for your benevolence.[56]

In exchange, Benedetti sent him a copy of his discussion on the relative sizes of the elements of earth and water, as witnessed by a letter from Patrizi (Ferrara, 18 January 1588).[57]

The two scholars shared views on cosmology that were to be censured by the Inquisition in the 1590s. It is thus expedient to briefly recall Patrizi's natural and cosmological views, as they are close to those Benedetti expressed in his writings, especially in the *Diversae speculationes*. Already in his *De rerum natura libri I priores. Alter de spacio physico, alter de spacio mathematico* (First Two Books on Nature, One on Physical Space and One on Mathematical Space, 1587), Patrizi embraced the neo-Stoic doctrine of the fluidity of the heavens, the infinity of space beyond the sphere of the stars, and the free motion of planets through cosmic space following an inner drive. He later expanded on that in *Pancosmia*, which is the cosmological section of his philosophical masterwork, *Nova de universis philosophia* (New Universal Philosophy, 1591). In it, he did not limit himself to asserting the infinitude of luminous space beyond the visible stars, to repeating the thesis of planetary self-motion, and to rejecting the existence of celestial spheres responsible for the transportation of the heavenly bodies. He also ascribed to Earth the daily motion around its axis without renouncing its central position in the cosmos and in

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[56]Patrizi 1975, XXVII, 53: "Molto Magnifico et Eccellentissimo Signore, mi rallegro con Vostra Signoria, che più tosto che non credea si è rilevata dal male, e li rendo moltissime gratie dell'haver presentato il mio libro a quel Serenissimo Prencipe, e ricevuto il favore, che Ella mi avvisi ciò che haverà detto, dopo che l'havrà letto. Et se per sorte per li molti negozii il libro andasse in oblio, spero da Lei il rimedio di un poco di ricordanza, la quale, se partorirà alcun segno che Sua Altezza Serenissima l'habbia havuto caro, mi sarà carissimo e tutto l'obbligo l'haverò a Vostra Signoria e all'amor suo verso me."

[57]Patrizi 1975, 57–58. In the letter Ottonaio is also mentioned as a common acquaintance and an intellectual partner.

the planetary system.[58]  All of these theses—which, as we shall see, are also present in Benedetti's *Diversae speculationes*—were censured by the Inquisition in the 1590s, in the course of an attack directed against Patrizi's *Nova de universis philosophia*.

The censure of Patrizi's philosophy occurred after Benedetti's death—he was lucky enough to avoid witnessing the prohibition of theses to which he subscribed. On October 8, 1592, the Master of the Sacred Palace, Bartolomé de Miranda, and his fellow, Pedro Juan Saragoza, wrote a document of censure, attacking many passages and doctrines of the *Nova philosophia*. The same Saragoza would later be one of the two censors of Bruno's work during his Inquisition trial in Rome.[59] The two censors of Patrizi did not limit their criticism to theology (especially Patrizi's theses on the Trinity) but also scrutinized natural philosophy and cosmology. They rejected the idea that there is only one unique heaven and judged this to be an opinion at odds with accepted philosophical doctrines and against theology (*In lib. 13 Pancosmias tenet unum tantum esse caelum*). Furthermore, Patrizi was accused of following Copernicus, whose doctrine of terrestrial motion was seen as incompatible with the Bible:

> In *Pancosmia*, Book 17, f. 103, p. 1, column 2a, he [Patrizi] states 'that the motion of the Earth is by far in better agreement with reason than the motion of the heavens or the uppermost celestial bodies.' And he refers to Nicolaus Copernicus's sentence according to which the sidereal heaven is immobile, along with the stars, while the Earth moves.[60]

Further theses to be censured were his vitalistic concept of celestial bodies and celestial infinity. The criticism of the latter point goes as follows:

> This [to sustain this view] is to dream in very deep obscurity and fall down a precipice after abandoning the common way. In fact, the best and greatest God created everything according to weight, number, and measure. Therefore, everybody agrees that no infinite body is possible in act and no existing multiplicity can be infinite in act. On the empyrean heaven see the Fathers and Thomas Aquinas.[61]

Patrizi replied with an *Emendatio in libros suae novae philosophiae* (Correction of the Books of His New Philosophy), written before Christmas 1592. As far as Earth's motion is concerned, he clarified that he advocated its motion but not its eccentricity (*Non tamen dixi, eam de medio suo et naturali loco exire*). Furthermore, he stressed that terrestrial motion was supported by many arguments and several philosophers, and claimed that this view does not contrast with theology. However, he declared himself ready to erase passages referring to terrestrial motion, if necessary (*Delebo tamen si iubetis*). He also defended his views about the infinity of space but was ready to renounce this thesis as well, if he was ordered to do so: "Delebo hanc etiam si iubetis."[62]

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[58] Seidengart 2006, 116–124 and Omodeo 2014a, 174–175.

[59] See Bruno 2000b, doc. 45, 225.

[60] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2216: "Lib. 17 Pancosmias fol. 103, pag. 1, col. 2.a ait *quod Terram revolvi longe videtur esse rationi consonantius, quam Coelum, vel suprema astra moveri. Et refert sententiam Nicolai Copernici dicentis Coelum sydereum stare simul cum* stellis, Terram vero moveri."

[61] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2219: "Hoc est somniare per altissimas tenebras, et a via communi declinando in praecipitia ruere, nam cum Deus opt. Max. omnia in pondere, numero, et mensura produxerit, nullum infinitum corpus actu dari nullamque rerum subsistentium multitudinem actu infinitam omnes viri fatentur. De Coelo empyreo consultat Patres, et sanctum Thomam."

[62] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 7, 2231.

The Jesuit Benedetto Giustiniani proved a more open-minded censor of Patrizi's natural views in 1593.[63] The same person, however, would be one of the eleven theologians who decided that the Copernican theory was not reconcilable with the Catholic religion, in 1616. As to Patrizi's work, it was downright (*omnino*) prohibited in 1594, and placed in the Clementine Index of 1596 as well as in later Indexes.[64]

Not only did Benedetti correspond with intellectuals engaged against the mainstream in Rome, but he himself had differences with Roman Aristotelians, as he mentioned in the preface to the second edition of the *Demonstratio motuum localium contra Aristotelem* (1555). On some occasions, Benedetti even allowed himself to be ironical about theological dogmas. For instance, in one of the letters of the *Diversae speculationes*, he accompanied his adherence to methodological Pythagoreanism (a mathematical approach to the investigation of nature) with a joke about reincarnation and his earlier life:

> If the souls' transmigration imagined by the father of Italian wisdom, Pythagoras, were true, I believe that your soul and mine were once the souls of hunting dogs.[65]

Another indicator of Benedetti's attitude towards the Counter-Reformation and the confessional quarrels of his time emerges from his approach to the calendar reform. This was a very divisive issue. The pope imposed upon all Christianity an emendation of the calendar in an age when it was affected by profound divisions. In this climate, the pope's political and religious legitimacy and his authority in such matters was cast into doubt by many, especially in the reformed countries. Reputed Lutheran astronomers such as Michael Maestlin opposed the calendar reform implemented by Christopher Clavius and promulgated by Gregory XIII in 1582. The elimination of ten days to make the spring equinox date correspond to its date at the time of the Council of Nicaea was particularly controversial, since it symbolically legitimized the universality of the Roman Church through Constantine I. Benedetti wrote a reform proposal on April 1, 1578, and Emanuele Filiberto sent it to Rome on May 31. The text was printed two times, first in 1578, and then reprinted in 1585 as the first of the letters in the epistolary section of the *Diversae speculationes*. The most striking feature of Benedetti's proposal is its technical radicalism and extreme rationalism. August Ziggelaar's perspicuous description of Benedetti's proposal here follows: "Benedetti prefers the Eastern date to be fixed according to the true motions of Sun and Moon rather than by cycles. He finds that the Prutenic tables are sufficiently exact for this purpose. Furthermore not ten days, not even 14, but 21 days should be left out in order to make the first day of January the winter solstice. The lengths of the months are to be adjusted so that they coincide with the presence of the Sun in each of the twelve zodiacal signs. Surely, these time intervals change their lengths in the course of time because of the motion of the perihelion of Earth, but Benedetti assures us that only after 24,000 years will an adjustment be necessary. The proposal is not only interesting and original but also very rational because, by eliminating all reference to the Moon, it makes the year entirely solar."[66] The extreme technicality of such a proposal, one can argue, was

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[63] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 10.

[64] For a reconstruction of the anti-Platonic reaction also affecting the reception of Patrizi, see Rotondò 1982. On the censure of 1616, see Bucciantini 1995, Bucciantini, Camerota, and Giudice 2011 and Omodeo 2014a, chap. 7.

[65] Benedetti 1585, 285: "Si vera esset animorum illa transmigratio quam sibi Italicae sapientiae Pater Pythagoras effinxerat; tuam, meamque existimarem animam canis, quandoque venatici fuisse."

[66] Ziggelaar 1983, 211 and 214.

also a means to de-ideologize the issue. Therefore, it was not only rejected for its "scientific radicalism" but also for its rationalistic distance from confessional struggles. This positioning is in line with late-humanistic universalism and signals Benedetti's distance from the Counter-Reformation and the militant cultural production of those years.[67]

## 2.6 Cultural Institutions: University, Academies, Collections, and the Press

The reformation of the *Studio* was a cornerstone in Emanuele Filiberto's and his successor's cultural policy. It was reopened in Mondovì in 1560 after the French closed it as a potential center of dissent and revolt. It was transferred to the new capital on October 12, 1566, after long discussions and a dispute between Turin and Mondovì. According to the new statutes, issued in 1571, the direction of the university was conferred to nine *reformatores*, among whom were the Archbishop, the ducal chancellor, the first and second presidents of the Senate of Turin, and the court physician. These men were responsible for the scientific and administrative direction, and for academic discipline.[68]

Most of the professors (about thirty people) were jurists. Among them, the most reputed was the professor of civil law Guido Panciròli. The physicians Francesco Valleriola and Giovanni Argenterio were also illustrious professors, known and appreciated by Conrad Gesner and Michel de Montaigne. The reopening of the university offered Argenterio the opportunity to come back to Piedmont after many years of practice as a physician in Lyon, and of teaching in Antwerp, Bologna, Pisa, Rome, and Naples.[69] Teodoro Rendio of Chio was appointed to teach Greek grammar and, as mentioned before, the poet Giraldi Cinzio became professor of humanities. At the University of Turin, the teaching of philosophy on the basis of the *corpus Aristotelicum* received high recognition, as the professors in this discipline had a better salary than their colleagues of medicine. Giacomo Castagneri taught on Aristotle's *Physica*, *De generatione et corruptione*, and *De Anima*. Other celebrated scholars in philosophy were Antonio Berga and Agostino Bucci, both Piedmontese educated at Padua.[70]

One of the most reputed professors appointed in Mondovì was Francesco Vimercato of Milan, with whom Benedetti later corresponded. Vimercato was a thoughtful Renaissance commentator on Aristotle, whose work he read in the original language. He published commentaries on *De anima* (1543), on *Metaphysics* (1551), in particular on book lambda and on *Meteorologica* (1556). However, his commentary on *Physics* has to be seen as his magnum opus. After his studies in renowned Italian universities, he was appointed in Paris as the first royal lector in philosophy. There, he was first involved as a judge in the Ramist disputes and later was a colleague of Petrus Ramus. In 1561 he came to Piedmont to serve the Savoy family as a university professor, a councillor, a tutor to Carlo Emanuele I, and, from 1567 to 1570, as diplomat in Milan.[71]

Bordiga argued that Benedetti might have taught at the reopened university, first in Mondovì and then in Turin, by relying on some indirect sources. In fact, the information about Benedetti's teaching activity at Mondovì was derived from the sixteenth-century biographer of Emanuele Filiberto, Giovanni Tonso, who included his name on the list of

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[67] Steinmetz 2011.
[68] See Bonino 1824–1825, Naso 1993, and Catarinella and Salsotto 1998.
[69] On Argenterio, see Temkin 1974, 141–144 and 149–152 and Mammola 2012, 185–193.
[70] On the philosophical culture of Turin of those years, in particular on Bucci, see Mammola 2013.
[71] See N. W. Gilbert 1965.

those who made that university eminent (*Qui viri insignes publice docuerint*).[72] Another contemporary of Benedetti, the mathematician Cristini, mentioned him as one of those whom Emanuele Filiberto called to Turin in order to increase the prestige of his university.[73] Still, the role that he could have played in the academic life of Turin remains obscure.[74]

We could find no direct evidence that Benedetti served as a professor in the documents preserved at the *Archivio di Stato di Torino*. We considered the acts gathered under the signature "Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701)," which include the statutes of 1571 and other precious sources concerning the first years of the univeristy. A dossier entitled "1571, Costituzione de' Riformatori dell'Università dello Studio di Torino, coll'Istruzione da osservarsi da medesimi, colle distribuzioni delle ore per la Lettura, e Rolli de' Stipendi de' Lettori" (*fascicolo 7 primo*) includes decrees concerning the reform of the university, the names of those responsible for accomplishing it, and a list of the chairs with the corresponding salaries and the names of the professors. These documents indicate that the professor of mathematics was Francesco Ottonaio of Florence, who had occupied the same chair at Mondovì. Other documents testify that the same person held the chair of mathematics in 1573 and in 1585/6. It is therefore hard to believe that Benedetti was professor in this subject before 1586, as several secondary sources contend.[75] As to his alleged position at Mondovì, that university was opened by Emanuele Filiberto in 1560 and then transferred to Turin in 1566, that is, before Benedetti's arrival at the Savoy court. Thus, it must be excluded as a possibility that he ever taught there, contrary to the claim made by Tonso. It is possible though, even likely, that he acted as an external advisor on matters concerning the university.[76]

Turin had fewer academies than other centers such as Rome, Naples, and Florence, although it was a typical Renaissance phenomenon to establish academies, circles of learned men who met to discuss issues pertaining to literature, the arts, or the sciences, and who published works representative of their common intellectual efforts. Apart from two minor academies ("*de' Solinghi*" and "*degl'Impietriti*"), Carlo Emanuele I conceived the project of forming an academy guided by the Jesuits around 1585. Bonifacio Vannozzi, a man of letters from Pistoia, described it as follows:

> His Highness, the very serene [duke] of Savoy, had the wish to found an academy in this august town of Turin. He charged three Jesuit Fathers of the renowned College with the task. Although they are generally sober of mind, in this case they were so intemperate as to entrust myself [with this endeavor] although the overwhelming responsibility [*machina da incurvar le spalle*] would be excessive for even the most competent person. His Highness has made himself Prince, Protector, and Head [of the academy], in order to attract a good deal of his courtiers [into the academy] who are so cultivated

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[72] Tonso 1596, 141: "Neque vero liberalium disciplinarum omniumque artium colendarum quam susceperat cogitationem unquam deposuit: nam et publicum earum Gymnasium pro tempore in oppido Monteregali instituit: et qui viri in quacunque scientia excellerent undique conquisuit. […] Mathematicos illustres Franciscum Othonarium, et Io. Baptistam Benedictum Venetum."

[73] Bordiga derived this information from a manuscript of Cristini's preserved in the Biblioteca Marciana in Venice. See Bordiga 1926, 596–597.

[74] The historian of Piedmontese Universities Silvio Pivano complained already in the 1920s about the lack of relevant documents. Pivano 1928, 19–22.

[75] See, e.g., Bauer 1991, 156–157.

[76] Roero 1997, 65, n. 5. Evidence for Benedetti's role as an advisor in university matters can be found in Patrizi's correspondence, as already mentioned.

and refined that, if one adds to it the splendor of the arts [lettere], there will be
no court in Europe more illustrious than this one. Our name is 'Incogniti.'[77]

In spite of the initial impetus, this academy was not particularly successful and did not
leave significant traces of its activities. Perhaps it was negatively affected by the fluctu-
ating relations between the Crown and the Jesuit order.

Emanuele Filiberto also created a *Theatrum omnium disciplinarum*, which was re-
alized for him by Ludovic Demoulin de Rochefort. It is not completely clear what this
theater contained. It was probably a *Wunderkammer*, as Mamino argued.[78]

In March 1572, duke Emanuele [Filiberto] established in Turin a museum
[*theatrum*] of all disciplines [organized] in marvelous order and at a very high
cost. Archbishop Gerolamo della Rovere and the philosopher Ludovic De-
moulin de Rochefort, the most educated men in all fields, cared for it.[79]

Moreover, the dukes supported editorial activities. Emanuele Filiberto first called the
Flemish printer Laurens Leenaertsz van der Beke, better known as Torrentinus, to Pied-
mont. Torrentinus had already initiated a printing house in Florence under Cosimo I., but
he died shortly after his arrival in Mondovì (1561). Hence, in 1570, Emanuele Filiberto
employed another printer, Niccolò Bevilacqua. This pupil of Manuzio founded the *Com-
pagnia della Stampa* (Printing Company), which benefited from ducal privileges (a sort
of editorial monopoly). The activity of Bevilacqua and his heirs culminated in 71 editions
produced between 1578 and 1580, most of them on juridical subjects. Scientific issues
were not neglected in Turin. Among the publications issued during the years of Emanuele
Filiberto, between 1563 and 1580, books on scientific and natural subjects constitute about
one fifth of the entire production, that is, almost as much as literary publications.[80] They
also printed several books by Benedetti, including the *Diversae speculationes*.

## 2.7   Scientific Debates

### 2.7.1   Courtly Conversations

Renaissance Turin was a center of lively cultural and scientific debates taking place in
different institutional settings, at court, at the university, and in town. The printing press
was a powerful means for public discussion, dissemination of ideas, and criticism. Printed
sources are evidently our main source of information about the intellectual debates or
polemics that animated Turin in Benedetti's time.

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[77]Tiraboschi 1824, 289–290: "L'Altezza di questo Serenissimo di Savoia ha desiderato, che si dia principio
a fondar un'Accademia in questa sua Augusta cittá di Turino, et n'ha data la cura a tre Padri del Gesù di
questo insigne Collegio, i quali, non so da che allucinati, soliti però a non s'abbagliare, hanno fatto gran
fondamento nella persona mia, caricandomi d'una macchina da incurvar le spalle, quantunque gigantesche.
S.A. se n'è fatto Principe, e Protettore, e Capo, per tirarvi buon numero de' suoi Cortigiani, tanto culti e
fioriti nel resto, che, se vi si aggiugne l'ornamento delle belle e delle pulite lettere, non sarà Corte in Europa
più rilucente di questa. Il nostro nome è degli Incogniti […]."

[78]Mamino 1992 and Mamino 1995. By contrast, Cibrario thought that it was an encyclopedic project. See
(Cibrario 1839).

[79]Pingone 1577, 88: "Anno Christi 1572 mense Martio, Emanuel Dux Taurini theatrum omnium disci-
plinarum miro ordine, nec minimis sumptibus instituit, curantibus Hieronymo Ruvereo Archibiscopo, et
Ludovico Molineo Rochefortio Philosopho, viris in omni doctrinae genere absolutissimis."

[80]On Renaissance publications in Piedmont, see Bersano Begey 1961, especially vol. 1. See also Merlotti
1998.

First of all, we should consider courtly debates. A circle of intellectuals gathered around the Savoy family. The professor of jurisprudence, Bernardo Trotto, depicted the relationship between the rulers and their learned courtiers as follows:

> These learned men, played by the Prince like well-tuned musical instruments, immediately give out their specific sounds with words. And they give it their best to be clearly understood in conversations, to please the others with good arguments and to convince them of their opinions. It is like the consonance of truth. In fact, everyone says what one knows or, at least, considers to be true. Hence they discuss natural issues and at times moral ones and mathematical ones. In conclusion, one can regard him [the prince] as Apollo surrounded by the Muses near the water spring that was born from the hoof of Pegasus.[81]

A reflection of the intellectual climate and the topics addressed in such informal meetings is a poem by the court physician Arma, *Proposte tenute co'l Serenissimo Prencipe* (Issues Discussed with the Very Serene Prince), printed in Turin in 1580. In this tiny book, addressed to Emanuele Filiberto, Arma reported a discussion on meteorological problems that took place between Carlo Emanuele I, Benedetti, Ottonaio, Berga, and himself during the prince's meal for three successive days. On the first day, Carlo Emanuele I asked the reason why the sun heats. Arma used Plato to argue that its body is not warm but that it heats through the percussion of its rays on terrestrial elements.[82] The next day, Arma had a quarrel with Benedetti, Berga, and Ottonaio concerning "solar attraction."

> The day after, Mister Benedetti
> And Mister Berga, along with Ottonaio
> Expressed opinions that are far from mine:
> That the Sun attracts everything to itself with its great brightness
> As if it had hands.[83]

Arma objected that, if the sun attracts other bodies, this would be very inconvenient for its inhabitants, the solarians (*solari*). The argument is not clear and the reader is only informed about the fact that Benedetti agreed with him.

During the conversation, Ottonaio mentioned the vapors brought upward by the sun's action. Accordingly, the next topic of discussion concerned natural places and elemental displacements with reference to the behavior of vapors.

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[81] Trotto 1625, 2–3: "[…] questi huomini saputi, tocchi dal Prencipe, come instrumenti musici bene accordati, subito rendono ciascuno il suo suono con le parole et quanto meglio possono procurare d'essere intesi discorrendo, e di dar diletto con le buone ragioni, et anco di tirare gli altri al suo parere, come ad una consonanza della verità: perché ognuno dice quello ch'egli sa o crede almeno sia vero. E quindi si veggono trattare hor cose naturali, hor morali, hor mathematiche. Sì che egli quasi come uno Apolline si può dire, che sta fra le Muse, intorno al fonte, che uscì dal colpo del piede del cavallo alato." On Trotto's teaching, see Vallauri 1846, 28 and 48–49.

[82] Arma 1580a, f. A2*r*: "Scalda co raggi […]/ Sbattendo la Terra di caldo priva. Sì com'il martel che bate l'incudine,/ Riscalda l'un e l'altr'in certitudine."

[83] Arma 1580a, f. A2*v*:

> "Il Signor Benedetti l'indomani
> Col signor Berga, insiem' a l'Ottonaglio
> Forn'in pensier' a me d'assai lontani,
> Che'l Sol tirass'a sé com grand'abbagio
> Ogni cosa si com'havesse mani."

On the third day, the prince asked about the origin of lightning, and why we perceive their light before we hear the thunder. Arma answered that our sight is much quicker than our hearing, but this remark was received with skepticism by his colleagues. No details are reported about the objections that should have concerned the theory of perception, light, and optics.

The next issue was colors and the rainbow; Benedetti asked about the center of the rainbow's arc and Arma offered the following answer:

> Benedetti, as an expert master of his art,
> Asked me about the center of the arc [of the rainbow].
> I answered that it was on the vertical line
> Descending downward from the center of the heavenly body,
> As was the opinion of Zoroaster.
> And with this answer I got rid of him.[84]

At the end of this three-day conference, all opinions were written down for the prince and signed by the ducal advisors:

> All of this was presented in written form
> To His Highness, reporting all speeches.
> Dr Berga confirmed it [the rightness of the report].
> Benedetti did the same.
> After that we discussed other issues,
> Occult things and their effects.[85]

Other publications also mention such table talks at court. For instance, the physicist and philosopher Bucci wrote in the preface to his book on physiology, *Disputatio de principatu partium corporis* (Disputation on the Superiority of Parts of the Body, 1583), that his discussion about whether the heart or the brain held superiority in the body, and about the localization of the soul, originated from a table talk. On a certain occasion, in fact, Carlo Emanuele I had gathered his learned courtiers and addressed these issues. Among the participants, including several physicians, Bucci also mentions the "mathematicians," Ottonaio and Benedetti.[86]

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[84]Arma 1580a, f. A4*r*:

> "Il Benedetti, come degno maestro,
> Mi dimandò d'il centro di tal arco.
> Dissi, che gliera col centro de l'Astro,
> Ne la medema linea giù scarco.
> Si com'anchora volse Zoroastro.
> E con tal dire di lui mi discarco."

[85]Arma 1580a, f. A4*r*:

> "E tutto quest'in scritti fu donato
> A Sua Altezza, con tutti soi detti.
> E fu dal Dottor Berga conformato.
> Il che fece'l signore Benedetti.
> Fu poi d'altre proposte ragionato
> E de gl'occolte cose, e soi effetti."

[86]Bucci 1583, 7–8. See Mammola 2013, 6–8.

### 2.7.2   Academic and Scholarly Controversies

Scholarly controversies and polemics on various issues and with very different tones were printed in Benedetti's years. While courtly debates had a polite and entertaining character, academic disputes could be more vehement. However, the two contexts were not always neatly divided. In 1572 two professors of philosophy, Berga and Bucci, held divergent opinions concerning the interpretation of Aristotle's *De anima*.[87] As Simone Mammola has argued, their disputes on that issue should be understood against the background of the Averroistic-Simplician debates on Aristotle's psychology started at Padua and continued at Turin by scholars such as Filateo and Vimercato. The Turin controversy over Aristotle's soul doctrine, which developed through academic disputations and publications, ranged from cognitive problems linked to the functions of the soul (such as the problem of the relationship between imagination and cogitation) to metaphysical and theological issues, e.g., the legitimacy of a reading of Parmenides's theory of being as a form of *prisca theologia*. The real issue at stake was the correct approach to the Aristotelian corpus based on different commentators.[88]

The court physician Arma was at the center of several public controversies as well. In 1575, he defended the scientific status of medicine, traditionally considered "only" an art, in the programmatic book *Quod madicina sit scientia et non ars* (That Medicine is a Science and Not an Art). This writing was considered worthy of a second edition in 1585. In 1579, Arma entered a dispute against the professor of medicine Giovanni Costeo over the healthiness of bread made out of rice ("*pane fatto col decotto di riso*"). The two parties then issued a series of publications on this controversial topic.[89]

Another polemic opposed Berga and Benedetti regarding the proportion between the earthly and the watery element of our globe. The debate was initiated by some questions Carlo Emanuele I asked his courtiers. While Berga stuck to the Scholastic view that the elements have an increasing quantity proportional to their distance to the center, Benedetti favored the merging of empirical and mathematical arguments as proposed by Alessandro Piccolomini. Piccolomini had come to the conclusion that the quantity of the earthly element is superior to that of the water element, as one can read in his *Della grandezza della terra e dell'acqua* (Venice, 1558).[90] Although Benedetti regarded Piccolomini's arguments as conclusive, Berga undertook to compose a Scholastic refutation of them, *Discorso… della grandezza dell'acqua e della terra contra l'opinione dil S. Alessandro Piccolomini* (Turin, 1579). Part of his strategy was to eliminate arguments derived from the "misure dei cieli e della terra, dalla Scuola dei matematici immaginate," that is, from mathematical and empirical methods applied to this issue ("measurements of the heavens and the earth imagined by the school of the mathematicians"). Benedetti reacted with his *Considerazione… d'intorno al discorso della grandezza della Terra, et dell'Acqua. Del Eccellent. Sig. Antonio Berga Filosofo nella Università di Torino* (Consideration… on the discourse concerning the dimensions of earth and water by the excellent Mr. Antonio Berga, philosopher of the University of Turin, Turin, 1579). In it, he applied mathematics to show that, even if the entire earth were covered by water, the volume of the terrestrial

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[87]Bucci 1572 and Berga 1573.

[88]See Mammola 2013.

[89]Merlotti 1998, 585: "Come s'è visto per la polemica fra Costeo e Arma […] non si trattava di isolati testi a stampa che generavano dibattiti destinati a rimanere manoscritti e chiusi nell'ambito degli eruditi, ma semmai del contrario: di discussioni, cioè, sorte in circoli ristretti di medici e scienziati, prima affidate a manoscritti e poi trasportate a stampa a vantaggio d'un più vasto pubblico."

[90]For an accurate reconstruction of the polemic and its cultural and scientific context, see Ventrice 1989, 103–145 and Mammola 2014.

element would not be inferior to that of water since the depth of seas and oceans is small
in comparison to the terrestrial radius. The oceanic navigations, the geographical explo-
rations, and the European colonization of the globe during the fourteenth and fifteenth
centuries had indisputably demonstrated that water and earth constitute one single globe,
a unique *globus terraquaeus*.[91] This cosmographical advance had been the basis for Pic-
colomini's considerations, which met with considerable success and agreement among
scholars. On such issues, as Benedetti stressed, experience and mathematical reasoning
should be joined:

> Very serene Prince, the discovery, after two thousand years, that the [ele-
> ment] earth is much more than the [element] water (for which we are greatly
> indebted to the very learned Mr. Alessandro Piccolomini) very much pleased
> the spirits of the most renowned philosophers of our time. In the past, they
> did not dare to depart from the false doctrine they had imbibed for many cen-
> turies, although it was sustained by implausible reasons. Today they are glad
> to embrace the opposite opinion [concerning water and earth], because both
> the senses and reason are in accordance with the [new] demonstration of the
> truth. The ancient mistake has been unveiled by the mathematical school
> with very certain proofs that offer a firm foundation of the measurement of
> the heavens and the Earth.[92]

The dispute continued with the Latin translation of Berga's writing by Francesco Maria
Vialardi (1580) and a skeptical intervention by Arma. The latter was a poem dedicated
to Carlo Emanuele I, entitled *Stanze del dottore Arma al serenissimo Carolo Emanuele di
Savoia et Piemonte Prencipe, etc. suo signore sempre osservandissimo. Che l'acqua e la
terra non si possono a modo alcuno misurar* (Doctor Arma's Stanzas to the Most Serene
Prince Carlo Emanuele of Savoy and Piedmont, His Perpetually Honored Lord, 1580).
The composition is poor both from a stylistic viewpoint and a scientific one. The courtly
physician could only point out the uncertainty of human knowledge and the wisdom in
measure gifted by God to creation, although he also stated we cannot grasp the latter:
"The Creator gave it a measure,/ Which cannot be grasped by any creature."[93] Benedetti
addressed the issue again in one of the letters of the *Diversae speculationes*.[94]

### 2.7.3 Astronomical-Astrological Polemics

In Renaissance Turin, astronomical and astrological issues were at the center of intense de-
bates and even polemics. In 1578 the protophysician Arma was the target of a denigrating
pamphlet in twenty points. An anonymous author attacked a booklet of his on the comet
that had just appeared. As we know from indirect evidence, Arma had written one or two
treatises on this issue. One was entitled *De significatione stellae crinitae*.[95] Perhaps it has

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[91]Vogel 1993.

[92]Benedetti 1579, 3: "[…] l'essersi doppò due mila e più anni scoperto con trionfo della verità, che la terra è
molto maggiore dell'acqua, (del che si ha da haver grande obligo tra gl'altri al dottissimo Signor Alessandro
Piccolomini) ha non poco rasserenato, Serenissimo Principe, l'animo de' più famosi Filosofi di nostra età; i
quali, sì come prima non intendeano dipartirsi dalla già imbevuta falsità, e per molti secoli adietro, benché
con inefficaci ragioni difesa, così hora si lasciano volentieri persuadere il contrario; poiché il senso, e la
ragione s'accorda alla dimostratione del vero. E nella scuola de Mathematici per certissime prove si scuopre
l'antico errore, puotendosi far fondamento stabile delle misure de cieli, e della terra."

[93]Arma 1580b: "Il Creator gli diede tal misura./ Che saper non si può da creatura."

[94]Benedetti 1579, 397–405, "Defensio nostra contra Antonium Bergam, et Alexandrum Piccolomineum."
See Ventrice 1989, 131–134.

[95]This information stems from Bonino 1824–1825.

to be identified with a composition in verses, referred to as πωγωνία (bearded comet) in the anonymous pamphlet. The only remaining evidence of the cometary polemics is a defence of Arma, *La Stravagantographia del Sig. Filosofo stravagante, in difesa de la πωγωνία d'il Dottore Arma* (The Stravagantography of Mr. Stravagant Philosopher in Defence of Dr Arma's Bearded Comet).[96] The apologist was one Monsignor Della Torre linked to the court. His apology offers an insight into the controversy: It concerned astronomical and meteorological issues relating to the nature of the comet, its location below or above the sublunary sphere, the criticism of astrological interpretations of the celestial phenomenon, as well as personal insults. The provocateur who had published against Arma is depicted as follows:

> I cannot stop wondering who this person is. I cannot understand why he sometimes presents himself as a scholar, sometimes as a cook, as a Roman courtier, or as a practicing friar [*frate osservantino*] (as he speaks about the *osservantini*). I cannot believe that he is a practicing [man of religion], as the ecclesiastics speak in a correct manner and not heedlessly like him (who behaved heedlessly). Moderation has always been praised. Therefore, moderate people will always damn this person. I will never believe that he is a scholar. In fact, today's scholars are well-educated and would never indulge in such excesses, especially against such a man [Dr. Arma] from whom they did not receive anything but pleasure, honor, and courtesy. Although he seems to come from the area of Rome, in the end he shows himself to be a dishwasher because even a cook would behave better than him. Whoever the hell he is, if he will not control himself better in the future, I will repay him as he deserves.[97]

Possibly the identity of this mysterious denigrator was the philosopher Giordano Bruno, who was in Turin in 1578 on his way to Chambéry, in Savoy. At that time he wore the Dominican habit and had just published, as mentioned above, a booklet on meteorology in Venice entitled *De' segni de' tempi* (On the Signs of the Times) that presumably dealt with the comet.[98] The reference to the provocateur as a Roman courtier could correspond to an episode of Bruno's life. In Paris, in 1585, he told the librarian Guillaume Cotin that he had been once received at the Roman court by Pius V and the Cardinal Rebiba to whom he demonstrated his technique of the art of memory.[99] Moreover, the Turin episode resembles a *querelle* that burst out in Geneva in 1579, after the publication of a pamphlet in twenty points ridiculing a professor of philosophy. The detractor, Giordano Bruno, was discovered and condemned to exclusion from communion. As one reads in a document (6 August 1579), " Philippe Jordan, dit Brunus, Italien était détenu pour avoir faict imprimer

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[96]It is preserved in the Biblioteca Reale of Turin, coll. G 25–67.

[97]Della Torre 1578: "[…] non mi posso quietare pensando chi possi esser costui. Non posso capire, perché quando fa d'il scuolaro, quando del cuogo, quando del corteggiano di Roma, quando del frate osservantino, poi che di osservantini parla. Di esser osservante, nol posso pensare, perché li religiosi parlano correttamente, e non si sgovernano nel parlare, come ha fatto costui, il quale mattamente si è sgovernato. Fu sempre lodata la modestia. Sarà donque dalli modesti dannato costui. Che sij scuolaro, non lo crederò mai, perché hoggi dì li scuolari sono ben creati e non farebbono tale scappate specialmente contra di un'huomo tale da cui mai hebbero altro che apiacere, honor e cortesia. Par bene che habbi del Romanesco nel principio, ma il fine dimostra più presto haver del sguattero, perché il cuogo si sarebbe meglio deportato che non fa costui. Sij chi diavol esser si voglia. Se esso per avanti meglio non si governarà, tale e tanto mi ritrovarà, quale e quanto mi ricercarà."

[98]See Omodeo 2008a. On Bruno's lost meteorological-cometary work, see Ernst 1992.

[99]Spampanato 1921, 654–655 and Ricci 2000.

certaines responses et invectives contre Mr. de la Faye, cottans 20 erreurs d' iceluy en une de ses leçons." [100]

Only one year later, between 1580 and 1581, Benedetti was involved in an astronomical-astrological quarrel with a certain Benedetto Altavilla of Vicenza concerning the reliability of ephemerides and astrological prognostication. The controversy was sparked by the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, 1580) and was continued with two further publications by the same author.[101] They cast the reliability of ephemerides' calculations and astrological forecasting into doubt, based on the alleged theoretical flaws of mathematical astronomy. Since they appear to have been directed against scientists linked to the court, Benedetti eventually intervened with a printed letter, *Lettera per modo di discorso… intorno ad alcune nuove riprensioni, et emendationi, contra alli calculatori delle effemeridi* (Turin, 1581). He later translated it into Latin as *Defensio ephemerides* and included it in the miscellanea of epistles of the *Diversae speculationes*. We will expand on this polemic later, in the section on Benedetti's astronomy.

### 2.7.4 Posthumous Criticism: Cristini on Benedetti

Benedetti died on January 20, 1590, two years before his own astrological prediction. This untimely death did not leave him the time to complete the astrological work that he announced at the end of the *Diversae speculationes*. What is worse, the fact that his own prediction was wrong awakened doubts and rumours about his scientific talent. The mathematician Cristini was quick to compose a critical essay, entitled "Examination of the mistake and emendation of the nativity of the very excellent mathematician, Mr. Giovanni Battista Benedetti, now deceased, to account for his [wrong] prognostication of his own death two years later than it in fact occurred" (*Essaminatione dell'errore, della rettificatione de tempo della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico eccellentissimo, per cagion del quale esso s'era pronosticato morte due anni appresso in circa al tempo che gl'è avvenuta*). Although the manuscript was lost in the fire of the Biblioteca Nazionale of Turin in 1904,[102] a transcription of significant parts by the eighteenth-century biographer of Cristini, Giuseppe Vernazza, is still extant in the Biblioteca Reale of Turin among the documents that Vernazza gathered for the composition of his *Notizie di Bartolommeo Cristini* (Notes on Bartolomeo Cristini, 1783).[103]

Cristini started his examination of Benedetti's errors with specific reference to his mistaken self-prognostication in the *Diversae speculationes*. He also reported that Benedetti himself acknowledged an error in his nativity, amounting to four minutes. After that, Cristini took upon himself the task of recalculating that horoscope on the basis

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[100]Spampanato 1921, 132.

[101]On astronomical-astrological quarrels in Renaissance Italy and Turin, see Omodeo 2008a and Tessicini 2013.

[102]Bordiga 1985, 609, n. 6. See Omodeo 2014c.

[103]Vernazza 1783. Two manuscript copies of Vernazza's biography of Cristini are still extant. One is preserved in the Turin State Archive (Archivio di Stato di Torino, coll. Miscellanea J.b.VIII. 9), the other is kept in the Biblioteca Reale of Turin (Vernazza manuscript, misc. 67.5). The latter is a good copy, ready for the printer. It contains an appendix of "documents" for the personal use of the author. These are transcriptions or translations of significant passages of documents by Cristini that were lost or seriously damaged after the fire at the Turin National Library in 1904. They comprise the dedication and table of contents of the *Revolutione trentesimaterza del Ser[enissi]mo Sig[nor] il Signor Carlo Emanuele duca di Savoia* (1596), notes from various astrological diaries, an Italian version of the beginning of *La rithmomachia o sia gioco di Pithagora* and, most importantly, a long extract from the *Essaminatione dell'errore… della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico*.

of the figure published by Gaurico and came to the conclusion that the mistake was even bigger. It amounted to eight minutes.

> Benedetti published his prognostication of the moment of his death in the work entitled *Diversarum speculationum mathematicarum et physicarum liber* (published in 1585), in a letter to the most illustrious Wolfhard Eisenstein [Volfardus Aisenstain], which is to be found at the end of this work. After a brief assessment of those things of the judicial art that he regarded as vain or false, and after announcing to Wolfhard that he would expand [on astrology] in that tract with his astrological observations, which he wished he could publish before his death, he added the indication of the time in which, according to him, [his death] was to happen (that is, [the date] before which he wished he could publish the aforementioned tract). These are his words: "antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti" [as indicated by my horoscope, before I meet the body of the adverse Mars. This is going to happen in 1592].
>
> As we can see, he was certain that he would die when the direction of his ascendant and Mars would meet. He calls [Mars] "anaereta," that is, giver or announcer of his death. He confirmed this when […], just before his death, he felt that the disease was attacking him and declared that he made a mistake of four minutes in the rectification of the time of his birth horoscope [*natività*]. This is as if he would say that, by augmenting by four minutes the time of his birth horoscope, he would have predicted the direction [of his ascendant sign meeting Mars] at about the time when he became sick. Hence, he believed he was dying, and this [his death] in fact occurred at the end of the ongoing year 1590, at 17:00 of 20th January according to [the calendar of] Gregory, which corresponds to the 10th of the old [calendar]. I had to know the time in which he believed he was born in order to assess by how much time he was mistaken in the rectification of his birth horoscope, so that the direction of his horoscope relative to Mars corresponded to the days when he left this world. Therefore, at Benedetti's death, I immediately began to compute the error of the aforementioned time, though only approximately, because I did not know Mars's latitude. And I found that it [the mistake] amounted to eight minutes […]. Later, when the same person who told me that Benedetti had acknowledged a mistake of only four minutes according to his calculations, openly accused me of not being able to do this calculation, as my mistake was two times [that of Benedetti], I began the calculation in the following manner. First, I determined the time attributed to his birth […] Etc.[104]

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[104] From Vernazza's papers accompanying his manuscript of his *Notizie di Bartolommeo Cristini*. Biblioteca reale di Torino, Misc. 67.5, *Vita di Bartolomeo Cristini con documenti*, "M.S. L.1.10, 11.493, di pag. 42." See Omodeo 2014b: "Ha pubblicato il Benedetti, il pronostico fattosi del tempo di sua morte nell'opera sua titulata *Diversarum speculationum mathematicarum et physicarum liber* stampata dell'anno '85 in una lettera scritta all'ill.mo Volfardo Aisestain, posta nel fine d'ess'opera, percioché appresso haver brevemente dichiarato quali cose egli stimava vere nella giudiciaria e quali vane o false, et detto com'esso Volfardo potrà veder poi meglio in quel trattato dell'osservationi sue astrologiche, quale sperava dar in luce avanti la sua morte, soggiunge il tempo il quale giudicava essa doverli avvenire, o sia avanti al quale desiava pubblicar detto trattato, con queste istesse parole: "antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti." Donde appare ch'esso teniva per certo d'haver a morire, quando giongerebbe alla

In his transcription of Cristini's *Essaminatione dell'errore*, however, Vernazza omitted numbers and calculations. These can be found in another astronomical-astrological assessment and criticism of Benedetti entitled "Thirty-first revolution of the very serene Sir, Duke Carlo Emanuele of Savoy, for the year 1592, very diligently and reliably calculated and explained by Bartolomeo Cristini, scholar of mathematical disciplines in the service of His Highness, according to the best opinion of the main judiciary astrologers" (*Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria per Bartolomeo Crestino studioso delle mathematiche discipline in servitio di Sua Altezza*). Although the manuscript was damaged by the fire of the Turin library in 1904, it is still readable. The dedicatory letter shows that, at the time of its composition (June 8, 1592), Cristini was striving to obtain a stable appointment at court:

> But your very generous Highness awoke in my spirit the desire of mathematical virtues and of undertaking the present endeavor. Your request woke up and unveiled in me the desire (which is always alive) to serve [Your Highness]. However, my desire has been impeded by the difficulties of my continuous poverty and adverse times owing to the fact that no treasurer (or any monetary and financial administrator) regards me as an ordinary servant of Your Highness. [I have been acknowledged as a servant] only in exceptional cases, when my capacity, readiness and knowledge in making calculations has proved useful—as has happened several times, when I was required to serve Your Highness. […]
>
> Therefore, I place growing hope only in Your Highness the more [you] require my services, the more efforts I make for You and the fewer are the number of [benefactors] by whom I can hopefully be supported[105]

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direttione del suo ascendente al corpo di Marte, quale chiama anaereta cioè datore, o promissore de la morte sua. Il che pare habbi volsuto confirmare quando che, come dice, poco avanti la sua morte ei si sentì carrigar dal male, disse d'essersi fallato di quattro minute nel rettificare il tempo di sua natività, perché questo è come s'havesse detto che quando egli havesse accresciuto tempo di sua natività per quattro minute havrebbe conosciuto la direttione predetta essere minore di quello [che] l'haveva fatta, et periciò il tempo della sua morte caggionata da essa direttione dover essere circa questo tempo, ch'egli s'era infermato, et credeva di morire come è pur avvenuto, essendosi occorso ciò fare dell'anno presente 1590 circa le 17 hore del 20 giorno di genaro secondo Gregorio, che viene ad essere il dieci dell'anno antico. Perciò volendo io essaminare di quanto tempo egli habbi fallato nella rettificatione di essa sua natività, accioché giustamente la direttione predetta dell'horoscopo suo al corpo di Marte venisse a cadere nel giorno istessi ch'egli partì da questo secolo, m'è stato necessario sapere il tempo ch'egli havea presupposto fosse quando nacque […]. Perciò mi posi subito seguita la morte del Benedetti a far conto dell'errore del tempo predetto, così alquanto alla grossa, per non haver nota la sopradetta latitudine di Marte, et ritrovai detto errore essere di minute otto in circa di hora […] Ma perché ho dipoi inteso che chi mi ha riferto il Benedetti haver confessato il detto fallo di min. 4 et haver solamente ritrovato tanto per calculo ha espressamente detto che io errava del doppio et non sapea far questo conto […] mi posi a calcolare di questa maniera. Prima ho ritrovato il tempo presupposto della natività […] Etc."

[105]Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 4*r–v*: "Ma V[ostra] Alt[ezza] benignissima sì come è stata cagione d'eccitar nell'animo mio il desio delle vertù matematiche, et di farmi fare la presente fatica; così ancora co'l chiamarmela adesso ha risvegliato, o riscoperto le sempre vive brame mie di servirla, le quali erano tenute sepolte dai disaggi che queste carestie et mali tempi mi causano maggiori giornalmente, percioché non sono conosciuto per servitore ordinario di V[ostra] Alt[ezza] da Tesoriere alcuno, né da ministro di suoi dinari o finanze; se non ne' casi che la vertù et prontezza, o cognizione mia ne' conti, può reccarli qualche giovamento come ha fatto più volte quando per servitio di

In this case, the allegation against Benedetti is for using the Alfonsine tables to determine the nativity of Carlo Emanuele I, instead of more reliable Copernican tables. Such inaccuracy invalidated his astrological judgments.

> I took into account the places where they [the planets] are to be found in the horoscope made according to the true time calculated on the basis of Copernicus, following the teaching of the major authors on astrology. In general, since scholars are in disagreement concerning the employment of different tables to compute their horoscopes [*revolutioni*] and although I have demonstrated (in the calculations at the beginning of my tract) that only one set [of tables] is true, I calculated the astrological figures of the heavens according to both tables—in fact, false ones were also in use by many and in particular by Benedetti—and I offered double astrological judgments depending on the places assigned according to the different figures. In this manner, your Highness will possibly compare them and see which ones are in better agreement with the truth.[106]

The terms of Cristini's polemics echoed those of the controversy of the years 1580–1581 between Altavilla and Benedetti. In fact, Cristini examined two astrological figures calculated by Benedetti: a nativity based on the "Copernican" tables of the German astronomer Erasmus Reinhold, *Prutenicae tabulae* (1551), and a prognostication based on the Alfonsine tables.[107] He pitted these figures against a "figure that is computed from Giovanni Antonio Magini's tables of the second celestial mobiles" (*figura della natività di novo da me calculata con le tavole de secondi mobili celesti di Antonio Magini*) and a "figure of the revolution that I calculated according to the time of the real motion indicated in Magini's's ephemerides" (*figura della revolutione da me calculata sotto il tempo che si trova per il moto vero insegnato nell'effemeridi del Magini*).[108] Finally, he discussed the differences between his and Benedetti's calculations and concluded with an accusation directed against Benedetti for being careless and opportunist:

> But I believe that he [Benedetti] followed the calculation of Alfonso X rather than the true one only owing to its simplicity. In fact, before [the publication of] the ephemerides of Magini it was very difficult to establish the true time of the revolution. Before him, nobody calculated the Sun up to the seconds in any ephemerides, which is the presupposition for more exact and true computations [...]. It is only in consideration of Benedetti's authority that I did not omit to compare his horoscope with the other one.[109]

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V[ostra] Alt[ezza] sono stato da loro richiesto [...].

    Et per questo sempre cresce maggiore la speranza mia, in solo vostra Altezza quanto ch'essa più m'incita a servirla, et che maggior è fatica che faccio per lei, et minor il numero di quelli in quali posso haver spernaza di soccorso."

[106] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 8*r*: "[Ho] havuto riguardo ancora ai luoghi ne' quali cadono essi [pianeti] nella figura della revolutione fatta secondo il vero tempo dato dal Copernico, come è insegnato da principalissimi scrittori dell'astrologia. Et nell'universal giudicio perché ho conosciuto tra scrittori essere certa diversità seguendo alcuni un tempo et altri un altro nel fare delle revolutioni delli quali ancor ch'io provi (come per i calculi di ciascuno posti al principio di questa opera) l'uno solo essere il vero, ho fatto le figure del cielo che si mostrano sotto ambi essi tempi (atteso che ancor la falsa era seguita da diversi et particolarmente dal Benedetti), ho radopiato essi giudici per i luoghi che diversi significati fanno havere esse figure. Acciochè V[ostra] Alt[ezza] provandole ambidue conosca ancor lei quale meglio secondi la verità."

[107] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 11*v*–12*r*.

[108] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 12*v*–13*r*.

[109] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 16*v*–17*r*: "Ma io tengo ch'egli seguisse più tosto il calcolo d'Alfonso che il vero; solo per causa della facilità d'esso percioché avanti

In this second criticism, Cristini continued to discredit Benedetti. This time he cast his capacity as both an astrologer and a mathematical astronomer into doubt. Cristini suggested, in fact, that Benedetti misused his prestige to disguise the lack of accuracy in his astrological computations. Mistakes affected not only the horoscope he carried out for himself but also those cast for his patrons.

Once he had established himself as an expert in the field, Cristini continued to prepare prognostications for the ruling family from 1592 to 1595, as testified to by the titles of several manuscripts, such as "*diari*" and "*revolutioni*," which are for the most part lost.[110] He obtained the position at court that he desired in 1594. Carlo Emanuele I designated him "as our and our princely children's mathematician, follower of Giovanni Battista Benedetti, who has recently passed away" (*per mathematico nostro et dei principi nostri figliuoli in luogo del fu Gio[vanni] Battista Benedetti ultimamente defonto*). He moreover accorded to the new court mathematician a "reasonable stipend" (*un ragionevole stipendio*) of three hundred scudi per year, "so that, according to our wish, he will cover the efforts of his studies, and will be in condition to serve us with more ease and comfort in all the duties we will entrust him" (*acciò che possi comportar alle fatiche delli studi, et trattenersi al servitio nostro più agevolmente et commodamente come desideriamo in tutti li carighi che ha da noi*).[111]

## 2.8  Strengths and Limitations of the Institutional Framework of Benedetti's Science

Benedetti's life, career, and work, as well as his legacy, fortunes, and misfortunes should be understood against the background of the Renaissance world he was part of, in particular the Italian and Turin environments. His case is paradigmatic of both the strength and the limitations of Renaissance science. On the one hand, the cultural and economic flourishing of centers such as Turin, new and challenging engineering and architectural projects, and the establishment of a court and of a modern state apparatus with its need for technical advice and cultural grandeur created an exceptional environment, favorable also to the pursuit of science and philosophical speculations. The constraints of Counter-Reformation culture did not affect the speculative freedom of Benedetti. This is especially due to the pragmatic cultural and religious politics of the Savoy dukes, who were trying to establish a balance between their state and international diplomacy and confessional tensions. On the other hand, however, the fragility of Renaissance knowledge institutions also comes into view. Universities were teaching institutions instead of research centers. Professors were concerned with the transmission of knowledge rather than with the implementation of new knowledge and theories. The intended mission of early-modern universities was preservation, namely the transmission of traditional knowledge to future generations, not producing change. Thus, epistemic processes in institutional settings were often imperceptible and transformations of knowledge often occurred against the explicit intentions of the historical actors. Lectures and commentaries on authoritative sources—the teaching of which was sanctioned by academic statutes and curricula—were not expected to alter the knowledge preserved in the classics and in the textbooks. The Savoy dukes tried to

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l'effemeridi del Magini molto difficil cosa era trovar il tempo vero della revolutione percioché nissuno avanti lui havea nell'effemeridi calculato il Sole sino alle seconde onde ne seguono i calculi più sottili e veri […]. Con tutto ciò solo per l'autorità d'esso Benedetti non ho volsuto lasciar del tutto la consideratione delle figura sua con l'altra come vedevasi."

[110] See Peyron 1904, 617–618.
[111] Vernazza 1783, 20–21.

attract prestigious professors to Turin and also supported, at least initially, the teaching of humanities according to the new standards set by humanistic philology. However, the place for free inquiry and innovation was outside universities. Benedetti's works, marked by original and unorthodox conceptions in physics, mathematics, and other disciplines, emerged from a courtly environment. Yet, this institutional frame proved ephemeral as it was dependent on patronage. It also had a strongly personal character, as it depended on informal exchanges within a system of unsystematic patronage. In Turin, modern scientific academies, with a stable body of investigators and statutes, had not yet made their appearance. Not even literary and artistic academies met with a particularly favorable environment.

Given this context, Benedetti's scientific activity, accomplished outside university and institutionalized settings, cannot but appear as occasional. In fact, it was linked to the contingency of courtly life, for instance to the requests for advice by the Savoy rulers or other patrons. This is the case with all of Benedetti's letters and with other publications, such as his writing on the calendar reform. He appears to have given expert advice on issues of cultural policy, such as university appointments, as well as on technical issues, and not least on matters of astrology. His construction of a fountain and of sundials, as well as his writings on technologies and gnomonics are directly connected to his role as a mathematical expert at the court. In the same function, he also entered debates and polemics animating Turin. Some of his interventions were friendly, for instance his exchanges on meteorology with other courtiers in the presence of members of the ruling family. His controversy with Professor Berga over the quantity of water and earth in our globe was more vehement but never harsh. Astrological polemics were the most virulent ones, as evidenced by Benedetti's publications against the critic of astrology, Altavilla. A constant feature of Benedetti's scientific work remains its occasional character. This is also reflected in the lack of systematic order in his *magnum opus*, the *Diversae speculationes*. Benedetti probably saw himself primarily as a courtier, participating in the cultural life of Turin as an exponent of the Savoy elite, and not as a scientist pursuing the immaterial glories of scholarly achievements. As a matter of fact, he did not primarily take upon himself the burden of a scientific effort going beyond the deliverance of brilliant booklets, short judgments, and advice on specialistic issues.

One astonishing aspect of Benedetti's intellectual activities is the lack of an enduring and explicit legacy. On the one hand, his conceptions clearly influenced contemporaries and followers in Italy and abroad. Among others, his impact is reflected in the positive opinions of Brahe and Kepler, in Galileo's reception of several insights of his mechanics and physics, and in the European circulation of his ideas on physics through Taisner's plagiarism. On the other hand, the lack of an enduring acknowledgment of his work is equally evident and seems to be linked to the fact that he was not able, and perhaps not even willing, to establish a school like the one set up by Commandino in Urbino, or by those later set up by Galileo in Padua and Tuscany. Not even in Turin did he benefit from lasting recognition. As we have seen above, his immediate successor as court mathematician, Cristini, even saw the denigration of his astrological and astronomical skills as an opportunity to obtain a visible position in town and start a courtly career.

In many ways, Benedetti is the mirror of his world, in particular of the courtly society he belonged to. His work can be seen as the embodiment of this context. His case is different from that of many other Renaissance scholars, who strongly identified themselves with their scientific work and output. For scholars like Galileo, for instance, the publication of their works had a functional aim in accessing the courtly milieus. In the case of Benedetti, he was already part of the patrician and aristocratic milieu for many reasons. His work is the product of courtly life rather than his entry ticket to it. The author disap-

pears (or almost vanishes) and leaves in his place a sort of collective author, which is not the scientific Republic of Letters, but rather the court itself, its institutions, its elites, its participants, and its networks. In this respect, Benedetti differs from the great protagonists of Italian Renaissance science. He is very far from the self-celebration of intellectuals like Cardano and Galileo. His work is no monument to himself but rather to his environment, ranging beyond the local boundaries of Piedmont and the Savoy. The Urbino school was also populated by scholars less concerned with their own ego than with science. However, in contrast with this school Benedetti conceived of himself as an innovator, rather than as a restorer of antiquity and classicism.

# Chapter 3
# Structure of the Book and Main Issues

## 3.1 Benedetti Introduces His Physico-Mathematical Speculations

The *Diversarum speculationum… liber* is composed of six books, indicated as follows in the table of contents:

1. "Arithmetic Theorems" (*Theoremata arithmetica*);
2. "Explanation of the Operations of Perspective" (*De rationibus operationum perspectivae*);
3. "Mechanics" (*De mechanicis*);
4. "Disputations on Some Opinions Held by Aristotle" (*Disputationes de quibusdam placitis Arist[otelis]*;
5. "On Euclid's Fifth Book [of the *Elements*]" (*In quintum Euclidis librum*);
6. "Epistolary Answers on Physics and Mathematics" (*Physica et mathematica responsa per epistolas*).

It should be noted that this partition of themes does not mirror the relative relevance of the issues according to the author. In fact, the length and the importance of the books does not exactly correspond to the subdivision indicated in the table of contents. Two sections are quite short: Book 2 on perspective and Book 5 on Euclid. By contrast, the first one, on arithmetic, and the last one are much longer. Chapter 6 is an extremely diverse collection of letters on the most different subjects.

The dedicatory epistle of the *Diversae speculationes* begins with an acknowledgment of the generosity of Emanule Filiberto of Savoy, with whom Benedetti had particularly good relations. As one reads, they often talked about mathematical issues pertaining to arithmetic, geometry, optics, music, and astrology.[1] Since the *Diversae speculationes* appeared after Emanuele Filiberto's death, it was dedicated to his successor, Carlo Emanuele I. Benedetti reports that both dukes encouraged his inquiries and their questions motivated his investigation of specific questions. This is the reason why Benedetti's *Speculationes* have an occasional character and are not ordered in a systematic manner. The *Theoremata arithmetica* (book one) clearly originated from conversations with Carlo Emanuele I as the theorems are presented as answers to the patron's questions. For instance, the first one is introduced as follows: "The very serene duke of Savoy asked me to prove by means of science and speculation (as one says) that the product of two fractions is inferior to either factors."[2] The curiosity the dukes held for mathematical matters was not idle but rather rooted in a deep comprehension of the importance of practical mathematics in military and civil affairs. However, courtly etiquette required that serious matters be

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[1] Benedetti 1585, f. A2*r*. Note that the pagination A1–A4 is doubled: the first installment A1–A4 contains frontispiece, table of contents, dedicatory epistle, and preface to the reader, whilst the second installment includes the first eight pages of the *Theoremata arithmetica*. Since the latter has also a pagination number (1–8) we will quote from the first fascicle indicating the folio and from the second giving the page.

[2] Benedetti 1585, 1: "Interrogavit me Serenissimus Dux Sabaudiae, qua ratione cognosci posset scientifice et speculative (ut dicitur) productum ex duobus fractis numeris, quolibet producentium minus esse."

concealed under the mask of aristocratic detachment and disinterestedness. The court was a refined cultural center that also functioned as a political headquarters, where the most important decisions had to be taken. "Reality and imagination—as has been remarked—prescribed that at court the weight of the duty of government be mirrored by the lightness of amusement."[3]

> Following his example [that of the Duke] [...] many asked my advice either in person or by mail on those mathematical problems. As I never avoid work in support of friends, it happened that, after so many years, looking at my paper boxes [*scrinia*] I found so many solved problems that they could be gathered in a fairly big volume.[4]

Benedetti expressed his admiration for the stimulating intellectual environment made possible by the magnificence of Carlo Emanuele I and the legacy of his father. The wide range of scientific interests shared by both dukes is emphasized in the concluding remark of the dedicatory letter:

> Therefore, the glory [of your Highness] will equal that of the ancient Persian kings, and we can expect great happiness in this century if Plato's prophecy is correct: the future State in which princes philosophize will be blissfully happy.[5]

Benedetti's preface to the reader,[6] following the dedicatory epistle, provides some more information related to his mathematical-physical work. The author repeats that the *Diversae speculationes* are a miscellanea of thoughts on various subjects brought about by his own curiosity or by that of patrons and friends. He declares himself confident that, in spite of their disordered format, his speculations will be considered useful, and makes a further claim for the absolute originality of his ideas: "*non dubitans quin illis* [*meis scriptis*] *in illis scientiis aliquid commodi atque utilitatis allatura sint, praesertim cum in eiusmodi quaestionibus investigandis atque perpendendis, nemo* (*quod sciam*) *hactenus elaboraverit*" (as I have no doubt that these [writings of mine] will bring something pleasant and useful, especially because (to my knowledge) nobody has so far sufficiently investigated and pondered such issues).[7] The enthusiasm for novelty that was to motivate the supporters of modernity in the *querelle des anciens et des moderns* can be sensed in these words. Moreover, Benedetti does not exclude the possibility that different people, at different times and in different places, could have made the same discoveries by treating similar problems:

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[3] Barberis 2017, xvii.

[4] Benedetti 1585, f. A2*r*: "Illiusque imitatione [...] non pauci aut praesentes, aut per litteras me de his, atque illis mathematicis quaestionibus consuluerunt. Cumque ego nunquam laborem amicorum causa defugerim, evenit ut post tot annorum curricula, mea scrinia scrutatus, invenerim tot absolutas quaestiones, ut ex eis corpus mediocre effici posse videretur."

[5] Benedetti 1585, f. A2*v*: "Quare, et veterum Persarum Regum gloriam [tua celsitudo] aequavit, et nos veluti in spem certam faelicitatis huius saeculi induxit, si verum est Platonis vaticinium, beatam eam futuram Rempublicam in qua Principes philosophentur."

The comparison between the Duke of Savoy and the ancient Persian kings is in line with an established Renaissance topos derived from classical sources, in particular Xenophon's *Life of Cyrus*, as discussed by Vester 2007, 228-229.

[6] Benedetti 1585, "Ad lectorem," ff. A3*r*–A4*r*.

[7] Benedetti 1585, f. A3*r*.

> In fact, I included [*traditum est*] nothing in these books that, as far as I remem-
> ber, I read or heard from others. If I picked up on the suggestions of others,
> either I offered a somehow different demonstration or I wrote on the same
> subject more clearly. In the case that somebody else wrote the same ideas
> [*eadem tradidit*], either I was not informed about this person's speculations
> or the memory of these readings has vanished. Aristotle himself remarked that
> it can easily happen that many come to the same ideas. It can even happen
> that, writing so much, I repeat a certain issue having forgotten that I already
> wrote on it. Indeed, this has happened to me a few times.[8]

Benedetti adds that very few people ever wrote books which are entirely and solely the
fruit of their own mind, except perhaps for Archimedes. Following in the footsteps of
this illustrious predecessor, he presents the results of his personal investigations as inde-
pendent from any authority. He only acknowledges some influence from Tartaglia and a
few other authors on whose works he relies: "Hieronymus Cardanus, Michael Stifelius,
Gemma Frisus, Ioannes Novimagius, Cuthebertus Tonstallus, caeterique huiusmodi." The
omission of Del Monte as a source on mechanics is striking and telling about the enmity
between the two men, which is well documented and will be discussed later.

Benedetti envisages possible criticism of his work and seeks to anticipate objections
with a remark inspired by a humanistic sense of relativism: "Quot capita, tot sententiae (As
many heads, so many opinions)."[9] Since a book will never receive universal approbation,
he writes, it must suffice to provide fruitful insights which will encourage others to under-
take further investigations. He mentions only three ancient models: Ptolemy, Euclid, and
Pythagoras. Whereas the reference to the first two men can be seen as an appreciation of
the most valuable ancient sources on geometry, mathematical astronomy (and astrology),
and geography, the mention of Pythagoras explicitly refers to astronomy and should be
understood as a reference to Copernicus's "restoration" of heliocentrism.[10]

The letter to the reader ends up with a reflection on and a rebuttal of the principle
of authority. Benedetti claims that mathematical and natural investigation should only be
inspired by the love for truth. All considerations which do not derive from an open-minded
philosophical attitude ought to be dismissed as unfounded:

> In order to establish the truth, I occasionally had to oppose the opinions of
> others in many places, but I do not want you to ascribe it to some vice of
> mine nor to call me a malevolent and a sycophant as I display the errors of
> others. They should rather be thankful to me, since I aim to erase wrong
> opinions while I am dealing with the same issues—according to Antisthenes,
> it is indeed necessary "to begin by unlearning errors." I show the truth, which
> all philosophers, beginning with Aristotle, should hold in higher esteem than
> any human authority or favor. As you will encounter something of this sort in
> my volume, I beg you to abandon all passions in your judgement, keeping in
> mind Sallust's admonishment: "Those who assess controversial issues should
> avoid all hate, friendship, rage, and compassion." Thus, always favor truth,

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[8]Benedetti 1585, f. A3*r*: "Nihil enim his libris a me traditum est, quod aut legisse, aut ab aliis audivisse
meminerim, nam si aliena attigi, ea, aut cum aliqua differentia demonstrationis, aut dilucidius scripsi, quod
si forte alius eadem tradidit, aut eius lucubrationes ad me non pervenerunt, aut earum perlectionis memoria
excidit. Ut etiam Aristoteles ipse sensit facile fieri potest, ut pluribus eaedem opiniones in mentem veniant.
Immo multa scribendi evenire potest, ut cum iamdiu aliquid scripserit, iam oblitus, idem repetat, quod mihi
etiam nonnunquam accidit."

[9]Benedetti 1585, f. A3*v*.

[10]Omodeo 2014a, "The Invention of the Pythagorean Cosmology," 167ff.

which is worthy of the greatest efforts, instead of some person, as too many do. Hopefully you will benefit from my work and, in case you will pick some fruit, in the first place you should be thankful to Him from Whom all sciences descend.[11]

## 3.2 Mathematical Sections

Benedetti proved his capacity as a mathematician in his early writings, especially in the *Resolutio omnium Euclidis problematum* (1553). The relevance of this publication for the ongoing debates of the time, involving Ferrari, Cardano, and Tartaglia has often been stressed. For instance, Moritz Cantor, in his classic history of mathematics, *Geschichte der Mathematik*, regarded Benedetti as "*ein wirklicher Geometer*" on account of his treatment of geometrical problems using a compass with fixed opening.[12] Special mathematical problems scattered in the *Diversae speculationes* have been summarized by Bordiga. We will deal with some of these problems in other sections, in connection with other aspects of Benedetti's work such as mechanics, natural philosophy, and astronomy. For the time being, we will limit our treatment to the two books of the *Diversae speculationes* specifically dealing with mathematics.

### 3.2.1 Geometrical Demonstrations for the Solutions of Arithmetic Problems

Two books of the *Diversae speculationes* are devoted to mathematics in the strict sense, Book 1 to arithmetics and Book 5 to proportions. Additionally, several letters in Book 6 are dedicated to mathematical problems and some metaphysical and physical issues are also treated as geometrical problems. In Book 4 Benedetti discusses examples such as the perfection of the circle, i. e., whether the circle, owing to its properties, has to be seen as the "first" or the "last" of the geometrical figures, and the possibility of a never-ending motion on a finite line or the possibility of an infinite motion on a finite line.

Book 1 of the *Diversae speculationes* has the form of a collection of mathematical exercises. Although Benedetti did not give them a systematic order, he claimed that his readers might find many useful explanations and remarks.[13] He chose to call his propositions "theorems" instead of "problems" to stress their originality, as Benedetti believed they deserved higher recognition than mere problem-solving. He also apologized for the brevity of his treatment by saying that he had confidence in the intelligence and expertise of his readers (the same remark can be generalized for many other sections of the work). In Book 1 he approached arithmetic problems geometrically following Euclid's example in Book 2 of the *Elements*. This geometrical approach finds a legitimization in epistemological considerations about the role of geometrical visualization for the comprehension

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[11]Benedetti 1585, ff. A3*v*–A4*r*: "Quoniam vero multis in locis accidit, ut veritatis iudicandae causa necesse mihi fuerit quorundam sententiis adversari nolim te hoc mihi vitio tribuere, meque hoc nomine carptorem maledicumque habere quod alienos errores aperiam, cum potius habenda sit mihi gratia, quod in iis interdum laborans (quae Antisthenes in disciplinis magis necessaria esse dixit, *ut mala scilicet prius dediscantur*) falsas opiniones evellere studeam, veritatemque ostendere, quam omnis philosophus, Aristotelis exemplo, pluris quam cuiusvis hominis authoritatem, aut gratiam facere debet. Cumque in hoc volumine aliquid eiusmodi legeris te oratum volo, ut in iudicando, affectum omnes exuas, Sallustianum illud prae oculis habens. *Omnes qui de rebus dubiis consultant, ab odio, amicitia, ira, atque misericordia vacuos esse decet.* Hinc fiet, ut non personae (ut multi solent) sed veritati, quae summo studio dignissima est, semper potius faveas. Vale nostrisque laboribus utere, si quem inde fructum, sicuti spero tuleris, illi precipue habes gratiam a quo omnes fluunt scientiae."
[12]Cantor 1892, 521–525.
[13]Benedetti 1585, f. A3*v*.

of truth: "images are necessary to the intelligent one in order to speculate" (*quoniam opor-tet intelligentem phantasmata speculari*). In Benedetti's eyes, the process of learning and thinking requires images (*phantasmata*).

Moreover, Benedetti informs his reader about the process through which knowledge is acquired. In other words, his treatment is not restricted to theory but intentionally expands on heurstics. He complains that ancient mathematical knowledge was often transmitted in a very concise manner. It often lacked demonstration and definitions, or a clarification of the fundamental concepts. For this reason modern readers are often forced to investigate the hidden reasons of "the numbers" and "their effects," with huge effort:

> As we know, ancient mathematician-philosophers discovered many properties of the numbers but transmitted them to posterity either without reasons or with too few ones. Hence, several mathematical problems emerged, which have been addressed by the duke of Savoy. I consider some of the ensuing reflections on the ancients' propositions to be worth transmitting to posterity. In this manner, my speculations will not fall in oblivion. Rather, I will offer to many an occasion to investigate abstruse themes which are entailed in problems and theorems and could hardly find an explanation so far.[14]

In order to examine Benedetti's geometrical approach to arithmetic problems, let us consider theorem 120. Here is the problem:

> The ancients already addressed this problem: three associates have an amount of money. The sum of the [money of] the first and the second is known, as well as the sum of the first and the third and the sum of the second and the third. From such three aggregates [the ancients] derived the particular [amount of money] of each one of them.[15]

Following a method of resolution, which Benedetti ascribes to Gemma Frisus, he offers the solution to a case chosen arbitrarily:

> Gemma Frisus solves this problem applying the *regula falsi* [rule of the false]. I will follow the same path. Suppose that the addition of the first with the second is 50, that of the second with the third 70 and that of the first with the third 60. From those sums take any two, for instance 50 and 70, whose addition is 120. Subtract from this sum the other one, that is, 60. The result is 60. Its half is 30. This is the amount of money of the second associate. It you subtract this number from 70 (which is the addition of the second with the third) you will get 40. This is the amount of the third associate. Finally, from this number taken away from 60 you will be able to derive the amount of the first associate.[16]

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[14]Benedetti 1585, 1: "Praeclare multa veteres mathematici philosophi de numeris eorum effectibus excogitata posteris tradiderunt, quorum cum vix ullam rationem reddiderint, aut certe per exiguam, occasione diversorum problematum mihi Serenissimo Sabaudiae Duce propositorum praebita, de iis quae ab antiquis proposita fuerunt contemplanda nonnulla occurrerunt, quae posteritate commendare non inutile arbitratus sum, ne heae meae cogitationes interciderent, et occasionem praeberem quamplurimis abstrusa haec indagandi, quae problematibus et theorematibus involuta, vix aliquem qui evolveret nacta sunt."

[15]Benedetti 1585, 81: "Supponunt etiam antiqui tres socios nummos habere, quorum summa primi et secundi cognita sit, item summa primi et tertii cognita et summa secundi et tertii item cognita, atque ex huiusmodi tribus aggregatis veniunt in cognitionem particularem uniuscuiusque illorum."

[16]Benedetti 1585, 81: "Gema Frisus solvit hoc problema ex regula falsi. At ego tali ordine progredior. Sit verbi gratia, summa primi cum secundo 50 et secundi cum tertio 70 et primi cum tertio 60; harum trium

If we express the problem in modern form, this is the set of equations Benedetti is dealing with:

$$x + y = 50$$
$$y + z = 70$$
$$x + z = 60$$

The algebraic solution devised by Benedetti is the following:

$$2y = 50 + 70 - (x + z)$$
$$y = \frac{120 - 60}{2} = 30$$

Thus,

$$z = 70 - 30 = 40$$

and

$$x = 60 - 40 = 20$$

After this solution, Benedetti offers a geometrical demonstration of the validity of this procedure, referred to as "Gemma Frisus's *regula falsi*." For this purpose he draws a triangle with an inscribed circle (Figure 3.1) and supposes that the three sides correspond to the sums that we have expressed as a set of equations.
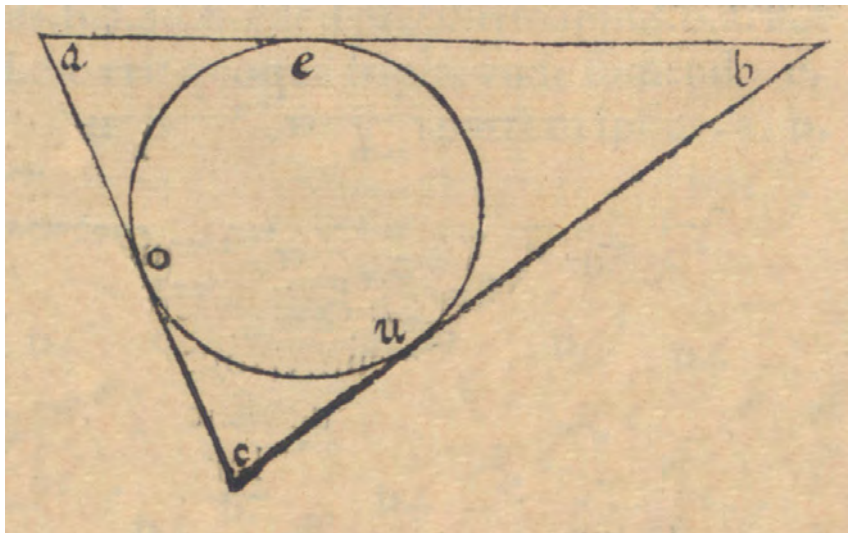


Figure 3.1: Geometrical demonstration for the solution of a particular set of equations. Benedetti, *Diversae speculationes*, Book 1, Theorem 120, p. 82. (Max Planck Institute for the History of Science, Library)

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summarum accipiantur duae quaevis, ut puta 50 et 70, quae coniunctae simul dabunt 120 a qua summa detrahatur reliqua, idest 60. Et restabit nobis 60, cuius medietas ergo 30, hoc est numerus nummorum secundi socii; quo numero detracto a 70 (hoc est a summa secundi cum tertio) remanebit 40, hoc est numerus tertii socii; et adhic numerus desumptus a 60 residuus ergo numerus primi socii."

Benedetti's demonstration here follows:

> In order to grasp this problem, let us consider the triangle here indicated as
> *ABC* each side of which corresponds to the addition of [the money of] two
> associates—for instance, assume that the side *AB* corresponds to the addition
> of the first with the second whereas the side *BC* to the addition of the sec-
> ond with the third and the side *AC* to the addition of the first with the third.
> Thereby *AE*, or *AO*, should be the number [i.e., the amount of money] of the
> first associate; *EB*, or *BU*, should be the second associate; and *CU*, or *CO*,
> should be the third. Furthermore, since *AE* is equal to *AO*, *BE* to *BU* and *CU*
> to *CO*, according to premises, if we derive the sum or side *AC* resulting from
> the addition of the side *AB* with *BC* corresponding to the other sums, we will
> know the addition of *BE* with *BU*. Hence, we will also know the half, which
> is *BE* or *BU*. Once we subtract it from the sum *BA* the resultant will give us
> the number [corresponding to] *AE*. By subtracting the number of *AE*, that is
> *AO*, from the sum (or side) *AC*, and [by subtracting] *BU* from *BC* we will
> know as a result *OV* and *CU*.[17]

We can (anachronistically) formalize Benedetti's reasoning in the following manner.

$$ab = x + y$$
$$bc = y + z$$
$$ac = x + z$$
$$ae = ao = x$$
$$eb = bu = y$$
$$cu = co = z$$

The geometrical visualization of Frisus's method, the so-called *regula falsi*, corresponds
to the addition of two sides of the triangle and the subtraction of the third one:

$$ab + bc - ac$$

This corresponds to the following addition:

$$ab + bc - ac =$$
$$= (ae + eb) + (bu + cu) - (co + ao) =$$
$$= ae + eb + eb + cu - cu - ae = 2eb = 2y$$
$$c.v.d.$$

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[17]Benedetti 1585, 82: "Pro cuius ratione consideremus triangulum adhic subnotatum *abc* cuius unumquo-
dque latus significet summam duorum sociorum, ut puta latus *ab* significet summam primi cum secundo,
latus vero *bc* summam secundi cum tertio, latus autem *ac* summam primi cum tertio, et *ae* seu *ao* sit numerus
primi socii, et *eb* vel *bu* sit secundi socii, et *cu* seu *co* sit tertii, cum autem *ae* aequalis sit *ao* et *be* aequalis *bu*
et *cu* aequalis *co* ex supposito si de<su>mpta fuerit summa seu latus *ac* datum ex aggregato laterum *ab* cum
*bc* reliquarum summarum, relinquet nobis cognitum aggregatum ex *be* cum *bu*. Quare et eius medietas *be*
sive *bu* nobis cognita erit, qua detracta ex summa *ba* relinquetur nobis cognitus numerus *ae*, detracto vero
numero *ae* hoc est *ao* ex *ac* summa, seu latus, aut *bu* ex *bc* remanebit *oc* seu *cu* cognitus."

### 3.2.2  Theory of Proportions

Book 5 of the *Diversae speculationes* deals with the fifth book of Euclid's *Elements*, on proportions. This book of the *Elements* was a notorious case in medieval translations, as the definitions were often misunderstood, inconsistent interpolations had been added, and flaws introduced in the demonstrations. In his Italian version, Tartaglia tried to emend these flaws. Commandino's and Clavius's attempts followed suit. Moreover, a certain number of Euclidian definitions (especially the fifth and the seventh) could not be simply taken for granted, but required analysis and justification.[18]

Hence, Benedetti proposed to introduce new postulates which he considered to be clearer (*intellectui commodiora*) in order to render the theorems that rely upon them more easily understandable (*quandoquidem iis nostri postulatis admissis, sequentia Theoremata perfacillima reddentur*). Actually, what he undertook was a revision of Euclid's theory of proportions, based on a reorganization of postulates and theorems. As Enrico Giusti stressed, despite its brevity, this booklet by Benedetti stands out as "the first modern attempt to accomplish an organic reform of the fifth book of Euclid's elements."[19] The difficulty that he and his contemporary readers of Euclid had to face was not technical but rather foundational, as it resided in the clarity of the definitions and the internal coherence of the theory. For the sake of intelligibility, Benedetti thus inserted two preliminary axioms and reversed the relation between the most simple of the Euclidean theorems and the less intuitive of the axioms in order to establish a gradual progression from the most simple to the more complex propositions. The first two postulates are derived from Euclid's common notions II and III concerning the addition and subtraction of equal parts to equal quantities.[20]

Benedetti rephrases Euclid's common notions translating the addition (or subtraction) of parts as the composition of ratios:

> [First:] All additions [*composita*] of an equal number of equal parts are equal. Similarly, all proportions are equal that result from the addition of an equal number of other proportions, which are equal among themselves. Euclid tries to demonstrate this in [*Elements*] V 22 and 23.

> Second: If one subtracts equal parts from equal wholes, the remaining parts are equal among them. Conversely, if one adds equal parts to equal wholes, the additions will be equal among them. One should consider these considerations as referred to proportions.[21]

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[18]In Commandino's Italian translation (Euclid 1575, ff. 63*r*–*v*), these definitions go as follows:

"V. Le grandezze si dicono essere nella medesima proporzione, la prima alla seconda, et la terza alla quarta, quando le ugualmente molteplici della prima, et della terza, o vero insieme avanzano le ugualmente molteplici della seconda, et della quarta secondo qual si voglia moltiplicatione, o vero insieme le pareggiano e vero insieme sono avanzate da loro."

"VII. Quando delle ugualmente moltiplici, la moltiplice della prima vanterà la moltiplice della seconda, et la moltiplice della terza non avanzerà la moltiplice della quarta, allhora la prima alla seconda si dirà haver maggior proportione che la terza alla quarta."

[19]Giusti 1993, 22.

[20]In Commandino's Italian translation, these definitions go as follows. See Euclid 1575, f. 7*r*:

"II. Se alle cose uguali, si aggiungono cose uguali, tutte sono uguali fra loro.

III. Se dalle cose uguali si traggono cose uguali, etiamdio le rimanenti sono uguali tra loro."

[21]Benedetti 1585, 198: "[Primum.] Quod tota composita ex aequali numero partium aequalium, sunt invicem aequalia. Ut si quis diceret omnes proportiones quae compositae sunt ex aequali numero aliarum proportionum invicem aequalium, sunt etiam invicem aequales, quod Euclides conatur demonstrare in 22. et 23. quinti libri.

The postulates that follow (3–9) are Euclid's propositions 7–13 with minimal changes in the order (the inversion of Euclid's propositions 8 and 9). Benedetti adds three additional postulates (10–12) before he tackles an explanation of Euclidian theorems on proportions. What is lacking is an explicit treatment of the definitions underlying Benedetti's "reform" of the theory of proportions.[22]

Bordiga underscored the didactic character of Benedetti's treatment of Book 5 of the *Elements* by remarking that his concern about clarity and simplicity might have emerged from the teaching of mathematics to his princely pupils in Turin.[23] This might be true; however, the theoretical relevance of this section and of Benedetti's effort to reform the theory of proportions should not be neglected. During the sixteenth century, geometry, in particular the theory of proportions, was gradually becoming the fundamental tool for the treatment of physics in the process toward a mathematical inquiry and explanation of natural phenomena. In this context, Book 5 of the *Elements* acquired a higher theoretical status by offering a sort of "meta-geometry" or *mathesis universalis*.[24]

The universal meaning of geometry as the foundation of rationality itself had been emphasized in the generation before Benedetti in an emphatic *Encomium geometriae* (*Eulogy of Geometry*) delivered by Girolamo Cardano at the *Academia Platina* of Milan in 1535. In this talk he presented geometry as the highest science, or as a sort of *prisca scientia*, by contending that geometrical rationality, based on quantity and proportion, is the source for all arts and disciplines. Indeed, the *modus geometricus* is the essence of rationality and even the *a priori* of God's Creation. In Cardano's eyes, geometry was also a practical discipline insofar as it included statics, mechanics, and architecture as subordinate disciplines. Actually, Cardano's list of arts and sciences that depend on geometrical rationality (committed to the study of quantities and proportions) is long. Geometry, as one reads in his *Encomium*, is fundamental for arithmetic, music, astronomy, and optics. It is the *conditio sine qua non* of architecture (*non aedificare sine illa licet*) and of the plastic arts, painting and sculpture. Geometry is necessary for the construction and understanding of clocks and machines (*horologiorum, machinarumque structura*). It is further presupposed by natural magic, by the science of weights, by aesthetics (*pulchritudo… tota geometrica ratione constet*), and by countless other fields of human activity and knowledge.[25]

Benedetti agreed on the fundamental relevance of geometry as the cornerstone of natural inquiry. The fact that his treatment of proportions, although it was very short and condensed, was printed as a book in its own right in the *Diversae speculationes* bears witness to the relevance he attached to this part of mathematics. In fact, it was crucial for his treatment of weights and thus a close examination of Book 5 of the *Elements* was an indispensable premise of his mechanics. As Giusti emphasized, Benedetti's treatment of composite propositions (relating to Euclid's proposition 17) was the most significant

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Secundum. Quod si a totis aequalibus detractae fuerint aequales partes, quae remanent eruntque partes invicem aequales. Et e converso si aequalibus aequalia addas composita erunt invicem aequalia. Quod in ipsis proportionibus hoc loco semper intelligendum est."

[22]Cf. Giusti 1993, 27 quoting from Antonio Nardi: "Il Benedetti, Geometra insigne non si accorse, che volendo riformare il 5° libro di Euclide, trascurò la definitione della uguale, e disuguale ragion, quale principio e fondamento dell'opera. Stupiscomi certo di tale inavertenza."

[23]Bordiga 1985, 629.

[24]Giusti 1993, 22.

[25]Cardano 1966, vol. 4, 440–445.

aspect of his theory insofar as it offered an elegant computational tool.[26] As a matter of fact, Galileo and his school would follow the same track by applying composite propositions to the analysis of functional relations in physics, in particular to motion. Hence, from the viewpoint of natural inquiry, Benedetti's Book 5 is not exclusively motivated by an abstract interest in pure mathematics but by the challenging problems of contemporary physics.[27]

## 3.3 The Geometrical Theory Underlying Linear Perspective

Book 2 of the *Diversae speculationes*, entitled *De rationibus operationum perspectivae* (Reasons of Perspective Operations), is a short treatise on linear perspective, that is, the optical discipline dealing with the construction of perspective to give the illusion of depth. Its main task was to recreate the "cone of vision." Historically, it originated from the practical problems of three-dimensional representation in the fine arts but also had relevance for architecture (e.g., in surveying or in theoretical treatises), the military art (e.g., the derivation of the structure of the enemy's fortification from a scout's sketches), and found special application in theatrical stage scenery.

Benedetti's book on perspective begins with a claim of originality: "To my knowledge nobody has so far taught the true and inner causes of the operations in perspective in an accomplished manner. Thus, I deemed it to be worth undertaking some reflection [*disputationem*] in this field."[28] The fact that "nobody" (*nullus*) has adequately treated perspective before is attested to by the great number of misunderstandings and widespread errors in this discipline: "In fact, many of those who prescribe the rules of such operations ignore the implications of the true causes, therefore they make various mistakes, as for instance in the following plane figure A [etc.]."[29] Benedetti's treatment was indeed different from the tradition of practical treatises (from Piero della Francesa's *De prospectiva pingendi* onwards) explaining *how* to construct a perspective picture owing to its higher mathematical sophistication and theoretical depth. Moreover, it took a different angle than Federico Commandino's discussion of perspective in connection with mathematical astronomy and geography in his edition of Ptolemy's *Planispherium* (1558), which treats it as mathematically equivalent to stereographic projection. Benedetti's focus was the explanation of the reasons underlying perspective, as well as errors that might occur in theory and practice. In a manner that is similar to his treatment of problems of arithmetic, geometry offers the conceptual tools to formalize knowledge embedded in practice.

Like most parts of the *Diversae speculationes*, this book on perspective originated from extemporary sources of inspiration. In fact, it begins *ex abrupto* with the examination of an error in linear perspective which leads Benedetti to more general considerations. Moreover, the dedication of its seventh chapter to the architect Giacomo Soldati strengthens the impression of the occasional character of the writing.[30]

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[26]Giusti 1993, 33: "D'altronde, l'interesse della teoria di Benedetti non sta nel risultato globale, ma soprattutto nel ruolo chiave della proporzione composta, e in particolare nella creazione di un algoritmo di calcolo agile ed elegante." See Benedetti 1585, 202.

[27]See Giusti 1993, chap. 2, section 2.

[28]Benedetti 1585, 119: "Cum nullus adhuc (quod sciam) veras internasque causas operationis perspectivae perfecte docuerit, operaeprecium existimavi aliqua de iis disputationem suscipere."

[29]Benedetti 1585, 119: "Multi enim eorum, qui huiusmodi operationis regulas praescribunt, cum eius effectuum veras causas ignorent, varios diversosque errores committunt, ut exempli gratia in subscripta figura superficiali A [etc.]." See Figure 3.2.

[30]Benedetti 1585, 133: "Superioribus diebus non diu postquam de perspectivis inter nos sermonem habuimus, dum animus totus adhuc in his esset. Illud in mentem venit quod eximius ille vir, et profundissimae

The error discussed in the *incipit* of the *De rationibus operationum perspectivae* is represented in a two-dimensional figure (Figure 3.2).
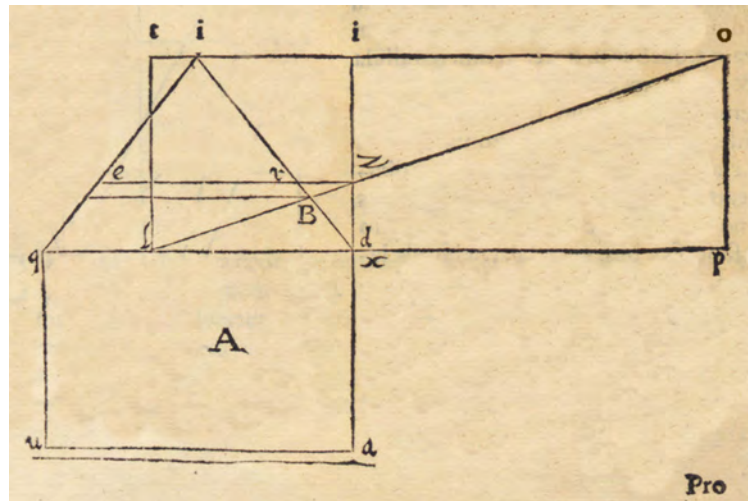


Figure 3.2: *Figura superficialis*, or plane diagram, pointing out an error of linear perspective. From Benedetti, *Diversae speculationes* II, p. 119. (Max Planck Institute for the History of Science, Library)

Benedetti explains how to correctly construct the linear perspective "inscribed" in the given triangle *IDQ*:

> […] in the plane figure *A* (here included), in order to 'degradate' (as one says) [i.e., to put it in perspective] the rectangle *QA* [*QXAU*] in the triangle *IDQ*, they [the practitioners] draw a line parallel to *QD* from point *B* (at the intersection of line *OL* with the side *ID* of the triangle) or, indifferently, they draw it from point *Z* (the intersection of the same *OL* with the perpendicular *XI*). They are not aware that the latter method is the only correct one, not the former. Conversely, if the former were correct, the latter could not be true. They might excuse themselves by arguing that one draws the aforesaid parallel line from *B* supposing that the plane *IDQ* is inclined horizontally relative to the rectangle *QA* according to the angle *IDQ*. But this excuse cannot be accepted because, according to their supposition, if one assumes that *IDQ* is inclined, the inferior angles of the rectangle put in perspective should not be as acute as *IDQ* and *IQD*. This can be easily understood considering their construction [*ratio*], which I show in the plane figure *A* here included. However, if one wants to see the rectangle in perspective, one should locate this plane relative to the eye in the same manner as the line *ID* relative to *O*. But this is too difficult [to represent].

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doctrinae, nec unquam satis laudatis Daniel Barbarus se accepisse profitetur a Ioanne Zamberto patritio Veneto, qui ad verbum omnia desumpserat a Ioanne Cusino Parisiense. Nec parum mirabar peritissimus illum Cusinum, quod in capite quarto secundae partis perspectivae, ut quod piam planum quadrilatam composuisse. […]." See Mamino 1989.

> To summarize, the correct manner [to put a rectangle in perspective] is to
> draw a line *ER* parallel to *QD* from point *Z*, which is common to *OI* and *XI*
> (perpendicular to *LP*).[31]

In order to visualize the construction, Benedetti produces an additional diagram (Figure
3.3) offering the tridimensional correspondent of the plane diagram he has just examined.
Note that point *O* is the point of departure of the cone of sight. The observer is thus lying
with his feet in point *P*.
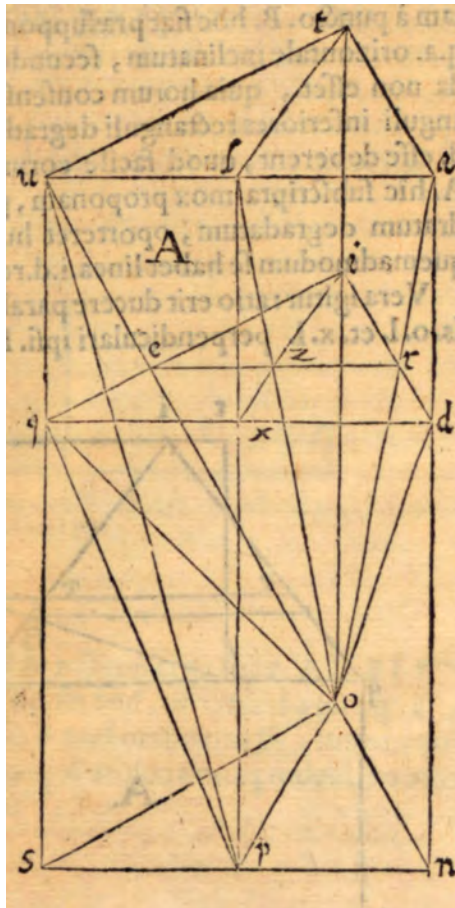


Figure 3.3: *Figura corporea*, or
tridimensional diagram,
visualizing the same
problem of linear
perspective as in Figure 3.2.
From Benedetti, *Diversae
speculationes* II, p. 120.
(Max Planck Institute for
the History of Science,
Library)

Departing from these considerations, Benedetti treats various cases (including the one in
which the projection plane is not perpendicular to the observer's line of sight) using the
same approach. That is to say, he first draws bi-dimensional diagrams showing the con-
struction and then offers tridimensional geometrical explanations providing an insight into
the physical reality underlying the construction. He even offers a sort of virtual instrument

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[31]Benedetti 1585, 119: "[…] in subscripta figura superficiali *A* volentes degradare (ut dicunt) rectangulum
*qa* in triangulo *idq* ducunt parallelam ipsi *qd* a puncto *B* intersecationis lineae *ol* cum latere *id* trianguli, et
(idem) indifferenter, eandem quoque a punctoque *Z* intersecationis ipsius *ol* cum perpendiculari *xi* ducunt,
nescientes hunc solum esse verum modum, non item alium, quia si alius, talis esset, hic, verus non existe-
ret, nam si vellent sese excusare, quod ducendo dictam parallelam a punctoque *B* hoc fiat praesupponendo
planum ipsius *idq* versus rectangulum *qa* orizontale inclinatum, secundum angulus *idq*, haec excusatio acci-
pienda non esset, quia horum consensu, praesupponendo planum *idq* inclinatum, anguli inferiores rectanguli
degradati, non tam acuti, quae sunt duo *idq* et *iqd* esse deberent, quem facile eorum ratione innotescet, quae
de figura corporea *A* hic subscripta mox proponam, praeter id, quod volentes deinde aspicere quadratum
degradatum, oporteret huiusmodi planum respectu oculi ita collocare, quemadmodum se habet linea *id* re-
spectu *o* quem factu nimis arduum esset.

Vera igitur ratio ergo ducere parallelam *er* ad *qd* a puncto *Z* communi ipsis *ol* et *xi* perpendiculari ipsi *lp*."

to guide constructions in linear perspective.[32]  As Judith V. Field has argued, Benedetti "shows the applied mathematician's concern with particular problems, rather than the pure mathematician's interest in theoretical developments. The significance of his work lies not only in its mathematical insight but also in the fact that it shows us innovative work in the purest of sciences, namely pure mathematics, being carried out within an applied tradition."[33]  As has been argued, it is plausible that Benedetti's constructions were included in later works on perspective, in particular by Simon Stevin and Guidobaldo del Monte, although none of them explicitly referred to Benedetti.  Apart from Del Monte's enmity towards him, it was typical of Renaissance mathematicians who wrote on optics to leave their sources unmentioned.[34]

## 3.4    Sections on Physics: Mechanics and Natural Philosophy

Book 3 of the *Diversae speculationes* deals with mechanics.  Benedetti remarks that many learned people have already written extensively on this issue, but that their achievements are not sufficient because nature and practice (*usus*) awaken many doubts concerning the validity and the completeness of previously acquired knowledge.  New problems have emerged. His intention is to present many novelties and to propose theses that either have never been treated or have never been adequately demonstrated (*nusquam ante hac tentata, aut satis exacte explicata*).  In the final words of his introduction to Book 3, we can see that he attaches great importance to his speculations on mechanics; he even states that he will be especially remembered after his death for his contribution to the advancement of this discipline:

> Many man have written a great deal about mechanics, and they have written most ably.  But nature and experience are always wont to bring to light something new or previously unknown.  And it is therefore incumbent upon a high-minded and grateful individual, if he happens to discover something previously shrouded in darkness, not to begrudge it to posterity. For he himself gained a great deal from the work of others. Now it is my desire to publish a few items that will, I believe, prove not unwelcome to those who concern themselves with mechanics, items which have never before been dealt with or have not been sufficiently well explained. I may thus either show my desire to be helpful or at least give some evidence of possessing a bit of talent and industry. And perhaps in this way alone may I leave behind me proof that I ever lived at all.[35]

Book 4 essentially deals with Aristotle's opinions on local motion in *Physica* and *De caelo*, although critical references to *Metaphysica* and *Meteorologica* are also present.  In this section, Benedetti seeks to provide new foundations to the theory of motion from a

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[32]Benedetti 1585, 131.

[33]Field 1987, 247.

[34]See Andersen 2007, 152

[35]Translation from Drake and Drabkin 1969, 166.  Cf. Benedetti 1585, 141: "Scripserunt multi multa, et quidem scitissime, de mechanicis, at cum natura ususque, aliquid semper vel novum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posteri invidere, si quid ei contigerit comperuisse priis tenebris involutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quaedam futura, ut reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exacte explicata in medium proferre voluit: quo vel iuvandi desiderium, vel saltem non ociosi ingenioli argumentum aliquod exhiberem: atque vel hoc uno modo me inter humanos vixisse testatum reinquerem."

philosophical perspective based on a mathematical approach to nature (*inconcussa mathematicae philosophiae basis*). He revises basic concepts of physics, such as place and time, as well as natural issues, for example the existence of a physical void. Eventually, he deals with cosmological aspects linked with the theory of motion, including a defense of the Copernican system.

We will treat books 3 and 4 of the *Diversae speculationes* separately and with particular attention (in chapters 5 and 6), owing to their historical and theoretical relevance.

## 3.5  The Epistles: Miscellanea Mirroring a Scientific Network

The last book of the *Diversae speculationes* is a large collection of letters, "Physica et mathematica responsa" (Epistolary Answers on Physics and Mathematics). The autonomy of this section is underscored by the insertion, at its beginning, of a second preface.

> To the reader.
>
> Nothing is more suited to virtue than being active and present through constant motion. Just like a shining star shimmering for the eyes of the spectators. It occurred to me that this or that person invited me with words or stimulated me with letters to dispute on those issues in which I am versed. In fact, I was devoted to mathematics and to highly philosophical speculations while I sojourned in the most splendid princely courts and illustrious cities, where many intelligent people displayed admirable curiosity, desire for knowledge and conversation. I collected part of those disputes and answers, because I judged them to be worth communicating. I planned to reread and revise them, if I had some spare time, [having in mind two goals]: on the one hand, that science itself circulates as much as possible and grows; on the other hand, that the names of those excellent men, who stimulated me with their questions, are made immortal, as far as it is in my power to do so, and that I can lead others to follow their example, abandon the idle sloth (which is able to corrupt even the most talented) and start investigating, exchanging, and discussing serious subjects that could prove useful at some point and worth disseminating. For the time being, please, approach our efforts with a benevolent and judicious attitude. Regards.[36]

Significant elements emerge from this passage: the courtly environment influencing Benedetti's activity, the interest in applied knowledge, the recognition of the importance

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[36]Benedetti 1585, 204: "Ad lectorem. Ut nil magis virtutis est proprium, quam agitari, et incessabili motu prodesse. Ac veluti fulgidum sydus ante oculus spectantium commicare. Ita mihi mathematicis iisque maxime philosophicis speculationibus dedito, saepissime, ut in principium summorum aulis, et amplissimis civitatibus degenti, ubi multa semper Nobilium mira curiositate, sciendi desiderio, et conferendi cupiditate referta, versantur, ingenia, contigit, modo ab his, modo ab illis, aut verbis tentari, aut literis provocari ad disserendum, de his, in quorum studiis versamur. Quarum concertationum et responsionum, quoniam non omnino indigna existimavi, quae memoria comendarentur, partem aliquam apud me conseruivi. Ubi vero per ocium licuit, relegi, ac tandem de manu mittere decrevi. Tum ut scientia ipsa quo magis diffundetur, crescat; et quicquid valeo, sine invidia in communem utilitatem conferam. Tum ut virorum praestantissimorum, qui me suis interrogationibus excitaverunt, quantum in me erit, gratitudine ergo, nomina reddam immortalia, et eorum exemplo alios, ocio sordidiore abiecto, quod solet iurialium praecipue excelsa ingenia corrumpere, ad sciscitandum conferendum, et disserendum, de rebus seriis, et quae usui aliquando esse possint, et quandoque evulgari mereantur, alliciam. Tu interim nostris laboribus fruere, et nostram diligentiam boni, et aequi consule, et Vale."

of dissemination, as well as the celebratory intention of the work (to render the dedicatees immortal). For us, this epistolary is the most tangible evidence of a network of social relations and its scope.

The epistles are not organized chronologically (actually the dates are almost always omitted) but rather according to the importance of the addressees, some of whom were already dead at the time of publication. The first letter was directed to the duke Emanuele Filiberto, the second to his son Carlo Emanuele I, and the following four letters to the powerful nobleman Andrea Provana de Leyní. The topics are linked to Benedetti's role as court mathematician and mathematical advisor.[37] The first letter is an expert report on the reform of the calendar, the second deals with a geometrical problem (the determination of the circle circumscribing a given square) that probably emerged from the lessons imparted to the prince, and the letters from three to six address practical problems of navigation (such as the use of astronomical instruments at sea).

Although it is not possible to identify all of Benedetti's correspondents, one can say with certainty that most of the epistles are tied to the northern Italian environment, specifically to Turin and to Venice. Many correspondents were in fact Subalpine or Venetian patricians or courtiers and artists linked to one of these two centers. Sometimes they were linked to both, as was the case with ambassadors such as Domenico Morosini (*Dominicus Moresinus*), Pietro Pizzamano (*Petrus Pizzamanus*), and Francisco Mendoza. There is great variety in the status and professions of the correspondents. Michela Cecchini and Clara Silvia Roero, in their accurate reconstruction, came to the following assessment: "The variety of themes that were discussed and of the professions of the participants in the debates shows that Giovanni Battista Benedetti was a man of culture and practice. He was ready to engage in a fruitful debate with exponents of the scientific world in the broadest sense (such as mathematicians, physicians, jurists, and philosophers) and with politicians, diplomats, and ambassadors, as well as with experts of military art and religion. Moreover, he did not dislike architects, artisans, constructors of instruments and fortifications, surveyors, and astrologers."[38]

Among his correspondents, the list of Savoy patricians and courtiers is long. Those who emerge most prominently are the orator Francesco Barbaro, who sojourned in Turin between 1578 and 1581, the Turin professor of jurisprudence Bernardo Trotto, the court historian Emanuele Filiberto Pingone, and the functionary and artillery general Giuseppe Cambiani. Benedetti advised Cambiani on ballistics, commenting upon Tartaglia to determine the most effective inclination for a cannon.[39] Another member of the Savoy milieu was Giovanni Battista Femello, philosopher, physician, and ordinary professor of practical medicine at Turin. Benedetti wrote to him on a very special geographical issue: the determination of the position of Iceland. By contrast, the mathematical advice directed to his friend, the jurist Francesco Ferrari, concerned ethics and justice. Benedetti explained

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[37]"Mathematics" has to be here understood in the wide and interdisciplinary Renaissance meaning. It comprised arithmetic and geometry, as well as practical mechanics, architecture and engineering, astronomy and meteorology, optics, physics, and even metaphysical and epistemological issues.

[38]Cecchini and Roero 2004, 32: "Dal quadro variegato dei temi discussi e dalle professioni dei protagonisti dei dibattiti emerge la figura di Giambattista Benedetti come uomo di cultura e di pratica, disposto ad instaurare un dialogo proficuo sia con esponenti del mondo scientifico in senso lato, come matematici, medici, giuristi e filosofi, sia con politici, diplomatici e ambasciatori, come pure con uomini d'arme e di religione, non disdegnando neppure architetti e artigiani, costruttori di strumenti e di fortificazioni, agrimensori e astrologi." In the following we will often rely on Cecchini and Roero for an overview of Benedetti's correspondents.

[39]Benedetti 1585, 258–259.

to him how to make use of proportions in order to explain "commutative justice."[40] An epistle on the quantification of the time necessary to spill the water out of a vase through three tubes, alongside another two epistles on geometry and the application of geometry to the study of solar radiation, are directed to the Savoy secretary Ludovico Niccolò Calusio. The letters on mechanics addressed to Giovanni Paolo Capra of Novara are better known to historians of science. They intermingle considerations on turning wheels with others on astronomy and the boundaries of the cosmos. Other noblemen belonging to the Turin milieu who corresponded with Benedetti were Franchino Trivulzio and Demoulin de Rochefort, responsible for the ducal *Wunderkammer*.

Benedetti had intense exchanges not only with patricians but also with artists, engineers, and practitioners. Four letters are addressed to the architect Gabriele Busca and their issues range from natural philosophy and meteorology to the use of instruments and topography. The Savoy clock-maker Jacopo Mayeto (perhaps Maletto) shared Benedetti's interest in gnomonics, sundials, and mathematical instruments, as witnessed by one of the letters in the collection.[41] Furthermore, a letter to the Savoy surveyor Angelo Ferrario is at once technical and playful: it is a discussion of the curve described by a hanging rope that Ferrario and Benedetti observed together in the park of the ducal castle of Lucento by Turin.[42] Additionally, Benedetti wrote on music to the Flemish madrigalist Cipriano de Rore, who had been appointed in Venice, Ferrara, and Parma and whom Benedetti had probably already met in his hometown Venice. Among the artists, the Venetian poet Girolamo Fenarolo is the addressee of two letters on geometry.

Some epistles tackle the philosophical and epistemological issues underlying mathematics and their cognition. For instance, those to his Venetian friend Domenico Pisani deal with the issues "de philosophia mathematica" (on mathematical philosophy) and "de imaginatione specierum" (on the imagination of the species).[43] Another philosopher corresponding with him was the Turin professor Francesco Vimercato to whom several letters are addressed. While these mainly deal with optics, theses on natural philosophy and cosmology are discussed in a brief letter to the theologian Gerolamo Cordero (*Hyeronymus Condrumerius*). It is entitled "Quod recte Arist[otelis] senserit coelum casu non esse productum" (Aristotle correctly saw that the heavens are not produced in a casual manner).[44] Benedetti moreover corresponded with the Paduan professor Pietro Catena, known as a keen supporter of the *certitudo mathematicarum* (see the next chapter) and the mathematical method applied to natural inquiry.

The identity, biographies, and work of many of Benedetti's correspondents are still obscure. This is especially true for foreign scholars, presumably Germans, whose names were latinized as follows: Theodosius Raisestaim, Paulus Aemilius Raisestaim, Volfardus Aisestain, and Conradus Neubart. Benedetti does not give these names any titles. An exception to this is the correspondent Anselm Rosenburg, who is designated as Imperial surveyor (*agrimensor Cesareus*).

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[40]Benedetti 1585, 284.
[41]Benedetti 1585, 423–425.
[42]Benedetti 1585, 361–363.
[43]Benedetti 1585, 298 and 298–299.
[44]Benedetti 1585, 298 and 298–299. For the identification of this person, see Cecchini and Roero 2004, 58–59.

# Chapter 4
# Epistemology

One of the most challenging aspects of Benedetti's endeavor was his attempt to merge mathematical and physical speculations, as is clearly stated in the title of the *Diversae speculationes mathematicae et physicae*. In order to understand his way to "physico-mathematics," we will discuss his mathematical epistemology starting from some statements scattered in his major work and then look at the premises implicit in his treatment of nature. We will briefly review the Renaissance reflections on mathematics linked to practical developments in technological fields as well as to eclectic reassessments of Pythagorean and Aristotelian debates on the certainty of mathematics and their applicability to natural philosophy. Focusing on the epistemological premises underlying Benedetti's mechanics, we will discuss medieval and early modern approaches to natural knowledge, which, in spite of their mathematical rigor, rested on a physics and metaphysics of contingency. For many centuries, it was assumed that the mathematical regularity of the phenomena does not imply their causal necessity.

## 4.1 The Certainty of Mathematics

In the letter to the Venetian patrician Domenico Pisani included in the collection of the *Diversae speculationes* and entitled *De philosophia mathematica* (On Mathematical Philosophy), Benedetti emphasized the philosophical dignity of his discipline, placing it at the same rank as physics, metaphysics, and ethics—if not higher than them, considering the certainty of its demonstrations (*certitudo suarum conclusionum*):

> I am surprised that, although you are well-versed in Aristotelian philosophy, nonetheless you make a distinction between the philosopher and the mathematician in your writings, as if the mathematician were not as much a philosopher as the naturalist and the metaphysician. In fact, as far as the certainty of his conclusions is concerned, he deserves the title of philosopher much more than them.[1]

This reference to mathematical *conclusiones* reveals Benedetti's methodological focus on the dignity and validity of his discipline. In his connection of mathematical and physical speculations, he seems to put the emphasis on the method rather than on ontology and to seek for the certainty of mathematics and its applications by way of its specific logic. This was the position of his correspondent, the Paduan professor Pietro Catena.[2] Along with him, Benedetti maintained that the certainty of mathematics has an extra-sensible and intelligible character.[3] As Benedetti added in his letter to Pisani:

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[1] Benedetti 1585, 298: "Miror quod cum in Aristotele sis versatus, in tuis tamen scriptis philosophum a Mathematico separes, quasi mathematicus non sit adeo philosophus, ut est naturalis, et metaphysicus, cum multo magis quam ii philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus."

[2] Benedetti includes a letter to Catena in Benedetti 1585, 371.

[3] See on this De Pace 1993, 228–229.

> Actually, you are not the only one who makes this mistake, but this is more grave in consideration of the fact that, although you [Aristotelians] even label ethics as a philosophical discipline, you do not acknowledge that the divine mathematical sciences also should be adorned with the name of philosophy. In fact, if we consider this name more attentively we will clearly see that it is in itself more suited to the mathematician than to anyone else, since none of the others is more certain in his affirmations than the mathematician. And no one is more driven by the love of science in his cognition. This is evident. In fact, [the mathematician] does not rely on the senses nor accepts any presupposition that is not so true and evident to the intellect that no power whatsoever could show that it is false.[4]

Benedetti was acquainted with scholars quarreling over the status of mathematics, its demonstrative methods, and its legitimacy in the treatment of natural issues.

In his time such debates on the foundations and status of mathematics were intense. As an instance of epistemological reflections on the philosophy of mathematics, historians often mention the controversial theses by the Paduan professor of philosophy Alessandro Piccolomini, with whose work Benedetti was familiar. Piccolomini authored, among other writings, a treatise *De certitudine mathematicarum* (On the Certainty of Mathematics, 1547) affixed to his paraphrases of pseudo-Aristotelian mechanics, *In mechanicas questions Aristotelis paraphrasis*. As one reads in this sort of appendix, one ought not to cast into doubt the certainty of mathematics. However, this does not depend on demonstrative methods but rather on the subject of inquiry: "Mathematical disciplines are certain not due to the force of their demonstrations but rather to their subject matter itself."[5] Their special subject is quantity, connected to matter. Hence, the certainty of mathematics, for an Aristotelian such as Piccolomini, rests on the fact that it deals with universal properties of nature that can be extracted from concrete reality by means of abstraction (*res mathematicae sunt ex abstractione*).

> The cause of the certainty of mathematics is evident from Aristotle's statements. Simplicius is of the same opinion when he states (in *De anima* I 11) that the cause of the certainty of mathematics is due to the fact that they refer to quantity. In fact, as he argues, quantities are sensible things, they have sensible causes and they are known to us as such.[6]

This consideration led Piccolomini to argue that motion can become a mathematical object, if one abstracts from materiality:

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[4]Benedetti 1585, 298: "Verum quidem est, te in huiusmodi errore solum non versari; sed gravius est, quod cum vos videatis etiam res morales sub philosophiae appellationem cadere, non animadvertatis divinas scientias mathematicas etiam philosophiae nomine ornandas esse. Quod si eiusdem nomen penitius considerare velimus, inveniemus aperte, mathematico magis illud ipsum quam cuilibet alio convenire, cum nullus ex aliis tam certo sciat id quem affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, et scientiae cupiditatem magis ducantur, ut aperte patet, cum nec etiam ipsi sensui det locum, neque aliquid praesupponat, quem non sit ita verum et intellectui notum, ut nulla quaevis potentia, illud esse falsum ostendere queat."

[5]Piccolomini 1565, f. 107*v*: "Mathematics disciplines esse certas non vi demonstrationis, sed ex subjecti ipsius ratione."

[6]Piccolomini 1565, 106*v*: "Patet igitur ex dictis Aristotelis causa certitudinis mathematicae. Hoc idem sensit Simplicius, qui primo de Anima 11. dicit causam certitudinis mathematicarum esse, quia versantur circa quantum. Quantitates enim ut dicit ipse, sunt res sensatae, et causas sensatas habent, et ideo nobis notas."

> One could argue that, just like magnitude, motion is a common sensible, too. Moreover, it has its effects and causes (see *Physics* V and VI). Thus, there can be a science of motion (a natural one), which is certain, similar to the science of quantity, that is, mathematics.

> We can answer to this [apparent objection], that if we consider motion in general, as separated from matter and insofar as it is a continuum […], our consideration will be mathematical. This is not in contrast with our principles.[7]

The "ontological" and not only "epistemological" dimension of mathematical physics would concern later scholars such as Kepler and Galileo, going beyond the shared Aristotelian discourse in their investigations of the mathematical properties of material processes.[8] Benedetti was rather concerned with mathematics as an intellectual tool, a sort of "logic of scientific inquiry." In the above-mentioned letter to Pisani on his mathematical philosophy, he stressed the certainty of mathematical reasoning rather than that of its "objects." Nonetheless, he was interested in the question raised by Piccolomini as to the usefulness of mathematics in the study of motion. As we will discuss, Benedetti's insight concerning the generalization of the methods already in use in mechanics, in the science of weights, established the premises for the conceptualization of problems in dynamics.

Benedetti's interest in mathematics as a conceptual instrument accords with the interest in the demonstrative power of mathematics shown by many scholars entering the debates about mathematical certainty. The publication of Piccolomini's *De certitudine mathematicarum* led to a series of negative or sympathetic reactions, among them the criticism made by the translator of Proclus's *Commentary* on Euclid, Francesco Barozzi, as well as those by the Paduan professors Pietro Catena and Giuseppe Moletti. Barozzi, in his 1560 *Quaestio de certitude mathematicarum*, and Catena, in his 1563 *Oratio pro idea methodi*, argued in favor of the demonstrative certainty of mathematics, contra Piccolomini's exclusive focus on mathematical objects. The theoretical discussion regarding the status of mathematics, the certainty of its demonstrations, their applicability to the investigation of nature, and the hierarchy between natural philosophy and mathematics continued for a while. It also produced frictions among Jesuit scholars such as the philosopher Benito Pereira and the mathematician Clavius, who were inclined to assign different levels of importance to the study and teaching of mathematics in the colleges of their order.[9]

As far as the institutional side of the defence of mathematics is concerned, it opposed scholars and intellectuals benefiting from varying social status, such as mathematicians, philosophers, and theologians. Benedetti's self-perception and, later, Galileo's self-presentation as "philosophers" involved polemical stances. They claimed for their math-

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[7]Piccolomini 1565, 107*r*: "Si vero adhuc replicaretur, quod motus etiam est sensibile quoddam commune, sicut magnitudo; habet autem motus suas passionet, et suas causas, ut patet 5. et 6. Phys. ergo ita erit certa de motu scientia, naturalis scilicet, sicut scientia de quantitate, quae Mathematica est. Ad hoc respondere possumus, quod si motum consyderabimus, in communi, abstractu a materia quatenus continuum quoddam est, […] tunc consyderatio erit mathematica, et nihil contra nos."

[8]As Ofer Gal and Raz Chen-Morris recently stressed: "It is not epistemology that worries the two court mathematicians here, but ontology. Neither of them questions the power of mathematics to provide the knowledge they seek; it is the objects that mathematics can be true about that they both feel forced to establish." See Gal and Chen-Morris 2013, 118–119.

[9]The literature on the Renaissance debates on the philosophical status of mathematics is wide. Among other sources, see Giacobbe 1972, Giacobbe 1973, Carugo 1983, Jardine 1990, 693–697, De Pace 1993, Cozzoli 2007, and Axworthy 2016, chap. 2. For the Jesuit debates on mathematics, see Romano 1999. For the seventeenth century, cf. Mancosu 1996, 8–33.

ematical and physical investigations a wide cultural meaning against critics who down-played such investigations as merely technical and specialistic.

Early polemics over the viability of the *mos geometricus* were not purely intellectual and academic but were also rooted in the rising recognition of the practical import of mathematics in engineering, architecture, mechanics, and warfare. A new class of intellectuals was emerging composed of "scientist-engineers," so to speak, both expert in practical disciplines and trained in letters.[10] Edgar Zilsel already remarked that the Renaissance exaltation of mathematics went far beyond purely Platonic and Pythagorean influences. At that time new mathematical writings were composed and published dealing with the practical problems of commerce, topography, architecture, and the arts.[11] Moreover, the emergence of mathematical and natural conceptions dependent on the advance of technology was reinforced by the growing self-consciousness of new social groups.[12] As an example of the awareness of the status of the practical arts one could mention Filippo Pigafetta's introduction to the Italian edition of Del Monte's work on mechanics. Here he reversed the assessment of craftsmen and practical knowledge, which had been marked by the contempt of aristocrats and traditional intellectuals, as follows:

> 'Mechanic' is a very honored title. According to Plutarch it refers to a profession linked with warfare. It is suited to a man of high rank who is also capable of using his hands and his intelligence to realize wonderful works of rare usefulness and pleasure for human life.[13]

This judgment well expresses the shifting opinion on practical knowledge which also marked Benedetti's environment. We have already stressed the centrality of practical mathematics for the Savoy dukes, in particular Emanuele Filiberto, in their construction of the new capital, Turin.

## 4.2 Physico-Mathematics

As a direct consequence of this mathematizing epistemology Benedetti dismissed the well-established separation between physics and mathematics in cosmology, that is, he refused to separate the investigation of "causes" and calculation.[14] This anti-fictionalist perspective implied a realist commitment related to the Copernican system and its embedding within a renewed cosmology. As we will discuss in the section on Benedetti's views on the universe, he praised the system "of Aristarchus and Copernicus" as it avoided the absurdities of an anthropocentric conception according to which the immensity of the firmament was created only for us. Rather, all planets are like Earth or, better, like moons reflecting the solar light. Among the direct consequences of the Copernican view was accepting that the fixed stars do not rotate around the center of the world within one day; rather, they are immobile.[15]

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[10] See Valleriani 2010 and Valleriani 2013.

[11] Zilsel 1942.

[12] See Lefèvre 1978.

[13] Pigafetta in Del Monte 1581, *Ai lettori*: "Mechanico è vocabolo honoratissimo, dimostrante, secondo Plutarco, mestiero alla Militia pertinente, et convenevole ad huomo di alto affare, et che sappia con le sue mani et co'l senno mandare ad esecutione opre maravigliose a singulare utilità et diletto del vivere humano."

[14] Hypotheses on conventionalism already emerged from the debate on the conflict between Ptolemy's geometrical models and Aristotle's homocentric cosmology. See Di Bono 1990 and Granada and Tessicini 2005.

[15] Most of these cosmological views are discussed in Benedetti 1585, Book 4. We deal with the details in chapter 6 as well as, partly, in chapter 7.

From this viewpoint, Benedetti's understanding of mathematics is not too removed from that of a mathematician such as Copernicus, who, in Book 1 of *De revolutionibus*, indicated that the mathematical superiority (simplicity and intelligibility) of his own planetary system was such that natural philosophy had to be subordinated to mathematical astronomy and not vice versa. The theologian who wrote the anonymous introduction to Copernicus's work, Andreas Osiander, tried to reaffirm the hypothetical character of mathematical astronomy, and its subordinate position as a discipline relative to physics and theology. By contrast, Renaissance scholars who appreciated the physical meaning of the Copernican system called it "Pythagorean" to underscore at once its natural philosophical and mathematical character.[16] As an extreme case one could mention Bruno's declarations during his Inquisition trial. In order to defend his cosmological views, and in particular the motion of Earth, he did not mention Copernicus but the ancient philosophical school of Pythagoras: "I affirmed [the existence of] infinite individual worlds [i.e., planetary systems] similar to that of the Earth. Following Pythagoras, I regard the latter as a celestial body. The Moon is similar to it, as well as other planets and stars, which are infinite [in number]."[17] Pythagorean cosmology was regarded with suspicion by the Inquisitors and the doctrine of the plurality of worlds became one of the allegations against Bruno, who would be eventually executed as a heretic in Rome. In the same years in which Bruno was a prisoner of the Holy Office in Rome and his works were examined for censure, the censors also attacked Patrizi for his natural views, including the doctrine of terrestrial motion. Although Benedetti shared similar views about the plurality of worlds and the possibility of terrestrial motion, he did not incur any censure. We dare say that he was one of the last Renaissance authors who could freely speculate on nature in Italy before natural philosophy became a highly ideological issue in the religious repression escalating in the 1590s.

Benefiting from his subalpine freedom, Benedetti reflected on Pythagorean cosmology in a section entitled *Pythagoreorum opinionem de sonitu corporum coelestium non fuisse ab Aristotele sublatam*, where he excluded the possibility that the "sound of celestial bodies" is the production of any physical sounds. Rather, he identified the Pythagorean doctrine of the world harmony with divine providence:

> As to motions, dimensions, distances, and influences there is nothing that corresponds to such proportions, but, since all of them depend upon the infinite Divine Providence of God, these velocities, those dimensions, distances, and influences must have the most perfect order and relations among them and relative to the universe.[18]

According to Benedetti's outlook, the harmony of the heavens does not correspond one to one to musical harmony in the strict sense. From this viewpoint, Kepler's later effort to translate heavenly geometries into musical melodies in the *Harmonices mundi libri V* (1619) can be seen as a radicalization of similar "Pythagorean premises."

Most significantly, Benedetti and Kepler shared a commitment in favor of the fusion of mathematical and physical accounts of nature in the frame of an early modern transfor-

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[16]Omodeo 2014a, 167–170.

[17]Bruno 2000b, doc. 13, 67: "Ho dechiarato infiniti mondi particulari simili a questo della Terra; la quale con Pittagora intendo uno astro, simile alla quale è la Luna, altri pianeti et altre stelle, le qual sono infinite."

[18]Benedetti 1585, 191: "Quod autem attinet ad motus, ad magnitudines, ad distantias et ad influxus, nihil est, quod hisce proportionibus conveniat, sed quia haec omnia dependent ab infinita et divina providentia Dei, necessario sit ut istae velocitates, eae magnitudines, distantiae et influxus, talem ordinem et respectum inter se ipsa et universo habeant, qualis perfectissimus sit."

mation of natural science in which the methods of the physico-mathematical disciplines gained a paradigmatic status. The epistemological shift also involved well-established disciplines such as astronomy. Kepler's astrophysics, first illustrated in the *Astronomia nova* (1609), was a significant step toward the derivation of celestial geometries from physical forces. Kepler translated a geometrical discipline (Ptolemaic and Copernican mathematical astronomy) into a physico-mathematical one. In fact, he explained the elliptical path of planetary orbits as the effect of interactions of forces. He emphasized the double bound of his astronomy, inseparably intertwining physics and mathematics, in the title of the work: *Astronomia nova αἰτιολογιτός seu physica coelestis de motibus stellae Martis* (New Astronomy Investigating the Causes, or Celestial Physics Concerning the Motions of Mars). As Kepler announced in the introduction: "In this work I mixed celestial physics with astronomy."[19] He meant to launch a new discipline, "celestial physics," that merged mathematical modeling with causal physics.[20] Kepler remarked that the ignorance of physical causes compels scholars to settle for conjectures since no choice can be made between mathematically equivalent hypotheses. By contrast, physical arguments are decisive in deciding between mathematically equivalent models. Therefore, celestial physics and astronomy should be unified. The result was a mixed science (*scientia mixta*) whose data came from the senses and whose demonstrations are expressed in mathematical terms. This physicalization is well shown in Kepler's physico-mathematical concept of "orbit" (*orbitae*) substituting that of orbs (*orbes*) (that is, the material spheres transporting celestial bodies). According to him orbit is "the path together with its physical causes— expressed as physical laws."[21] Shape and velocity of astronomical orbits depend on the force (*vis*) emanating from the sun, that is, on a physical cause of geometrical effects.[22]

Descartes's *Traité du monde et de la lumière* (completed in 1632–1633, but printed posthumously, in 1664) and the *Principia philosophiae* (1644) marked a culminating point in the move toward the reduction of natural disciplines (such as optics and astronomy but also physiology) to material interactions of corpuscles in motion. Descartes's philosophy was particularly influential as it legitimized a mathematical treatment of nature with the advances of physics in his time. At the same time, he connected his explanations to views on matter and causality irreconcilable with the qualitative, essentialist, and teleological accounts of the Scholastic tradition. In particular, his mechanization elevated the results of Renaissance mechanics to a higher and more generalized level.

Benedetti's place is rather at the beginning than at the end of this process. As the title of his major work hints, he was committed to a mathematical-physical investigation of nature. He did not limit his application of a mathematical method to those fields where this approach was already established, but extended it to the treatment of all realms of natural inquiry.

## 4.3   The Contingency of Nature and Mechanics

Benedetti's mathematical approach to nature did not lead him to the belief that physical phenomena are ruled by necessity. Rather, he shared a medieval and early-modern ontology and epistemology of contingency enabling a particular cohabitation of mathematized physics and indeterminism (in other words, formal determination without causal neces-

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[19]Kepler 1937–2001, vol. 3, 19.
[20]Gingerich 1975, 261–278.
[21]Goldstein and Hon 2005, 76.
[22]On Kepler's discovery, see Donahue 1988, Donahue 1993 and Wilson 1968.

sity). In order to better understand it one has to look at Scholastic motives informing
his physics, in particular his mechanics, and the scientific and philosophical work of his
successors. This will require a short excursus.[23]

### 4.3.1 Scholastic Treatments of Nature as the Realm of Contingency

It would be misguided to think that a mathematical approach to nature in Renaissance sci-
ence implies the assumption that natural causation is ruled by necessity. This was indeed
not the case for well-established medieval and Renaissance views. Only in the course of
the seventeenth century would contingency be banned from the realm of natural causa-
tion in the developments of post-Cartesian mechanism. For philosophers such as Baruch
Spinoza and Gottfried Wilhelm Leibniz contingency marked the limitations of our knowl-
edge and not an ontological limitation of nature. As one reads for instance in Spinoza's
*Ethica ordine geometrico demonstrata* (Ethics, demonstrated in geometrical order) I 29:
"There is no contingency in nature. All natural beings are determined by divine necessity
to exist and operate in a special manner." (*In rerum natura nullum datur contingens, sed
omnia ex necessitate divinae naturae determinata sunt ad certo modo existendum et ope-
randum*). By contrast, in the Renaissance a mathematical treatment of natural phenomena
underlaid no principle of sufficient reason, hence it did not imply the necessity of natural
causation. In particular, mixed mathematical disciplines that had received a Scholastic
embedment or systematization rested on a well-established Aristotelian conception, ac-
cording to which sublunary phenomena are determined without necessity.

Historically, *contingentia* is the Latin variant translation of the Aristotelian concept
of "possibility," both as modal logical *endechomenon* as well as physical-metaphysical
*dynamis* within a hylemorphic framework. In the context of the Christian reception, this
terminus received an onto-theological connotation in a frame of creationist theology. In
late Scholasticism, *contingentia* came to signify the worldly reality, or nature as Creation.
Nature was deemed to be contingent. It exists *de facto* but could also not exist because it
depends on God's will. As John Duns Scotus put it,

> So then, the first issue has become clear: how there is contingency in things—
> because it comes from God—and what is in God which is the cause of this
> contingency—because it is his will.[24]

In Aristotle, there was a tension between two meanings of "possibility." According to
*Analytica Priora* (13: 32 a 18–20) the possible is that which is "neither necessary nor
impossible," whereas according to *De interpretatione* (13: 22 a 14–13 a 26) possibility is
exclusively that which is opposed to "impossibility" and therefore includes also that which
is necessary. As a reminiscence of this original tension, one can find in Scholastic phi-
losophy two different definitions of contingency either as "quod est nec impossibile nec
necessarium" (that which is neither impossible nor necessary) or "quod non est impossi-
bile" (that which is not impossible).[25] Both meanings were kept in the Latin rendering
of the Aristotelian possibility as *contingentia* by Gaius Marius Victorinus (III–IV cent.

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[23] We have first discussed contingency and mechanics in the Renaissance in Omodeo and Renn 2015. A vol-
ume entirely devoted to ontological and epistemological contingency in the natural debates of early moder-
nity is Omodeo and Garau 2019.

[24] Duns Scotus 1994, 140: "Sic igitur apparet primum, quomodo est contingentia in rebus, quia a Deo, – et
quid est in Deo quod est causa huius contingentiae, quia voluntas eius."

[25] Cf. Vogt 2011, 52. The entire first chapter is relevant for a historical overview of the reception and trans-
formation of the Aristotelian concept of "possibility" as "contingency" in the Latin tradition.

CE) and Boethius (IV–V cent. CE), but the Latin expression also suggested affinity between that which is contingent (*contingit*) and that which occurs (*evenit* or *accidit*).[26] This third connotation would eventually prevail through the late-Scholastic differentiation between *contingentia* and *possibilitas* and its reception in the philosophical systems of the seventeenth century (and most notably by Leibniz).[27] Unlike abstract (purely logical) possibility, contingency referred only to that which is real but not so by necessity: "id, quod [est sed] potest non esse" (that which [is but] could not be). In the Christian perspective of the Almighty's Creation, contingency happened to include all that is not God himself, that is to say, nature, or the universe.

This background is fundamental to understand not only theological disputes but also natural philosophical and scientific developments during the Middle Ages and the Early Modern Period. The connotation of nature as contingent—as that "which could not be"—is theological and metaphysical in its essence, since it points to the dependency of the world on God. However, from the point of view of natural conceptualizations, not only the "vertical" dimension of metaphysics is relevant but also the "horizontal" dimension of causality within nature. On the horizontal plane of the interrelation among finite beings, contingency refers to a degree of indetermination, and a certain unpredictability in the connection between causes and effects. Moreover, whereas a theological perspective focuses on the radical contingency of that which exists as created being, natural philosophy addresses the *relationship* between contingency and necessity within nature, that is, between divine order and phenomenal imperfection. This relationship between that which is not necessary and that which is necessary *had to* be conceptualized and indeed was conceptualized as the relationship between the *absolutum* and the *conditionale* or *secundum quid*.

In Book 1 of the *Summa contra gentiles*, Thomas Aquinas defined contingency through its distinction from necessity. In the case of the contingent beings, as one reads in *Summa contra gentiles* I 67, a cause can produce its effect or not, whereas in the case of necessary beings, their cause cannot not produce them:

> The contingent differs from the necessary according to the way each of them is found in its cause. The contingent is in its cause in such a way that it can both not-be and be from it; but the necessary can only be from its cause. […] Just as from a necessary cause an effect follows with certitude, so it follows from a complete contingent cause if it be not impeded.[28]

A contingent cause, as one reads, will fulfill its tendency to produce a certain effect "si non impediatur," that is, if no impediment hinders its realization.

In Book 2 of the *Summa contra gentiles*, Thomas dealt extensively with the contingent being ("*omne quod est possibile esse et non esse*" and "[*id quod*] *ad utrumlibet se habet*").[29] According to him, the world is contingent insofar as it is created. In this general sense, "God is to all things the cause of being" (*Summa contra gentiles* II 15).[30]

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[26]Vogt 2011, 50.

[27]Schepers 1965.

[28]Aquinas 1975, 221f: "Contingens a necessario differt secundum quod unumquodque in sua causa est: contingens enim sic in sua causa est ut non esse ex ea possit et esse; necessarium vero non potest ex sua causa nisi esse. […] Ex causa necessaria certitudinaliter sequitur effectus, ita ex causa contingenti completa si non impediatur."

[29]Thomas, *Summa contra gentiles* II,15. Cf. Aquinas 1975, 48: "everything that can be and not-be" and "it is indifferent to either."

[30]Aquinas 1975, 46: "Deus est omnibus causa essendi."

In particular, God's free will is the origin of this world. Nonetheless, Thomas does not exclude that natural reality is populated by both necessary and contingent beings. Absolute necessity (*necessitas absoluta*), he writes in *Summa contra gentiles* II 29, does not pertain to God, since His decision and action is independent from any constriction (*debitum*). Rather, absolute necessity pertains to the immaterial, or "separated" beings as well as to those bodies in which the form fulfills all potentialities of their matter, as is the case with the heavenly bodies transported in circles. As for terrestrial (sublunary) bodies, their forms are imperfectly realized. Matter, as the potentiality to take different forms, is at the origin of their contingency, that is, it is the source of the possibility to realize or not to realize a certain effect: "But in things whose form does not fulfill the total potentiality of the matter, there still remains in the matter potentiality to another form."[31] For the low realm of birth, corruption, and change, Thomas speaks of conditional necessity (*necessitas conditionalis*). In the sublunary sphere, contingency cohabits with absolute necessity (e.g., the inevitability of death for all animals and the hylemorphic composition of all bodies). Whereas necessity pertains to the formal determinations of natural phenomena, contingency is the partial fulfillment of necessary tendencies.[32]

According to Scholastic terminology, there is always a "quid" producing the deviation of material phenomena from their formal rule. We will call this outlook an "ontology and epistemology of contingency."[33] The Pythagoreanism of many Renaissance scholars such as Benedetti did not depart from a view stressing the contingent character of natural phenomena in general. As we will argue, one encounters in Benedetti's physics and mechanics a systematic use of theoretical tools implying natural contingency in the form of a distinction and interrelation between formal mathematical necessity and its material realization. In order to understand Benedetti's mathematical treatment of contingency it is useful to consider the medieval approaches to contingency, especially the science of weights (*scientia de ponderibus*) he relied upon.

The idea of contingency informing physics and mechanics was related to its use in other disciplines, even ethics. Whereas there can be no obstacle impeding the realization of God's will, which is therefore "absolute" (*voluntas absoluta*), human will, or *voluntas secundum quid*, is conditioned by circumstances. In other words, the realization of the highest aims of humankind is intrinsically contingent, as Dante expressed in the *Divine Comedy*:

> But utterance and feeling among mortals,
> For reasons which are evident to you,
> Have different feathers making up their wings.
> I, too, as man feel this disparity […].[34]

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[31] *Summa contra gentiles* II 30: "In quibus [rebus] vero forma non complet totam potentiam materiae, remanet adhuc in materia potentia ad aliam formam." Cf. Aquinas 1975, 87.

[32] *Summa contra gentiles* II 23: "Omnis enim agentis per necessitatem naturae virtus determinatur ad unum effectum. Et inde est quod omnia naturalia semper eveniunt in eodem modo, nisi per impedimentum: non autem voluntaria. Divina autem virtus non ordinatur ad unum effectum tantum […]. Deus non agit per necessitate naturae, sed per voluntatem." Cf. Aquinas 1975, 68: "For the power of every agent which acts by natural necessity is determined to one effect; that is why all natural things invariably happen in the same way, unless there be an obstacle; while voluntary things do not. God's power, however, is not ordered to one effect only […]. Therefore, God acts, not out of natural necessity, but by His will."

[33] Omodeo and Renn 2015.

[34] Alighieri 1984, 94; Dante Alighieri, *Paradiso* XV 79–83:

> "Ma voglia e argomento ne' mortali,
> per la cagion ch'a voi è manifesta,

Apart from ethical contingency, Scholastic authors also used *secundum quid* in logic. For instance, Petrus Hispanus explained the meaning of the so-called *secundum quid* fallacy in his *Tractatus sive summule logicales*, commenting on Aristotle's *On Sophistical Refutations* V (166b36–167a14).[35]

In logic, *secundum quid* meant either a "diminution" of a concept through restriction of its definition (*secundum quid et simpliciter*), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*). A *secundum quid* fallacy occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (or *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument "est animal pictum, ergo est animal" is not correct. In this case, there is a quid signalizing the gap between universal necessity and particular or concrete contingency.

### 4.3.2   Contingent Causation in the *scientia de ponderibus*

The *scientia de ponderibus* heavily drew on the idea of the conditional limitation of natural necessity depending on circumstances (*secundum situationem*, also literally meant as "depending on the position"). In particular, the concept of *gravitas secundum quid*, or positional heaviness, had a powerful explanatory function, most notably in the Aristotelian treatment of weights by Jordanus Nemorarius, and continued to be essential during the Renaissance in the reflections on mechanics by scholars such as Tartaglia, Cardano, and Benedetti himself.[36]

In mechanics the "limitation" or "determination" *secundum quid* implied that the dynamic tendency of a body was reduced or enhanced depending on intervening constraints or circumstances, in particular mechanical ones. The rotations of a lever around a pivot or of a balance around its fulcrum were conceptualized as constrained motions. In such displacements, the inherent ("necessary") vertical tendency of a weight resulted in a circular motion due to external constraints. Similarly, the heaviness (*gravitas*) of the bodies suspended at the extremities of a simple machine varied in relation to their changing positions within the system. In such cases, a "necessary" straightforward motion in accordance with natural order resulted contingently in a circular one. The implicit mental model for this kind of displacement was that "circular motion is constrained rectilinear motion." This means that, in the sublunary sphere of contingency, straightforwardness and rectilinear tendency had a higher onto-epistemological status than circularity since straightforwardness was necessarily rooted in natural order. By contrast, circularity, as the deviation from such order, had to be explained. As a consequence, circularity (in the elementary sphere) was allotted a derived and subordinated onto-epistemological status. In other words, circularity was an instance of nature departing from necessity owing to some rather elusive factor or *secundum quid*. From this viewpoint, it was seen as a deviant realization of given potentialities similar to moral deviation from the necessary laws of uprightness. In order to stress that the mechanical treatment of the *scientia de ponderibus* was embedded in the framework of contingency, we could also formulate the principle in this way: "circular motion is rectilinear motion modified by a contingency."

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diversamente son pennuti in ali;
ond' io, che son mortal, mi sento in questa
disagguaglianza […]."

Also, see *Paradiso* IV, 87, IV, 109, IV, 113, and *Purgatorio* VII, 57.
[35] Hispanus 1972, 157–158.
[36] See Renn and Damerow 2012, especially the sections from 3.6 to 3.8.

Almost at the beginning of his small treatise "on the weights," Nemorarius stressed his Aristotelian commitment. In fact, his approach was based on the opposition between the *natural* vertical motion of the elements and the *violent* hindrances producing circular deviation. At the same time, he introduced the key concept of *gravitas secundum quid* (in some cases, also *levitas secundum quid*), which we will refer to as "positional heaviness."

> [...] if equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter, to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously. I then say that since the longer arc of the circle is more contrary to a straight line than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the fall which the heavy body would have along the straight line than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc than over the shorter one; otherwise the motion would become heavier. Since something moves with more violence in the ascent [along the arc], it is apparent that there is more positional heaviness [*gravitas secundum situm*] and, as it is like that depending on position [*secundum situationem*], one can aptly call it 'positional heaviness' [*gravitas secundum situm*].[37]

In its circular descent along a circular path, a weight deviates from its natural tendency, or *intentio*, the more the arm of the balance departs from the horizontal position. Therefore, the "violence" is greater when the arc of displacement is longer, while the weight progressively loses its weight insofar as the vertical component in its motion is reduced.

According to Nemorarius, a weight that reaches the bottom of the circular arc described by the arm in its displacement is not "at rest" but only "lighter." In fact, a natural being is at rest only if it is fully accomplished, that is, once it has realized the aim, or act, toward which its power is directed teleologically. By contrast, a body is always in motion, or striving to move, until it has reached its end: "All motion strives toward its aim—indeed the whole nature strives towards actuality and is realized [in it]—hence the opposition occurs against [a displacement] contrary [to the natural tendency]."[38]

A body on one arm of the balance becomes lighter during its downward motion than an equal one located on the other extremity. Thus, as Nemorarius assumes, or tries to demonstrate, a balance removed from its state of equilibrium will tend to restore the original state. As one reads in the *propositio secunda* (with reference to the diagram in Figure 4.1), which is the second of a series of propositions developing the details of Nemorarius's doctrine of weights,

> Suppose now that the descent occurs on the side *B* and the ascent on the side *C*. I say that both will go back to the [horizontal] position of equality. In fact,

[37]Nemore 1533, f. A3*v* (emphasis added): "[...] si sumantur de circulo maiori et minori arcus aequales, corda arcum maioris circuli longior est. Propeterea posset ex hoc ostendi, quod pondus in libra tanto sit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod maior arcus circuli plus contrariatur rectae lineae quam minor, et casus gravis per arcum maiorem, plus contrariatur casui gravis, qui per rectam fieri debet, quam casus per arcum minorem. Patet ergo *maior est violentiam in motus secundum arcum maiorem*, quam secundum minorem. Aliter enim fieret motus magis gravis. Cum ergo plus in ascensu aliquod movetur violentiae, patet, quam maiore est *gravitas secundum situm, et quia secundum situationem* talium sic sit, dicatur gravitas secundum situm."
[38]Nemore 1533, ff. A3*v*–A3*r*: "In termino enim cuiscunque motus intenditur, intenditur et viget tota natura in actu, qui in motu sit quasi in potentia, secundum quem fiebat contrarietatis suae oppositio."

*B* will not further descend, because its descent towards *D* is more oblique than
the ascent of *C* towards the [horizontal position of] equality; in fact, *B* and *C*
are equidistant from the place of equality.[39]



Figure 4.1: Diagram accompanying preposition two in Apianus's 1533 edition of Nemorarius's
*Liber de ponderibus* (1533, f. B2*r*). (Bayerische Staatsbibliothek)

Nemorarius's reasoning becomes clearer in light of propositions four and five:

> Fourth [proposition]: It is positionally heavier, insofar as its descent, in the
> same position, is less oblique.

> Fifth [proposition]: But a more oblique descent partakes less of the straight
> [descent], for the same quantity [of the path].[40]

In proposition five, it is suggested that the vertical components of the potential descents
of the two beams could be identified and compared. This was the source of the idea
that the variation of heaviness could also be determined by comparing the straightness of
the descents. A similar procedure was later taken up and explained in detail in Niccolò
Tartaglia's considerations in the *Questiti et inventioni diverse* (1546) about the manner of
ascertaining the positional heaviness of two weights on the basis of the so-called angles
of contact. These are the "curvilinear" or "mixed" angles between the circular path of the

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[39]Nemore 1533, ff. B2*r–v*: "Ponatur nunc, quod fiat descensus a parte B, et ascensus a parte C, dico quod
redibunt ad situm aequalitatis. Non enim ulterius descendet B, eo quod descensus eius versus D magis
obliquus est, quam ascensus C ad aequalitatem; B enim et C iam aequaliter distant a situ aequalitatis."
[40]Nemore 1533, f. A3*r*: "Quarta [propositio]: Secundum situm gravius esse, quanto in eodem situ minus
obliquus est descensus. Quinta [propositio]: Obliquiorem autem descensum minus capere de directo, in
eadem quantitate." Translation from Renn and Damerow 2012, 63. For proposition four, see Nemore 1533,
f. B3*v*–B4*r* and, for proposition five, Nemore 1533, f. B4*r*–C2*v*.

arms of a balance and the vertical lines connecting the weights to the cosmological center of gravity (see Figure 4.2). Tartaglia compared the angles of contact of two equal weights located on the extremes of a balance, and argued that the lifted one is always smaller than the lowered one. Thus, the lifted weight would face a descent that is more oblique. It would acquire a greater positional heaviness than its lowered counterweight and, as a further consequence, the inclined system would reestablish its horizontal balance, if not hindered to do so.
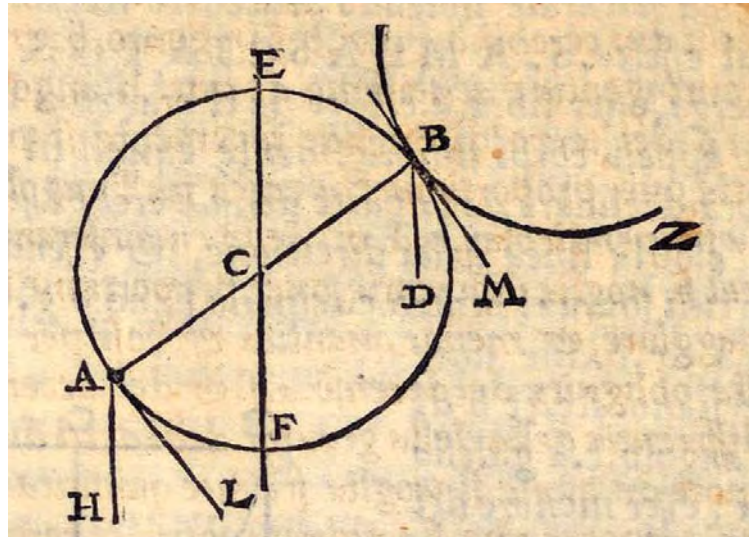


Figure 4.2: In the *Quesiti et invenzioni diverse*, Tartaglia argued that the relative positional heaviness of the weights A and B on a balance could be determined on the basis of the "mixed" angles of contact HAF and DBF. Since it is argued that DBF < HAF, the weight B will be heavier than A. Thus, the inclined system will strive toward the restoration of a horizontal equilibrium. (Max Planck Institute for the History of Science, Library)

In spite of his attempt to quantify the *quid* accounting for the alleged restorative motion of the inclined balance, Tartaglia's geometrical quantification maintained a margin of indeterminancy. As he stated, the ratio between the two mixed angles is less than that between any determined quantities. Therefore, it is impossible to stabilize the system in its inclined position by adding a small (no matter how small) weight on the lowered side of the balance. According to Tartaglia, it is impossible to counterbalance the positional heaviness of the lifted weight. Quite on the contrary, any additional weight added to the lowered side would make the balance rotate and reach the vertical position.[41]

## 4.4 The Epistemological Import of Benedetti's Generalization from Weights to Forces

As we have argued so far, in the medieval *scientia de ponderibus* circular motion is conceived of as constrained linear motion. Yet, within an Aristotelian cosmology, this mental model is restricted to the sublunary sphere, where motions cannot fulfill their nature. This is indeed the sphere of contingency, where a gap is to be witnessed between the necessary

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[41]Tartaglia's approach was controversial, already in his time. See Renn and Omodeo 2013, sec. 3.6.

order of things (or "nature" as actuality) and the effective phenomena (subjected to "violence" or to external constraints). The four elements naturally tend toward their places through a straightforward descent or ascent. Heavy bodies, for instance, strive toward the center of gravity, which is, at the same time, the center of the cosmos. If their motion is hindered, as is the case with mechanical constraints, a certain factor or *quid* has to be taken into account, which explains the deviation from the rule. In this theoretical context, contingency is the concept expressing the relationship between the natural law and phenomenal reality, which follows a norm while deviating from it. The *secundum quid* is that which explains this deviation. Possibly, it has to be expressed through geometrical means, although it might prove unintelligible or infinitesimal, as was the case with Tartaglia's ratio between mixed angles accounting for the *gravitas secundum quid* of the weights of a balance. In the treatment of weights, in particular of those on a balance, Nemorarius and his followers made a limited use of the mental model of curvilinear motion as constrained linear motion. In fact, they employed it to account for phenomena linked to gravity (i.e., the vertical fall of bodies explained in Aristotelian terms). It was Benedetti who made the decisive step toward the generalization of this model in the direction of inertial dynamics. Let us consider his application of it first to balances and then to centrifugal forces.

In the section on mechanics of the *Diversae speculationes*, Benedetti picked up and revised the Scholastic concept of *gravitas secundum quid*. Guidobaldo del Monte had already criticized Nemorarius's and his followers' conclusion that an inclined balance hinged on its fulcrum as its center of gravity would return to the horizontal position, but his criticism went so far as to renounce the concept of positional heaviness altogether.[42]

Relying on the Archimedean concept of the center of gravity of a body, Del Monte concluded that an equal-arms balance hinged on its fulcrum would remain stable in any position (a correct conclusion only if it is assumed, in modern terms, that the gravitational field is homogeneous): "*Propositio IV: Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus, ubicunque relicta manebit.*" (Fourth Proposition: Take a balance that is equidistant from the horizon and that has weights in its extremities which have the same weight and equally distant from the center (the latter being located in the balance itself). Whether it is displaced or not, it will remain in the same position in any position.)[43]

Benedetti shared the criticism of Nemorarius and Tartaglia with regard to their specific argumentation about the tendency of such an inclined balance to reach the horizontal position but based his judgement on a novel treatment of positional heaviness. The first chapter of Benedetti's *De mechanicis* begins with the statement: "Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself."[44]

Hence, he clearly committed himself to a mechanical theory of equilibrium based on positional heaviness. Benedetti's technical terms are not always employed in a rigorous and consistent manner. He treats the *pondus* at times as the varying quantity to be taken into consideration, as is shown by expressions like "*proportio ponderis in C ad idem pondus in F*" and "*unde fit… pondus magis aut minus grave,*" in *De mechanicis* II (Benedetti 1585, 142). Given these semantic fluctuations, we will translate *pondus* as "body" or as

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[42] Renn and Damerow 2012, 86–92. We will discuss the divergent interpretations of Benedetti and Del Monte later, in chapter 5.

[43] Damerow and Renn 2010, 65.

[44] Drake and Drabkin 1969, 166. Benedetti 1585, 141: "Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet."

"weight" and *gravitas* as "heaviness" or as "weight," depending on the context. At the beginning of chapter 1 of his book on mechanics, Benedetti talks of a varying quantity of heaviness, or gravity (*gravitas*), belonging to a weight (*pondus*) or a body placed on a balance beam. Hence, he makes a terminological distinction between *pondus*, as a kind of absolute weight or heavy thing, and *gravitas*, as a downward tendency that can act with more or less force on the body (depending on the inclination of the beam). In this case (as in most cases in the text), *pondus* has the essentialist meaning of a substance (a substratum or ὑποκείμενον). It is the body or weight on the balance, whose special property of being heavy, namely the *gravitas*, varies depending on a *quid*. This *quid* is the position, or *situm*.

Benedetti seeks to quantify it by means of a method he invented. He considers the line, which he calls *linea inclinationis* or *linea itineris*, connecting a weight on an inclined balance beam to the cosmological center of gravity. Note that Benedetti calls the elementary downward tendency an *iter* from a merely kinematic viewpoint, but also an *inclinatio* from a physical and more proper one. According to him, the major or minor heaviness of the weight can be assessed through the projection of the *linea inclinationis* on the horizontal line passing through the fulcrum (Figure 5.1). The more distant it is from the fulcrum, the heavier the positional heaviness becomes. Thus, the weight reaches a maximum of heaviness when the balance is horizontal, and its minimum when it is vertically resting (*nititur*) on the fulcrum or hanging (*pendet*) from it. Notably, this approach anticipates the one based on the determination of the torque in classical physics, and comes to the same conclusions.[45]

Additionally, Benedetti equates the heaviness to a *virtus*, *vis*, or *vigor*, i.e., a force, which might also act in different directions (in *De mechanicis*, Ch. 3) and is applied to the extremity of a constrained mechanical system, like a lever or a balance. This is a significant generalization from weights to forces, but for our present discussion the most important generalization relates to rectilinear tangential tendencies in systems set in circular motion.[46]

The relevant treatment is the epistle to Capra and is included in the *Diversae speculationes*. It deals with the rotation of a millstone and the question of whether its motion could be perpetual. Benedetti denies this by arguing that the rotation is impeded first by the friction of the air and, second and more importantly, by the resistance of the millstone's parts. The latter have a straightforward tendency, an *inclinatio recte eundi*, along the tangential lines of their rotation (Figure 4.3). As one reads, this rectilinear inclination or impulse (*impetus*) can be bent only by violence. Moreover, the centrifugal tendency grows in proportion to the augmentation of the velocity, as witnessed by other cases, among them the rotation of a catapult or a sling (*machina missilis*). A centrifugal tendency is seen as a rectilinear natural inclination (*naturalis inclinatio recte eundi*).

> You ask me this question in your letter. Suppose a millstone rested on a virtually mathematical point and was set in circular motion, could that circular motion continue without end, assuming that the millstone is perfectly round and smooth?
>
> I answer that this kind of motion will certainly not be perpetual and will not even last long. For apart from the fact that the wheel is constrained by the air which surrounds it and offers resistance to it, there is also resistance from

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[45]Renn and Damerow 2012, 138. We will deal with the details of Benedetti's mechanics in the next section.
[46]Cf. Büttner 2008.

the parts of the moving body itself. When these parts are in motion, they have by nature a tendency [*impetus*] to move along a straight path. Hence, since all the parts are joined, and any one of them is continuous with another, they suffer constraint in moving circularly and they remain joined together in such motion only under compulsion. For the more they move, the more there grows in them the natural tendency to move in a straight line, and therefore the more contrary to their nature is their circular motion. And so they come to rest naturally: for, since it is natural to them, when they are in motion, to move in straight line, it follows that, the more they rotate under compulsion, the more does one part resist the next one and, so to speak, hold back the one in front of it.[47]
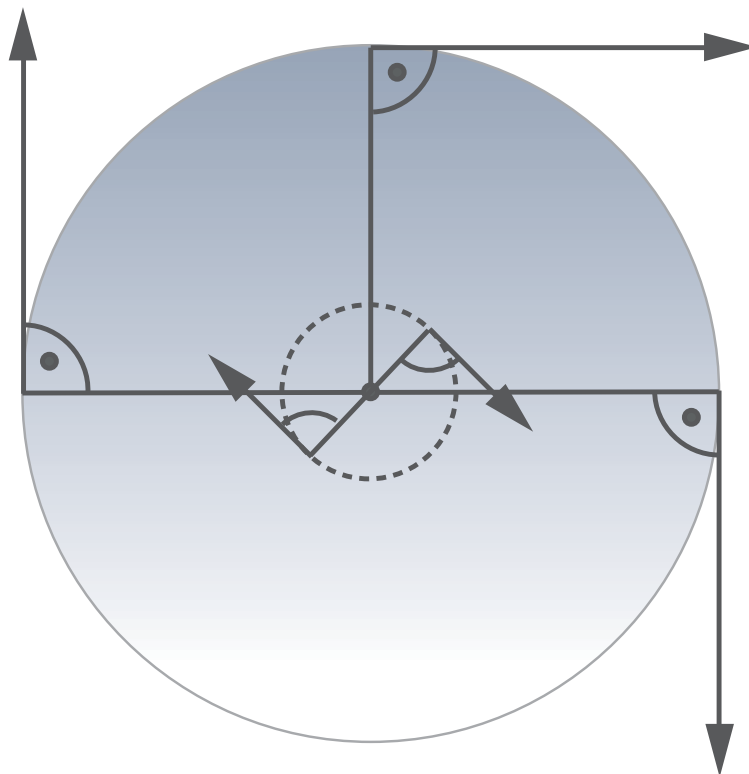
Figure 4.3: A diagram showing Benedetti's considerations on the rotating millstone stressing the centrifugal tendencies of its parts. (Drawing by Irina Tupikova)

The mental model of circular motion as constrained straight motion receives in Benedetti's treatment a higher degree of generalization. In this case, he argues that, since it contrasts

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[47]Drake and Drabkin 1969, 229. Benedetti 1585, 285 (emphasis added): "Quaeris a me literis tuis, an motus circularis alicuius molae molendinariae, si super aliquod punctum, quasi mathematicum, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfecte rotundam, et laevigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia praeterquam quem ab aere qui ei circumcirca aliquam resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quae cum motae sunt, natura, impetum habent efficiendi iter directum, unde cum simul iunctae sint, et earum una continuata cum alia. *Dum circulariter moventur patiuntur violentiam, et in huiusmodi motu per vim unitae manent, quia quanto magis moventur, tanto magis in iis crescit naturalis inclinatio recta eundi, unde tanto magis contra suammet naturam volvuntur*, ita ut secundum naturam quiescant, quia cum eis proprium sit, quando sunt motae, eundi recta, quanto violentius volvuntur, tanto magis una resistit alteri, et quasi retro revocat eam, quae antea reperitur habere."

with a natural inclination, it cannot be eternal. Note that this assumption (violent motion cannot be eternal) is Aristotelian but emerges in a context in which this legacy is meant to be rejected.[48]

Another Aristotelian echo looms over Benedetti's statement that the linear tendency makes a body "lighter," since if it were freed from the constraint hindering its projection, it would not fall vertically but rather travel through a more or less rectilinear trajectory tangent to the circular motion of the constrained rotation. In the conclusion of his reflection on the natural rectilinear striving of the parts of a body set in circular motion, Benedetti stressed the originality of his treatment "without precedents" and its opposition to Aristotelian dynamics (according to which the projection of a body through a medium presupposes the support of the medium itself).

> But if you wish to see this truth more clearly, imagine that while the body, i.e., the top, is spinning around very rapidly, it is cut up or divided into many parts. You will observe not that those parts immediately fall toward the center of the universe, but that they move in a straight line, and, so to speak, horizontally. No one, so far as I know, has previously made this observation on the subject of the top.
>
> From such motion of the top or of a body of this kind it may be clearly seen how mistaken are the Peripatetics on the subject of the forced motion of a body. They hold that the body is driven forward by the air which enters [behind it] to occupy the space left by the body. But actually the opposite effect [that is to say, resistance] is produced by the air.[49]

We have so far observed two instances in Benedetti's work on mechanics in which a tension between mathematical laws of nature and their empirical realization emerges: his treatment of the rotation of a beam about its pole and that of a turning wheel. In both cases, natural straightforward tendencies are constrained and deviated into violent circular ones. The epistemological meaning of these concepts lies in the possibility of a geometrical treatment of natural contingency seen as the connection between the *necessity* of the rules and of the principles and their *necessitation*, that is, their deviation, as witnessed by the empirical reality of curvilinear motions.

## 4.5 From *inclinatio* to *inertia* and Beyond: Mechanistic Perspectives

René Descartes generalized the insights implicit in the idea that curvilinear motion is contingent rectilinearity at an epistemic level (through the expansion of their realm of application) as well as at an epistemological and ontological level (giving them a foundational meaning). In *Le Monde*, circular motion is treated as a deviation from rectilinear motion.

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[48] On Benedetti's anti-Aristotelianism, see Maccagni 1983.

[49] Drake and Drabkin 1969, 229–230. Benedetti 1585, 285: "Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, trochum scilicet, dum velocissime circunducitur secari, seu dividi in multas partes, unde videbis illas omnesque, non illico versus mundi centrum descendere, sed recta orizontaliter, ut ita dicam, moveri. Id quem a nemine adhuc (quem sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clare perspicitur, quam errent peripatetici circa motum violentum alicuius corporis, qui existimant aerem qui subintrat ab occupandum locum a corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur."

Descartes develops a general theory of the world in which circularity is the main characteristic of the motions of both the particles of matter as well as of planets revolving about the centers of their orbits.[50]

> […] when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is, the inclination they have to move, is different from their motion.[51]

This is the third of Descartes's three laws of nature (*loix or règles de la Nature*) as exposed in chapter 7 (" *Des loix de la nature de ce nouveau Monde*" ). It follows the inertial law of conservation of the state of the bodies and that of the conservation of the quantity of motion. The third law is particularly relevant from the viewpoint of our epistemological inquiry into mathematics without necessity, since it clearly expresses the gap between law and effective reality, between the straightforward tendency of all bodies and their real circular motions, in a manner that is akin to medieval and Renaissance predecessors such as Benedetti. Note that Descartes calls the rectilinear tendency " *inclination*" just as Benedetti called it "*inclinatio recte eundi*." This terminological choice is apt to express its character as a natural inner tendency. The examples that Descartes choses to illustrate his claim are familiar to readers of Renaissance sources on mechanics: the wheel (*une roue*) and the sling (*fronde*) (Figure 4.4).

In the *Études galiléennes*, Koyré affirmed the complete independence of the law of inertia, which is only *in nuce* in Galileo's physics, from experience, since rectilinear motion is never observed in nature. "Contrairement à ce qu'on affirme bien souvent, la loi d'inertie n'a pas son origine dans l'expérience du sens commun et n'est ni une généralisation de cette expérience, ni même son idéalisation. Ce que l'on trouve dans l'expérience, c'est le mouvement circulaire ou, plus généralement, le mouvement curviligne. On n'est jamais—sauf le cas exceptionnel de la chute, qui n'est justement pas un mouvement inertial—en présence d'un mouvement rectiligne."[52]

In light of our reconstruction, this statement proves quite inaccurate. As we have seen, the vertical fall of a heavy body is not the only observable straight motion: the beginning of the trajectory of a projectile thrown with great speed also looks rectilinear. Slings and catapults are in fact the instruments with which turning wheels and rotating millstones were compared, and it was from these instruments that Benedetti, Descartes, and also Galileo in the Second Day of the *Dialogo sopra i massimi system del mondo*, derived the centrifugal tendencies of the parts of rotating objects. Is this not a generalization from experience? Such generalization went so far as to include the explanation of the behavior of bodies on a rotating Earth, in the case of Galileo, and the conceptualization of corpuscular and planetary motions, as was the case for Descartes. Moreover, before the classical law of inertia was defined, what took center stage was the observation of rectilinear motions—either the vertical fall or centrifugal tendencies—and of their circular deviations. A major physical problem faced by Scholastic and post-Scholastic mechanics

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[50]On the Cartesian cosmos, see Aiton 1972, 30–64 and Gaukroger 2006, 304–317.

[51]Descartes 1998, 29. Descartes 1986, 43–44: " Lors qu' un corps se meut, encore que son mouvement se fasse plus souvent en ligne courbe, et qu' il n e s' en puisse jamais faire aucun, qui ne soit en quelque façon circulaire […], toutesfois chacune de ses parties en particulier tend toujours à continuer la sien en ligne droite. Et ainsi leur action, c' est à dire l' inclination qu' elles ont à se mouvoir, est differente de leur mouvement."
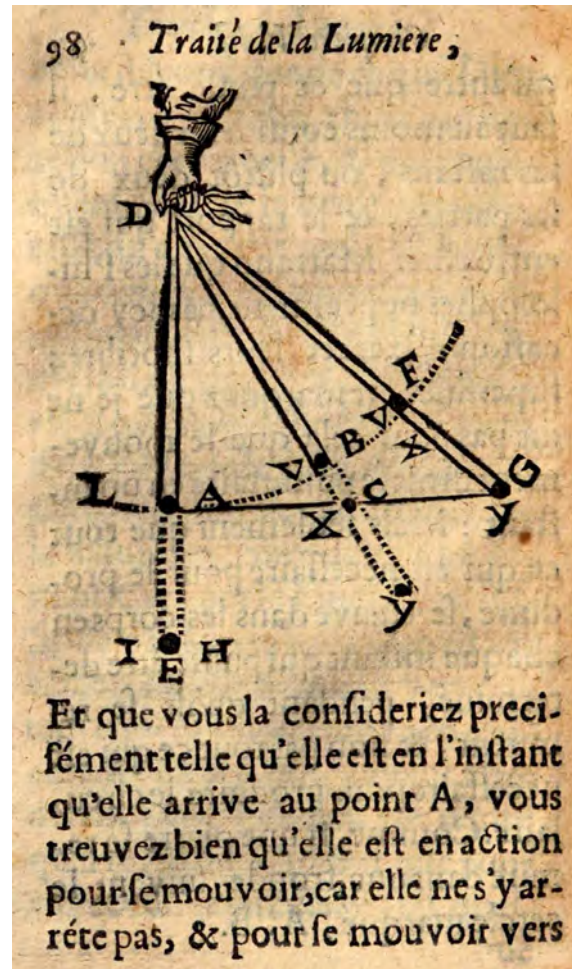
[52]Koyré 1986, 206.

Figure 4.4: Descartes's visualization of the centrifugal tendency of bodies thrown by a sling, in *Le Monde*, Ch.7. (Bayerische Staatsbibliothek)

was precisely that of conceptualizing the relationship between curves and straight lines. In particular, against the backdrop of Aristotelian philosophy, curvilinear motion appeared as constrained. It was a derived displacement resulting from a *violent* external intervention bending the straightforward *natural* tendency of a moving body. In such an Aristotelian and post-Aristotelian context, circular motion was seen as contingent. That is to say, it was the deviation from natural order depending on an obstacle which was called the "secundum quid." As we have argued, the concept of "secundum quid" is embedded in the Scholastic reflections upon natural necessity, order, and contingency. It was referred to as a model of causality in which the observed phenomena represent a partial fulfillment of an underlying order, or of natural laws. Accordingly, elementary bodies express their necessary laws in a limited manner, that is, they have to be explained through the so-called *necessitas conditionata* or *necessitas secundum quid*. Contingency is the relation between necessary order and phenomenal reality. The gap has to be explained, and was explained with a quid, a factor, or a determination. Accordingly, a quid was introduced into mechanics to account for circular motions in terms of mechanical constraints.

In the medieval *scientia de ponderibus*, two determinations were considered for the equilibrium of a balance: first, the circle resulting from the inclusion of the vertical motions of the weights in a mechanical system, and second, the *situm* (location) of the weights in a mechanical system determining a variation in heaviness. The reflection on *gravitas secundum situm* (positional heaviness) from Nemorarius to Benedetti presupposes this twofold *quidditas* and focuses on the latter aspect (the variation of the heaviness).

The conviction that circular motion, as a violent motion, requires an explanation is based on the mental model that "circular motion is constrained (or *contingented*) straight motion." Although they were embedded in the medieval discourse on contingency, the several attempts to quantify the *quid* accounting for the deviation testify to the common effort to overcome the qualitative and indeterminable characterization of contingency as a form of causality. What was maintained, for instance in Descartes, was the idea of a gap between law and phenomenon. Yet, if the deviation from the law can be perfectly quantified, then the separation between the order of nature and its realization is virtually eliminated, that is, the fracture between absolute necessity and conditional necessity is recomposed. To be sure, this step toward the necessitation of nature, resulting from the abandonment of contingency in both senses (causal and epistemological), was accomplished only later, in the course of the seventeenth century.

The work of Benedetti and his onto-epistemology of contingency are representative of an age of transition from Scholastic and Renaissance natural philosophies to the various instantiations of the classical science of the next century. Benedetti's Pythagorean commitment to mathematics, seen as the most powerful logical means applied to all fields of knowledge and to nature in particular, is an illustrative case of the complex and non-linear history of scientific thought. His efforts to overcome Aristotelian conceptions could not really renounce the crucial assumption of the Aristotelian outlook under attack. This particularly concerns the ontology and epistemology underlying his scientific theories and practices. Mathematical determination, both in science and nature, did not imply necessity, neither at the level of material causation nor of explanation. The gap between the laws of nature and the effective processes reflected a Scholastic distinction between formal necessity and material imperfection. Such philosophical assumptions underpinned medieval treatments of phenomena, including statics, and Renaissance developments, especially in the line connecting Tartaglia and Cardano to Benedetti and Descartes. The distinction between formal necessity and phenomenal contingency offered them a horizon within which they could conceptualize general laws as well as their empirical instantiation. In particular, Benedetti could extend the area of application for the mental model that circular motion is a constrained (violent) deviation from the law of rectilinear motion. He did this by applying a model originating from statics to the area of dynamics, thus paving the way for the classical concept of inertia. However, we should not neglect the practical roots of his work in a Scholastic-embedded science of weights, which generalized observations of mechanical systems in order to make universal statements about nature.

# Chapter 5
# Mechanics

The book on mechanics, *De mechanicis*, the third of the *Diversae speculationes*, is divided into twenty-five chapters. Mechanical issues and references to mechanics can also be found in the epistles. As to the discussion of the motion of fall through media and of hydraulic problems, these are not part of this book. *De mechanicis* begins with a brief preamble in which Benedetti claims that he treats topics that have never been dealt with before or have not been sufficiently explained. In this section we will discuss the positioning and controversies implicit in this strong statement in an age when mechanical studies were very lively in the Italian peninsula and abroad. We will first offer an overview of Book 3 of the *Diversae speculationes*. Second, we are going to look more closely at the first foundational chapters of the treatise. Third, we will consider the rivalry with Del Monte, emerging from the latter's harsh criticism of Benedetti and, in part, his misunderstanding of some crucial elements of Benedetti's theory. The context of these lively disputes is the reaction to the publication of Tartaglia's eclectic work on this subject, the *Quesiti, et inventioni diverse* (1546), and his re-issue of the medieval classic on the science of weights. Benedetti, as a critical pupil of Tartaglia, could not sympathize with the absolute rejection of Tartaglia and the medieval tradition his approach rested upon. At the same time, he felt the need to distance himself from several aspects of Tartaglia's treatment, as we will reconstruct in detail in this section. The debates between Benedetti and Del Monte arguably culminated with Galileo's work, which stands out as a sort of synthesis of earlier positions. Understanding these historical developments, as well as the intellectual triangle Benedetti-Del Monte-Galileo, is fundamental in order to trace Benedetti's influence on his contemporaries and on the young Galileo.[1]

## 5.1 An Overview of *De mechanicis*

### 5.1.1 The Foundations of the Theory of the Balance

Chapters 1 to 6 of *De mechanicis* contain a systematic account of the foundation on which Benedetti built his mechanics. Chapter 1 clarifies qualitatively how the variable weight changes depending on the obliqueness of the balance beam. While a body attached to the end of the beam has a maximum weight if the beam is in a horizontal position, it vanishes when the beam is in a vertical position. Benedetti explained this behavior as a consequence of the different extent to which the attached weight rests on the center of the balance. If the position of the beam is close to the vertical, the weight of a body attached to the end of the beam is close to zero since it rests nearly completely on the center of the balance.

Chapter 2 clarifies the positional changing of the weight quantitatively. Benedetti related the balance with an oblique position of the beam to a bent lever with one horizontal and one oblique arm, thus providing the precondition for a generalization of his result. A

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[1]Section 5.1 is derived from Renn and Damerow 2012, chap. 6.1–6.3 and section 5.2 from Renn and Omodeo 2013.

generalization of this kind is indeed required if the lines of inclination of the bodies at the end of a balance are conceived as being directed to the center of the earth and hence no longer as being parallel to each other. Benedetti mentioned this possibility at the end of this chapter, but considered the angle between the two directions as being too small to be measured and thus not necessary to be taken into account.

In chapter 3 Benedetti generalized from the downward inclination of a body attached to the balance beam to forces acting upon the body not vertically but making an acute or obtuse angle with the horizontal beam. Accordingly, he replaced the bodies at the end of the balance beam with two weights or two moving forces (*duo pondera, aut duae virtutes moventes*), as he formulated somewhat ambiguously. His derivation of their quantities was based on a reinterpretation of the horizontal distances between the center of the balance and the vertical projections of the bodies at the end of a beam in an oblique position (Figure 5.1). He interpreted these distances as perpendicular distances from the center of the balance to the lines of inclination, and was thus also able to apply the result he achieved for vertically descending weights to lines of inclination caused by forces that are not vertical.
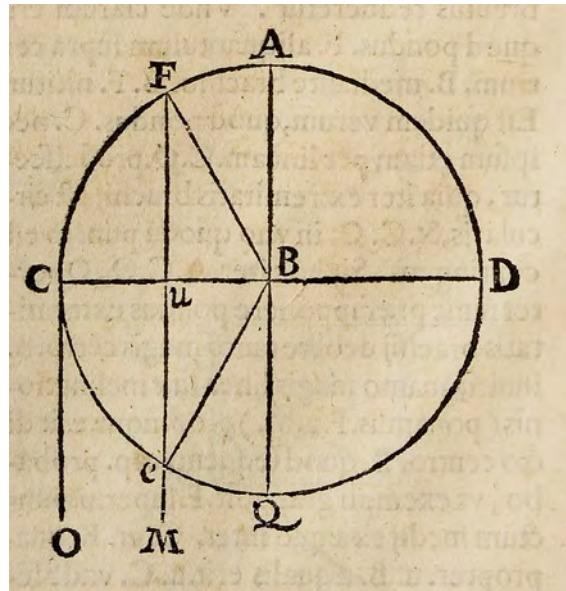


Figure 5.1: Benedetti's diagram showing a balance CBD or FBD. The lines CO and FUEM are the so-called lines of inclination connecting the weights C and F with the center of the elements. The length of the projection on the horizontal is proportional to the positional heaviness. (Max Planck Institute for the History of Science, Library)

Benedetti maintained that his arguments in chapters 1 to 3 clarify all the causes operating on balances and levers. To demonstrate this, he discussed in chapters 4 and 5 the validity of his results if applied to material balances and levers, taking into account that they have a beam with finite extension. This, however, does not imply that he calculated the influence of the weight of the beam itself. His discussion was rather restricted to a justification of his claim that the geometry of a rectangular beam does not require a modification of his propositions. In chapter 5 he treated the case of a lever whose fulcrum is at one of its ends.

Finally, in chapter 6, Benedetti added the description of an instrument used in bakeries for treating the dough. He explained the function of the instrument by applying his proposition from chapter 3.

The systematic approach used by Benedetti in this first part of his treatise is comple-mented by chapter 9, in which he explained the division of the scale of a steelyard into equal intervals.

### 5.1.2 Criticism of Tartaglia and Nemorarius

In chapters 7 and 8 Benedetti criticized the theorems of his former teacher Tartaglia, in particular those that Tartaglia adapted from Jordanus Nemorarius. Both chapters deal ex-clusively with some propositions of Book 8 of Tartaglia's *Quesiti, et inventioni diverse*,[2] which is concerned with the science of weights and is entitled, accordingly, *Sopra la scien-tia di pesi*. In those cases in which Tartaglia's propositions are adapted from Nemorarius, Benedetti mentioned explicitly the corresponding proposition in the edition of Nemorar-ius' *De ratione ponderis*, corrected and illustrated by Tartaglia, and published under the title *Iordani opusculum de ponderositate Nicolai Tartaleae studio correctum novisque fi-guris auctum*.[3]

Chapter 7 starts with some brief critical remarks on Tartaglia's propositions 2 to 5. Tartaglia's proposition 2 essentially paraphrases and modifies the Aristotelian claim that the speed of moving bodies is proportional to the driving force. Following Nemorarius, Tartaglia maintained that the velocities of descending heavy bodies of the same kind are proportional to their power (*potentia*), while in the case of ascending bodies their veloci-ties are inversely proportional to their power. For bodies of the same kind their power is conceived here as proportional to their sizes, that is, to their weights. Descending bodies are thus simply falling bodies with velocities proportional to their weights, while in the case of ascending bodies their weight acts as a resistance. Tartaglia's proposition 3 gener-alizes proposition 2 for bodies with equal weights but unequal positional heaviness. His proposition 4 maintains that in the latter case the power of bodies attached to a balance is proportional to the distances from the center.

Benedetti's critical remarks are somewhat eclectic. He argues that Tartaglia, in his second proposition, does not take into account the quantity of external resistance (*quan-ti momenti sint extrinsecae resistentiae*). With regard to Tartaglia's third proposition, Benedetti points to its assumptions, namely that the bodies have to be homogenous and must have the same shape. He criticizes Tartaglia's proof as it does not actually require these assumptions, but would also be true for heterogeneous bodies or for bodies with dif-fering shapes. Concerning the fourth proposition, he criticizes Tartaglia for not proving what he claimed to prove. Instead, he should have followed Archimedes's proof of the law of the lever.

Benedetti's chapter 7 continues with a detailed discussion of the second part of Tartaglia's proposition 5 and the following two corollaries and is thus directly concerned with the equilibrium controversy, that is, the controversy about whether or not a balance in equilibrium removed from its horizontal position will automatically return to this position. Tartaglia maintained in this proposition that a balance that is in equilibrium in a horizontal position will necessarily return to this horizontal position when moved into an oblique position. In a first corollary, he claimed that the more the balance beam is brought into an oblique position, the more the bodies attached to it become positionally lighter. In a second corollary, he claimed that while both bodies in this case become positionally lighter, the lifted body loses less of its positional heaviness than the body moving down. He concluded that the beam will return to a horizontal position. Benedetti

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[2]Tartaglia 1546.
[3]Nemore 1565.

questioned Tartaglia's approach by referring to the first three chapters of his own treatise, arguing in particular that Tartaglia's second corollary must be wrong. He discussed once more the balance beam in an oblique position, but now without the assumption that the lines of inclination of bodies attached to the balance beam are parallel. Rather, he considered the case that these lines are directed to the center of the world, showing, as we have discussed, that it is not the lifted body, but rather the body that is moved down, which loses less of its positional heaviness.

Benedetti continued in chapter 8 with critical comments on Tartaglia's propositions 6, 7, 8, and 14. Tartaglia's proposition 6 contains the proof of his fallacious claim that the lifted body of an oblique balance beam loses less of its positional heaviness than the body moving down, now modified by the further claim that the difference is smaller than any finite quantity. Tartaglia claimed:

> […] that the differences between the heaviness of these two bodies is impossible to give or find between two unequal quantities.[4]

Like Del Monte had done before him, but with different results, Benedetti criticized Tartaglia for not taking into account that the lines of inclination are not parallel.

Tartaglia's proposition 7 contains the simple statement that if the arms of a balance are unequal and bodies with equal weights are attached to the ends of the beam the balance will tilt on the side with the longer arm. Benedetti criticized Tartaglia again for not taking into account that the lines of inclination are not parallel, and claimed that in any case Tartaglia did not give the correct cause of the effect.[5]

Tartaglia's proposition 8 formulates, following Nemorarius, the law of the lever in terms of positional heaviness, stating that if the lengths of the parts of the balance beam with unequal arms are inversely proportional to the weights of the bodies attached to them, their positional heaviness will be equal. Benedetti criticized that this proposition is much better demonstrated by Archimedes.

Finally, Tartaglia's propositions 14 and 15 concern Nemorarius's proof of the law of the inclined plane, which from a modern perspective is essentially correct. Benedetti criticized Tartaglia's argument by attributing to it an interpretation of the inclined plane as a balance, with the top of the plane being its center. His criticism, based on the propositions of his chapters 1 to 3, thus completely missed the point of Tartaglia's argument.

### 5.1.3   Criticism of Aristotle's Mechanics

Benedetti's treatise on mechanics continues mainly with critical notes on the Aristotelian *Mechanical Problems*.[6] His notes are as diverse as the Aristotelian *Mechanical Problems* themselves.

Before he embarked on this criticism, Benedetti dealt with the problem of why a steelyard carries a linear gradation in chapter 9.[7] He took into account the weight of the beam and that of the scale by postulating the equilibrium of the balance when no extra weight is added. Then he added weights of one pound on both sides, arguing that, by

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[4]Tartaglia 1546, 91*r*: "[…] che la differenzia ch'è fra le gravità de questi dui corpi egli è impossibile a poterla dar, over trovar' fra due quantità inequali." Translation in Drake and Drabkin 1969, 130.

[5]We will discuss Benedetti's criticism in more detail later.

[6]Aristotle 1980. See Rose and Drake 1971 and also the introduction to Nenci 2011.

[7]Benedetti 1585, 152. See Drake and Drabkin 1969, 178.

common science (*scientia communis*),[8] the balance stays in equilibrium if they are placed at equal distances from the fulcrum. He had thus found the mark on the beam that indicates a magnitude of one pound. He then successively placed further weights onto the scale, now arguing from the law of the lever that they must be compensated by distances proportional to their number. He thus avoided the problem of applying the law of the lever directly to a material steelyard, just as one does in practice when gauging such a balance.[9]

In chapters 10 and 11 Benedetti started with critical remarks on Aristotle's first problem. Aristotle asked why larger balances are more accurate than smaller ones.[10] Actually, this concrete physical question is not the focus of the extensive answer the author gave to this problem. Instead, he provided a long proof of the basic explanatory principle which plays a major role in the whole treatise. At the end of the proof Aristotle argued that the same load will move faster on a larger balance, thus making such balances more accurate.[11]

The criticism Benedetti applied to Aristotle's argument has two parts. In chapter 10 Benedetti began by rejecting Aristotle's claim that the circumference of a circle combines concavity with convexity. He then argued against a specific part of Aristotle's proof of his principle which involves the superposition of motions. In this part Aristotle showed that:

> […] whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line.[12]

He concluded:

> […] if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line.[13]

For the Aristotelian author this proposition served as a means to describe circular motion as a result of two movements with no fixed ratio. Benedetti, however, did not relate his criticism to this context. He argued only that Aristotle's inference concerning movements in two directions is not sufficient since a straight movement can result from two quite different motions. This criticism does not really relate to the Aristotelian argument, other than by showing that his entire attempt to derive the behavior of a balance from a principle of circular motion is misguided.

In the same vein, Benedetti's criticism in chapter 11 then deals directly with Aristotle's answer to the question of why larger balances are more accurate than smaller ones. He argued that Aristotle's argument is not well founded since the greater accuracy has nothing to do with the motion of the balance beam but only with the geometrical constellation.[14] To conclude he added a consideration of material balances, arguing according to his own principles that a weight on the larger balance will be positionally more effective.

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[8]In the sixteenth century the term *scientia communis* was used to designate knowledge common to all mathematical sciences, its core being the Euclidean theory of proportions. See Sepper 1996, 153–154.

[9]See the discussion in Damerow, Renn, et al. 2002.

[10]Aristotle 1980, 1, 848 b 1–850 a 2 (337–347).

[11]Aristotle 1980, 1 (347).

[12]Aristotle 1585, 507: "Quandoquidem igitur in proportione fertur aliqua id, quod fertur, super rectam ferri necesse." Translation in Aristotle 1980, 1, 848 b 11–848 b 13 (337).

[13]Aristotle 1585, 508: "Si autem in nulla fertur proportione secundum duas lationes nullo in tempore, rectam esse lationem est impossibile." Translation in Aristotle 1980, 339.

[14]Benedetti 1585, 153; Drake and Drabkin 1969, 180–182.

Benedetti's chapter 12 concerns problems 2 and 3 of the Aristotelian *Mechanical Problems*.[15] Problem 2 raises the question that forms the starting point of the equilibrium controversy:

> If the cord supporting a balance is fixed from above, when after the beam has inclined the weight is removed, the balance returns to its original position. If, however, it is supported from below, then it does not return to its original position. Why is this?[16]

Aristotle implicitly assumed that the balance beam has a certain thickness and weight. It follows as a result of the geometry of the balance in an oblique position that if the beam is fixed from above, a greater part of the beam is on the lifted side of the perpendicular line across the suspension point. Consequently the beam will move back by itself into the horizontal position. The opposite is true for a beam fixed from below. In this case, the greater part of the beam is on the lower side so that it cannot move back into a horizontal position by itself.

Benedetti criticized the first case by arguing that it is not only the weight of the beam that causes it to return to the horizontal position, but also the different distances of the weights in an oblique position from the vertical through the point where the beam is fixed. According to his theory of the dependency of the weight on the obliqueness of the beam, the weights must be different on both sides. Benedetti thus generalized Aristotle's argument to the case of a balance without a material beam carrying weight itself.

In the second case of a beam supported from below, he argued that Aristotle is completely mistaken. Benedetti maintained that the beam will not remain in its oblique position, but that the lower part will move down until the beam is in the vertical position.

Problem 3 of the Aristotelian *Mechanical Problems*[17] concerning an explanation of the effect of a lever is, for Benedetti, not worth the effort of a detailed criticism. He only briefly notes that Aristotle did not give the true cause, which one will find in his own theory presented in chapters 4 and 5.[18]

In the very short chapter 13, Benedetti criticized problem 6 of the Aristotelian *Mechanical Problems*:

> Why is it that the higher the yard-arm, the faster the ship travels with the same sail and the same wind?[19]

The Aristotelian answer provided in the *Mechanical Problems* is based on an interpretation of the yard-arm as a lever that has its base at the point where the yard-arm is fixed as the fulcrum. Benedetti maintained that this interpretation of the yard-arm as a lever:

> […] does not give the true explanation. For on this kind of explanation the ship would have to move more slowly rather than more swiftly. For the higher

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[15] Aristotle 1980, 347–355; Drake and Drabkin 1969, 182–183.

[16] Aristotle 1585, 511: "Cur siquidem sursum fuerit spartum, quando deorsum lato pondere, quispiam id admovet, rursum ascendit libra: si autem deorsum constitutum fuerit, non ascendit, sed manet?" Translation in Aristotle 1980, 347–349.

[17] Aristotle 1980, 353–355.

[18] Benedetti 1585, 154; Drake and Drabkin 1969, 183.

[19] Aristotle 1585, 515: "Cur quando antenna sublimior fuerit, iisdem velis, et vento eodem celerius feruntur navigia?" Translation in Aristotle 1980, 361.

> the sail that is struck by the force of the wind, the more the ship's prow will
> be submerged in the water.[20]

Benedetti added one sentence with his own explanation, according to which the ship with a higher sail moves more swiftly because the wind blows more strongly in the higher region.

Chapter 14 provides a long discussion of problem 8 of the Aristotelian *Mechanical Problems*. The question posed in this problem is why round and circular bodies are easiest to move. Three examples are mentioned and later discussed: the wheels of a carriage, the wheels of a pulley, and the potter's wheel. Benedetti claimed that Aristotle's answer to the question he posed is not sufficient. Nevertheless, Benedetti himself argued essentially in a similar manner, only somewhat more extensively. Both of them argued that the circle, contrary to differently shaped bodies, touches a plane only at one point which can be considered as the fulcrum of a lever. But Benedetti added a further argument which is not given by Aristotle. He argued that a circle can be pulled along a plane without difficulty and resistance:

> [...] because in such a case the center will never change its position by moving
> upward from below, i.e., will never change its position with respect to the
> distance or interval which lies beween it and line *AD*.[21]

At the end of the chapter, Benedetti discussed the question of why a potter's wheel set into motion by an external force will continue to rotate for a time but not forever. In his response he took into account the friction with the support of the wheel and with the surrounding air. But he also discussed reasons that are more deeply concerned with the nature of such motion, as we have discussed above. He claimed, in particular, that the rotational motion is not a *natural motion* of the wheel, evidently making reference to the Aristotelian distinction between natural and violent motions. He also claimed that a body moving by itself because an *impetus* has been impressed upon it by an external force has a natural tendency to move along a rectilinear path. This statement seems to comes close to the principle of inertia of classical physics, but it actually deals with rectilinear motion as a forced motion and does not involve any assertion about its uniformity. Benedetti seems to suggest, in any case, that this natural tendency is in conflict with the forced rotational motion of the wheel, which in turn slows it down. The smaller the wheel and the more its parts are constrained to deviate from the rectilinear path, the greater the decrease in speed will be.[22]

In chapters 15 and 16 Benedetti dealt with issues of scale as they are brought up by the Aristotelian *Mechanical Problems*. In chapter 15, consisting merely of one short sentence, Benedetti referred to his own earlier treatment of Aristotle's question of why larger balances are more exact (erroneously citing chapter 10 instead of chapter 11 of his treatise) in order to deal with the ninth problem of the Aristotelian *Mechanical Problems*, which reads:

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[20]Benedetti 1585, 155: "[...] verum non est. Huiusmodi enim ratione navis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsum, tanto magis proram ipsius navis in aquam demergit." Translation in Drake and Drabkin 1969, 183.

[21]Benedetti 1585, 155: "[...] quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiae seu intervalli, quae inter ipsum lineamque *AD* intercedit." Translation in Drake and Drabkin 1969, 184.

[22]For the historical context, see Büttner 2008.

> Why is it that we can move things raised and drawn more easily and more
> quickly by means of greater circles?[23]

In chapter 16 he discussed the tenth problem of the Aristotelian *Mechanical Problems*,
which reads:

> Why is a balance moved more easily when it is without a weight than when
> it has one?[24]

In his detailed response to this problem—indeed much more detailed than the one found
in the Aristotelian text—Benedetti compared balances that are alike with different sets of
weights on their scales, one with two weights of one ounce, the other with two weights of
one pound. He then added a half-ounce weight on one side of each balance and observed
that the balance with the smaller weights moves more rapidly. He explained this effect
by referring to the dynamical assumption that one always has to consider *the ratio of the
moving force to the body moved*.

In chapter 17 Benedetti addressed the twelfth problem of the Aristotelian *Mechanical
Problems*, which reads:

> Why does a missile travel further from the sling than from the hand?[25]

Benedetti's response is based on the concept of *impetus*, conceived as an intrinsic cause
of motion originally acquired by the action of an external force that then gradually de-
creases after separation from the original mover. He argued that a greater impetus can be
impressed by the sling due to the repeated revolutions which evidently lead to an accumu-
lation of this intrinsic force. He observed that the impetus would lead, if not impeded by
the sling or the hand, to a straight motion of the projectile along the tangent to the circle of
its forced motion. He also noted—distancing himself from a claim made by Tartaglia—
that the motion due to the impressed force can mingle with the projectile's natural motion
downward, thus leading to a curved trajectory. It may well be the case that it was this
claim that later convinced Galileo and Del Monte to perform their experiment on projec-
tile motion from which they drew the conclusion that such a mixture of motions indeed
takes place.[26]

In chapter 18 Benedetti considered problem 13 of the Aristotelian *Mechanical Prob-
lems* dealing with the question of why larger handles can be moved more easily around a
spindle than smaller ones.[27] In his short response Benedetti simply referred to the fourth
and fifth chapters of his own treatise, stressing that everything depends on the lever. He
was evidently convinced that the Aristotelian reduction of such problems to properties of
the circle is superfluous, if not misguided.

In chapter 19 he handled problem 14 of the Aristotelian *Mechanical Problems* in the
same way. It reads:

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[23] Aristotle 1585, 517: "Cur ea, quae per maiores circulos tolluntur et trahuntur, facilius et citius moveri
contingit […]?" Translation in Aristotle 1980, 365.

[24] Aristotle 1585, 517: "Cur facilius quando sine pondere est, movetur libra, quam cum pondus habet?"
Translation in Aristotle 1980, 365.

[25] Aristotle 1585, 518: "Cur longius feruntur missilia funda, quam manu missa […]?" Translation in Aristotle
1980, 367.

[26] See the discussion in Renn, Damerow, and Rieger 2001.

[27] Aristotle 1980, 367.

> Why is a piece of wood of equal size more easily broken over the knee, if one
> holds it at equal distance far away from the knee to break it, than if one holds
> it by the knee and quite close to it?[28]

Again, Benedetti just referred to the earlier chapters of his treatise.

In chapter 20 Benedetti reconsidered problem 17 of the Aristotelian *Mechanical Problems*, which reads:

> Why are great weights and bodies of considerable size split by a small wedge,
> and why does it exert great pressure?[29]

In the Aristotelian text, the answer is based on interpreting the wedge as two levers opposite to each other, their fulcra being placed at the entry points of the wedge into the wood. Benedetti, however, disagreed with the identification of the two levers allowing the action of the wedge to be interpreted in terms of force, fulcrum, and resistance. He claimed that the fulcrum is actually placed just underneath the deepest point of the opening produced by the wedge entering a block of wood.

In chapter 21 Benedetti claimed to provide the true explanation of compound pulleys. He reduced a compound pulley to a chain of balances by appropriately identifying forces and fulcra, each wheel of the pulley corresponding to one balance.

In chapter 22 Benedetti discussed Aristotle's wheel, i.e., problem 24 of the Aristotelian *Mechanical Problems*, which reads:

> A difficulty arises as to how it is that a greater circle, when it revolves, traces
> out a path of the same length as a smaller circle, if the two are concentric.[30]

While the author of the *Mechanical Problems* referred to dynamical reasons in explaining this apparent paradox, Benedetti resorted to a kinematic argument, a pointwise reconstruction of the trajectory of the motion of a point on the circumference, arguing that it results from a superposition of two motions. In the case in which the motion is controlled by the larger circle, a point on the circumference of the smaller circle traverses a path resulting from an *addition* of two motions. In the case in which the motion is controlled by the smaller circle, a point on the circumference of the larger circle traverses a path resulting from a *subtraction* of two motions.

Chapter 23 of Benedetti's treatise does not exist.[31] In chapter 24 Benedetti discussed problem 30 of the Aristotelian *Mechanical Problems*, which reads:

> Why is it that when men stand up, they rise by making an acute angle between
> the lower leg and the thigh, and between the trunk and the thigh?[32]

In his response Benedetti suggested that the reason for this behavior is to create an equilibrium of the body with regard to the line that serves as support underfoot.

In chapter 25 Benedetti addressed the last problem, problem 35 of the Aristotelian *Mechanical Problems*, which reads:

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[28] Aristotle 1585, 518: "Cur eiusdem magnitudinis lignum facilius genus frangitur, si quispiam aequi diductis manibus extrema comprehendens fregerit, quam si iuxta genu?" Translation in Aristotle 1980, 369.

[29] Aristotle 1585, 520: "Cur a parvo existente cuneo magna scinduntur pondera, et corporum moles, validaque sit impressio?" Translation in Aristotle 1980, 371.

[30] Aristotle 1585, 525: "Dubitatur quam ob causam maior circulus aequalem minori circulo convolvitur lineam, quando circa idem centrum fuerint positi." Translation in Aristotle 1980, 387.

[31] In Drake and Drabkin 1969, 193; chapter 22 is erroneously numbered as chapter 23.

[32] Aristotle 1585, 532: "Cur surgentes omnes, femori crus ad acutum constituentes angulum, et thoraci similiter femur, surgunt?" Translation in Aristotle 1980, 403–405.

> Why do objects which are travelling in eddying water all finish their move-
> ment in the middle?[33]

Benedetti's answer simply referred to the fact that whirlpools are depressed in their mid-
dle without giving an explanation of this phenomenon. He could thus restrict himself to
arguing that the motion of an object to the center of such a whirlpool is simply its natu-
ral downward motion. The final comment by Benedetti is a remarkable conclusion to his
criticism of Aristotle as well as his treatise on mechanics:

> But in the case of all those other problems that I have omitted, Aristotle's
> explanations are correct.[34]

## 5.2    The Beginning of Benedetti's Mechanics

After our overview of Benedetti's book on mechanics, we concentrate on the theses he
expounded in the first chapters because they have a foundational character and proved
particularly controversial, at least in light of Del Monte's criticism, which we are aware of
from the comments he made in one of his notebooks and from marginal notes in his own
copy of Benedetti's book.

### 5.2.1    *De mechanicis* I: "On the different positions of balance beams"

In chapter 1, Benedetti notes that "a body (*pondus*) […] acquires a larger or smaller weight
(*gravitas*) depending on the different ratio of the beam's position" (*pondus… maiorem, aut
minorem gravitatem habet, pro diversa ratione situs ipsius brachii*). According to him,
a body has the greatest heaviness when the beam at whose extremity it is loaded is in
the horizontal position. His idea is based on a simple common-sense intuition: if one
considers an equal-arms balance suspended at its center, the weight of a loaded body is:

  - borne entirely by the fulcrum when resting vertically upon it,
  - entirely hanging on the fulcrum when suspended vertically below it,
  - not supported in any way by the fulcrum when the beam is in the horizontal position.

In the first case, the body completely rests or leans on the center (*nititur*), and the center
in turn hinders (*impellet*) the downward tendency of the weight. In the second case, the
body is suspended vertically (*pendet*) and the center "attracts" it (*attrahet*), in the sense
that it hinders its natural tendency to fall down (*inclinatio*). Hence, the body attains its
maximum weight in the third case. If the balance beam moves upward, departing from the
horizontal position, the weight slowly decreases and reaches its minimum at the top when
the beam is in the vertical position. If the rotatory motion around the fulcrum continues,
now downward, the weight increases again until it reaches its maximum in the horizontal
position. It then diminishes until it is suspended entirely below the fulcrum. Benedetti
visualizes these variations of weight depending on the position (*situs*) in a diagram com-
paring the lines connecting the weight to the center of the world in different cases, more
specifically if the beam is:

  - horizontal,

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[33]Aristotle 1585, 533: "Cur ea quae in vorticosis feruntur aquis, ad medium tandem aguntur omnia?" Trans-
lation in Aristotle 1980, 409.
[34]Benedetti 1585, 167: "[…] a quo aliarum omnium quaestionum, quas ego omisi rationes sunt bene propo-
sitae." Translation in Drake and Drabkin 1969, 196.

- raised upward, or
- moved downward with the same angle as in the second case (which is equivalent to 2).

The parallel lines, called *lineae inclinationis* or *lineae itineris*, indicate the direction in which a body would fall if it were free. The closer these lines are to the center of the beam, Benedetti says, the "less heavy" the body becomes.

In his own copy of Benedetti's book, Del Monte wrote a brief annotation in the margin of chapter 1: "this first chapter is derived entirely from our treatise on the balance in the *Mechanicorum liber*."[35] Clearly, he sought to assert the relevance of his treatise for Benedetti's speculations, in spite of the latter's claims of originality. It should be remarked, however, that Del Monte's treatment of the balance, based on the concept of center of gravity, was significantly different from Benedetti's, which was based on an original reworking of *positional heaviness*. Del Monte merely reassessed a concept received from authors such as Jordanus Nemorarius, Tartaglia, and Cardano, all of whom he personally opposed. In his book on mechanics, Del Monte had in fact criticized the concept of *positional heaviness*. Downplaying Benedetti's theory as a repetition of his predecessor's theories, he could therefore claim that his own treatment already included a summary (as well as a criticism) of Benedetti's approach.

### 5.2.2 *De mechanicis* II: On the proportion of weights at the extremities of a balance beam in a position other than the horizontal

In chapter 2, Benedetti deals with the proportions of a weight placed at the extremity of a balance beam if its position is not horizontal (*De proportione ponderis extremitatis brachii librae in diverso situ ab orizontalis*). The thesis to be demonstrated is the following: "The proportion between [the weight of] a body (*pondus*) at $C$ and [the weight of] the same body (*pondus*) at $F$ corresponds to that between the whole beam $BC$ and its part $BU$, which is [set on the beam $BC$ and is] delimited by the fulcrum and the [intersection between the beam and the] inclination line $FUM$ that connects the weight at $F$ to the center of the world" (Benedetti 1585, 142). For the sake of simplicity, we will represent these relations symbolically in modern terms:

$$C : F = BC : BU$$

where $C$ is the weight in the horizontal position and $F$ in the inclined position; $BC$ is the beam and $BU$ the part of the beam $BC$ between the center $B$ and the perpendicular line drawn from $F$.

Benedetti's demonstration is as follows. He imagines placing a weight $D$ on the other extremity of the balance that has the same proportion to $C$ as $F$, that is, the following proportion expressed in modern terms:

$$D : C = BU : BC.$$

In accordance with Archimedes's *De ponderibus* I. 6, the balance will be stable if the weight $C$ is loaded at $U$, since weights and distances from the fulcrum are proportional by supposition.

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[35]"Hoc primum caput to[tum] desumptum est a n[ostro] *Mechanicorum libri* tractatu de lib[ra]."

The next step is to show that $F : C = BF : BU$ (where $BF$ is the beam, hence $BF = BC$). In order to demonstrate this, Benedetti resorts to the mental model (*imaginemur*) of a string hanging vertically from $F$, to which a weight equal to $C$ is suspended. He claims that it is visually evident that the weight has the same effect at $F$ as at $U$. The same is valid for the case in which the weight is suspended from $U$ and intersects the circumference described by the rotation of the beam at a point $E$. In both cases, the balance would remain horizontal since the weight $C$ at $F$, $U$, or $E$ would balance the weight at $D$. Benedetti further argues that the balance under consideration can be treated like a bent lever with a horizontal and an inclined arm (*FBD* or *EBD*): "si brachium BE consolidatum fuisset […]" (If the beam *BE* was made solid […]).

The author concluded that his reasoning has satisfactorily demonstrated his thesis: "A body (*pondus*) is more or less heavy (*grave*) the more or less it hangs from (*pendet*) or rests on (*nititur*) the fulcrum" (Benedetti 1585, 142). And he deems this resting on or hanging from the fulcrum to be the most direct cause (*haec est causa proxima, et per se*) of the positional changing of a weight.

As an additional commentary, Benedetti remarks that in his diagram he supposes the inclination line $CO$ to be perpendicular to $CB$ and parallel to $BQ$, whereas $CO$ and $BQ$ in fact converge at the center of the sphere of the elements (*centrum regionis elementaris*), that is, the earth. But for the sake of his present argumentation, this angle is negligible and one may simply assume perpendicularity and parallelism. Benedetti thus developed a method to quantify positional heaviness that corresponds to the modern concept of "torque."

## 5.3  Del Monte's Criticism Concerning the Non-Negligibility of the World's Center

As will be shown in the following section, it was only in his initial treatment of the inclined balance, in chapter 1 of *De mechanicis*, that Benedetti neglected to consider the convergence of the inclination lines to the center of the elements. This omission gave rise to criticism. Del Monte severely criticized both this assumption and Benedetti's reasoning in general in *De mechanicis*, in his handwritten notes on scientific and technical matters known as *Meditatiunculae de rebus mathematicis*. In his notes he assessed Benedetti's arguments from his perspective, relying on the concept of the center of gravity as it was developed in his own book on mechanics.

In a marginal note to the *Diversae speculationes* (Figure 5.2), Del Monte expressed his disagreement with Benedetti's conclusion: "Thus, in this manner, a weight (*pondus*) more or less hangs from or rests on the center; this is the next cause and the [cause] in itself [of the variation in heaviness]."[36] His disagreement reads as follows:

> because that [that is, the greater or smaller extent to which a weight rests at
> the center] is neither the next [cause] nor the [cause] in itself. For the weight
> at $F$ of the arm $BF$ is not equally heavy as the weight $U$ of the arm $BU$; nor
> is the weight at $E$ of the arm $BE$ equally heavy as the weight at $U$ of the arm
> $BU$. Thus, this entire demonstration is false.[37]

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[36]"[…] unde fit ut hoc modo pondus magis aut minus a centro pendet aut eidem nititur: atque haec est cause proxima, et per se […]."

[37]See Renn and Damerow 2012, 207: "non est neque proxima neque per se; nam [pond]us in *F* brachii [*BF*] non est equegrave ut pondus in *U* brachii *BU*; [nec] pondus in *E* brachii *BE* est equegrave ut pondus [in] *U* brachii *BU*. Unde tota haec demonstratio falsa est."

Figure 5.2: Del Monte's marginal note to *De mechanicis*, II. (Max Planck Institute for the History
of Science, Library)

This means that Del Monte did not accept the claim that a weight is equally heavy in different positions on the balance beam, provided the projections of the beam along the horizontal are the same length or rather, as Benedetti writes, the distances between the projections of the beam on the horizontal and the center have the same lengths.

To find Del Monte's counter-arguments, one must look to the *Meditatiunculae*, f. 145, *Contra Cap. 2 Jo. de Benedicti de Mechanicis*. As mentioned, he basically rejected Benedetti's perspective by objecting that he did not take into due account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other, as Benedetti assumed in this part of his treatise.

In his diagram (Figure 5.3) Del Monte compared the line *LUS* (parallel to the line *AQ*, connecting the fulcrum *B* of the balance with the center of the world *M*) with the line *FM* (connecting the upper weight *F* and the lower weight *E* with the center of the world *M*). *S* is the point at which the line *LUS* meets the circle that the beam makes around the fulcrum, which is above the position of the lower weight *E*.



Figure 5.3: Del Monte's critical reworking of Benedetti's diagram in *Meditatiunculae*, f. 145.
(Bibliothèque Nationale de France)

Next, he considered a bent lever made of the oblique arm *BS*, rigidly connected to the straight arm *BD*, assuming that *BU* is half *BD*. If a weight is now placed at *S* that is double the weight at *D*, the bent lever will be in equilibrium, as Del Monte showed with reference to his book, because the center of gravity of the weights at *S* and at *D* will be at the point *R*, which will be in its lowest place on the vertical line *BQ*. He therefore concluded that it is the weight at *S*, but not the lower weight *E*, that will be equally heavy as the weight at *U*.

He proceeded to demonstrate this in greater detail by considering the proportions into which the line connecting the two weights is cut by the perpendicular *BQ* for the two cases, that is, the weight placed at *S* and the weight placed at *E*. Del Monte concluded that the same weight is heavier at *S* than at *E*. He then turned to a closer consideration of the upper weight *F*. Again he constructed a bent lever *LBD* in equilibrium in order to compare it with the bent lever formed with the upper weight *F*. Again he showed that the weight is heavier at *L* than at *F*.

Del Monte concluded by summarizing that the entire fallacy is due to Benedetti assumption that the weight at *F* would gravitate in the same way as at *U*, which would only be the case, according to Del Monte, if it were to hang freely.

## 5.4   Benedetti on Weights and Forces Acting on a Balance

Chapter 3 of Benedetti's *De Mechanicis* contains a generalization of the results of chapter 2 or, rather, presents a general rule concerning the action of forces (*virtutes*) on balance beams, including in the case that they do not act vertically downward but also with an acute or obtuse angle. Benedetti moves forward from the result of the previous chapter as follows: the length of the line perpendicularly connecting the center to the line of inclination (the line *BU* in the diagram) allows the quantity of the positional force (*quantitas virtutis… in… situ*) of a weight (*F* in the diagram) to be established. Thus, Benedetti calls the positional weight a force, and this is the presupposition that allows him to generalize from *gravitas* the action which he calls *virtutes moventes*, or "moving forces." The thesis of this chapter is summarized in its title: "That the quantity of any given weight (*pondus*) or moving force in relation to another quantity can be determined thanks to the perpendicular projections connecting the center of the balance to the line of inclination."

Benedetti draws two diagrams showing a balance at whose extremities two weights or forces act in different directions (Figure 5.4). At the left extremity *B*, a weight *E* has a downward tendency, while at the right extremity, a weight *C* acts making an acute or an obtuse angle. According to Benedetti, the length of the perpendicular projection drawn from the center to the inclination line, *OT*, permits the determination of the distance *OI* on the beam at which the same force acting vertically downward produces the same effect. Given this equation, Benedetti can determine how much the force acting in a non-perpendicular direction has to be augmented in order to balance an equal weight acting perpendicularly on the opposite beam. This measure is given according to the following proportion (expressed in modern terms):

$$E : C = BO : OI$$

where *E* is the weight acting vertically on the extremity *B*; *C* is the *virtus movens* acting on the opposite extremity *A* at an angle; *BO* is the left beam and *OI* the part of the right beam *OA* determined as explained above.

In his argumentation, Benedetti thus equates a balance (*BOI*) with a bent lever (*BOT*). Accepting this equation, he concluded that, according to commonly shared knowledge (*communi quadam scientia*), the weights or forces that are required to obtain a perfect balance can easily be calculated.

The chapter ends with a cosmological corollary: "The closer the center *O* of the balance is to the center of the elementary sphere, the less heavy (*minus grave*) it becomes." In fact, the angles between the beam and the inclination lines become progressively smaller.
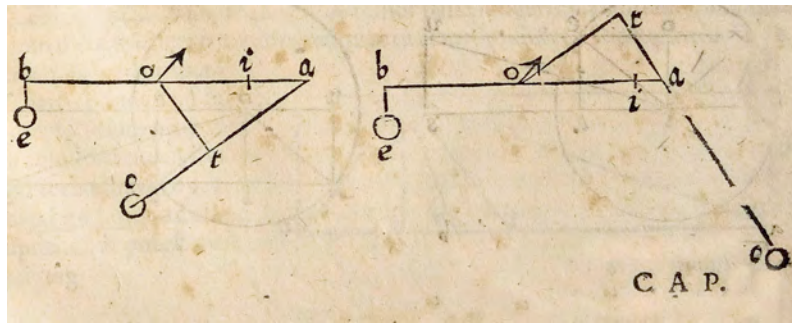


Figure 5.4: Benedetti's representation of forces acting on a balance in arbitrary directions. (Max Planck Institute for the History of Science, Library)

## 5.5   Del Monte's Misunderstanding

In his notes on folio 146 of the *Meditatiunculae*, Del Monte grappled with Benedetti's instructions on how to determine positional heaviness in the case of forces acting in an arbitrary direction. These he refuted at length under the erroneous assumption that Benedetti had claimed forces can be indiscriminately replaced by weights. Like Benedetti, Del Monte considered a bent lever *BOAC* with fulcrum *O*, weights *E* and *C*, a straight arm *BO*, and a bent arm *OAC* to discuss the two cases of an acute and an obtuse angle *BAC* (Figure 5.5).
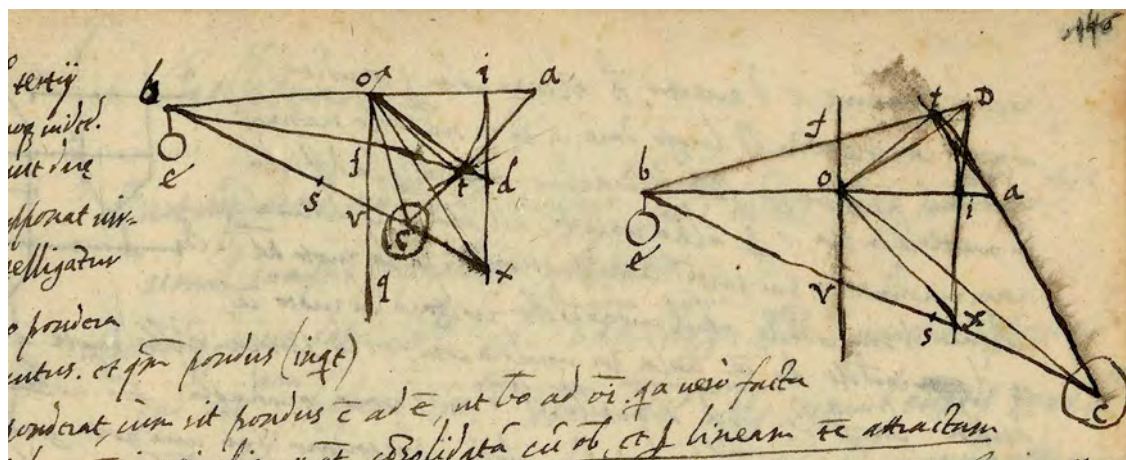


Figure 5.5: Del Monte's critical reworking of Benedetti's representation of forces acting on a balance in arbitrary directions. (Bibliothèque Nationale de France)

He first recapitulated Benedetti's procedure, assuming that a vertical line *OT* drawn from the fulcrum to the line *AC* represented the oblique arm of the bent lever. He stated that when the weight *C* is placed at the end of the horizontal line *OI*, whose length is the same as that of the perpendicular *OT*, according to Benedetti it will be in equilibrium with the weight *E* if the weight *C* is to the weight *E* as is *BO* to *OT* or *OI*. Del Monte then summarized Benedetti's claim that when a force represented by the weight *C* acts along the line *TC*, the bent lever formed by the straight arm *BO* and the oblique arm *OTC* will also be in equilibrium, which he doubted.

Del Monte reformulated this claim by stating that the same weight *C* will be in equilibrium with the weight *E* whether it is placed on the straight balance *BOI* or on the broken bent lever *BOTC*. He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and as a result arrived at absurd conclusions.

Del Monte then showed that the same weight will be heavier on the horizontal at point *I* than along the bent lever at *T*, demonstrating that the bent lever *TOB* will not be in equilibrium if the straight lever *BOI* is in equilibrium. To show this, Del Monte again proceeded by finding the center of gravity of the weights *E* and *C* placed at *T*. More precisely, Del Monte determined a position for the weight *C* where the bent lever is in equilibrium, a position, however, that is distinct from *T*. Thus it follows that *T* cannot be the position of equilibrium. For this purpose, he extended the line *BT* to *D*, just beneath *I*, so that it is immediately evident that if the weight *C* is placed at *D*, the center of gravity of the two weights will be just beneath the fulcrum.

Using the same pattern, he continued by showing that the bent lever *BOC* cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum. Finally, he applied this argument to the broken bent lever *BOTC*. Del Monte next addressed the case in which the bent lever is characterized by an obtuse angle *BAC*, showing that the weight at *T* is lighter than the weight at *I*. In his concluding remarks, however, he began to waver. Once again, he stated that Benedetti is completely mistaken when applying his procedure to weights. But he did admit that this may be true when dealing with a force.

As an afterthought, Del Monte once again criticized Benedetti's appeal to common sense: he did not feel this to be worthy of an expert mathematician. And as a second afterthought, he constructed an extreme case in which it is immediately clear that the broken bent lever cannot be in equilibrium if weights are attached to it rather than forces.

The following considerations enable Del Monte's marginal annotations to Benedetti's *De Mechanicis* III to be understood. These are not perfectly legible, but nonetheless their meaning becomes clear in light of the *Meditatiunculae*:

> If we understand that a weight is at *C*, as we can assume from his own words, then *CT* must also be understood as being solid [and connected with] the solid lines *TO* […] If we hence understand that *C* is a weight and not moving, [the proposition] is false. If it is understood that *C* moves as […] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing […] therefore as is evident in chapter 7. All demonstrations of the author are founded on these two chapters inasmuch as they are the first fundaments of mechanics; once their falsity is recognized, everything is rejected.[38]

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[38]See Renn and Damerow 2012, 213: "si intelligamus p[ondus] in *C*, ut supponi p[otest] ex verbis ipsius, intelligendum est *C*[*T*] quoque consolidatam consolidatis *TO* […]. Unde si intelligamus *C* pondus et non

## 5.6    Diverging Approaches to Tartaglia

Del Monte's and Benedetti's criticisms of Tartaglia's conception of positional heaviness help us to understand where these two scholars converge and diverge on the issue of the equilibrium (or lack of equilibrium) of a balance deflected from its horizontal position, and also the reasons for the presumed equilibrium or tendency to restore it. Moreover, their arguments reveal a different attitude toward the medieval tradition of the *scientia de ponderibus* and the *gravitas secundum situm*.

### 5.6.1    The Tradition of Nemorarius, Tartaglia, and Cardano

The concept of *gravitas secundum situm*, or positional heaviness, was extensively employed in Jordanus Nemorarius's *Liber de ponderibus*. Del Monte owned and annotated a sixteenth-century Nuremberg edition of the book, commented, and illustrated by Petrus Apianus. Del Monte's handwritten annotations document his general disagreement with the approach of this medieval scholar, who did not know the Archimedean concept of the center of gravity and therefore tried to develop a deductive science of weights relying solely on the Aristotelian theory of motion and its development in the Arabic tradition of the science of weights. We have already hinted at the Aristotelian framework underlying the concept of *gravitas secundum situm*. In his book, Jordanus stated that a deflected balance would return to the horizontal position (his second proposition) (Nemore 1565, B2 *r*). According to Jordanus, the upper weight acquires more positional heaviness than the lower one due to the fact that its descent is less oblique. In fact, he postulated that positional heaviness depends on the obliqueness of descent of a weight (his fourth postulate) and that "a more oblique descent partakes less of the straight [descent] for the same quantity [of path]" (fifth postulate) (Nemore 1533, A4 *r*). The determination and possibly the quantification of obliqueness was therefore essential to establish the behavior of a deflected balance.

In the sixteenth century, Tartaglia in *Quesiti, et inventioni diverse* (1546), and Cardano in Book 1 of *De subtilitate* (first edition, 1550) and in *Opus novum de proportionibus* (1570), expounded their own versions for determining descent and reinforced Jordanus's second proposition (that the deflected balance returns to the horizontal position). A brief account of three ways to determine positional heaviness is given in the following pages. The first two are derived from Tartaglia and the last from Cardano.

*Descent*: A first method of dealing with positional heaviness consisted in comparing the lengths of the projections of the equal arcs described by the motion of opposite balance beams—one ascending and one descending—on the vertical line of descent to the center of the world.

As Tartaglia's diagram in Figure 5.6 shows, the vertical component of descent of the upper weight is always larger than that of the lower. Thus, the former acquires more heaviness (*secundum situm*) than the latter and the balance returns to the horizontal position.

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movens, falsa est i[ta]que si intelligatur *C* movens ut homi[…] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i…] ipse [vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil […] ut patet in 7 cap. In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur]."
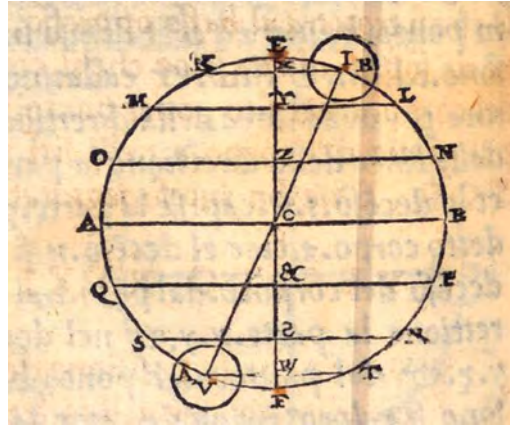
Figure 5.6: According to Tartaglia, the body at *I* is positionally heavier than the body at *V*, since
the projection of the arc *IL* on the vertical *XY* is greater than the projection of *VF*,
*WF*. (Max Planck Institute for the History of Science, Library)

*Angle of contact*: Tartaglia's second method of determining positional heaviness consists
in comparing the angles between the circular path of the beams and the perpendicular lines
connecting the weights to the center of the elements (as already mentioned in chapter 4).
These angles "of contact" are also called "curvilinear angles" or "mixed angles" since they
result from the intersection of a straight line downward and a curved line, that of the circle
circumscribing the balance (Figure 5.7).
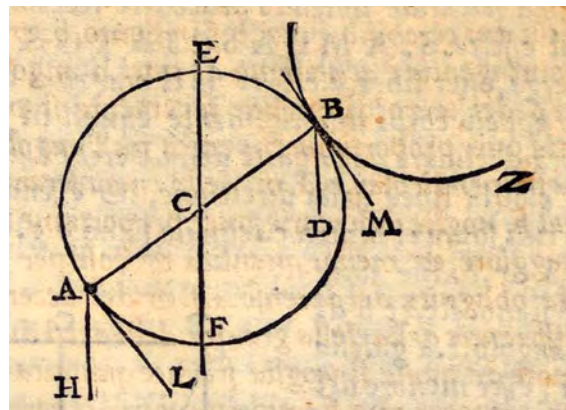


Figure 5.7: Tartaglia's representation of the angle of contact for the determination of positional
heaviness. (Max Planck Institute for the History of Science, Library)

By comparing the angles of contact of the two weights, Tartaglia could establish that the
higher angle is always smaller than the lower; therefore, the higher weight has a straighter
descent and is positionally heavier. The inclined balance would therefore return to the hor-
izontal position. It should be noted that Tartaglia perceived the comparison of curvilinear
angles as problematic. He considered the ratio of two such angles to be less than any ratio
between determined quantities. As a consequence, no weight placed on the positionally
lighter side of the deflected balance could compensate for the other weight and keep the
balance inclined. On the contrary, any additional weight—no matter how small—would
have produced an opposite displacement of the balance beam toward the vertical.

*The angle between the support and the beams*: We have so far considered two ways of determining positional heaviness on the basis of Tartaglia's *Quesiti*. Assuming that positional heaviness depends on the obliquity and straightness of descent, positional heaviness can be determined either from the projections of the descents on the vertical, or the curvilinear angles that are produced by the intersection of the descent arcs and the lines connecting the weights to the center of gravity. Cardano considered three criteria for establishing positional heaviness which he mistakenly regarded as equivalent: first, the distance of the beam from the vertical; second, its distance from the horizontal; and third, an angle that he called *meta*. This was the angle between the support of the balance and the beam. Commenting on the diagram that is reproduced here as Figure 5.8, he explained:

> Aristotle says that this happens when the support is above the balance, because the angle *QBF* of the *meta* is larger than the angle *QBR*. And similarly, when the support is *QB*, the *meta* will be *AB*, and thus the *RBA* will be larger than the angle *FBA*, but the larger angle will render the weight heavier. […] The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.[39]



Figure 5.8: According to Cardano, there are three ways to determine positional heaviness. The positional heaviness at point *F*, for instance, may be determined by the horizontal *FP*, by the vertical *FL*, or by the angle *QBF*. (Max Planck Institute for the History of Science, Library)

Given these premises, Cardano contended that a weight will reach its maximum positional heaviness in the horizontal position. He therefore shared Nemorarius's and Tartaglia's opinion about the return of an inclined balance to the horizontal position.

### 5.6.2 Del Monte's Critical Remarks on Positional Heaviness

Del Monte's criticism of Benedetti, in the *Meditatiunculae* as well as in the marginal remarks of his copy of *Diversae speculationes*, are closely related to his criticism of Nemorarius, Cardano, and Tartaglia in his *Mechanicorum liber* (1577). Here he dealt

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[39]Cardano 1550, 17–18: "Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus *QBF* metae, maior est angulo *QBR*. Et similiter quum trutina fuerit *QB*, erit meta *AB*, et tunc angulus *RBA*, maior erit angulo *FBA*, sed maior angulus reddit gravius pondus. […] Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora."

extensively with the balance and provided a detailed discussion of the theories of these scholars which he judged to be irremediable. These theories supported the idea that an inclined balance returns to the horizontal and were thus at odds with his own treatment of the matter, which he based on the Archimedean concept of center of gravity. Del Monte believed that an ideal balance would remain in any position as long as it had equal arms, was hinged on its fulcrum and was loaded with equal weights. The only difficulty in testing this theory, he asserted, was the technical difficulty in constructing a perfect balance. It should be noted, moreover, that he assumed that a center of gravity meeting the requirement of his (and Pappus's) definition of the center of gravity always exists:

> The center of gravity is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotation by that motion.[40]

Apart from the conceptual irreconcilability between his own approach and that of the Nemorarius school, Del Monte tried to demonstrate the inconsistencies of positional heaviness also within the conceptual framework of his adversaries. One of his main objections was based on a consideration of the cosmological context, which he considered relevant to correctly treat the inclined balance, at least with regard to positional heaviness. Of course, this aspect indeed matters when considering Tartaglia's remark that the difference in positional heaviness is infinitesimally small and cannot be compensated by any finite weight resulting from the infinitesimal difference between curvilinear angles.

Contrary to the assumptions of Nemorarius and his successors, Del Monte noted that the downward tendencies of the weights are not parallel but converge at the center of the world. Since the directions toward the center of the world from different points on the circular path of the end of the beam cannot be parallel, they are inappropriate for representing positional heaviness. From the fact that those lines converge, he argued further that the lower weight should actually become positionally heavier than the higher one. His idea is clearly illustrated by the diagram in Figure 5.9.

Del Monte objected that, from the point of view of positional heaviness, it is not in the horizontal position that a body weighs the most but at that point where a straight line drawn from the center of the world touches the circle described by the balance arm. Certainly, if the center of the world were infinitely distant and all lines of direction converging at it were perpendicular and parallel to each other, then the extreme point would mark the horizontal position of the balance arm. Still, for a finite distance from the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum. Del Monte even demonstrated that the closer the balance is to the center of the world, the further this "extreme point" (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum).

Del Monte's crucial objection to the Nemorarius school was that one should not consider both weights separately, but rather in terms of their connection by the balance beam. He drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. With regard to their positional heaviness the two weights are then equal. Thus Del Monte could claim, using the premises of his adversaries, that the deflected balance does not return to the horizontal.

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[40]Del Monte 1577, 1*r*: "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latione circumvertitur." Translation in Drake and Drabkin 1969, 259, revised in Damerow and Renn 2010, 57.

Figure 5.9: According to Del Monte, if *S* represents the center of the world, then the mixed angle *SEG* between the circular path of the weight at *E* and the direction to the center of the world is less than the mixed angle *SDG*. Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at *E* must be heavier than that at *D*. (Max Planck Institute for the History of Science, Library)
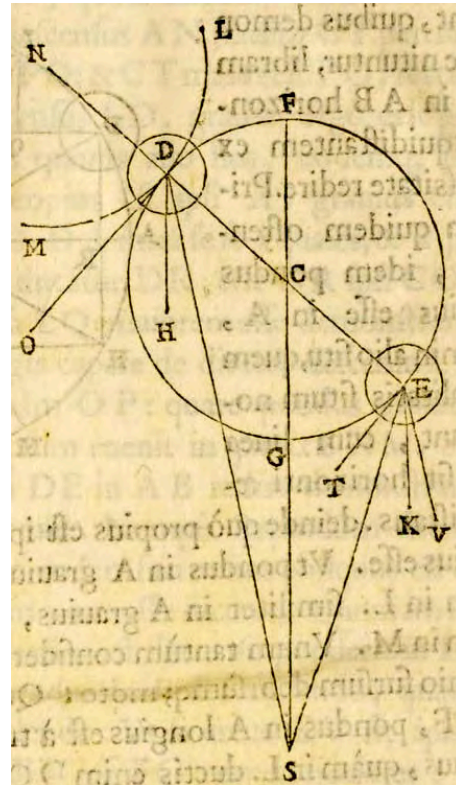
### 5.6.3 Benedetti on Tartaglia's and Nemorarius's Shortcomings

Benedetti addressed the ideas of Tartaglia and Nemorarius on positional heaviness in section seven of his *De mechanicis*. There, Benedetti stressed that his approach to positional heaviness, focusing on the distance from the fulcrum to the line of inclination, was distinct from and superior to Tartaglia's approach in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central thesis that when a balance is moved from its horizontal position, it will return to this position because the body that has moved upward will attain greater positional heaviness than the body which has moved downward. As we have seen above, Jordanus's and Tartaglia's arguments were based on a comparison of the descents of the two weights. In other words, the balance would have to break in the middle to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by Del Monte, that when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and positioned in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he came to the conclusion that when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

> Therefore the weight of *A* in this [lower] position will be heavier than the weight of *B*.[41]

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[41] Benedetti 1585, 148: "Pondus igitur ipsius *A* in huiusmodi situ, pondere ipsius *B* gravius erit." Translation in Drake and Drabkin 1969, 176.

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line corresponding to the weight that had been lowered is longer than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally, so that one would expect the balance to tilt into a vertical position.

Benedetti added some more critical remarks on Tartaglia's consideration of positional heaviness. As we have seen, Tartaglia had argued in *Quesiti* that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight. This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument, Benedetti again emphasized that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just as Del Monte had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even such a small deviation from the parallel must be relevant. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as being parallel to each other.[42]

In summary, Benedetti introduced a way of determining the positional effect of a weight or a force that, in the cases he considered, essentially produces the same results as the application of the modern concept of torque. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

## 5.7    The Triangulation Benedetti-Del Monte-Galileo

In this chapter, we have dealt with Del Monte's and Benedetti's different approaches to mechanics emerging from their reflection on the balance and their treatment of earlier authors. Relative to the issue of positional heaviness, Del Monte's self-positioning was essentially external whereas Benedetti positioned himself (albeit critically) within the tradition of the Nemorarius school. He explicitly mentioned Tartaglia and Cardano as relevant sources for his treatment, whereas he omitted any mention of Del Monte.[43] In spite of their opposite intentions and mutual suspicion, Benedetti and Del Monte shared several opinions and sometimes reached the same conclusions, albeit following different paths: both considered the cosmological center of gravity relevant for an evaluation (and criticism) of Tartaglia's concept of positional heaviness, and both remarked that one cannot treat the two balance beams separately, but rather emphasized that they must be considered simultaneously. Moreover, both stressed the ambiguity of the concept of a mixed angle

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[42]Benedetti 1585, 150: "Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit." Translation in Drake and Drabkin 1969, 177.
[43]Benedetti 1585, f. A3*r*.

and the difficulty of its determination. Nevertheless, their approaches were quite different. As mentioned, Benedetti still worked within the framework of the *gravitas secundum situm*, while Del Monte renounced it in favor of the concept of *centrum gravitatis*. For Del Monte, the displacement of the balance toward the vertical position was an absurdity that revealed the untenability of Tartaglia's premises. Benedetti deemed this vertical tilt to be the consequence of a correct analysis of the balance based on a concept close to the modern idea of torque, in consideration of the cosmological context. Furthermore, one should stress the importance of Benedetti's attempt to determine the quantity of positional heaviness, a fact that distinguishes him from his predecessors. Additionally, unlike Del Monte, he treated the balance by also taking into consideration the general case of forces acting arbitrarily on the beams.

In conclusion, it may be useful to recall the problems linked to the triangulation Benedetti-Del Monte-Galileo, on which the equilibrium controversy sheds new light. The remarkable proximity of these authors on several issues is well known in the history of mechanics. Nevertheless, recent accounts tend to neglect or even deny a possible influence of Benedetti on Galileo.[44] By contrast, the influence of Benedetti on Galileo was assumed and underscored by earlier scholars like Raffaello Caverni, Pierre Duhem, Emil Wohlwill, and Ernst Mach.[45] It is helpful to mention the most important issues common to these authors: the attempt at a theory of motion based on Archimedean hydrostatics, the treatment of the acceleration of fall and its causes, the formulation of what in hindsight appear as proto-inertial principles, a similar treatment of the bent lever, the analysis of the relation between vibrating strings and musical tones, their views on the irradiation of surfaces and on thermal and hydrostatic phenomena, and, last but not least, their support of the Copernican world system.[46] Although many of these themes and ideas belonged to the shared knowledge of preclassical mechanics, in some respects the agreement of their approaches is so striking that one may suspect that this is not mere coincidence.[47] Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*. In any case, the strongest evidence of Galileo's acquaintance with Benedetti's insights is provided by Del Monte's *Meditatiunculae*.

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[44] See the discussion by Ventrice in Bordiga 1985, 732–736. He mentions Drake, Drabkin, Fredette, and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. Notable exceptions are the commentaries by Carugo and Geymonat in their edition of Galileo's *Discorsi*, see Carugo and Geymonat 1958. Bertoloni Meli even considers the possibility of Del Monte and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on Galileo's thinking because that influence supposedly would have arrived too late, see Bertoloni Meli 2006, 61–65.

[45] Cozzi and Sosio 1996. For an overview of such potential connections, see the discussion in Bordiga 1985, 732–736 who also mentions Mersenne, Clavius, and Cardinal Michelangelo Ricci as possible intermediaries.

[46] For an overview, see Bordiga 1985.

[47] See, for instance, Drake and Drabkin 1969, 36. Yet, the question of Benedetti's direct impact on Galileo remains unclear, in particular as Benedetti's work was never mentioned by Galileo.

There are several possible connections between Benedetti and Galileo that have been considered in the past. For instance, Benedetti is referred to by Galileo's Pisan colleague Jacopo Mazzoni in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597. See Mazzoni 1597. He is often mentioned in the Galileo Studies as the addressee of a famous letter by Galileo arguing for the Copernican system (May 30, 1597). See Galilei 1968, vol. 2, 194–202. In his book Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous. See Benedetti 1585, 183–184. For a historical discussion of the context of this argument in contemporary technology, see Freudenthal 2005, a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also refers explicitly to Copernicus. See Mazzoni 1597, 193 and Galilei 1960, 326. It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni, and Del Monte during Del Monte's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.

An important clue is page 145*bis* of the *Meditatiunculae* (Figure 5.10), which is the page opposite the one containing the detailed criticism of Benedetti dealt with in this chapter. This page shows Galileo's construction of the inclined plane, reducing it to a bent lever.



Figure 5.10: Del Monte, *Meditatiunculae*, p. 145*bis* showing Galileo's construction relating the bent lever to the inclined plane. (Bibliothèque Nationale de France)



Figure 5.11: Del Monte's construction related to the inclined plane on p. 64 of his notebook. The construction was adapted from Pappus's erroneous solution. (Bibliothèque Nationale de France)

This fact is all the more noteworthy since Del Monte's notebook, on an earlier page, also contains his own problematic adoption of Pappus's analysis of the inclined plane (Figure 5.11).[48] In his writings, Galileo had criticized this analysis, substituting it with his own solution of the problem, which makes use of the bent lever conceptualized in the same way as Benedetti had done.[49] Del Monte therefore must have learned about this proof from Galileo, and he must also have seen the connection to Benedetti's methods. In any case, it is likely that the two scientists discussed this connection and quite plausible that Galileo became familiar with Benedetti's work through Del Monte. Galileo began to correspond

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[48] Del Monte 1587, 64.
[49] Galilei 1960, 172.

with Del Monte in 1588, three years after the publication of Benedetti's *Diversae speculationes* and shortly before he embarked on the writings that later became known as *De Motu*.[50] Galileo first wrote a dialogue version of *De Motu* and then an essay in twenty-three chapters. Only the second essay version of these writings contains his proof of the law of the inclined plane, the argument about continuity of motion along a straight line, and a mention of Copernicus. This version was most likely written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time discussed explicitly the problem of the effective lever arm, was written much later, certainly after he had visited Del Monte in 1592 during his journey to Padua. Hence, it seems most likely that Galileo was already familiar with Benedetti's key ideas at the time of writing these works.

Recent research into Del Monte's biography has shown that Del Monte and Galileo must have met as early as 1589 in Tuscany.[51] They might even have met jointly with Galileo's teacher, Mazzoni, who, as mentioned earlier, cited Benedetti in his work. Thus, Del Monte, Mazzoni, and Galileo may have discussed Benedetti's *Diversae speculationes*, leading Galileo to reconsider his work in progress on motion and, in particular, his treatment of motion along inclined planes, making use of Benedetti's theory of the bent lever that was mentioned in Del Monte's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may have started taking the Copernican hypothesis much more seriously after his encounter with Benedetti's work, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597, Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they had meanwhile reached an agreement, and also tried to press him on the Copernican hypothesis.

In particular, Galileo's concept of *momento*[52] and his analysis of the bent lever—crucial to both his mechanics and his theory of motion—evidently emerged from the midst of the controversy about positional heaviness. In that debate, Galileo took a position much closer to Benedetti than to Del Monte. Rather than *gravitas secundum situm*, Galileo used the concept of *momento* or *momentum* that Del Monte had introduced in his book by quoting Commandino's definition of the center of gravity. But while Del Monte made no further use of this in his mechanics, Galileo took this concept from the respected Urbino school, gave it a new meaning that was taken from Benedetti, and made it a pillar of his own conception, which included Commandino's definition of the center of gravity:

> Center of gravity is defined as that point in every heavy body around which
> parts of equal moments are arranged.[53]

The evidence for this claim concerning Benedetti's legacy in Galileo's work derives from the marginal notes Del Monte made in his copy of Benedetti's book, as well as from his entries in the *Meditatiunculae* which contain traces of Galileo's intervention in this controversy.[54]

According to Benedetti and Galileo (and contrary to Tartaglia and Del Monte), the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum

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[50]Galilei 1960. For a thorough discussion of the chronology of these writings, see Giusti 1998.

[51]Menchetti 2012.

[52]See the extensive discussion in Galluzzi 1979.

[53]Galilei 1968, vol. 2, 159: "Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti." Translation in Galilei 1960, 151. See also Galilei 2002.

[54]Del Monte 1587.

of the balance to the line of inclination, determines the effectiveness of a weight or a mechanical constellation. In his *Mechanics*, Galileo later stressed how important it is to carefully define the effective distances of weights from their support:

> There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.[55]

In his analysis of the inclined plane using the bent lever, Galileo also made clear that this procedure is critical for determining the *momento* of a given weight.[56] As discussed earlier, in his *Diversarum speculationum [...] liber*, Benedetti convincingly demonstrated the efficacy of this method for determining the magnitude of a force or weight according to its position.

In conclusion, the very existence of Del Monte's annotations on his copy of Benedetti's *Diversae speculationes* provides a definitive answer to the question of whether Del Monte had read this book or not.[57] It is also difficult to imagine that he did not discuss his views on Benedetti's mechanics with Galileo, views that he considered both misguided and profoundly challenging, as is made evident in his handwritten notes. It was most probably Del Monte, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo. At the same time, he also made it virtually impossible for Galileo to openly admit to Benedetti's influence if he did not also want to jeopardize the protection of the most important patron of his early career.

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[55] Galilei 1968, vol. 2, 164: "Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi." Translation in Galilei 1960, 156–157.

[56] See Galilei 1968, vol. 2, 181. Translation in Galilei 1960, 173.

[57] The knowledge that he had read it, however, is not entirely new. See Renn, Damerow, and Rieger 2001, 74.

# Chapter 6
# Astronomy

## 6.1 Benedetti as an Astronomer

Benedetti's astronomical considerations are not systematic. They are scattered throughout the volume in different sections. In spite of the difficulty of ordering them and obtaining an overview, they were very much appreciated among his contemporaries. Apart from Kepler's eulogy of Benedetti's ingenuity, the broad European success of the astronomical parts of this work is documented in other references. A few years after the publication of the *Diversae spaeculationes*, Brahe must have had a copy of it in Denmark, as he quoted it extensively and accurately on two occasions. In his correspondence with Landgrave William IV and the Hesse-Kassel court mathematician Christopher Rothmann, he referred to Benedetti's observation of the light of Venus reflected on the part of the lunar disc not presently enlightened by the sun:

> In fact, I sometimes saw that Venus illuminated in a rather sensible manner that part of the Moon that was most distant and opposed to the Sun, although the Moon is by far more distant from Venus's circuit than the comet. I remarked that the Venice patrician Giovanni Battista Benedetti, the most excellent philosopher and mathematician, noted something similar in that erudite work which he wrote on mathematical and physical speculations. At the end of an epistle to a certain Savoy baron, Filiberto, he says: "[…] that the part of the Moon which is deprived of the Sun's light is sometimes partially illuminated by Venus's light. I observed this often and showed it to many people."[1]

Brahe quotes this passage correctly from Benedetti's letter to Baron Emanuele Filiberto Pingone "*De Luce, Lumine, et Colore, De obiectu oculi, De lumine Lunae, et Rubedine nubium*" (On light, lumen, and color; on the eye's object, on the lunar lumen, and the redness of the clouds).[2]

A second long direct quotation of Benedetti can be found in Brahe's book on the nova of 1572, which was part of the *Astronomiae Instauratae Progymnasmata*, posthumously published in Prague in 1602.[3] The Danish astronomer here praised Benedetti as a "philosophus et mathematicus inprimis excellentem," and his work as "praeclarum Opus." He entirely reproduced Benedetti's letter and diagrams on the star in Cassiopeia.[4] This letter

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[1]Brahe 1919, 172: "Veneris enim Stella, visa est mihi aliquando eam partem Lunae, quae a Sole aversa erat, et ipsi obiecta, satis sensibiliter illuminare, utut Luna longe remotius a Veneris circuitus distiterit, quam Cometa. Simile quid Ioannem Baptistam Benedictum, Patricium Venetum Philosophum et Mathematicum inprimis excellentem, animadvertisse reperio, in erudito illo Opere, quod de Mathematicis et Physicis speculationibus inscripsit. Sic enim in fine Epistolae, ad Baronem quendam Sabaudarum Philibertum scribens, ait: '[…] quod pars Lunae lumine Solis destituta, a lumine Veneris aliquantulum illustratur, quod ego saepe vidi, et multis ostendi.'"
[2]Benedetti 1585, 256–257.
[3]Brahe 1916, 251–253.
[4]Benedetti 1585, 371–374.

was directed against Annibale Raimondo—an author whom Brahe also criticized—and demonstrated that the nova appeared above the sublunary sphere. Brahe commented:

> Here follows the epistle which I referred to. It is taken from the aforementioned book by [Giovanni] Battista Benedetti alongside the demonstrative diagrams offered by the same author. Afterwards I will consider others, who discussed that star [i.e., the nova of 1572] in an extraordinarily incompetent manner. This [quotation from Benedetti] (as mentioned) will cast light on these issues through a synthetic and wise geometrical truth, so that no significant doubt will survive.[5]

Another reader of the *Diversae speculationes* was the English scholar of magnetism William Gilbert. In *De mundo nostro sublunari philosophia nova* (New Philosophy on Our Sublunary World, written about 1600 but published long after the author's death, in Amsterdam in 1651), he in fact discussed Benedetti's views on the spots on the surface of the moon, in a chapter trying to determine which parts of it were seas and continents.[6] It is evident that the *Diversae speculationes* had a wide European circulation, and that the astronomical part attracted the attention of many scholars dealing with mathematical and physical issues.

Benedetti's treatment of astronomical matters ranges from the calendar reform to the nova of 1572, sundials, and astrology. We would like to focus on a special issue: Benedetti's defense of ephemerides, *Defensio ephemerides*, and the quarrel that motivated its writing. This defense of ephemerides figures as one of the epistles of the *Diversae speculationes*. It is the Latin translation of an Italian letter, *Intorno ad alcune nuove riprensioni… contra alli calculatori delle effemeridi* (Letter in the Form of a Discourse… Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators, Turin, 1581), addressed to Trotto, which Benedetti had already published when a heated quarrel on the reliability of ephemerides burst out in Turin between 1580 and 1581. In the following pages we will give an account of these facts.[7]

## 6.2  The Controversy over the Reliability of Ephemerides

The ephemerides controversy began with the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, Turin, 1580). This lesser-known author from Vicenza intended to denounce the inexactitude of all existing astronomical computations.[8] For this purpose he compared predictions and horoscopes cast using different sets

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[5]Brahe 1916, 251: "Nunc igitur epistolam, quam pollicitus sum, subiungam, verbotenens e praedicto Baptistae Benedicti libro desumptam, una cum demonstrationum delineationibus, quas ipse author assignavit. Deinde ad caeteros qui de hac stella nimis incompetenter, sententiam tulerunt, calamum dirigam. Ex quo (uti dixi) haec adeo succinte et scite geometricam veritatem redoleant, ut nullum, quod alicuius sit momenti, super esse queat, dubium."

[6]W. Gilbert 1651, 173: "Luna maculas quasi ostendit substantiae et peripheriae differentia: ita Tellus erga Lunam maculas repraesentat, terrarum continentium minus relucentium; aquarum vero et Oceani, propter laeviorem et luminis apprehensivam naturam magis splendentem. […] Non enim maculae Lunae existunt a partibus Lunae magis perspicuis, ut Iohannem Benedictus contendit, in quibus lumen non reflexum sed penetrans nobis occultatur." See Pumfrey 2011, 193–203.

[7]Section 6.2 is a revision of Omodeo 2014a, chap. 3.8–9 and chap. 6.3 of Omodeo 2014a, chap. 4.7.

[8]This Benedetto Altavilla could be the same person involved many years later, in 1606, in a gunpowder plot in Venice; he pretended to have discovered it by astrological means and was tortured by the Venice authorities in order to obtain information about the perpetrators. Cf. L. P. Smith 1907, vol. 1, 364–365.

of tables and ephemerides. In particular, he pointed out that ephemerides diverged from each other even more than the astronomical tables, Alfonsine or Copernican, from which they were derived. In his opinion, this fact undermined the reputation of astronomy in general, regardless of whether its cause was the inaccuracy of the compilers (*calculatores*) or the inexactitude of the tables themselves: "We consider nothing to be more odious than an unreliable person who is regarded by many as trustworthy."[9] Altavilla declared himself unwilling to decide between Alfonsine or Copernican computations. However, he himself was probably interested in the cosmological issue, judging by the fact that the *Animadversiones* were introduced with a poem by Pandolfo Sfondrati in favor of a new world system with the earth in motion.[10]

Altavilla had established by observation that both Alfonsine ephemerides and Johannes Stadius's Copernican computations were in disagreement with the heavens. Still, Stadius's computations proved to be in better agreement with the heavens. The reference to Stadius is not casual, since the Flemish astronomer had been a protégé of duke Emanuele Filiberto of Savoy, as one can read in the *Ephemerides novae* of 1556, where the author gave himself the title "mathematician to the King [of Spain] and the Duke of Savoy" (*Regius et Ducis Sabaudiae mathematicus*). Altavilla listed predictive errors of Ptolemaic astronomers (Regiomontanus, Stöffler, Leowitz) as well as those of post-Copernican ephemerists (Stadius and Giuntini). This led him to skepticism toward predictions in general: "You see, dear reader, how reliable ephemerides are."[11] Altavilla invited scholars (*magistri*) to trust only their eyes and to correct astronomy through observational campaigns with no regard for any authority: "Posterity should learn how dangerous it is to blindly adhere to the opinions of the ancients without [perfecting the art through] daily observations of the heavens, and to prefer their opinions to truth."[12]

The *Animadversiones* were soon followed by a second publication in Italian: *Breve discorso intorno gli errori dei calculi astronomici* (Brief discourse on the mistakes of astronomical calculations, 1580). A poem by a certain Francesco Onto of Pinerolo, inserted as a preface to the *Breve discorso*, made its polemical target explicit: "Altavilla has unveiled the astrologers' fallacy, as they think to cast certain [astrological] judgments about our lives relying on flawed ephemerides."[13] Altavilla's criticism was directed mainly against astrology, whose validity he considered to be doubtful due to the inaccuracy of predictions. His argumentative strategy was no different than that of Pico della Mirandola in books 8 and 9 of the *Disputationes in astrologiam divinatricem* (Disputations against divinatory astrology, 1496): an attack on mathematical astronomy aimed to discredit astrological forecasting. Altavilla even claimed that astrologers and ephemerists should renounce their activity, as they were not capable of superseding the flaws of their discipline: "Since it is impossible for the scholars in those sciences (especially those who are not capable of using the tables) to renounce ephemerides, and they know that they will encounter irremediable errors, they should be forced to abandon their studies."[14]

In his second publication, the *Discorso*, Altavilla complained that many scholars (who were not named) pretended to ignore his criticism. He explained that the decision to write another booklet, this time in Italian instead of Latin, originated from the desire to reach readers outside academic and scholarly circles, probably also at the Savoy

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[9] Altavilla 1580a, f. A2*r*.
[10] See Omodeo 2008b and Omodeo 2012a.
[11] Altavilla 1580a, *Conclusio*.
[12] Altavilla 1580a.
[13] Altavilla 1580b, 2.
[14] Altavilla 1580b, 4–5.

court: "In these few pages, I aimed at demonstrating not only to the learned man, but also to everybody else, that the errors [of the ephemerides] are worthy of consideration."[15] He first reassessed the inadequacy of Alfonsine tables and Alfonsine ephemerides (those of Peuerbach, Prugnerus, Bianchini, Regiomontanus, Stöffler, Schöner, Gaurico, Pitati, Simi, Carelli, Moletti, Leowitz, and others). He moreover stressed the superiority of the Copernican tables in order to show the inconsistency of some unnamed Turin ephemerists who used Alfonsine ephemerides for their predictions although they claimed to prefer Copernicus. To illustrate this inconsistency, he analyzed some astrological figures on the basis of Stadius's and Giuntini's tables. In the last section Altavilla turned on the Copernican ephemerists, denouncing the excessive difference between computations based on Stadius and Giuntini: "And the difference between one computation and the other is really great and monstrous."[16]

This attack on the reliability of astronomical computations and astrology provoked negative reactions both at the university and at the court. Altavilla thus felt compelled to challenge his critics to an academic debate on August 14 and 15, 1581, announcing it through a broadside that is still preserved in the libraries of Turin, along with copies of his *Animadversiones*.[17] The public dispute concerned the theory of Mars for which, as one reads, some scholars blamed him. He maintained, in fact, that Mars cannot stay in a zodiacal sign for more than two months, considering that its entire revolution lasts twenty-four months. He argued that ephemerides are wrong if they forecast that it would spend six or even seven months in the same zodiacal constellation. This incorrect opinion presented the court mathematician and philosopher Benedetti with an occasion to intervene and criticize Altavilla on this and other issues related to astronomical theory, computation, and astrological prediction.

Soon after Altavilla's public dispute, Benedetti published an epistle "on some recent remarks and emendations directed against ephemerists" (Turin, 1581). At the beginning, Benedetti indicated Altavilla's intentions: "I assume […] that his intention was only to demonstrate that [different] ephemerides assigned a different place to the planet at the same point of time […] and that, as a consequence, they offer no certain ground on the basis of which the future can be judged or predicted."[18] In his account, Benedetti rejects Altavilla's complaint that Copernican and Alfonsine ephemerides diverge from each other more than the tables from which they are derived. He assures the reader that "the people who calculated have been very accurate and trustworthy" (*i calcolatori sono stati diligentissimi e fedeli*) and they are exact in their calculations, although some minor and accidental mistakes can occur.[19]

Moreover, he accuses Altavilla of misunderstanding Ptolemy's astrology, interpreting it in light of Abu Ma'shar and Al-Qabisi (*Alcabitius*). In particular, Altavilla draws from these sources the rule of the "triplicity" of the conjunctions of Jupiter and Saturn, according to which these planets meet four times in the same three astrological signs, or trine, before they can meet in the next trine. However, although the mean motions of two planets should meet in the triplicity sign, nonetheless their "real" motions (those observed and calculated by the ephemerides upon which astrological predictions rely) may meet elsewhere. This is an obvious consequence of planetary theory. In fact, it distinguishes between "mean" motions, which correspond to the revolutions of the deferents, and "real" motions, which

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[15]Altavilla 1580b, 3.
[16]Altavilla 1580b, 6.
[17]In Turin: Biblioteca Nazionale di Torino, coll. Q.V.191, and Biblioteca Reale di Torino, coll. G.25.12.
[18]Benedetti 1581, 5.
[19]Benedetti 1581, 6.

correspond to observable phenomena and are the product of moving epicycles. Benedetti calculates the period of triplicity to be 794 years and 138 days, whereas the Arabs on whom Altavilla relies overestimated it at 960 years.[20] He furthermore remarks that Altavilla neglected planetary theory by criticizing those who let Mars run too fast or too slowly along the signs of the zodiac. Simple observations would show the correctness of the theory according to which the planet can remain in the same sign for six or even seven months. Benedetti explains that the amplitude of Mars's epicycle accounts for its complex phenomenology, in particular the long period of retrograde motion. On this account, he reports an observational campaign accomplished between 1565 and 1566 in order to check Stadius's ephemerides:

> Yet, he [Altavilla] dared too much, seeking to reprimand so many talented an-cient and modern men who, as is required by diligent observers of the heavens, checked with their own eyes these appearances of Mars as well as of the other [planets]. From those [observations], they were forced to "imagine" such a large [Martian] epicycle. By contrast, he has never observed the motions of either this or any other planet, but rather limited himself to look at what is written in the ephemerides. In fact, if he had at least said that he observed Mars's journey for a certain period, and that he found that the others' opinion was false, he would have at least given some "color" to his opinion. In my assessment, however, if he had made an observation of the path of Mars, he would not have held the contrary view. In fact, the truth is the following: in every revolution of its epicycle, Mars in the lower part of its epicycle always stays many months (six or seven, or more) in a twelfth [*duodecatemerio*] of the zodiac. I observed this many times, for instance, in the years 1565 and 1566. First, consulting Stadius's ephemerides, I found that Mars would finish its retrograde motion on about 12 January 1566, in 16° of Gemini, and that, equally, Mars would be in the same place on the last day of August 1565, be-fore it began its retrograde motion. Second, I found that, after that retrograde motion, on 11 April 1566, Mars would be in 16° of Cancer, so that it would take [Mars] seven months and eleven days [to move] those thirty degrees, from 16° of Gemini to 16° of Cancer. After these computations, I took the instruments and got ready to make a test. And I found that the last night of August of the year 1565 Mars was in the aforesaid 16° of Gemini, as Stadius had noted. I then made observations every week, in order to see the retro-grade motion, and I saw that, at about the end of October, the [planet] began its retrograde motion and that retrograde motion lasted until January (or about January) 1566. I later observed the position of that planet on 11 April, and I found it in 16° of Cancer, that is, the place where Stadius had located it. Thus, my experience confirmed Stadius's computations and I found that he was not mistaken. In the same manner, everybody can ascertain the truth every two years by carrying out observations.[21]

Benedetti thus demonstrated not only the theoretical incompetence of his opponent, but also his lack of empirical verification. Altavilla's appeal to base astronomy using obser-vation backfired. Benedetti challenged his opponent to observe Mars's backward motion in Cancer which, according to Stadius's tables, would begin on November 20, 1582 and

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[20]See Bonoli 2012, 49–55.
[21]Benedetti 1581, 17–19.

last until the end of February 1583. He furthermore observed that everyone familiar with planetary theory would understand the reasons for the orbit of Mars and other planets. For the theory, he added, it did not matter whether one relied on Ptolemy's *Almagest* or on the "*Rivolutioni de gl'orbi celesti* dell'eccellentissimo Copernico."[22] Of course they were only equivalent as far as the understanding of a system of deferents and epicycles was concerned, but not in their general hypotheses, since Benedetti himself tended toward heliocentrism.

As to the difference between Leowitz's and Stadius's computations, Benedetti traced this back to the contrast between the theories underlying the Alfonsine and the Copernican tables. Nonetheless, he ensured that ephemerides never diverged by more than three degrees. Thus, if Altavilla detected greater discrepancies, this was due only to false computations. Benedetti added that Stadius's superiority over Leowitz was a consequence of him employing better parameters. He advised Altavilla to always rely on the most recent observations and tables.[23] In fact, he judged the progress of astronomy to be such that more recent tables would inevitably be superseded by new ones, augmented and perfected through new observations, just as Copernicus had superseded Alfonso's astronomers. Divergence between ephemerides was not a shortcoming, but a necessary and desirable sign of the advancement of knowledge and predictive accuracy.

As a courtier expert of mathematics, Benedetti defended the validity of some astrological figures that Altavilla criticized in his second published work, *Breve discorso*. These horoscopes had probably been cast by somebody that he knew well. Altavilla complained that some astrological figures had not been calculated on the basis of Copernican tables. Benedetti replied that it was not always necessary to use the best tables for predictions, especially if a generic horoscope was expected and if the astrologer had no Copernican tables to consult. He showed, moreover, that Altavilla himself was not able to employ Giuntini's tables properly and made mistakes of computation. He concluded: "And such monsters [those denounced by Altavilla] are not generated by different tables or ephemerides but, instead, they are the offspring of this author."[24] He added as a remark: "As to the difference of the Sun according to Copernicus and Alfonso, no learned man, [expert] in these sciences, ignores it, and, as a consequence [everybody knows] the different place [assigned to it] in the heavens during the annual revolutions."[25] In 1581, the general views of *De revolutionibus* were so well known in Benedetti's environment that he deemed it unnecessary to expand on them in the context of a polemic on the accuracy of heavenly computations. The cosmological implications of these different hypotheses were not addressed explicitly in this dispute. However, the defense of mathematical astronomy could not avoid a reference to Copernicus as a source for tables (Reinhold, Stadius, Giuntini) and theory. In this context, "Copernican" and "not Copernican" are expressions that merely mean "based on Copernican tables" or not. Altavilla's criticism would have been more effective if it had been directed against astrological beliefs as such, rather than attempting to show the inconsistency of the mathematical basis of astrology without sufficient preparation. On the other hand, Benedetti, in his *Lettera*, focused on the mathematical aspects and cautiously avoided expanding on ethical issues related to astrology.

Altavilla never responded to the court mathematician who had rebutted his arguments so forcefully. The epilogue to their quarrel was the inclusion of a Latin translation of the

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[22]Benedetti 1581, 20.
[23]Benedetti 1581, 32–33.
[24]Benedetti 1581, 37.
[25]Benedetti 1581, 37–38.

*Lettera*, as *Defensio ephemeridum* (A defense of Ephemerides), in Benedetti's *Diversae speculationes*.[26]

## 6.3   The System of the World

Benedetti did not limit himself to considering astronomy from a computational point of view, but also expanded on cosmological aspects. The epistle "De fine corporum coelestium, et eorum motu" (On the Aim of Celestial Bodies, and their Motions),[27] addressed to Pingone, bears witness to his interest in cosmology and his realist interpretation of Copernicus's hypotheses. Benedetti remarks that it is not reasonable (*si [...] humanam rationem sequi volueris*) to believe that the heavens were created only for the sake of terrestrial life, "as these [celestial] bodies are divine, uncountable, and endowed with the greatest dimensions" (*cum ea corpora sunt divina, in numero incompraehensibilia, maximis magnitudinibus, et motibus velocissimis praedita*).[28] This absurdity can be avoided, as Benedetti claims, if one accepts the planetary doctrine of Aristarchus and Copernicus:

> [...] this will hardly be believed by those who embrace the doctrine of
> Aristarchus of Samos and Nicolaus Copernicus. Following their approach it
> is impossible to make them believe that the rest of the universe has no other
> aim than to rule over this center of the lunar epicycle [the earth] (to use their
> way of speaking).[29]

Although he speaks in the third person, as if he were reporting the views of someone else, these are his own views. He is inclined to accept the Copernican system or some variation of it, as the following pages of the letter and the force of the arguments show. Firstly, he assumes a principle of cosmological homogeneity according to which there is no reason why other planets should not be subjected to alterations (*ab ortu, et interitu*), as the Aristotelians suppose. The peripatetic argument that no change in the heavens was ever observed is not valid, because the distance does not permit verification of whether there is any life or alterations on distant bodies (*unde etiam fieri potest, ut in coelo sint particulares alterationes, quae a nobis tamen, qui ab illis longe distamus, non compraehendantur*).[30] Benedetti even surmises that other planets are moons reflecting the solar light to dark planets invisible to us.[31] He ascribes this opinion to the followers of Copernicus. This is a free interpretation on his part. Perhaps he aimed to explain the epicyclic motions of other planets through an analogy with the lunar epicycle around the earth. Benedetti also rejects Ptolemaic and Aristotelian arguments against terrestrial motion. Following Copernicus (*De revolutionibus* I 8), he stresses that the axial rotation avoids the otherwise enormous motion of the fixed stars: "which is eliminated by the rotation of the Earth about its axis (as they say) as it is sufficient to receive the light and the influences of the [celestial] bodies."[32] Moreover, the annual revolution respects the dignity of the "divine body of the

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[26] Benedetti 1585, 228–248, "Defensio ephemeridum."

[27] Benedetti 1585, 255–256.

[28] Benedetti 1585, 255.

[29] Benedetti 1585, 255: "[...] id etiam minus putabunt hii, qui opinionem Aristarchi Samii, et Nicolai Copernici sequuntur, quorum ratione fieri non potest, ut credant eius, quod ex universo reliquum est, alium finem non habere, quam regimen huius centri [Tellus] epicycli Lunaris, ut illorum more loquar."

[30] Benedetti 1585

[31] The same thesis is presented in Benedetti 1585, 195–196.

[32] Benedetti 1585, 255–256: "quae quidem omnia [phaenomena], cum simplici gyro terrae circa suum axem (ut dicunt) tolluntur, quod sufficit ad recipiendum lumen, et influentias illorum corporum."

Sun" (*divinum corpus solare*), which stands still at the center of the planetary circles.[33]
Note Benedetti's astrological concern. In the final passage of his letter, he reassesses
Copernicus's objection to Ptolemy's view of how bodies suspended in the air are affected
by terrestrial motion:

> Ptolemy's objections are not valid for them [astronomers who assume that
> the earth moves]. As they say, every part maintains the nature of the whole,
> apart from the fact that the air and water circumscribing the earth receive the
> same natural impulse of motion [*impetum motus*]. This is slower the further
> the air is distant from the earth. According to the same doctrine, there is
> no necessity that the place of the fixed stars has (either convex or concave)
> superficial boundaries.[34]

According to this passage, the air close to the earth is transported by the motion of the
planet and slows down the more it is distant from it. The fixed stars are placed in a mo-
tionless air whose place (*locus*) has no boundaries, either convex or concave.

In a letter to the courtier Capra, Benedetti confronts the issue of the form of the heav-
ens.[35] This is said to be a sphere encompassed by infinite space. Accordingly, Benedetti
distinguishes between *spacium* (space) and *coelum* (heavens), a distinction that can be
traced back to Stoic cosmology or to the more recent views of Marcellus Palingenus Stel-
latus. The idea of the infinity of space beyond the starry vault can be found also in Patrizi's
*Nova de Universis Philosophia* (1591).[36]

Furthermore, Benedetti rejects the existence of material spheres with the role of trans-
porting the planets:

> That you do not accept that distinction of spheres, which was well-established
> in the past, but rather that you believe that the whole is a continuum accom-
> modating the stellar bodies, this is not new. In fact, some philosophers of
> solid doctrine were of the same opinion.[37]

The motion of celestial bodies is accompanied by that of transparent bodies similar to
vapors (*fumi*). Their motion is the cause of the apparent sparkling of the most distant
stars.[38] The sparkling of the new star in Cassiopeia in 1572 bears witness to its great
distance above the moon, which Benedetti also demonstrates through geometry.[39]

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[33] Benedetti 1585, 256.

[34] Benedetti 1585: "Rationes autem a Ptolomeo in contrarium adductae apud ipsos, nullae sunt, quia quaelibet
pars (ut inquiunt) retinet naturam totius, praeterquam quod aer, et aqua, quae ipsam terram circundant, plane
eundem naturalem impetum motus obtineant, qui tanto lentior est, quanto longius distat aer, ab ipsa terra,
secundum etiam talem opinionem, nulla necessitas, ut locus fixarum terminaretur aliquibus superficiebus,
convexa scilicet, et devexa."

[35] Benedetti 1585, 285–286, "De motu molae, et trochi, de ampullis, de claritate aeris, et Lunae noctu ful-
gentis, de aeternitate temporis, et infinito spacio extra Coelum, Coelique figura."

[36] For Benedetti's correspondence with Patrizi, see Claretta 1862.

[37] Benedetti 1585, 411: "Quod eam distinctionem orbium, quae iam invaluit, non teneas, sed putes totum esse
quoddam continuum excipiens corpora stellarum, novum non est, nam nonnulli solidae doctrinae philosophi
idem confuerunt."

[38] Benedetti 1585, in the section entitled "Disputationes de quibusdam placitis Arist[otelis]," n. 38: "Occul-
tam fuisse gravissimo Stagiritae causam scintillationis stellarum," 186: "Scintillatio ergo stellarum, neque
aspectus nostri ratione, neque alicuius mutationis earundem stellarum, sed ab inaequalitate motus corporum
diaphanorum mediorum nascitur, quemadmodum clare cernitur, quod si inter aliquod obiectum, et nos, ali-
quis fumus, qui ascendat, intercesserit, videbimus obiectum illud quasi tremere. Hoc autem tanto magis fiet,
quanto magis distabit obiectum ab ipso fumo; unde admirationi locus non erit, si stellas fixas magis scin-
tillare, quam errantes cernamus. Lumen stellae ad oculum nostrum accedens, perpetuo per diversas diapha-
neitates penetrat, medio continuorum motuum corporum mediorum, unde continuo eorum lumen variatur,
et hoc in longitudinis magis, quam in propinquis stellis apparet."

[39] Benedetti 1585, 371–374.

One of the books of the *Diversae speculationes* entails a discussion and a refutation of Aristotelian physical and celestial theses *de motu*. It has the rather neutral title *Disputationes de quibusdam placitis Arist[otelis]* (Disputations on Some Opinions Held by Aristotle) but it is indeed an attempt to revise basic concepts of natural philosophy such as *locus* (place) and *tempus* (time). We shall deal with this issue in detail in the next section. For now, it is important to anticipate that this anti-Aristotelian section entails Benedetti's most explicit defense of Copernican planetary hypotheses. Another remarkable thesis of these *Disputationes* on Aristotle is the statement of a principle of relativity according to which planets appear to us as we appear to them:

> Aristotle did not consider that one could affirm the same about the Earth as seen from great distance. There is no doubt that, even if the Earth had the light of the Sun and somebody tried to observe it from the eighth sphere, he would not be able to perceive it. In fact, those celestial bodies that are said to be of the first magnitude and that are believed to be more than a hundred times bigger than the Earth look just like points.[40]

Benedetti supports the plurality of worlds as well (*Minus sufficienter explosam fuisse ab Aristotele opinionem credentium plures mundos existere*). Every planet should be regarded as another Earth with its elements and natural places: "If those worlds existed, each of them would have its own center and its own circumference and the earths and fires would have an inclination towards the centers and the circumferences of their worlds, respectively."[41]

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[40]Benedetti 1585, 197, "Disputatio XXXIX, Examinatur quam valida sit ratio Aristotelis de inalterabilitate Coeli: Aristo[teles] non consideravit, quod similiter de terra dici posset, quando ipsa ita eminus prospiceretur, imo absque dubio putandum est, quod si terra luce Solis praedita esset, et aliquis ipsam ab octavo orbe vellet videre, nullo pacto cerneret, cum sidera illa quae primae magnitudinis vocantur, et quae plusquam centies maiora ipsa terra putantur non nisi ut puncta videantur."

[41]Benedetti 1585, 195: "Si essent dicti mundi, eorum quilibet suum proprium centrum, suamque propriam circunferentiam haberet, terraeque et ignes haberent inclinationem ad centra circunferentiasque suorum mundorum."

## 6.4    Appendix: An Assessment of Benedetti's Horoscopes (by Günther Oestmann)

For the recalculation of a historical horoscope, the same methods and means the author had at his disposal must be employed, that is, the use of modern parameters or tables is not allowed.[42]  In the following disposition, planetary positions are rendered in ecliptic longitude (degrees ; minutes) for each zodiacal sign (0–30°), geographical coordinates likewise in degrees ; minutes, and time in hours ; minutes. Latitude is denoted as φ.

### 6.4.1    Nativity Cast by Benedetti for Duke Carlo Emanuele I of Savoy

January 11, 1562 (Julian date), 16;23 p.m., φ = 45°; Planetary positions according to the *Prutenicae Tabulae* by Erasmus Reinhold (1551).
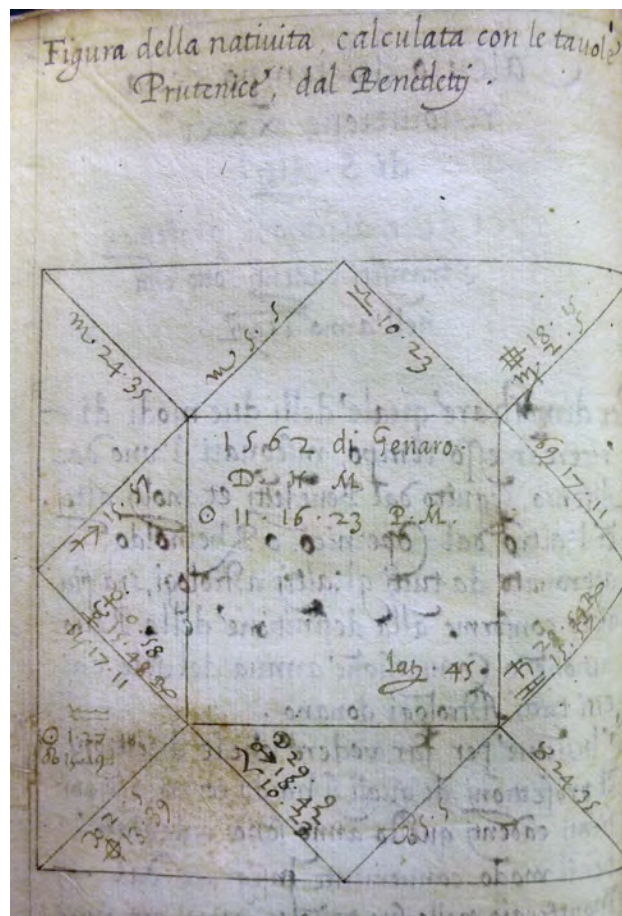


Figure 6.1: The horoscope cast by Benedetti for Duke Carlo Emanuele I of Savoy. This was calculated with the Prutenic tables, as transcribed by Bartolomeo Cristini in *Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria*, Turin. (Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 11*v*)

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[42]Here a convenient, unfortunately little-known computer program created by Peter Schiller especially for the needs of historians has been used.  See Schiller 2001.  There is not sufficient room here for a detailed analysis of the choice of appropriate historical parameters; for a concise description, see Oestmann 2002 and Eade 1984, 1–103.

Carlo Emanuele I of Savoy was born in the Castle of Rivoli (c. 15 km west of Turin) on January 12, but here the "noon epoch" is indicated in the manner commonly used by astronomers/astrologers: the date changes at 12:00 local time, and the hours are counted from there to 24—contrary to civil use, where sunrise or sunset often marked the change of day. With the proliferation of mechanical clocks in the late Middle Ages, the date change at midnight gradually became predominant and hours were counted from 1–12.[43]

The geographical coordinates of the nearest town to Turin listed in the *Catalogus locorum* in Reinhold's *Prutenicae Tabulae* is Venice, which is 0 h 50 m (12;30) west of Königsberg, the reference meridian of the Prutenic tables. In Petrus Apianus's *Cosmographicus liber* (1533), the following specifications are given: Turin 30;30 and Königsberg 42;16 east of the island Porto Santo near Madeira (f. XLII*r*, XXXIX*r*) → the difference of longitude is 11;46 (modern value: 12;46). In the following recalculation, a longitude of 12;00 west of Königsberg has been assumed:

Table 6.1: Planets

| Planets | Original Source | Recalculation |
|---------|-----------------|---------------|
| Sun | 1;27 Aq | 1;27 Aq |
| Moon | 29;09 Ar | 29;16 Ar |
| Saturn | 28;54 Ge retrograde | 28;55 Ge |
| Jupiter | [missing] | 21;02 Ta |
| Mars | 18;42 Ar | 18;41 Ar |
| Venus | 0;58 Cp | 0;58 Cp |
| Mercury | 15;48 Cp retrograde | 15;48 Cp |
| Lunar node (asc.) | 15;19 Aq | 15;16 Aq |

Table 6.2: Houses

| Houses (Regiomontanus) | Original Source | Recalculation |
|------------------------|-----------------|---------------|
| X | 10;23 Li | 10;16 Li |
| XI | 5;05 Sc | 4;59 Sc |
| XII | 24;35 Sc | 24;32 Sc |
| I | 15;57 Sa | 15;54 Sa |
| II | 17;11 Cp | 17;07 Cp |
| III | 2;05 Pi | 1;58 Pi |
| | | |
| Lot of Fortune (Night) | 18;15 Vi | 18;05 Vi |
| Lot of Fortune (Day) | 13;39 Pi | 13;43 Pi |

The Lot of Fortune (*Pars Fortunae*; named for the Roman goddess of luck and wellbeing) is calculated in diurnal charts by subtracting the ecliptic longitude of the sun from the

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[43] For details, see Bilfinger 1888, 262–286 and Ginzel 1914, 94–96.

longitude of the moon. Then the difference is added to the longitude of the Ascendant: Lot of Fortune = Ascendant + Moon – Sun. For nocturnal charts, the calculation is Ascendant + Sun – Moon.

Although this is a night-time birth chart, Benedetti has marked the Lot of Fortune for night and day.

The sign and degree occupied by the moon when crossing the ecliptic from southern to northern latitude is the ascending node (*Caput Draconis*). When the moon is moving in the opposite direction (crossing the ecliptic from north to south), the point of intersection is called the South Node (*Cauda Draconis*). The nodes are not fixed, but have a retrograde movement (a complete revolution of the nodes in the ecliptic takes 6798 days/18.61 years). To both points (which are important in the interpretation of a chart), the strength of a planet has been assigned. The Dragon's Head is considered beneficial, the Dragon's Tail malefic. (In Hindu astrology, the ascending node is called *Rāhu* and the descending node *Ketu*; both are considered malefic planets.[44].)

Benedetti forgot to inscribe Jupiter. Apart from this flaw everything has been calculated accurately.

### 6.4.2   Revolution or Solar-Return Horoscope

January 21, 1592 (Gregorian Date), 23 h 15 m 30 s p.m., φ = 45°.
Geographical coordinates of Turin according to Petrus Apianus in *Cosmographicus liber* (1533): f. XXXV*r* – Toledo 9;04 East of Porto Santo; f. XLII*r* –Turin 30;30 → 21;26 East of Toledo (the reference meridian of the Alfonsine tables).

Table 6.3: Planets

| Planets | Original Source | Recalculation (Alfonsine tables) | Recalculation (Prutenic tables) |
|---|---|---|---|
| Sun | 1;27 Aq | 2;08 Aq | 1;12 Aq |
| Moon | 9;27 Ta | 11;20 Ta | 8;12 Ta |
| Saturn | 7;29 Ca retrograde | 10;30 Ca | 7;30 Ca |
| Jupiter | 11;44 Sa | 10;31 Sa | 11;42 Sa |
| Mars | 3;05 Ar | 4;23 Ar | 3;00 Sa |
| Venus | 25;09 Sa | 23;00 Sa | 25;08 Sa |
| Mercury | 9;36 Aq | 5;43 Aq | 9;25 Aq |
| Lunar node (asc.) | 4;54 Ca | 5;05 Ca | 4;53 Ca |

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[44]See Hartner 1938, 131–134

Table 6.4: Houses

| Houses (Regiomontanus) | Original Source | Recalculation I ($\varphi$ = 45;00; 23;15 p.m.) | Recalculation II ($\varphi$ = 45;00, 23;17 p.m.) |
|---|---|---|---|
| X | 21;00 Ca | 20;30 Cp | 20;59 Cp |
| XI | 1;00 Aq | 10;36 Aq | 11;11 Aq |
| XII | 17;00 Pi | 16;26 Pi | 17;18 Pi |
| I | 10;49 Ta | 9;56 Ta | 10;43 Ta |
| II | 15;00 Ge | 14;36 Ge | 15;08 Ge |
| III | 5;00 Ca | 4;18 Ca | 4;45 Ca |
| | | | |
| Lot of Fortune (Day) | 18;49 Le | 16;55 Le | 17;44 Le |
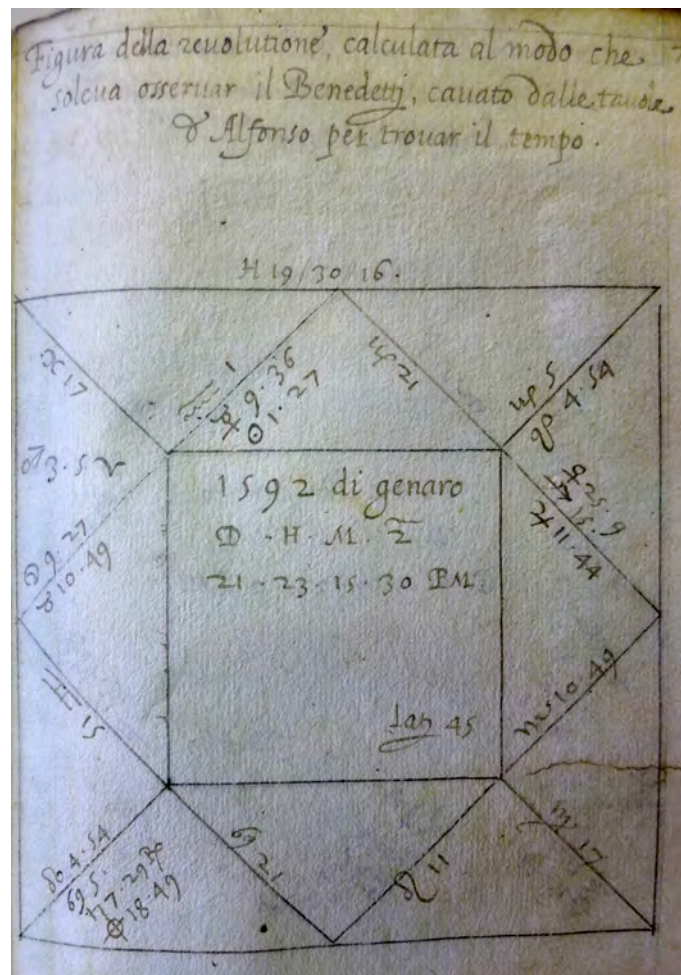


Figure 6.2: Benedetti's horoscope for Carlo Emanuele I, calculated with the Alfonsine tables, as transcribed by Cristini in *Revolutione trentesima prima* (1592), f. 12r. (Biblioteca Nazionale Universitaria di Torino, coll. N VII 10)

The second of Benedetti's horoscopes to be considered is also taken from Bartolomeo Cristini, *Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria*, Turin, Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 12*r* (Figure 6.2).

This is a chart constructed for the moment in which the sun returns to the degree and minute of its longitude at nativity (i.e., transiting the position of the "natal" sun) for the respective location. A revolution horoscope indicates the course of events during the ensuing year.

Contrary to Benedetti's caption (*Figura della revolutione [...] cavato dalle tavole d'Alfonso per trovar il tempo*), he has obviously used the Prutenic tables for calculating the planetary positions of this chart. But the moon's position is off by c. 1°, and the cusps of the houses deviate somewhat. Calculating with a time of 23;17 p.m. gives a reasonably good compliance, however. The cusp of house XI (1;00 Aq instead of 11 Aq) is most likely a scribal error.

It is noteworthy that minutes for an arc are only provided for the first house (i.e., the ascendant). For the other cusps, only whole degrees are noted. Benedetti simply cut off the minutes, which was a common rounding procedure at his time.

### 6.4.3   Natal Horoscope of Giovanni Battista Benedetti

August 14, 1530 (Julian Date), 13 h 13 m p.m., Venice; planetary positions according to the Alfonsine tables. Geographical coordinates of Venice according to Petrus Apianus (1533): Toledo 9;04 East of Porto Santo (f. XXXV*r*); f. XLII*r*: Venice 32;30, Latitude φ = 44;50 → 23;26 East of Toledo (the reference meridian of the Alfonsine tables).
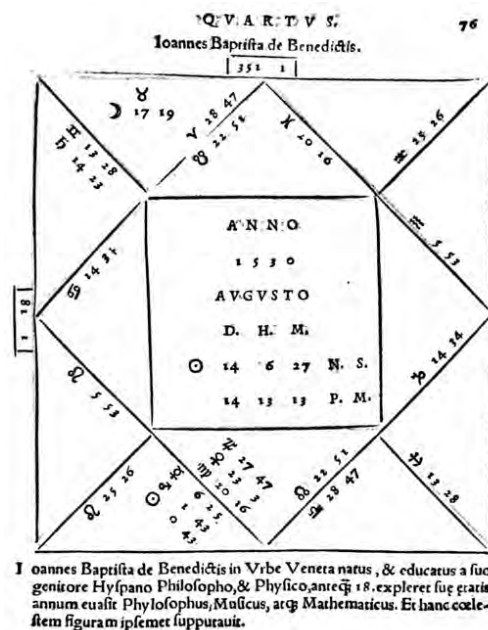


Figure 6.3: Benedetti's own horoscope, detailed in Luca Gaurico's *Tractatus astrologicus* (1552, f. 76*r*). (Bayerische Staatsbibliothek)

Table 6.5: Planets

| Planets | Original Source | Recalculation |
|---|---|---|
| Sun | 0;43 Vi | 0;42 Vi |
| Moon | 17;19 Ta | 15;02 Ta |
| Saturn | 14;23 Ge | 14;23 Ge |
| Jupiter | 27;47 Vi | 27;46 Vi |
| Mars | 1;43 Vi | 1;45 Vi |
| Venus | 23;03 Vi | 23;02 Vi |
| Mercury | 6;25 Vi | 6;25 Vi |
| Lunar node (desc.) | 22;51 Ar | 22;52 Ar |

Table 6.6: Houses

| Houses (Regiomontanus) | Original Source | Recalculation |
|---|---|---|
| X | 20;16 Pi | 20;14 Pi |
| XI | 28;47 Ar | 28;36 Ar |
| XII | 13;28 Ge | 13;17 Ge |
| I | 14;34 Ca | 14;23 Ca |
| II | 5;53 Le | 5;46 Le |
| III | 25;26 Le | 25;23 Le |

Apart from the moon's position (which is about 2° off) the horoscope is correct. In all three horoscopes the houses have been constructed according to the so-called "rational method," commonly—but erroneously—attributed to Regiomontanus:[45] Circles of position joining in the north and south point of the observer's horizon are laid at distances of 30° through the celestial equator, thus giving unequal sections of the ecliptic. This method of house division was widely used by astrologers during the sixteenth and seventeenth centuries.

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[45] It was already known in the Maghreb in the eleventh century, see Kennedy 1996, 543. For a profound treatment of the history of house division, see North 1986, although the way this text coins new designations is awkward and may lead to confusion.

# Chapter 7
# Foundations of Physics

In this chapter we analyze Book 4 of the *Diversae speculationes*, entitled *Disputationes de quibusdam placitis Aristotelis* (Disputations on Some Opinions Held by Aristotle). We will refer to this section as *Physical Disputations*. Benedetti here developed his theory of motion and clarified his physical conceptions by means of a discussion and criticism of Aristotle's physics. He dealt with fundamental concepts such as place and time. Moreover, it is here that the interdependency of physics and cosmology in his conception most clearly emerges.

## 7.1    Sections of the *Physical Disputations*

The *Physical Disputations* are a discussion of and an objection to Aristotle's theses on local motion and cosmology as presented in *Physics* and *De caelo*, and partly also in *Meteorologica* and *Metaphysica*. Benedetti does not limit himself to criticism but rather seeks to provide a new approach to and foundation of physics and cosmology, beginning with the theory of motion. He calls his approach mathematical (*inconcussa mathematicae philosophiae basis*). In actual fact, his conceptual tools for the treatment of motion are principally derived from Archimedes's *Floating Bodies* and Euclid's Book 5 on proportions. As we have seen, the reader of Benedetti's *Diversae speculationes* is provided with a brief book dedicated to Book 5 of Euclid's *Elements*, directly following the *Physical Disputations*.[1] Benedetti moreover reworks and transforms basic concepts of physics, such as place and time, and propounds anti-Aristotelian natural views such as spatial infinity and the existence of the void. His treatment culminates with cosmological speculations, including a defense of the Copernican system.

One can conveniently subdivide the *Physical Disputations* into five sections (see table 7.1 below). The first twelve chapters are a lengthy discussion of Aristotle's *Physics* IV 8. This section deals with the ratio of velocities of bodies moving through different media or the void. Secondly, from chapter 13 to chapter 18, Benedetti challenges *Physics* VII 5 on further problems linked with the theory of motion. The third subdivision (chapters 19 to 22) deals with basic philosophical matters (the void, infinity, place, and time). This section is extremely important for an overarching understanding of Benedetti as it connects the investigation of motion with a general reform of natural philosophy. Whereas Drake and Drabkin, in *Mechanics in Sixteenth-Century Italy*, translated the first two subdivisions (chapters 1–18), they neglected the foundational one, except for chapter 19 on the void. Their choice to expunge this part of the *Physical Disputations* deprived the English-speaking readership of some of the most daring pages in Benedetti's work.

Another subdivision (chapters 23–26, entirely translated in Drake and Drabkin 1969) deals with local motion and the shortcomings of the Aristotelian theory of natural places. The fifth and last subdivision, only partly included in the English edition, presents cosmological ideas. It deals with the "sphere" as a geometrical-cosmological figure, as well

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[1] Benedetti 1585, 198ff.

as with the (apparent) motion of the sun, with stars, meteorological aspects linked to the sun, the propagation of light in the cosmos, and other issues connected with astronomy in a broad sense. The Copernican system is discussed in the second part of this last section (chapters 35 to 39), along with other innovative theses such as the plurality of inhabited worlds akin to the earth and the reciprocity of the observational points in the universe.

We will concentrate on sections 1 (on motion), 3 (on the foundations of physics), and 5, part 2 (on cosmology). As one sees from our overview (table 7.1), the *Physical Disputations* are no less complex and heterogeneous than the volume of which they are part, the *Diverse speculationes*. Therefore, we will review Benedetti's arguments in enough depth to enlighten the thematic interdependency and his approach in general, deliberately leaving aside an excessive analysis of details that would not help to understand his intellectual endeavor as a whole.

Table 7.1: An overview of the *Physical Disputations* and of their English translation in Drake and Drabkin 1969.

| Sections and their subjects | Chapters | Details | Presence in Drake and Drabkin 1969 |
|---|---|---|---|
| 1. Phys. IV 8 | §1–12 | Discussion on the void and down- and upward motion in different media | ✓ |
| 2. Phys. VII 5 | §13–18 | On the proportions of velocities, volumes, and surfaces | ✓ |
| 3. Foundations of physics revised | §19–22 | On the void (XIX ≈ Phys. IV 8), place (XX ≈ Phys. IV 4), the infinite (XXI ≈ Phys. III 5, De Caelo I,9 etc.,), and time (XXII) | Only §19 |
| 4. Violent and natural motion | §23–26 | Rejection of the Aristotelian theory of natural places | ✓ |
| 5. The sphere (geometrical and cosmological) | §27–34 | On the sphere (§29), on starry light (§30), on infinite motion (§31), on the Sun, its warmth and seasonal changes (§30–31 and §34), celestial music, and harmony (§32–33) | Only §28 and §29 |
| | §35–39 | On the composition of circular and rectilinear motion and Copernicus's hypotheses (§35), plurality of worlds (§36), cosmic propagation of light (§37), geometrical aspects relative to elements (§38), and relativity of the point of observation (§39) | ✓ (only §38 missing) |

## 7.2    An Archimedean Theory of Motion

The Renaissance rediscovery of Archimedes's work can be seen as a crucial contribution to the theoretical advance of modern physics.[2]  The wide dissemination of this ancient work produced a renewed interest in mathematical methods for the investigation of nature.  In the *Questiti et inventioni diverse* of 1554, Tartaglia's Archimedean perspective implied a critical approach to Aristotle's mechanics.  By contrast, Del Monte had a rather syncretistic approach merging Archimedean and Aristotelian elements.  He even argued for the possibility of deriving terrestrial motion from Aristotelian premises (probably referring to geological motions rather than to an astronomical phenomenon).[3]  Benedetti's feelings toward Aristotelian philosophy are rather hostile; nonetheless, it is clear that his anti-Aristotelian theory of motion is embedded in the Renaissance discourse on natural philosophy, mechanics, and motion among mathematically-trained scholars.[4]

### 7.2.1    Aristotle's *Physics* IV 8

Benedetti's considerations on motion are presented as a criticism of Aristotle's *Physics* IV 8.  In that section Aristotle argued against the existence of the void and infinity of space and presented some reflections on the motion of bodies through different media. According to Aristotle, the void and infinity would undermine any reasonable theory of natural motion (indeed, his own theory of the natural and violent motions).  His polemic is directed against "those," probably Democritus and his followers, who held the void to be the condition of motion.  Aristotle claims that in an infinite and void space there would be no absolute directions; an up- or downward motion would be conceptually impossible. By contrast, he keeps to the "observation" that the elements display natural tendencies upward or downward (water and earth downward, air and fire upward), which, according to him, falsifies the thesis of an infinite and void space.  Additionally, this assumption would lead to "absurd" consequences:

> But in vacancy […] nothing could go on moving unless it were carried.  Nor (if it did move) could a reason be assigned why the projectile should ever stop—for why here more than there?  It must therefore either not move at all, or continue its movement without limit, unless some stronger force impedes it.[5]

As far as motion is concerned, Aristotle remarks that the difference of speeds between two bodies depends either on their different weight or lightness or on the different density of the media through which they travel.  He adds that the ratio of the speeds is reciprocally proportional to that of the densities, whereas it is directly proportional to that of the times.

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[2]Paolo D'Alessandro and Pier Daniele Napolitani have emphasized the impact of its sixteenth-century reedition in their studies on the Latin reception of Archimedes: "Nel 1544 usciva a Basilea l'*editio princeps* greco-latina di gran parte dei testi di Archimede oggi noti. Senza tema di esagerare, si può sostenere che questo avvenimento – al pari della pubblicazione del *De revolutionibus orbium coelestium* di Niccolò Copernico, apparso l'anno precedente a Norimberga – abbia fornito impulso alla nascita della scienza moderna." D'Alessandro and Napolitani 2012, 9.

[3]See Omodeo 2015.

[4]As has been argued in *Exploring the Limits of Preclassical Mechanics*, the wider conceptual framework of Archimedean theories of motion was in fact deeply rooted in Aristotelianism. See Damerow, Freudenthal, et al. 2004.

[5]Aristotle 1995, IV 8, 215a 17–22 (351).

For the sake of simplicity, we could represent these relations symbolically, in a modern way. Heath, in his study on *Mathematics in Aristotle*, put it as follows:[6]

$$\frac{speed\ in\ B}{speed\ in\ D} = \frac{density\ of\ D}{density\ of\ B}$$

Furthermore:

$$\frac{time\ taken\ in\ B}{time\ taken\ in\ D} = \frac{density\ of\ B}{density\ of\ D}.$$

On this basis, Aristotle concludes that motion in the void would be impossible, since "a body would move through the void with a speed beyond any ratio."

According to Aristotle, differences depending on the weights and on the dimensions of the bodies would disappear *in vacuo* with "very unpleasant" consequences:

> What reason can be assigned for this greater velocity [of a heavier falling body]? If the passage is through a medium, there must be such a difference [in the velocity of fall between heavier and lighter bodies]; for when there is anything there to cleave, the body superior in force of its thrust will necessarily cleave the medium faster, since either its more suitable shape or the natural thrust it exercises, whether following its natural movement or being thrown, makes it cleave the better. Where there is nothing to cleave, therefore, all bodies will move at the same velocity; which is impossible.[7]

Aristotle assumes that the speed of falling bodies is proportional to their weight (or dimension). For instance, one reads in *De caelo*:

> His must surely be a careless mind who does not wonder how it is that a small particle of the earth, if raised to a height and then set free, should refuse to remain where it was but begin to travel, and travel the quicker the bigger it is, whereas if one held the whole Earth in the air and let it go, it would not move. But in fact, for all its weight, it is at rest.[8]

This argument was repeated by Ptolemy in *Almagest* I 7: "If the Earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size."[9]

In summary, *Physics* IV 8 provides Benedetti with a series of entangled problems involving the void, infinity, and motion. He begins his reform of physics by dealing with the last issue and then moving to a treatment of the other ones. In relation to motion through a medium, he enlists some commonly accepted assumptions (*primo verissima et obiecta intellectus per se cognita*) in chapter 2:

> Therefore, whenever two bodies are subjected to or receive one and the same resistance to [the motion of] their surfaces, [the speed of] their motions will turn out to be to each other in precisely the same proportion as their motive forces. And, conversely, whenever two bodies have one and the same heaviness or lightness, but are subject to different resistances, [the speed of] their

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[6]Heath 1949, 116.
[7]Aristotle 1995, IV 8, 216a 17–21 (357).
[8]Aristotle 1986, II 13, 294 a (223–225).
[9]Ptolemy 1984, 44.

> motions will have the same ratio to each other as the inverse ratio of the resistances.[10]

The cases that have to be considered are basically two: first, different bodies in the same medium, and second, equal bodies in different media. A decisive difference between two bodies is their *gravitas* (gravity, weight) or *levitas* (lightness). For Benedetti, *gravitas* and *levitas* act as moving forces. He calls them *virtutes moventes* or *facultates moventes*. For the sake of brevity, we will refer to them as *virtus/virtutes*.

If we use V like *velocitas* for speed, P like *pondus* for *virtus*, and R like *resistentia* for resistance, we can formalize the previously mentioned general assumptions in the following way:[11]

I. Case in which R is constant:

$$\frac{V_1}{V_2} \sim \frac{P_1}{P_2}$$

II. Case in which P is constant

$$\frac{V_1}{V_2} \sim \frac{R_2}{R_1}$$

### 7.2.2 Speed in Different Media

Benedetti regards the Aristotelian theory of motion as inadequate and prefers to rely on Archimedes. He conceives of motion as analogous to the behavior of bodies in water. Following the *Floating bodies*, he holds that weight and lightness are not absolute but relative properties because they depend on the medium: *"quia in medio se densiore si poneretur, non grave esset, sed leve, quemadmodum Archimedes ostendit"* (in fact, if it is put in a denser medium it is not heavy but light, as Archimedes demonstrates).[12] Thus, the direction of a vertical motion and, as we will see, the speed depend on the matter of a body as well as on the fluid (air, water, or whatever) in which it is merged.

In the second *disputatio*, Benedetti declares that the *virtus* (weight or lightness)[13] of a body varies depending on the *densitas* (density) of the medium. Here he has in mind the three different cases considered by Archimedes in Book 1 of the *Floating bodies*: a body the weight of which is equal to that of the fluid, lighter, or heavier. Archimedes's seventh proposition, concerning the last case, might illustrate Benedetti's mental model for motion in a medium:

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[10]Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 169: "Quotiescunque igitur duo corpora unam eandemque resistentiam ipsorum superficiebus, aut habebunt aut recipient, eorum motus inter seipsos eodem plane modo proportionati consurgent, quo erunt ipsorum virtutes moventes: et e converso, quotiescunque duo corpora unam eandemque gravitatem, aut levitatem, et diversas resistentias habebunt, eorum motus inter seipsos eandem proportionem sortientur, quam habebunt eorum resistentiae converso modo […]."

[11]It should be remarked that this algebraic rendering of Benedetti's thoughts does little justice to contemporary efforts to represent functional dependencies among different magnitudes with the help of the available mathematical techniques, such as compound proportions, as will become clear from the following discussion; see also Damerow, Freudenthal, et al. 2004.

[12]Benedetti 1585, 170.

[13]In this passage, Benedetti employs the word *pondus*, but in the discussion he also talks of "lightness"; therefore, we prefer to use the term *virtus movens* or, more briefly, *virtus*, which is the term used by Benedetti.

> Solids heavier than the fluid, when thrown into the fluid, will be driven down as far as they can sink, and they will be lighter in the fluid by the weight of a portion of the fluid having the same volume as the solid.[14]

The actual *virtus* (either weight or lightness) of a body results by subtracting from the total virtue (*virtus totalis*) a quantity which Benedetti calls *resistentia extrinseca* (resistance, for instance in disp. 9) and depends on the *densitas* (density, for instance in disp. 3), which we could regard as an expression indicating the specific weight:[15]

$$virtus\ in\ medio = virtus\ totalis - resistentia$$

Accordingly, he holds the position against Aristotle that the ratio of the speeds in different media is not inversely proportional to the densities of the media but directly proportional to the *virtutes* in different media.

$$\frac{speed\ in\ A}{speed\ in\ B} = \frac{virtus\ totalis - resistance\ of\ A}{virtus\ totalis - resistance\ of\ B}$$

Given this equation, Benedetti shows (chapters 3–4) that the thesis of *Physics* IV 8, namely that the ratio of the speeds is reciprocally proportional to that of the densities of the media, is only true in a special case, which can be constructed *ad hoc*: if the ratio of an object's weight (or the corresponding speed) in one medium (for instance air) to its weight in another medium (for instance water) is equal to the ratio of the first medium (air) to the second (water). However, it is incorrect to claim that the ratio of the speeds of all bodies moving through different media remains the same. In chapter 6, Benedetti demonstrates moreover that the ratios of the weights (or speeds) of a body through different media are not maintained as constant in different media (*Quod proportione ponderum eiusdem corporis in diversis mediis proportiones eorum mediorum densitatum non servant. Unde necessario inaequales proportiones velocitatum producuntur*).[16]

We should now add some considerations on Benedetti's argumentative strategy. He begins chapter 2 with a reference to principles generally taken for granted: *primo verissima et obiecta intellectus per se cognita*. The adverb "primo" can have two meanings in this context: it could indicate either that the author is proposing the "first" principles of the motion theory or, quite on the contrary, that he is presenting theses that are universally true "only at a first glance." These "very true assumptions," these *primo verissima*, are in fact the Aristotelian statements concerning the proportion of the ratios of velocities and weights as well as the proportion of the ratios of velocities and resistances. In the second part of chapter 2, Benedetti brings forward his own conception of the proportion between the ratio of velocities and that of weights in a medium. He begins the paragraph on his theory with these words: "Aliud quoque supponendum est." This *incipit* can be interpreted in two different and rather opposite ways: as "Another proposition must also be presupposed" (which is Drake and Drabkin's translation) or as "We must presuppose a different proposition." As a matter of fact, the thesis that follows these words, that on the subtraction of the resistances from the total *virtutes*, is meant as a general truth. Benedetti himself intends to show that the Aristotelian laws are not true universally but only for

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[14]Dijksterhuis 1956, 376.

[15]See Benedetti 1585, 174: "dictis corporibus subtrahitur proportio resistentiarum extrinsecarum." For a discussion of Benedetti's employment of the termini 'pondus,' 'densitas,' and 'resistentia,' see Helbing 1987, 155–168.

[16]The details, the text, and the notes in the English edition by Drake and Drabkin are clear enough, so we will not expand on these aspects here.

special cases. Benedetti presents his theory of motion in opposition to that of Aristotle, so we tend to interpret chapter 2 as an opposition between two contrasting perspectives. Yet a certain ambiguity in his words cannot be denied. Perhaps it is a rhetorical strategy consciously employed by Benedetti: he first presents to the reader some propositions with which natural philosophers should be familiar, and then leads his reader to reject these common statements as not universally true rather than absolutely false.

### 7.2.3   Motion *in pleno* and *in vacuo*

After these considerations on motion in different media, Benedetti deals with the motion of bodies with different dimensions (but of the same material) through a medium as well as through the void. Chapter 7 considers the case in which two bodies A and O, made of the same matter and having the same figure, traverse the same medium. According to Aristotle the ratio of their speeds should be directly proportional to that of their weights. Benedetti demonstrates that this is false, since one could imagine a third body U, made of a different material, with the same *virtus* (here: *gravitas*) as O and the same dimensions (*magnitudo et figura*) as A. If V is the speed, M the *magnitudo* corresponding to a certain weight (or more in general, a *virtus*), then:

$$\frac{VA}{MA} = \frac{VO}{MO}$$

(according to Aristotle)[17]

$$\frac{VA}{MA} = \frac{Vu}{Mu}$$

(case of two bodies with equal dimensions and different weights).

Since by assumption MO = Mu, it follows that Vu = VO, but this is not consistent with the assertion that two bodies of equal weights and different dimensions will move with different speeds in a medium because a bigger body needs more "force" to penetrate the medium than a smaller one:

> The body which, when compared to the other, is of equal weight or lightness, but is subject to smaller resistance, moves [in natural motion] more swiftly than the other in the same proportion as its surface is subject to a smaller resistance than that of the other body, by reason of its being able more readily to divide the continuity of the air or water.[18]

Therefore, the ratio of the velocities of two bodies having equal material composition and figure but different dimensions is untenable, *pace* Aristotle.

Chapter 9 introduces the issue of motion through the void, asserting, against Aristotle, that the ratio of two bodies *in pleno* and *in vacuo* cannot be the same:

> In a plenum the ratio of the external resistances in the case of these bodies is subtracted from the ratio of the weights, so that the ratio of the speeds

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[17]It corresponds to the above-mentioned equation: $\frac{V1}{P1} = \frac{V2}{P2}$.

[18]Drake and Drabkin 1969, 198. See Benedetti 1585, 169: "Corpus illud quod alteri comparatum, aequalis erit ponderis, aut levitatis, sed minoris resistentiae, existet velocius altero, in eadem proportione, cuius superficies resistentiam suscipit minorem ea quae alterius est corporis, ratione facilioris divisionis continuitatis aeris, aut aquae […]."

remains. And this last ratio would be annuled if the ratio of these resistances were equal to the ratio of the weights.[19]

The argumentation is not immediately intelligible to the modern reader. Contrary to appearance, in fact, this passage does not describe the following subtraction: $\frac{P_1}{P_2} - \frac{R_1}{R_2}$ (V is the speed, P the weight, and R the resistance).

$$\frac{V_1}{V_2} \not\sim \frac{P_1}{P_2} - \frac{R_1}{R_2}$$

According to the theory of proportions, the addition and subtraction of a ratio to or from another ratio can be conveniently represented in a modern fashion as follows:[20]

I. Meaning of ratios addition:

$$\frac{a_1}{a_2} + \frac{b_1}{b_2} \sim \frac{a_1 b_1}{a_2 b_2}$$

II. Ratios subtraction:

$$\frac{a_1}{a_2} - \frac{b_1}{b_2} \sim \frac{a_1 b_2}{a_2 b_1} \sim \frac{a_1}{a_2} \frac{b_2}{b_1}$$

Given this, the relation indicated by Benedetti in the above-mentioned passage can be rendered through the following symbolic expression:

$$\frac{V_1}{V_2} \sim \frac{P_1}{P_2} \frac{R_2}{R_1}$$

This relation expresses in a concise form the Aristotelian perspective concerning the relation of velocities, weights, and resistances, as already taken up by Benedetti at the beginning of the *disputatio* number two. Benedetti notices that the ratio of the velocities is annuled (*nulla esse*) if the ratio of the *virtutes* is equal to that of the resistances ($\frac{P1}{P2} \sim \frac{R1}{R2}$). In fact, in this case (and only in this case) the speeds of bodies with different weights would be the same. As a consequence, there would be "no proportion" between different quantities but rather an equality of speeds. Benedetti indicates that this special case of motion *in pleno*, in which bodies with different weights have equal speeds, is the rule for motion *in vacuo*.

Chapter 10 expands on motion *in vacuo*. It presents the famous thesis that bodies of the same material move with the same speed *in vacuo*, although they might have different dimensions. Benedetti remarks that a body O can be divided into halves A and E, each of the same weight (or *virtus*). If an ideally weightless bar connects them, the weight at the center of the connection should be the sum of the partial weights and thus equal O. Therefore, bodies composed of the same material will fall with the same speed, independently of their weight. In chapter 11, one reads that motion *in pleno* is different as a consequence of the friction of the medium, except for the special case in which the parts travel through

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[19]Drake and Drabkin 1969, 205. See Benedetti 1585, 174 (emphasis is ours): "In pleno dictis corporibus *subtrahitur* proportio resistentiarum extrinsecarum a proportione ponderum, ut velocitatum proportio remaneat, quae nulla esset, si dictarum resistentiarum proportio, ponderum proportioni aequalis esset, et hanc ob causam diversarum velocitatum proportionem in vacuo haberent ab ea, quae est in pleno."

[20]The same concept of addition and subtraction of ratios can be found in the manuscripts of Thomas Harriot, who called them "compositio" and "subductio" (or "compositio contraria"). See for instance Schemmel 2008, 635, reference letters *g* and *o*.

media whose resistances have "the inverse proportion of the weighs" (*si duo corpora…
suas resistentias ad invicem proportionatas haberent, ut sunt eorum pondera*).[21]

### 7.2.4 A Note on the Historical Relevance of Benedetti's Reflections on Motion

Our analysis of the first part of the *Physical Disputations* should be sufficient to under-
stand Benedetti's approach to motion and the scope of his investigation. Subdivision two
tackles Aristotle's *Physics* VII 5 and deals with the proportions of velocities, volumes,
and surfaces.[22] We are not going to analyze in further detail Benedetti's views on mo-
tion. Instead, we deem it important to consider the "philosophical" part of the *Physical
Disputations* (which we have indicated as section 3), as it introduces novel prospects on
the foundations of physics. It extends from chapter 19 to chapter 22 and is an attempt to
revise basic concepts of physics from an anti-Aristotelian perspective.

Before we tackle these foundational aspects, we would like to recount the influence
that Benedetti's theory of motion exerted on the young Galileo. We have already hinted
at Benedetti's triangulation with Galileo via Del Monte. At a theoretical level, one of the
strongest pieces of evidence of this influence is the affinity between Galileo's early specu-
lations on motion and the theses that Benedetti propounded in the *Diversae speculationes*.
This link is evident and well known, beginning with the hydrostatic analogy to explain the
motion through a medium, the relativity of heavy and light, and the subtraction of resis-
tance from weight, which allows motion through a vacuum to be accepted and makes it
physically plausible.[23] For instance, several chapters of Galileo's first manuscript among
those gathered by Favaro under the title of *De motu* are very close to Benedetti's treat-
ment; above all Galileo's chapter 8, "in which it is shown that different bodies moving in
the same medium maintain a ratio [of their speeds] different from that attributed to them
by Aristotle"; chapter 10: "in which, in opposition to Aristotle, it is proved that, if there
were a void, motion in it would not take place instantaneously, but in time"; chapter 12: "in
which, in opposition to Aristotle, it is concluded that the absolutely light and the absolutely
heavy should not be posited; and that even if they existed, they would not be earth and fire,
as he believed"; and chapter 15: "in which, in opposition to Aristotle, the conclusion is
reached that rectilinear and circular motions have a ratio to each other." This connection
between the work of the young Galileo and Benedetti's insights on motion is significant,
the more so since Galileo's patron Del Monte was skeptical about the possibility of quanti-
fying motion.[24] Actually, in his manuscripts, Del Monte took some annotations on falling
bodies in different media. This might have been an issue of discussion between him and
Galileo.[25]

Moreoever, it should be remarked that the Copernican cosmological element is absent
from Galileo's early manuscript *De motu*, although this would become a crucial aspect of

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[21] See Drake and Drabkin 1969, 206, n. 119: "Benedetti asserts that the speeds are equal *only in the void*, on
the ground that in a plenum there would be an additional frictional resistance that would disturb this equality
(unless this resistance itself were proportional to the weights of the bodies: Ch. 11)."

[22] See Helbing 1987, 162.

[23] Apart from Koyré 1986, see also Drabkin, "Introduction" to Galilei 1960, 9–10.

[24] The ongoing debate between Galileo and Del Monte on the possibility of developing a mathematical dy-
namics is seen in Galileo's letter of November 29, 1602: "V.S. Ill.ma scusi la mia importunità, se persisto
in voler persuaderle vera la proposizione dei moti fatti in tempi uguali nella medesima quarta del cerchio;
perché essendomi parsa sempre mirabile, hora viepiù mi pare, che da V.S. Ill.ma vien reputata come impos-
sibile: onde io stimerei grand'errore e mancamento il mio, s'io permettessi che essa venisse repudiate dalla
di lei speculazione, come quella che fusse falsa." See Galilei 1968, 97.

[25] See Tassora 2001, 281–283.

his later investigations. Also, the alliance of mechanics and Copernican astronomy, which emerged only later in Galileo, bears witness to Benedetti's influence on his work.[26]

## 7.3    On the Void: Atomistic Prospects

Benedetti's considerations on motion are followed by a treatment of the void which, from the perspective of Aristotle's *physics*, is directly connected with the former issue. His theory of motion through media and through the void is the basis upon which he casts Aristotle's rejection of the void into doubt. Chapter 19 of the *Physical Disputations*, *Quam sit inanis ab Aristotle suscepta demonstratio quod vacuum non detur*, is a transition from the Archimedean theory of motion (chapters 1–18) to the reconceptualization of natural philosophy in general. The discussion on the void is directed against Aristotle's *Physics* IV 8, in which the theory of motion serves to reject the physical void and infinity. Benedetti claims that the void is conceptually possible as a consequence of his previous demonstration that the Aristotelian assumptions on the relation between speed and density of the medium are wrong or, at least, not universally valid. "*Ex iis, quae superius demonstravimus facile cognosci potest irritam esse eam rationem, quam Aristoteles 8 cap. lib. 4* Physicorum *ad destruendum vacuum, confinxit*" (From the demonstrations above it is easy to see that the argument that Aristotle devised in Physics VIII 4 is vain).[27] According to Benedetti's Archimedean dynamics, motion through a void is not absurd at all. He explains that such a motion is simply quicker than one taking place through any medium, because no resistance has to be subtracted from the body's *virtus*.

Hence, Benedetti reintroduces the concept of the void into physics, which was excluded by Aristotle as a criticism of Democritean atomism.[28] Benedetti's move is in line with the rehabilitation of atomistic philosophy during the Renaissance and the early modern period. The void and atoms are the two ingredients of the same matter theory inspired by the rediscovery of Lucretius and a reassessment of Epicureanism.

Among Benedetti's contemporaries the most committed natural philosopher supporting atomism was Bruno. Although they probably developed their theories independently of each other, they have many points of contact. In his Italian philosophical dialogues (published in London in the years 1584–1585), Bruno widely drew on ancient atomistic doctrines. He called Democritus and Epicurus those "who contemplated nature with open eyes" (*con occhi più aperti han contemplata la natura*).[29] He derived from them (often relying on Lucretius) a vision of the universe as infinite, the idea of a countless number of worlds (similar to the solar system), as well as the principle of cosmological homogeneity and the atomic theory of matter and of minima moving through a void. In *De l'infinito universo e mondi*, he celebrated Epicurus's theory of matter, summarizing it as follows:

> Epicure similarly nameth the whole and the universe a mixture of bodies and of the void; and in this universe and in the capacity thereof to contain the void and the empty, and furthermore in the multitude of the bodies contained therein he maintaineth that the nature of the world, which is infinite, doth exist.[30]

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[26]Drabkin 1964, Drake 1976, and Renn and Damerow 2012, 142–155.
[27]See Benedetti 1585, 179.
[28]On the medieval debates on the void and on the historical place of Galileo, and Benedetti before him, see Grant 1981, 60–66.
[29]Bruno 2000a, 374.
[30]Bruno 1968, 272–273.

However, the concept of the void assumes in Bruno a very special meaning since, in his philosophical terminology, the terms 'space,' 'void,' and 'aether' are used as synonyms. The ethereal void is the medium that makes the motion of bodies possible.[31] In the second dialogue of *De l'infinito*, he recapitulates the following three meanings of *vacuo*:

- First, the void refers to all which is not bodily and thus does not have the property of resisting penetration. In this sense, there is a "boundless ethereal region" (*eterea regione inmensa*), in which the worlds are plunged. The countless worlds populating the universe are themselves composed of matter and the void.[32]
- Second, the void has the meaning of infinite space (*spacio infinito*). In it everything is included. It is the container that cannot be included in anything else.
- Third, space can be assumed to be nothingness, in accordance with Aristotle. Bruno calls this a metaphysical meaning, or rather lack of meaning. According to him, this extra-physical meaning was a sophism that served Aristotle to refute the possibility of the void in nature. For Bruno, only the first and the second meaning of the void make sense. They could and should be embraced in natural philosophy. By contrast, the metaphysical void has to be rejected as meaningless.

Bruno's reflections on the physical void are part of his program to revive an atomistic conception of nature. He combined it with cosmological infinity and the post-Copernican idea that a plurality of worlds exists, each one of them a solar system centered on a star similar to ours. This vision, which shows similarities to Benedetti's, paved the way for seventeenth-century cosmologies, in particular for the Cartesian multi-centric universe.

Other Renaissance scholars supported combinations of atomism and heliocentric astronomy as well. A case in point is the circle of scientists gathered around Henry Percy of Northumberland in London. Just like Benedetti and Bruno, they brought together heliocentrism, atomism, an empirical and mathematical approach to nature, Renaissance naturalism, and anti-Aristotelianism.[33] Thomas Harriot, for example, was an atomist and a supporter of the infinity of the universe.[34] Nicolas Hill, another member of the "Northumberland circle," authored an apology of Epicureanism entitled *Philosophia Epicurea* (1601), which was directly inspired by Bruno. It included Copernican arguments in favor of terrestrial motion and many others in favor of the earth's magnetism, in accordance with Gilbert.[35] Following ancient and modern atomists, Hill affirmed the boundlessness of the universe and the plurality of worlds.[36]

Atomism was to be embraced by celebrated exponents of sixteenth-century science and philosophy such as Galileo and Pierre Gassendi. At the same time, corpuscular alternatives were spread by Descartes and his followers.[37] Atomism and, more generally, corpuscular theories of matter encountered much censorship, especially owing to theological difficulties, since they appeared to be irreconcilable with Scholastic accounts of the

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[31] See Del Prete 1999, 61 and Michel 1962.

[32] Bruno 2000a, 348: "[…] perché questo spirito, questo aria, questo etere non solamente è circa questi corpi, ma ancora penetra dentro tutti, e viene insito in ogni cosa."

[33] Kargon 1966.

[34] On his intellectual stature and achievement, see Schemmel 2008, who stressed that Harriot's unpublished manuscripts reveal that his research activity was similar to that of Galileo in the same years. Moreover, Harriot's reflections on infinity and the minimum bear witness to his familiarity with the work and ideas of Bruno. See Fox 2000 and Henry 1982.

[35] N. Hill 2007, 155–157. See Plastina 1998, Plastina 2004 and Omodeo 2014a, 372–377.

[36] For an overview of English science in that period, see C. Hill 1997, 15–76 and 118–200.

[37] On medieval and early-modern corpuscular matter theories, see Lüthy, Murdoch, and Newman 2001. On Galileo, see Galluzzi 2011.

Eucharist.[38]  Nonetheless, many modern scholars were attracted, just as Benedetti was, to the prospect of connecting the advances of mechanics with a physics and a theory of motion rooted in corpuscularism alongside a post-Copernican cosmological outlook. As Galluzzi has recently pointed out, this was a guiding idea for Galileo already in his *Iuvenilia*.[39] Theories of motion, atomism, and Copernican astronomy are three interconnected issues in his as well as Benedetti's work. In *De motu*, Galileo was committed to the homogeneity of matter and reflected on it in connection with motion, in particular with the Benedettian thesis that "*si vacuum esset, motuum in instant non contingere, sed in tempore*" (If the void exists, motion does not occur instantaneously but rather in time).[40] In the tenth section of this manuscript, Galileo followed in Benedetti's footsteps and came to the same conclusion: "Hence it follows, not that motion in a void is instantaneous, but that it takes place in less time than the time of the motion in any plenum."[41]

Benedetti shared many ideas with contemporary and later scholars in his criticism of Aristotle's natural philosophy as well as in his effort to construct a new physics. Whereas he developed his conceptions on matter and the cosmos independently of Bruno, and probably did not directly influence the English circle of Harriot and Hill, nonetheless he must have influenced the views of Italian scholars such as Galileo who were familiar with the *Diversae speculationes*. Benedetti showed them that a revision of dynamics could not be fulfilled independently of a broader program of philosophical reform.[42]

## 7.4   On Place: Space as *intervallum corporeum*

Chapter 20 of Benedetti's *Physical Disputations* deals with the concept of place. Aristotle, in *Physics* IV 4, defines it as the surface in which a body is included and denies that it could be conceived as an *intervallum*. By contrast, Benedetti defines it precisely as *intervallum corporeum*, an expression which could be roughly translated as an inter-bodily gap. Aristotle says that *locum* (the place) and *locatum* (the placed) must be coextensive but, as Benedetti notices, a spherical surface contains more "extension" (*intervallum*) than any other figures with an equal surface. As a consequence, one can imagine two places (in the sense given to the term by Aristotle) occupied by bodies of different dimensions or, the other way round, two bodies of different dimensions which suit the same surface. By contrast, an *intervallum* of space contains only bodies of equal volume, thus respecting the bijective relation between the volume of the place and that of the placed body: "But equal bodily extensions [*intervalla*] delimitated by any figure will always contain equal bod-

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[38]See Redondi 1983, chap. 7, 203–226. Also see Ariew 2013.

[39]Galluzzi 2011, 9.

[40]Galilei 1968, vol. I, 276.

[41]Galilei 1960, 47. See Galilei 1968, vol. I, 282: "[…] et ita non est necessarium, motum in vacuo fieri in instanti, sed in tempore minori quam sit motus in quovis pleno."

[42]Thus, Galluzzi's remarks on the scientific production of the young Galileo could be conveniently extended to Benedetti (Galluzzi 2011, 19): "Il laboratorio del *De motu antiquiora* servì a Galeo per prendere definitivamente coscienza che la cosmologia e la fisica aristoteliche non potevano essere riformate con interventi limitati ad ambiti di indagine circoscritti. In particolare, lo scritto pisano evidenziava la raggiunta consapevolezza dell'importanza strategica di una radicale riforma della concezione aristotelica del movimento. Era questo, tra l'altro, l'ostacolo più serio da superare perché potesse essere accolta la visione copernicana, intesa non come semplice ipotesi matematica, ma come dottrina fisica. […] Né si può escludere che abbia tratto anche da esse [da tali problematiche] l'impulso per dedicarsi, con l'impegno proprietario esibito dai documenti dell'attività giovanile, all'impresa ambiziosa di una radicale riforma della concezione tradizionale del movimento e della struttura della materia."

ies."[43] The definition of place as *intervallum* has the advantage that it allows us to assign a place to every body, "in the world or outside it, *pace* Aristotle."[44] This remark anticipates the cosmological treatment of infinite space, or of the possibility of space beyond our worldly system (*mundus*). In chapter 20, on place, Benedetti remarks that Aristotle is wrong when he rejects the concept of place as *intervallum* because it could lead to infinity. Benedetti objects that this is not inconvenient at all, since one could conceive of *infinita loca*. To grasp this infinity is no more difficult than to understand the never-ending process which Aristotle calls "potential infinity" (*infinitum in potentia*) and concerns the division of a body *ad infinitum*:

> That infinite places must exist, if place is [conceived of as] *intervallum*, is not inconvenient. In fact, just as any body can be [divided into] infinite bodies (which he [Aristotle] would call "*in potentia*"), so can any *intervallum* be [divided into] infinite *intervalla* as well.[45]

Benedetti is always brief in his treatment of philosophical issues and leaves much implicit. Therefore, the reader is challenged to reconstruct the complete vision implied in his short statements and critical points. Reference to Renaissance philosophical sources discussing the same theses from an Aristotelian viewpoint can help us to better understand Benedetti's conception. In this case, Alessandro Piccolomini's refutation of the conception of space as a receptacle of bodies can shed light on the opposite viewpoint defended by Benedetti. The following quotation is taken from the first part of Piccolomini's *Della filosofia naturale* (On natural philosophy), a very successful introduction to natural philosophy in Italian, which Benedetti might have known:

> Other philosophers (and not unimportant but very reputed ones) asserted that there is void space among natural beings and that it is not only distinguished in essence [*per sua natura*] from the bodies it accommodates, but also it is so separated from them that it can remain completely naked and really deprived of them. Furthermore, many supposed that such a space (which is completely void, without any bodies) is mixed and situated between the beings of this world and their parts. Additionally, they believed that it infinitely extends outside the heavens. Thus, these philosophers considered the void to be that being which not only can be deprived of all bodies and substances […] but also is apt and capable of receiving the bodies, giving them a place (something which does not occur to lines, points, and other accidents). One of the main reasons for holding such doctrines was, as they said, that, if there is no space in nature (or a void place without bodies), the motion from place to place would be impossible, although the motion of alteration would be possible. In fact, all beings, in order to move to some other place, need some in-between space through which they can move. If this space were not void, it would be necessary that, as it is filled with other bodies, different bodies

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[43]Benedetti 1585, 180: "Sed intervalla corporea aequalis a quavis figura terminata, continebunt semper corpora aequalia."

[44]Benedetti 1585, 181: "Et hoc modo nullum est corpus, quod in mundo aut extra mundum (dicat autem Aristoteles quicquid voluerit) locum suum non habeat."

[45]Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 180: "Quod si locus intervallum esset, infinita loca existerent […] inconveniens [non] existit, quia eodem plane modo quo aliquod corpus potest esse infinita corpora (quod ipse diceret in potentia), sic etiam intervallum aliquod posset esse infinita intervalla."

would penetrate each other while crossing that full space. And this is impossible.[46]

Thus Piccolomini presented the views of the supporters of a natural void in order to reject them. Yet his discussion is helpful as it indirectly presents us with the views of such thinkers as Benedetti who defended void and infinity. Benedetti was in fact favorable to the notions of the void and infinity. For him, space is prior to and independent of bodies. It should be remarked that, in Benedetti's perspective, infinite space does not imply cosmological infinity or the boundlessness of the material universe, precisely because space is independent of matter. In this respect, his conception is different from the one that Bruno defended in those years. Rather, it is closer to that of his correspondent Patrizi. As De Risi has pointed out, Patrizi's conception of space is marked by its ontological autonomy from bodies. It is in fact an "incorporeal and immaterial extension, tridimensional and infinite, which receives and precedes all created bodies."[47] Patrizi saw space as a sort of Plotinian *hypostasis* (*extensio hypostatica*), that is, a pre-corporeal quantity quantifying reality. He saw this idea as the lever to be employed in order to radically reform Scholastic metaphysics, a project that he developed in a series of publications, *De spacio physico et mathematico* (On physical and mathematical space, 1587), *Della nuova geometria* (On the new geometry, 1587), and eventually in the *Nova de universis philosophia* (1591).[48] It also served him to set the foundations of an epistemology of mathematics according to which the ancient science dealing with figures had to be substituted for a new science of space itself. This is the concept of his *New Geometry*. Patrizi sent a copy of this book to Benedetti, probably before the publication, to gain the favor of Carlo Emanuele I, to whom the book was dedicated.

To sum up, Benedetti defines the *locus*, against Aristotle, as *intervallum corporeum*, or bodily extension. He regards it as an invariable empty extension capable of being occupied by material bodies, an idea that becomes clearer from the related discussion on time. He basically rejects the Aristotelian definition of place as the *superficies corporis ambientis* (surface of the containing body), remarking that the place is a measure, actually an *intervallum*, and not a surface. Moreover, he explains that only his understanding of locus as a kind of objective space guarantees that two bodies of equal material constitution necessarily occupy the same quantity of "place" as *intervallum* or empty extension, independently of their form. By contrast, the definition given by Aristotle is inconsistent with the assumption of a bijective relation between *locus* and *locatus*. In fact, as geometry shows, if one accepts Aristotle's definition "a great absurdity [*maximum inconveniens*]

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[46]Piccolomini 1585, I 3, chap. 5, f. 44*r*: "Altri filosofi sono stati poi, et non già minimi, ma di gran fama, che han voluto che tra le cose della natura, si trovi spatio voto, non solamente distinto per sua natura dai corpi ch'egli riceve; ma intanto separato da quelli, che ignudo in tutto, et attualmente spogliato ne può restare. Né son mancati molti, che cotale spatio, voto in tutto d'ogni corpo, han posto, non solo meschiato, et interposto tra le cose che sono al mondo, e tra le parti loro; ma ancora fuor dei Cieli, infinitamente han creduto che si distenda. Questi filosofi adunque per il voto intendevano quella cosa, che non solo fusse privata d'ogni corpo et d'ogni sostanza […] ma fosse ancora atta, et capace, a ricevere i corpi, et dar lor luogo, il che delle linee, et de punti, et altri accidenti, non avviene. Tra le ragioni principali, che gli spingeva a credere una tal cosa, l'una era questa nella qual dicevano, che se non si potesse trovare tra le cose della natura spatio alcuno, o luogo voto d'ogni corpo; allora, se bene il movimento dell'alteratione potrebbe forse restare al mondo, tuttavia il movimento di luogo a luogo, non potrebbe senza 'l voto restar giammai. Conciosia c'havendo bisogno quella cosa, che ha da muoversi ad alcuno altro luogo, di alcuno spazio di mezo, per il quale si muova; se cotale spatio non fosse voto, saria forza che essendo pieno di altro corpo, nel passar per quello spatio pieno, venissero a penetrarsi più corpi insieme; adunque è cosa al tutto impossibile."
[47]De Risi 2014, 282.
[48]De Risi 2014, 276–277.

would follow, namely that equal places can contain unequal bodies or equal bodies can occupy unequal places."[49]  Additionally, Benedetti remarks that his concept of place admits an infinite universe, since it is capable of containing *infinita corpora* (infinite bodies).

## 7.5   In Defense of Infinity

Chapter 21 is devoted to infinity: *Utrum bene Aristoteles senserit de infinito* (Whether Aristotle judged correctly about infinity).  Needless to say, Benedetti holds that Aristotle's arguments on this issue are idle.  The Greek philosopher rejects the possibility of an infinite body (*infinitum corpus*), that is to say, the infinity of the universe because—as the argument goes—there is no *locus infinitus* which could include it.  Benedetti protests that this is a *petitio principi* since this rejection is based on a questionable definition of *locus*: "*cum Aristoteles debuerit beneficio loci destruere infinitum, ordine perverso de infinito prius*" (Since Aristotle had to destroy infinity with the benefit of [a correct understanding of] place, he wrongly started with [a discussion of] infinity).[50]  By contrast, Benedetti's redefinition of place as *intervallum* entails no conceptual hindrances to the acceptance of the infinity of the universe.  As one reads: "*hoc modo nullum inconveniens sequeretur, quod extra caelum reperiri possit corpus aliquod infinitum*" (In this manner it is not inconvenient to assume that one can find an infinite body outside the heavens).[51]

Additionally, Aristotle writes (*Physics* VIII 8 and *De caelo* I 9) that a continuum can include infinite parts only *in potentia* (potentially) and not *in acto* (in act).  Benedetti does not agree with him.  He argues that, if the continuum is *in acto*, its infinite parts should be *in acto* as well, because it is foolish (*stultum*) to believe that something actually existing could be composed of parts which exist only potentially (*quae potentia existunt*).

The weakest argument brought forward by Aristotle is, according to his critic, that the infinite cannot be considered to be a quantity (*Physics* III 5), because only a finite quantity is thinkable, for instance the dimensions of a square or a cube.  Benedetti objects that the definition of number (which he does not provide, however) does not include finitude (*necessitas terminorum*).  Numbers are not necessarily terminated (*determinati*) and one can conceive an infinite multitude exactly like a finite one:

> Aristotle's arguments in the same part of *Physics* III 5 are even worse.  He denies that infinity can be regarded as a quantity by saying that only one defined quantity is intelligible, such as a cubit, a three-cubit, etc.  Thereby, he does not consider that in the same manner the quantity of infinite cubits is intelligible as well.  Moreover, the definition of quantity does not imply limitation by necessity.  For instance, the definition of number does not imply the necessity of any determined number, since an infinite multitude is not less intelligible than a finite.[52]

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[49]Benedetti 1585, 180: "maximum inconveniens sequeretur, scilicet aequales locos capere inaequalia corpora, aut corpora aequalia, locos inaequales occupare."

[50]Benedetti 1585, 181.

[51]Benedetti 1585, 181.

[52]Benedetti 1585, 181: "Sed peius etiam sensit Aristotles eodem loco capitis quinti lib. 3 Physicorum, negando infinitum posse connumerari inter quantitates, dicens unam aliquam quantitatem intelligi ut cubitum, tricubitum, et cetera; ubi non considerat eadem etiam ratione intelligi posse aliquam quantitatem infinitorum cubitorum, et in quantitatis definitione nullam esse necessitatem terminorum, ut exempli gratia in definitione numeri, non est necessitas alicuius determinati numeri, quia multitudo, non minus infinita, quam finita, intelligi possit."

The last false Aristotelian assumption on infinity is the equation of *infinitum* and *vacuum* in *Physics* IV 8. Benedetti's commentary is harsh: "Later, in *Physics* IV 8, he says that there is no difference between infinity and the void. Indeed, he could not assert and imagine anything more absurd than this."[53]

Like Bruno, the infinitist *par excellence*, in *De l'infinito universo e mondi* (1584), Benedetti remarks that Aristotle's arguments are not compelling. Aristotle denied the possibility of an unbounded space on the basis of a finite cosmology (i.e., the theory of the spherical and geocentric heavens along with the theory of the natural places), which excludes cosmological infinity by definition. Yet his assumptions are not self-evident.

In summary, in chapter 21 of the *Physical Disputations*, Benedetti defends the mathematical and physical possibility of the infinite. The question of the title "Whether Aristotle Judged Correctly about Infinity" is rhetorical. It calls for a negative answer.

## 7.6   On Time: Toward an Absolute Frame for Physics

Benedetti's definition of *tempus* (time) is closely connected with that of *locus* (place). He deals with it in *Physical Disputations*, chapter 22, *Exagitatur ab Aristotele adducta temporis definitio* (Rejection of Aristotle's Definition of Time). Benedetti questions the definition of time as *motus mensura numerusque* (the measure and number of motion) and offers an alternative conceptualization. But before we discuss his opinion, we will recount standard conceptions of time from antiquity.

It should be noted as a preliminary remark that the understanding and definition of time was regarded as a natural as well as a psychological issue. This should not be surprising, since the doctrine of the soul, or *psychologia*, was an integral part of natural philosophy, or *physica*, in standard university curricula. A standard reference on time was the *Timaeus*, where Plato called time, rather poetically, the "moving image of eternity" (εἰκὼ κινητός αἰῶνος).[54] According to his myth, the divine Demiurge created time together with the heavens (οὐρανὸς), making them inseparable. Plato's pupil Aristotle, in Book 4 of the *Physics*, then proposed the definition of time as the "dimension of movement in its before-and-afterness."[55] He also noticed that χρόνος (time) had generally been connected with the motion of the celestial sphere and was referred first and foremost to the heavens because all measurements of motion and time depend on celestial regularities:

> Neither qualitative modification nor growth nor genesis has the kind of uniformity that rotation has; and so time is regarded as the rotation of the sphere, inasmuch as all other orders of motion are measured by it, and time itself is standardized by reference to it.[56]

For both philosophers, Plato and Aristotle, the tie between time and cosmic order was unquestionable. This interconnection was later refuted by a third authoritative source. At the downfall of the ancient world, Augustine, in Book 11 of the *Confessiones*, denied the commensurability of time and local motion: "The motion of a body is one thing, and that by which we measure how long it is, another thing."[57] He pointed out the transient

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[53]Benedetti 1585, 181: "Ubi postea cap. 8 lib. 4 Physicorum ait nullam esse differentiam inter infinitum, et vacuum, revera nihil absurdius hoc dicere fingereve poterat."

[54]Plato Timaeus 37C–39E

[55]Aristotle 1995, IV 11, 220 a 25–26 (395).

[56]Aristotle 1995, IV 14, 223 b 21–22 (425).

[57]Augustine 1988, XI 14, 265: "aliud sit motus corporis, aliud quo metimur quamdiu sit."

character of the temporal flux and noticed that the three dimensions of time are a product of our mind (*animus*). In the mind, time is constituted as the memory (*memoria*) of the past, the intuition (*attentio*) of the present, and the expectation (*expectatio*) of the future.[58] In other words, Augustine underscored the subjectivity of time, conceived of as a *distensio animi*, a "stretching" of the mind independent of heavenly cycles.

According to Benedetti's criticism of Aristotle, the definition of time as "the measure and number of motion" is intrinsically wrong because measuring presupposes commensurability. But because time and motion are heterogeneous, they cannot be compared. A line is measured by a line, a surface by a surface, and a *corpus*—that is, a three-dimensional body—by a *corpus*. Similarly, motion is measured by motion and not by time: "Time cannot be the measure of motion, but only motion can measure motion, precisely a faster one [measures] a less fast one, and a shorter one [measures] a longer one, whereas a number is measured by a number, and time by time inasmuch as it is long or short, and not inasmuch as it is fast or slow."[59] Hence, time can only be measured extrinsically (*per accidens*) through motion, as is the case with common expressions like "two hours, or two days, or two years," referring to astronomical displacements. These are only metaphors that refer to heavenly motions as "placed" in the interval of time that corresponds to their motion. In the following passage, presenting these reflections, time is called the "place of motion" (*locus motus*):

> It could seem to somebody that, to indicate [*significare*] a certain quantity of motion, one has to assume as much time as if one says, for instance, that a certain operation has been carried out in the space of two hours, or two days, or two years. Still, it should be remarked that this is not literally [*simpliciter*] true, since the imagination conceives the interval of two hours, days, or years as the motion of celestial bodies without which neither years, not days, not hours would exist, even though time is placed, so to speak, in time, just as a body in a place. Thus, motion is measured by motion and time by time, and not the one by the other.[60]

Benedetti claims that time, unlike motion, is eternal: "Necessarily, from a philosophical point of view, time is eternal and motion not."[61] Whereas a motion can be extinguished and a body can be at rest after a displacement, time goes on inexorably. It is always present to our senses and always escapes them because it is the never-ending passing of a single instant. Benedetti makes clear that he intends the *instans* to mean "one in species" (*unum in specie*), i.e., the essence underlying the vanishing flux of time that cannot be experienced in itself as a given and measurable succession (*non in numero*).[62] This Augustinian

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[58] Augustine 1988, XI 28.

[59] Benedetti 1585, 183: "Tempus non erit mensura motus, sed motus quidem potest mensurare motum, videlicet velocior minus velocem, et brevior longiorem; et numerus mensuratur numero, et tempus tempore in quantum longum est, aut breve, non in quantum velox, aut tardum."

[60] Benedetti 1585, 182: "Si alicui videtur, quod ad significandam aliquam quantitatem motus, dicere huiusmodi operationem duarum horarum, aut duorum dierum, aut duorum annorum spatio completam esse, sit ponere tantum tempus, animadvertere debet hoc simpliciter non esse verum, quia horarum, dierum, et annorum intervalla, imaginatione concipiuntur ut motus corporum caelestium, sine quibus neque anni, neque dies, neque horae existerent, etiam si omnis motus sit (ut ita dicam) locatus in tempore, ut corpore in loco, unde motus motu, et tempus tempore, non autem aliud ab alio mensuratur."

[61] Benedetti 1585, 182: "Tempus ex necessitate, philosophice tamen loquendo, res est aeterna, motus non item."

[62] Benedetti 1585, 182: "Tempus igitur potius locus motus erit dicendum, quam numerus aut mensura eius, et tale est, ut consumatum videatur a continuo quodam fluxu unius instantis […] et cum dico ab uno instanti,

paradox helps Benedetti to stress the heterogeneity of time and motion. Still, he does not renounce an objective meaning, which is essential to his dynamics. Motion and time, he writes, are *continua successiva*, continuous and successive quantities. Their relation can be conveniently described as that between a place and the bodies that it contains. His explanation sheds light on the concept of *locus* as well as on that of *tempus*:

> Just as a dense body occupies a lesser interval of place than a less dense [*rarus*] body, similarly a quick motion is accomplished [*peragetur*] in a shorter space of time than a slower motion.[63]

From this passage it is clear that Benedetti's idea of place as *intervallum corporeum* goes in the direction of an empty homogeneous space which can be occupied by bodies of greater or lesser density. Time has the same absolute character as place. It can contain more or less rapid motions without being affected by them. Space and time or, more precisely, *intervallum corporeum* and *tempus* are objective and independent coordinates of natural phenomena.

In order to understand Benedetti's considerations on time, it is useful to delve into the views of his immediate forerunners, in particular Cardano and Scaliger.

### 7.6.1 Cardano's Subtleties on Time

In the letter to the reader of the *Diversae speculationes*, Benedetti indicated Cardano as one of his principal sources.[64] However, concerning the concept of time their opinions are not in agreement. Cardano tackles the issue *tempus quid sit* (What is time?) in Book 18 of *De subtilitate*. Unlike Benedetti, he does not explicitly question Aristotle, but rather quotes his definition as a common truth: "in fact, motion is the measure of time" (*motus enim tempus est mensura*).[65] Still, he reworks the Aristotelian concept within a rather Augustinian perspective, which leads to original results.

He introduces the problem of time in connection with his treatment of the dream (*somnium*), beginning with the question: "Why does time appear much longer while dreaming than in reality?"[66] He reports that once he dreamed that he visited an unknown city far away from his home in Milan. He traveled through mountains, valleys, and fields. In order to cover that distance six days of travel would be barely sufficient. Thus, when he woke up, he thought that he had slept for a long time but in actual fact his nap had taken less than one hour. The reason for this expansion of time, Cardano explains, is due to the fact that dreamed activities (*operationes*) are accomplished independently of any bodily effort (*absque corporis labore*) and therefore very rapidly. A correct judgment about time depends on bodily movement. This is why the perception of time is not altered in our mind when we imagine long-lasting processes while awake. "During sleep, time is contracted in the opposite manner than if we are awake: in fact, motion is the measure of time."[67] Surprisingly, Cardano's Aristotelian conclusion, "motion is the measure of time," does

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unum in specie, et non in numero intelligo, quod a sensibus nostris percipi non potest, neque etiam notari, quia novum semper instans nobis occurrit."

[63] Benedetti 1585, 183: "Quemadmodum corpus densum occupat minus intervallum loci, contra quam fiat in corpore raro; sic etiam motus velox breviori temporis spatio peragetur, quam tardus."

[64] Benedetti 1585, *Ad lectorem*, f. A3*r*.

[65] Cardano 1966, vol. 3, 651.

[66] Cardano 1966, vol. 3, 651: "Cur somnium tempus longius multo ostendat quam sit."

[67] Cardano 1966, vol. 3, 651: "Contraria ratione tempus in somnio contrahitur, cum vel non somniamus […]: motus enim tempus est mensura."

not refer to heavenly motions, but to corporeal activity. In other words, he extrapolates and isolates the peripatetic definition from its original context. In fact, from an objective, physical, and cosmological meaning, time acquires a rather subjective meaning, connected with physiology and perception. Time, as Augustine would say, is an "expansion of the mind."

It should be added that there is a passage of the *Physics* where Aristotle also hinted at the subjective dimension of time, even at how the soul grasps it. This passage might have been a source of inspiration for Cardano:

> Time cannot be disconnected from change; for when we experience no changes of consciousness, or, if we do, and are not aware of them, no time seems to have passed, any more than it did to the men in the fable who 'slept with the heroes' in Sardinia, when they awoke; for under such circumstances we fit the former 'now' to the later, making them one and the same and eliminating the interval between them, because we did not perceive it. So, just as there would be no time if there were no distinction between this 'now' and that 'now,' there appears to be no time between two 'nows' when we fail to distinguish between them. Since, then, we are not aware of time when we do not distinguish any change (the mind appearing to abide in a single indivisible and undifferentiated state), whereas if we perceive and distinguish changes, then we say that time has elapsed, it is clear that time cannot be disconnected from motion and change.[68]

These considerations are not aimed at isolating a subjective meaning of time by eliminating any objective references. Still, it is plausible that Cardano took into account such passages in order to freely speculate on time, in Book 18 of *De subtilitate*, from a perspective that owes more to Augustine than to Aristotle.

This Augustinian influence is particularly evident from the following passage, in which Cardano tries to better define time:

> But what is time? Although nothing of it is ever [given], nonetheless every-thing is in it and it accompanies [*assistit*] everything always. It itself generates and destroys everything; it is the source [*auctor*] of life and death. Its expec-tation is always very long, while its memory is always very short. Although it is always with us, we never grasp it. Even though there is such an abundance of it, nonetheless no restoration [*reparatio*] of it is ever conceded, thence the waste of no other thing is greater or worse.[69]

In this passage, Cardano brings together ideas derived from erudite lectures, as well as from commonsense, experience, and even trivial commonplaces. Additionally, he re-counts the Augustinian paradoxes: time is everywhere and nowhere, it is made out of expectation and memory, and it is for us the most familiar and unknown mental presence. To quote from the *Confessiones*: "What is time then? If nobody asks me, I know: but if I

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[68] Aristotle 1995, IV 11, 218 b 20–219 a 1 (383).

[69] Cardano 1966, vol. 3, 651: "Sed quid tempus est? Cuius cum nihil unquam sit, omnia tamen in illo sint et semper omnibus assistit. Illud idem omnia generat et occidit, auctor vitae ac mortis. Utque illius expectatio longissima, ita semper memoria brevissima. Cumque nos semper comitetur, nunquam ipsum tamen agno-scimus. Nec cum eius tanta sit copia, reparatio tamen ulla conceditur: unde fit, ut nullius alterius rei iactura sit maior et vilior."

were desirous to explain it to one that should ask me, plainly I know not."[70] Cardano adds to this paradox a popular sense of the caducity of life, according to which a discourse on time is a kind of *memento mori*. Time itself is said to be the cause of life and death.

Hence, for Cardano, the assumption that "time is the measure of motion" does not mean maintaining the cosmological dependency of time on astronomical cycles. Quite to the contrary, time transcends motions and changes. In fact, we do not perceive it in itself but rather that which happens in it. What we know about time is a product of the mind, precisely of its imaginative faculty:

> Thus, we do not comprehend time but rather that which occurs, or occurred, and endures in it. But time itself per se is out of reach [*ignotum*] to the senses. In fact all that we know [about it] is constituted through imagination.[71]

Like Benedetti, Cardano denied the interdependency of cosmological space and time, although from a different standpoint. He was not concerned with reformulating the physical space and time framework of motion in mathematical terms. Instead, he concentrated on the psychological and physiological dimension of time as a construction of the *imaginatio*. Therefore, this treatment was connected with that of the mind and was introduced by considerations on sleeping and dreaming. This even led to considerations on altered states of mind such as ecstasies and hallucinations, as well as the divine and demoniac visions of the hermits—Cardano went so far as to report some hallucinations that he had when he was a child. His mental treatment of time, as well as this excursus on altered states of mind, were both harshly criticized by the Flemish humanist Julius Caesar Scaliger, as we shall now discuss.

### 7.6.2 Scaliger's Aristotelian Restoration

Scaliger conceived of his *Exotericae exercitationes* as a critical confrontation with Cardano's *De subtilitate*, although he formally presented them as a reverent discussion of some points that arose from the lecture on the "subtleties" of that *doctissimus vir*, as one reads in the dedicatory epistle, "who will never be praised enough" (*nunquam satis laudatus*). Among the most notable theses indicated in the *Index acutiorum sententiarum* at the end of the book, one is directly relevant to our discussion: the *exercitatio* number 352.2, which explains why "time is not the measure of motion." That *exercitatio* deals with the passage of *De subtilitate* on time that we have discussed, but reverses Cardano's viewpoint.

Scaliger tries to answer the question of "whether time is the measure of time," remarking that most people just repeat this definition without properly understanding its meaning.[72] The allusion is clear: Cardano repeats a commonplace without any thorough reflection on its meaning. In fact, even though time might be regarded as the measure of "our motion," it is definitely not the measure of worldly motion, in particular not of the first motion of the heavens, i.e., the daily one.[73] Scaliger therefore denounces Cardano's

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[70]Augustine 1988, XI 14, 238: "Quid est ergo tempus? Si nemo ex me querat, scio; si querenti explicare velim, nescio."

[71]Cardano 1966, vol. 3, 651: "Ergo nos non tempus, sed quod in eo fit, factumque est, atque manet, comprehendimus. Tempus vero ipsum per se est sensui ignotum: nam imaginatione constat quod a nobis cognoscitur."

[72]Scaliger 1557, f. 458*v*: "Hoc quidem ab omnibus iactatur: pauci introspectant."

[73]Scaliger 1557, f. 458*v*: "Nostri sane motus mensura sit: primi motus mensura non erit."

subjective reading of the Aristotelian definition. According to him, the objective dimension of time cannot be dismissed. From a cosmological-ontological perspective it is in fact a dependent (*affectus*) of heavenly motion;[74] to be precise, it descends from the "first motion," or the daily rotation of the starry sphere. The first body (*primum corpus*), that is, the heaven of the fixed stars, is the measure of all bodies. Similarly, its motion, the *primus motus*, is the yardstick of all motions. In accordance Book 12 of the *Metaphysica*, where Aristotle advocates a spherical and geocentric cosmology of concentric spheres, Scaliger states that "time is either the same thing as motion or its affection."[75] In other words, he intends to restore an Aristotelian objective conception of time in accordance with a metaphysical perspective that attaches an ontological priority to motion over time.

Scaliger acknowledges that Aristotle ambiguously defined the interrelation between time and motion by accepting both possibilities: "time is the measure of motion and, in turn, motion is the measure of time."[76] In his attempt to dispel this paradoxical interdependency, Scaliger distinguishes between two fundamental dimensions of time: the objective and the subjective dimension or, as one reads, the understanding "according to nature" (*a Natura*) and that "according to us" (*mensura nostra*). In nature, motion is the measure of time as well as its source. By contrast, time is the measure of motion only for us, which is an aspect that Cardano allegedly overemphasizes. This is clearly documented by expressions like "the equinoxial circle accomplishes its motion in this much time."[77] In actual fact, things are the opposite of what our way of speaking suggests: time is derived from the translation from "here" (*ubi*) to "there" (*ubi*) in space (*in loco*). The *ubi* (where), to which people improperly refer in order to express a quantity of time, is a naive way of thinking that reduces time to certain spatial determinations. As to the definitions: time is a transient "now" (*nunc*), whereas place (*locus*) is a continuous "where" (*ubi*):

> Since the quantity of time corresponds to that of a motion between a 'here' [*ubi*] and a 'there' [*ubi*], the 'where' [*ubi*] that we use for time is deduced [*transumptum est*], without inconvenience, from that 'where' [*ubi*] which, in fact, pertains to [a determination of] place [*locus*]. Actually, time is a transient *now*, whereas place is an enduring *where* [*ubi*].[78]

Scaliger's criticism of Cardano is not limited to a vindication of the objective meaning of time, its interconnection with space (or more accurately, place), and the priority of cosmological motions over time, i.e., time as something derived from celestial motions. He additionally criticises Cardano's hint that time is the cause of generation and corruption, dealing with the question of "Whether time generates and corrupts [things]" (*An tempus generet et corrumpat*). In Scaliger's assessment—and according to the Aristotelian authority on which he relies—this is impossible. Time cannot generate or corrupt anything, since it is not a substance but a quantity, that is, a property of a substance. Therefore, it cannot produce any effects.[79] Scaliger opposes his "very subtle" (*subtilissime*) opinion to Cardano's superficiality: "Our life is the act of the soul. In it, time has neither jurisdiction

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[74] Scaliger 1557, f. 458*v*: "At tempus est posterius motu primo. Est nimirum affectus eius, ab illo pendens."

[75] Scaliger 1557, f. 458*v*: "Tempus, inquit, aut est idem, quod motus, aut affectus eius."

[76] Scaliger 1557, f. 458*v*: "Tempus esse mensuram motus, et vicissim motum mensuram temporis."

[77] Scaliger 1557, f. 458*v*: "[…] dicimus tot partes aequinoctialis tanto moveri tempore."

[78] Scaliger 1557, f. 458*v* (punctuation and emphases have been standardized and modernized): "Quia tempus tantum est, quantus est motus inter 'ubi' et 'ubi' in loco. Iccirco 'ubi' quod pro tempore usurpamus, transumptum est, haud iniuria, ab eo 'ubi' quod est vere loci. Erit ergo tempus nunc fluens; et locus erit ubi continuatum."

[79] Scaliger 1557, f. 458*v*: "Cum enim tempus sit quantitas, nihil agit."

nor power."[80]   Scaliger adds that Cardano's words "are suited to the vulgar" (*vulgo pro-prior*), not to philosophers. What's more, his references to visions and alternative states of mind should be regarded as only insignificant phenomena which concern children and *melancholici* like Cardano himself.[81]

### 7.6.3   Benedetti and the Renaissance Concepts of Time

Our excursus on Renaissance concepts of time is far from exhaustive, but it helps us to grasp the scientific context out of which Benedetti's own position emerged. It also permits us to point out some major problems in the conceptualization of time, in particular its subjective and objective dimensions. The interrelation between *tempus* and *locus* was at the center of the reflections, the debates, and even the polemics of scholars investigating nature from various angles. The debate on time and on its relation to motion has meanings that are, at the same time, physical (Benedetti), psychological-physiological (Cardano), and cosmological-metaphysical (Scaliger). Finally, for a more traditional cosmological perspective on time as the measure of celestial motions and of the motion of the first mobile as the standard measure for all other measurements of time, one can refer to Alessandro Piccolomini's *Della filosofia naturale*, where one finds the following definition:

> Hence, time, which is the measure of all movements, mainly has to measure a motion that is the most regular and enables the measurement of all other [motions] that do not have the same regularity in their components. As this motion is that of the first heaven, one has to conclude that time (which is like [a property] of a substance [*in proprio soggetto*],) is first of all measured by it through that motion of the first mobile. Through it all other motions are ruled. Hence, although time can be called the rule and measure of any motion, it will be reasonable not to pluralize it alongside the plurality of motions. Rather, it will remain one and the same for the whole world, just as the first motion, which is its proper and true subject [*soggetto*], is singular.[82]

In the generation before Benedetti, Cardano affirmed that time is independent of cosmological space on the basis of his assumption that time is a "stretching of the mind" connected with the perception of corporeal activity. Cardano formally accepted the traditional Aristotelian definition of time as the measure of motion, but only as the consciousness of physiological motion. Scaliger criticized this perspective and accused Cardano of misunderstanding Aristotle. In the *Exercitationes* he tried to restore a peripatetic metaphysical conception of time as a product of celestial motions. In a sense, his conception of time has a "conservative" character for his commitment to the Aristotelian tradition. However, the distinction of the subjective and objective dimensions of the issue permits us to highlight a problematic aspect of the Renaissance definition of time. According to Scaliger, time can

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[80] Scaliger 1557, f. 458*v*: "[…] vita nostra est actus animae: in quem nihil habet tempus aut iuris, aut imperii."
[81] Scaliger 1557, f. 459*r*, *Exercitatio* 353, "De tua visione."
[82] Piccolomini 1585, I 3, chap. 7, ff. 52*v*–53*r*: "Di qui è, che'l tempo, c'ha da esser la misura di tutti i movimenti, bisogna che principalmente si applichi a misurare alcun movimento regolato sopra tutti gli altri, mediante il quale possa poi far da misura de gli altri tutti, che tal regola […] non hanno nelle parti loro. E tal movimento […] essendo […] quello del primo Cielo: si deve concludere, che'l tempo sia come in proprio soggetto da lui primamente misurato, nel detto movimento del mobil primo: mediante il quale, regolandosi tutti gli altri movimenti: ragionevol cosa sarà, che'l tempo, se ben si può chiamar regola, et misura d'ogni movimento, come di sopra habbiam lungamente detto, tuttavia egli non si multiplichi con la multiplicatione delli movimenti, ma resi uno stesso per tutto il mondo, sì come uno solo è quel movimento primo, che è il proprio, et vero soggetto suo."

be regarded as the measure of motion only from a subjective viewpoint, not from a natural one. According to nature, the relation between time and motion is the reverse: motion is the measure and time the measured thing.

For Benedetti, as a mathematician advancing a new Archimedean theory of motion and a post-Aristotelian physics, time is the *locus motus*, the place of motion, that is to say, an objective—we dare say, absolute—measure independent of its content and of spatial determinations. He shared with Cardano the independence of time from matter but not his subjectivism, while he maintained with Scaliger the objectivity of time without assuming the Aristotelian dependency of time on motion.

## 7.7  Natural and Violent Motions Revisited

After his revision of the physical concepts of the void, infinity, space, and time, Benedetti moves on to discard the Aristotelian theory of natural and violent motions in the section from chapter 23 to chapter 26. Chapter 23, "*Motuum rectum esse continuum, vel dissentiente Aristotele*" (Rectilinear motion is continuous although Aristotle is in disagreement), shows that continuity is not exclusive to circular motion, therefore Aristotle's distinction between circular and rectilinear motion must be corrected. In *Physics* VIII 8 one reads that "the motion of a body moving on a finite straight line cannot be continuous."[83] By contrast, Benedetti demonstrates that the projection of a circular motion has the same continuity as the circular motion itself, although it is a forward and backward motion on a line. This is the case with planetary appearances produced by the displacement onto an epicycle as seen from the earth.[84] In a way, this section is a rehabilitation of the epicyclic model against Averroist-Aristotelian criticisms concerning the physical tenability of a non-concentric model of heavenly circles. This contrast between mathematical and physical astronomy received much attention from sixteenth-century Aristotelian scholars who received their education at Padua. In the 1530s Giovan Battista Amico and Girolamo Fracastoro promoted a fleeting rebirth of homocentric astronomy through the publication of *De Motibus corporum coelestium iuxta principia peripatetica, sine eccentricis et epicyclis* (Venice, 1537 and Paris, 1540) and *Homocentrica* (Venice, 1538), respectively.[85] More directly, Benedetti's assertion that a continuous rectilinear motion is possible seems to be directed at the opposite statement in Piccolomini's Aristotelian *Della filosofia naturale* (Book 2, Part 2, chapter 3, "*Come esser non possa infinito corpo alcuno di quelli che per natura loro si muovano per retta linea*" (There can be no infinite body among those that move straightforeward following their own nature)). Just as in the *Diversae speculationes*, this section is complemented with diagrams.[86]

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[83] Aristotle 1995, VIII 8, 261b 32–33 (369).

[84] Benedetti 1585, 183: "[…] ut accidit lineae directionis, & retrogradationis planetarum, unde circulus uan erit ut epiciclus et b ut terrae centrum."

[85] Pierre Duhem pointed out the tension between physical and mathematical astronomy, albeit in a very inadequate way. See Duhem 1908. For a better understanding of the matter, see Di Bono 1990.

[86] Piccolomini argues as follows: "Non è a punto manco sconvenevole il concedere cotale infinità in qual si voglia corpo semplice, che o salendo, o scendendo si muova per retta linea; che si sia veduto disopra esser di quello che in circolo si rivolge. Percioche essendo il partirsi salendo dal mezo dell'universo, e' l venir discendendo a quello, che movimenti di luogo a luogo tra di lor contrarii: e ricercando li contrarii movimenti, luoghi contrarii parimenti; confessaremo esser opposti tra di loro il luogo disopra e questo qua giù da basso. E perché sempre tra due contrarii, se l'uno e per natura sua finito, e determinato, non sopporta il giustissimo governo della natura, che l'altro sia infinito e senza termine, secondo che discorrendo per tutte le contrarietà si può vedere." See Piccolomini 1585, II 2, chap. 3, f. 24*v*.

Chapter 24 is a refutation of a series of Aristotelian assumptions on natural and violent motion. In the title of this chapter, Aristotle is called *vir gravissimus*, but this attribution sounds quite ironic, since the theses of this "very grave man" are here refuted. The first criticism concerns the idea that a projectile is transported by air once separated from its thrower. According to Benedetti, it is rather the contrary: air is a hindrance to motion because it resists the penetrations. Secondly, Aristotle writes in *De caelo* I 8 that a body accelerates the closer it is to its aim. Instead, one should say that a body moves (e.g., falls) quicker the further it is from its aim (e.g., the ground). In fact, the longer the distance it covers, the more it is pushed (*maior fit semper impraessio*) by its inner *impetus*, which is due to the spontaneous inclination toward its proper place (*inclinatio ad locum suum eundi*). Chapter 26 is a rejection of Aristotle's statement that a body is not "heavy" in its proper place. In fact, air in air, or water in water, has no weight at all, except for when one artificially compresses an element (for instance inflating air in a bladder). The difference in density of the compressed element produces a difference in weight. Chapter 25 denies that vertical motion could legitimately be called natural. In fact, only perpetual circular motion is natural. An entire (i.e., spherical) body and its parts spontaneously move in circles. By contrast, rectilinear motion is that of a part separated from its whole. The cosmological significance of these remarks should be stressed. It is implicit but can be demonstrated by comparison with Copernicus's *De revolutionibus* I 8, where the same distinction between the motion of the whole and that of its parts accounts for the difference between the natural circular motion of a planet, basically the earth, and the vertical fall of heavy bodies.[87]

Copernicus presented his considerations on natural and violent motions in *De revolutionibus* I 8, which is the chapter aimed at rejecting Ptolemaic and Aristotelian arguments against terrestrial motion. There Copernicus attacked the Aristotelian theory of natural and violent motion and sought to abandon the doctrine of natural places. Benedetti's undertaking is very close, even though the cosmological theme has not emerged yet.

## 7.8 The Cosmological Perspective of the *Physical Disputations*

The cosmological dimension of Benedetti's anti-Aristotelian discussion is documented in the last part of the *Physical Disputations*.

### 7.8.1 Physico-Mathematical Astronomical Issues

The astronomical-cosmological section begins (chap. 28) with a reflection on the sphere that goes against the opinion of Aristotle, ironically called *Princeps Peripateticorum*. Whereas the ancient philosopher regarded the circle as the "first plane figure" (*prima figurarum superficialium*) and the sphere, the form of the heaven, as the "first body" (*prima corporearum figurarum, that is, the first three-dimensional figure*), Benedetti claims that they are the "last" figures. In fact, they can be regarded as polygons with infinite sides:

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[87]One can compare the text of Benedetti, *Diversae speculationes*, disp. XXV, with that of Copernicus, *De revolutionibus*, I 8. The latter wrote: "Igitur quod aiunt, simplicis corporis esse motum simplicem (de circulari in primis verificatur) quamdiu corpus simplex in loco suo naturali, ac unitate sua permanserit. In loco, siquidem non alius, quam circularis est motus, quo manet in se totus quiescenti similis. Rectus autem supervenit iis, quae a loco suo naturali peregrinantur, vel extruduntur, vel quomodolibet extra ipsum sunt. Nihil autem ordinationi totius et formae mundi, tantum repugnat, quantum extra locum suum esse. Rectus ergo motus non accidit, nisi rebus non recte se habentibus, neque perfectis secundum naturam, dum separantur a suo toto, et eius deserunt unitatem."

"the triangle is the first plane figure and the circle the last one."[88] He adds that the *principium*, the beginning and the origin of everything, is its center and those figures which equally encircle it in all directions can be said to be perfect. The author concludes as follows: "That which is perfect, although it is [qualitatively] first as to its essence [*natura*], is last as to its generation."[89] The circle, according to him, is perfect because it is, in a sense, an "infinite figure." If one considers it as a polygon of infinite sides, one can say that the sum of its angles is equal to an infinite number of right angles. One can interpret this statement as follows: every polygon inscribed in a circumference can be divided into several triangles whose vertices touch the center of the circumference and whose bases coincide with the sides of the polygon. In the case of the circle, taken as the "last" polygon, the triangles decomposing it are infinite in number. Since the angles at the vertices are zero and the sum of all internal angles must be 180°, it follows that the angles at the base must be two square angles. Thus, Benedetti feels vindicated: "The circle and the sphere are not constituted of one single angle, as Aristotle believes [...]. Rather, these are figures of infinite right angles. For that reason I call them last and perfect, because one cannot add anything to infinity."[90] To sum up this reasoning, Benedetti shares Aristotle's opinion that the sphere is the perfect figure, but adduces geometrical-metaphysical reasons. For both authors the sphere is the form of the world (Benedetti would say, "of the *caelum*" surrounded by an infinite empty space) for aesthetic and metaphysical reasons.

In the following chapters, Benedetti reviews a series of astronomical and meteorological issues on which he accused Aristotle of being wrong. Chapter 28 deals with the sparkling of the stars, which is, according to Benedetti, the effect of the motion of heavenly transparent media (*ab inaequalitate motus corporum diaphanorum mediorum nascitur*).[91] Among other things, Benedetti denies (chap. 30) that the warmth of the sun can be produced by its motion rather than by its light and subsequently (chap. 31) explains the seasonal variations. Chapter 33 reassesses, against Aristotle, the plausibility of the Pythagorean doctrine of celestial harmony. This has nothing to do with the production of sounds, nor with any harmonic proportions between the "aspects" of the heavenly bodies. Rather, it is the secret order imparted by to the world by divine providence.[92] Chapter 33 comprises a lengthy discussion on meteorology, in which atmospheric phenomena are essentially explained through the variations of air density.

### 7.8.2 The Copernican Conclusion of the *Physical Disputations*

Chapter 35, *Motum rectum curvo posse comparari* (Straight and curvilinear motions are comparable), is a crucial chapter for our analysis, since it is here that Benedetti, almost

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[88] Benedetti 1585, 186.

[89] Benedetti 1585, 185: "Quod autem perfectum est, licet natura sit primum, est tamen ultimum generatione."

[90] Benedetti 1585, 185–186: "Circulus sphaeraque non ex uno solo angulo recto constat, ut idem Aristoteles putat [...] sed sunt figurae infinitorum angulorum rectorum, et hanc ob causam a me dicuntur ultimae et perfectae, quia infinito nihil addi potest." The authors would like to thank here Irina Tupikova for suggesting this interpretation of Benedetti's argument.

[91] Benedetti 1585, 189.

[92] The Pythagorean harmony was ridiculed by Alessandro Piccolomini among others. See Piccolomini 1585, II 4, chap. 12, "Del suono, et armonia, che i pitagorici pensavano, che nascesse per li movimenti de' corpi celesti," f. 105*v*: "[...] Quando i corpi celesti movendosi facesser suono avvenir dovrebbe a noi poscia che da si alto, et soverchio strepito, et sproportionato all'odito nostro quasi assordati, né quello né altro suono che qua giù si faccia, odire in modo alcuno dovremmo giamai. Per la qual cosa essendo manifesto che tante diversità di strepiti, che tra questi corpi inferiori si fanno, ancora che piccolissimi sieno, son da noi odite distintamente, è forza dire, per la ragione ultimamente fatta che né suono, né armonia, non può causarsi per li movimenti de gli Orbi, o delle stelle che volgendosi faccin la su in cielo."

at the end of his *Physical Disputations*, introduces the Copernican theory. Although we have already dealt with Benedetti's astronomical views in the previous chapter, it is useful to recount here the most important features of his cosmology in the context of the philosophical section of the *Diversae speculationes* (IV) and add some more details.

The subject matter is the comparability of rectilinear and circular motion: "[Aristotle] is wrong when he says that straight motion cannot be compared to the curvilinear (*Physics* VII 4), where he mistakenly also says that one cannot find any lines equal to the circumference of a circle."[93] It is directed against Aristotle's denial that a straight and a circular motion could be compared, thus hinting at the qualitative difference between celestial circular motions and the vertical tendency of the elements in the sublunary sphere. From a Copernican perspective, Aristotle's words could be considered to be an implicit rejection of terrestrial motion. In fact, if the earth rotates, one should assume that the trajectory of a falling body is rectilinear for an observer on the earth but has a circular component as well, if considered in relation to the outside world.

Benedetti first appeals to Archimedes's *De quadratura circuli* (On the quadrature of the circle) to argue that the circle and the straight line are comparable: "If, then, this quadrature can exist, there can also exist, for the reason already given, a straight line equal to the circumference of that circle."[94] Thus, a geometrical problem, the squaring of the circle, attains a direct cosmological meaning. If the issue at stake is the distinction between celestial and elementary motions, they are of course different, but this difference does not lie in the circularity of the former and the straightness of the latter, but rather in the uniformity of speed opposed to acceleration.

These considerations offer Benedetti the occasion to expand on the velocity of celestial motions. According to the commonly held opinion (*secundum opinionem cummunem*), the heavens would have to cover an immense distance within the 24 hours of the daily rotation. Close to the equator, the sun would cover 1,000 Italian miles per minute and Saturn 260,000 miles per minute, not to speak of the rapidity of the fixed stars. The assumption of this inconvenient velocity would of course be avoided if one assumed "the most beautiful theory" (*pulcherrima opinio*) of Aristarchus, "divinely" restored by Nicolaus Copernicus:

> And as for the speed of the fixed stars situated near the equator, one may make one's estimate, and, in fact, this will seem very difficult to some. But this difficulty does not occur in the most beautiful system of Aristarchus of Samos that has been so divinely expounded by Nicolaus Copernicus.[95]

From a Copernican perspective, the sun would cover "only" 48 miles per minute and Saturn 24, whereas the heavens would be stationary.

In the subsequent chapter (chap. 36), Benedetti reworks the doctrine of the *doctissimus Aristarchus*. It is entitled "*Minus esse explosam ab Aristotele opinionem credentium plures mundos existere*" (The view of those who hold that many worlds exist was not adequately refuted by Aristotle) and deals with the plurality of worlds. According to Aristotle,

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[93] Benedetti 1585, 194: "[Aristoteles] recte dicere non potest motum rectum ad curvum comparabilem non esse 4. cap. lib. 7 *Physicorum* ubi errat quoque dicens reperiri non posse lineam aliquam rectam alicuius circuli circunferentiae aequalem."

[94] Drake and Drabkin 1969, 220. Cf. Benedetti 1585, 194: "Si igitur dicta quadratura dari potest, potest etiam dari una recta linea aequalis circunferentiae eiusdem circuli."

[95] Drake and Drabkin 1969, 221. Cf. Benedetti 1585, 195: "Et amplius de stellis autem fixis circa aequatorem posits quivis cogitet; quod revera difficillimum quibusdam videbitur, quod quidem non occurrit secundum pulcherrimam Aristarchi Samii opinionem, divinitus a Nicolao Copernico expressam, contra quam nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae."

a universe with a plurality of worlds similar to the earth would be unstable and eventually collapse, since the earthly parts of the other worlds would fall toward the cosmological center and the fiery parts would eventually become part of the fiery sphere of our sublunary world. This Aristotelian objection is based on an *a priori* assumption of the theory of the natural places. It is therefore easy for Benedetti to contradict him by arguing that all worlds (that is, planets) would have their elements and their places.[96]

Apart from that, as we have already seen, Benedetti proposes a bizarre transformation of the Copernican system based on an analogy between the moon and the other planets. Like our satellite, all these light-mirroring and wandering bodies are supposed to turn around dark earths which, in turn, spin about their axis:

> If the system of the learned Aristarchus is correct, it will be perfectly logical for that which takes place in the case of the Moon to take place also in the case of any of the five other planets. Thus, just as the Moon with the help of its epicycles revolves around the Earth as if on the circumference of a certain other epicycle of which the Earth is like a natural center (i.e., in the middle), carried around the Sun by the sphere of annual motion, so too may Saturn, Jupiter, Mars, Venus and Mercury revolve about some body situated in the center of their major epicycle. And this body, also having some motion about its axis, may be opaque, possessing conditions like those of the Earth, with conditions on the epicycle in question similar to those on the lunar epicycles described.[97]

This conception could provide an explanation for the existence of planetary epicycles, whose physical tenability has been already stressed in the *disputatio* 23. We could also regard these views of Benedetti as a cosmological reading of Copernicus focused on possible cosmological and physical consequences of the planetary theory. The plurality of worlds and the analogy between the moon and the planets are not the only innovative elements in comparison with the theses of *De revolutionibus*. After a section on the motion of light through the cosmic void (chap. 37) and one on the geometry of the elements (chap. 38), the conclusive section of the *Physical Disputations* (chap. 39) attacks a Peripatetic dogma: the unalterability of the heavens. In *De caelo* I 22 Aristotle remarked that no change in the heavens was ever observed. This is, according to Benedetti, not a valid argument. One should rather assume a principle of relativity of the point of observation. In fact, the earth would be invisible from the eighth heaven (that of the fixed stars), even though, by supposition, it was endowed with a light equal to that of the sun. The distance hinders us from perceiving changes that occur on other worlds.[98]

With this rejection of the distinction betwee a sublunary and a heavenly realm, Benedetti's criticism of Aristotelian physics is complete. It should be noticed that this final objection hits at the core of the Peripatetic natural philosophy, since it is a denial of the fundamental distinction between a terrestrial and a celestial physics, on which the entire physics and cosmology of the Aristotelians relies.

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[96]Benedetti 1585, 195.

[97]Drake and Drabkin 1969, 222. Cf. Benedetti 1585, 195–196: "Si doctissimi Aristarchi opinio est vera, rationi quoque consentaneum erit maxime, ut quod Lunae contingit, cuilibet etiam ex aliis quinque planetis eveniat, idest, ut quemadmodum Luna suorum epicyclorum ope circum terram volvitur, quasi per circunferentiam alterius cuiusdam epicycli, in quo terra sit instar centri naturalis (idest sit in medio) delati ab orbe annuo circa Solem; sic etiam Saturnus, Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio sui epicicli maioris, situm habens, volvantur; quod quidem corpus, et aliquem quoque habeat motum circa suum axem, sit opacum, iis conditionibus, quae terrae sunt similes, praeditum existat, et in dicto epyciclo sint res similes istis lunaribus."

[98]Benedetti 1585, 197.

## 7.9   An Evaluation: Benedetti's Path to Natural Philosophy

The *Disputationes de quibusdam placitis Aristotelis* is a complex book within the larger book. It concerns at least three main fields of investigation: motion, the foundations of physics, and astronomy, in particular cosmology. It begins with a rejection of the theory of the natural places (violent and natural motion) based on an Archimedean relativization of heaviness and lightness as well as on a mathematical approach derived from the Euclidean theory of proportions. It deals subsequently with basic concepts of physics. It defines space and time anew as an absolute framework for the investigation of natural phenomena, in particular motion. This part of the *Physical Disputations* also aims at demonstrating actual infinity and the void, which are Democritean theses rejected by Aristotle in *Physics* and *De caelo*. The astronomical part then follows, which confronts many special issues and illustrates what we shall call a "post-Copernican cosmology." Benedetti advocates the heliocentric system (albeit modified relative to the model of Copernicus's *De revolutionibus*), the plurality of worlds, the inter-changeability of the observational viewpoint in the universe, and, last but not least, the homogeneity and continuity of the sublunary and the heavenly realm, contrary to one of the most fundamental assumptions of Aristotelian philosophy.

Let us summarize the Copernican considerations that could have influenced Benedetti and consider the extent to which he went beyond them. First of all, Copernicus abandons the theory of natural and violent motions because, "if anyone believes that the Earth rotates, surely he will hold that its motion is natural not violent."[99] Additionally, the daily rotation of the heavens is more absurd than that of the earth because it would be excessive compared to that required of the relatively small earth. A third Copernican remark concerns the infinity of space. It is directed against the Aristotelian assumption that there is nothing, "no space, no body, no void," outside the heavens (*dicunt quod extra caelum non esse corpus, non locum, non vacuum*). Copernicus remarks that the axial rotation of the earth undermines the strongest argument in favor of cosmological finiteness: "For the chief contention by which it is sought to prove that the universe is finite is its motion."[100] As to the objections against the earth's motion based on considerations of the effects to be expected for flying and thrown objects, Copernicus assumes, against Aristotle's claim for the simplicity of motion, that things on Earth participate in the planetary motion and, therefore, the vertical displacement of light and heavy bodies (*cadentium vero et ascendentium*) is a composite motion (*duplicem*) relative to the whole (*mundi comparatione*), with a rectilinear and a circular component. Copernicus holds that only circular motion is natural and it does not only pertain to celestial bodies but also to the elements in their natural place. As we have seem, he defines rectilinear motion as the tendency of bodies to reach their whole if they have been separated from it. This vertical appetency is not uniform but accelerated. Copernicus also criticizes Aristotle's opinion that bodies are heavy (or light) in their proper place, since weight depends exclusively upon the tendency of the part towards their whole.

Many of these Copernican ideas and suggestions appear also in Benedetti's *Physical Disputations*: the rejection of the theory of natural places and of violent and natural motions, the excessive rapidity of the rotation of the heavens, the void, infinity, the naturalness of circular motion against the unnaturalness of the vertical motion of the parts separated from their whole, and the criticism of Aristotle's assertion about the weight of

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[99]Copernicus 1978, 15
[100]Copernicus 1978, 15–16.

the bodies in their natural place. However, it should be remarked that Copernicus does not expand on these ideas for the most part and cursorily presents them only for the sake of his apology for terrestrial motion. Benedetti's treatment is much more explicit and, what is more, his motivations and presuppositions appear to be quite different. His Archimedean and Euclidean treatment of motion is the basis of his rejection of the distinction between natural and violent motions. No consideration of this kind is present in Copernicus's work. Moreover, the reference to spatial infinity in *De revolutionibus* is limited to a remark. Copernicus himself does not explicitly support this thesis and leaves the discussion to the natural philosophers or, as he calls them, the *physiologi*. Actual infinity receives a substantially different treatment in Benedetti since it is closely related to the attempt to define space anew as *intervallum corporeum*. It is precisely this broad, natural philosophical dimension which is absent in Copernicus's work and which, in our opinion, Benedetti did not derive from his reading of *De revolutionibus* or from general astronomical concerns. It seems, by contrast, that he was primarily interested in the physical issue of a mathematical treatment of motion and that the criticism of the Aristotelian philosophy led him in a quite natural way to also confront cosmology. Nor could issues like the void and atomism be reasonably derived from Copernicus. Even the planetary theory of Benedetti departs from *De revolutionibus* as it includes theses like the plurality of worlds and the corruptibility of the heavens. However, it is clear that the heliocentric and geokinetic theories fit perfectly into Benedetti's worldview. In light of his general theory, as he writes, Aristotelian and Ptolemaic arguments against Copernicus's theory appear extremely weak: "contra quam [doctrina] nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae."[101] Koyré wrote that Bruno's *La cena de le Ceneri* (London, 1584) was the best defense of the Copernican system from a natural and physical point of view before Galileo's *Dialogo sopra i due massimi sistemi del mondo* (Florence, 1632). However, this statement seems to underestimate the force of Benedetti's *Physical Disputations*, which are perhaps less speculative than Bruno's dialogues but should nonetheless be regarded as an extremely strong apology for the physical tenability of the Copernican system. A reciprocal influence between Bruno and Benedetti cannot be excluded, since the wandering philosopher from Nola stayed for a period in Turin and the Savoy around 1578 and probably participated in a debate concerning the comet of 1577.[102] At any rate, the *Diversae speculationes* encountered much more acknowledgment among astronomers of the time than Bruno's work. As we have seen, Brahe extensively quoted Benedetti both in his *Epistolae astronomicae* of 1596 and in his book on the nova of 1572. Kepler's admiration for Benedetti was no less and was only equaled by his admiration for Commandino and Clavius.[103] The proximity of many themes of the *Diversae speculationes* and those of the young Galileo are a well-known issue in the history of mechanics; in light of our discussion, it is plausible to assume that Benedetti's influence on Galileo also concerned the insight into the close relation between the heliocentric theory and a new mechanics.[104]

Our analysis has shown that the heliocentric system is not the main issue at stake in the *Physical Disputations*, although that theory becomes part of a general program of reform for natural philosophy. Far from being a mere "Copernican enterprise," Benedetti's visionary project is much more complex. It is an ambitious attempt to build a new physics, in the wide Renaissance meaning of the term, out of a criticism of Aristotelian physics. Concerning Aristotle, it is clear that the *princeps peripateticorum* provides him with a

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[101] Benedetti 1585, 195.
[102] Omodeo 2008a.
[103] Kepler 1937–2001, 390.
[104] Damerow and Renn 2010.

model, albeit a negative one, in which the theory of motion, cosmology, astronomy, meteorology, natural philosophy, and metaphysics are closely interrelated. Benedetti's undertaking is precisely a revision and a restructuring of these interdisciplinary ties on the basis of new insights and a mathematical approach. Although his investigation intentionally and explicitly departs from Peripatetic physics, it is historically possible only in the form of a thorough confrontation with Aristotelian themes. Indeed, the *Physical Disputations* have the form of a dispute on Aristotelian places. Benedetti's familiarity with Aristotle's *Physics*, *De caelo*, and *Meteorologica* should also be underscored. This apparently contradictory aspect of early modern physics in its ambiguous relation to Aristotelianism has already been stressed by Anneliese Maier in her studies on the medieval contributions to classical science.[105] In a sense, the development of a new physics required a thorough confrontation with Aristotle and his concepts, as also the examples of Bruno and Galileo bear witness to in different ways.

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[105]Maier 1951, 304–305.

# Concluding Remarks

Giovanni Battista Benedetti, the Renaissance scientist, has received ambivalent historical judgements by scholars in the past. The historian of medieval science and philosophy Anneliese Maier, for one, viewed him with mixed feelings. To her, Benedetti appeared to be a sort of intellectual companion of Galileo Galilei, at the same time his "forerunner" in mathematical physics and an epigone who was disrespectful to his own medieval *Vorläufer* or predecessors. Maier wrote that Parisian scholastics such as Nicole Oresme and Jean Buridan had provided Benedetti and Galileo with the concepts they needed to inquire into physics—she particularly had the concept of *impetus* in mind—which they did not acknowledge in their fierce attacks on Aristotelian philosophy.[1] Maier shared Koyré's view that modern mechanics was constructed around a few central concepts and authors relevant for Newton's *Principia mathematica*. They were perplexed by the concomitant reception and rejection of medieval physics by Renaissance scientists. In our view, however, it is too narrow a point of view to just consider individuals and sets of ideas and their genealogies. Instead, one should consider the wider intellectual currents and the shared knowledge they generated. The incipient *querelle des anciens et des moderns*[2] is an example of a debate transcending specific questions and problems, even approaches and methodologies, towards a larger reflection on the relation between past and present. The problems inherent in this gap between the individual perception of change and the intellectual transitions of the time are exhibited by the astronomy of Nicolaus Copernicus, a sort of "unaware revolutionary,"[3] who saw himself (or at least presented himself) as a Renaissance restorer of planetary theories defended in antiquity by the legendary Pythagoreans. By contrast, his scholastic counterpart, the Padua-trained physician and natural philosopher, Girolamo Fracastoro, presented his homocentric reform of mathematical astronomy as a radical innovation, comparable with Amerigo Vespucci's discovery of the New World.[4] Fracastoro's work was based on the modeling of all celestial motions through concentric spheres (in line with a well-established Aristotelian tradition). In Benedetti's case, the rejection of the *philosophia naturalis* taught in the universities was achieved with intellectual means descending from that very philosophical tradition. Rather than viewing this fact as a paradox, it should be regarded as a sign of a profound tension in Renaissance science between past and present and a hallmark of what we have called preclassical mechanics.[5] The *intention* to outdo traditional authorities in order to move beyond their legacy had to rely on the shared knowledge of the time, which was marked by Aristotelian thought. In our introduction we delved into Benedetti's conceptions and reconstructed their socio-cultural coordinates, characterized by the Renaissance tension between conceptual heritage and novelty. Maier's perplexity thus rests upon a lack of reflection on the embedment of in-

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[1] Anneliese Maier established a connection between Benedetti's treatment of motion and that of Galileo in Maier 1951, 304–305.

[2] Lehner and Wendt 2017.

[3] Copernicus's revolutionary role *malgré soi* already puzzled Thomas S. Kuhn, who called him at once "radical" and "conservative" and regarded *De revolutionibus orbium coelestium*, the book propounding the first modern heliocentric theory in mathematical astronomy, "revolution-making" rather than "revolutionary." Cf. Kuhn 1959, 135 and 148.

[4] Goddu 2010 and Granada and Tessicini 2005. Also see Omodeo 2017.

[5] See Renn, Feldhay, et al. 2018.

tellectuals and their theories in socio-cultural processes. Benedetti in particular ought not to be seen as a link in a chain, but rather as one representative of a complex and comprehensive knowledge economy.[6]

In order to correctly locate Benedetti in the knowledge economy of the Renaissance, it is expedient to consider him against the background of the material and intellectual conditions of early-modern science, and as a figure between the intentions and identities of a new genre of intellectuals who formed the archetype for modern scientists. Benedetti's case helps us to reflect upon the social position and intellectual identity of these new types of scholars as well as on the way socio-cultural coordinates penetrated science, as far as its demarcation, content, form, and justification are concerned. With social coordinates we refer to the institutional setting involving Benedetti's role as a courtier and thus to his function as a court mathematician, which, in turn, was linked to the wider socio-economic interests of a Renaissance territorial state.[7]

In his seminal work on the sociological roots of modern science, Zilsel discussed the scientific relevance of the social transformations taking place in the late Middle Ages and the Renaissance. In particular, he argued that the emergence of modern science depended on the rise of capitalism. We could aptly refer to this phase as a pre-capitalistic or early-capitalistic "knowledge society." Technical knowledge proved to be a key element in the organization of life and production while the status of the artisans, those whom Zilsel called the "artist-engineers," increased and received high recognition among civil and political authorities. The town of Florence is prototypical for these changes, as Leonardo Olschki has forcefully demonstrated in his studies on science and vulgar literature.[8] A wide range of artist-engineers transformed Florentine society and its mentalities. Filippo Brunelleschi, most representatively and symbolically, forever changed the skyline of the same town in which, at the end of the Italian Renaissance, Galileo composed works that irreversibly modified the landscapes of science and scientific culture.[9] In Florence and Europe more generally, in the passage from the Middle Ages to early modernity the "artisan-practitioners" were confirmed as a new class. The codification of their experience and knowledge profoundly changed epistemology and science, most evidently in mechanics. This practical art was first codified as a physico-mathematical discipline, and then as a science in its own right, and was later adopted as a methodological and ontological point of reference in the shift toward the mechanistic world views of the seventeenth century.[10]

According to a corollary of the Zilsel thesis about the social origins of modern science, scientific culture was reshaped by the merging of three intellectual strands: the artisanal/technical, the scholastic/logical, and the humanistic/rhetorical. This fusion was accomplished by mediators, who were social actors with an in-between status bridging different intellectual and social realms. "Hybrid experts" became increasingly necessary because of their capacity to bring together the technical and the theoretical dimensions of knowledge. Their socio-cultural relevance would never diminish from the late Middle Ages to the Industrial Revolution and beyond.[11]

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[6]For a recent study accomplished in this vein, see Trzeciok 2016.

[7]For further considerations on Benedetti in light of a discussion on methodological and historiographical approaches, see Renn, Feldhay, et al. 2018.

[8]Olschki [1919–1927] 1965.

[9]On the Florentine prototype, see Renn 2014. Cf. Zilsel 2000, 941.

[10]On artisanal knowledge and its codification, see P. Smith 2004 and Long 2001. On practical knowledge, see Valleriani 2017. On the elevation of mechanics to a worldview, see Renn and Damerow 2010.

[11]Ursula Klein has made this point most forcefully in Klein 2015.

During the Renaissance, this mediation was secured by a new group of "scientist-engineers," a series of court mathematicians of which Galileo is the best-known figure and which also included his protector, Guidobaldo Del Monte. Actually, the description of the Renaissance figure of the "scientist-engineer" suits the intellectual and social profile of Benedetti very well.[12] Galileo and his like were well versed both in the technical as well as in the intellectual dimensions of knowledge production. Renaissance "scientist-engineers" underwent a period in apprenticeship of practical mathematics, in some field of application like architecture or the art of war, but later distanced themselves from artist-practitioners as they aspired to gain higher social recognition, especially as courtiers. They had a high degree of education, as they mastered theoretical mathematics, the language of the learned, Latin, as well as the courtly language, for instance by acquainting themselves with the elegant Italian of the literature of the time. Scientist-engineers thus acted as mediators connecting the centers of power and decision on the one hand and the workshops and building sites on the other. As was the case with Benedetti, these experts could supervise artisanal work or give advice on technical issues.[13] As courtiers they were additionally required to participate in the refined dialogical and literary culture of the elite, to serve as educators as well as to use their astronomical expertise to cast horoscopes for the rulers.

The most specific socio-political aspect of Benedetti's time is the affirmation of court society as a particular social formation whose features show continuities and fractures both with the earlier aristocratic setting of the feudal society and the later capitalist one. A distinguishing feature is the centralization of power and administration around the court. As Norbert Elias argued, this formation culminated in the absolutism of the Ancien Régime but was preceded, on a smaller scale, by early attempts at territorial centralization.[14] Although such social formations apparently gravitated around an individual sovereign who made all decisions (as much of the literature of the time on the *Principe* and its privileges boasted), it was in fact a hierarchical system in which the group of experts surrounding the princely ruler constituted an oligarchy who operated the complex organization of modern states. The Duchy of Savoy is one such case. The dukes strove to create a "modern" capital city partly by following the model of Florence, insofar as culture and prestige are concerned, but also the Spanish and French models, insofar as the suzerainty of the ruling family is concerned. Other models played a role, too, for instance the Netherlands for military technology and Switzerland for military conscription and discipline.

Benedetti shared the enthusiasm of his patrons (especially Emanuele Filiberto) for mathematics and its perceived powerfulness as an instrument for successful navigation in war and peace. He also shared the aristocratic values of the court such as disinterest and prestige. Adherence to these values largely explains his bias toward theory despite the practical origins of his knowledge and the fields of application of his mathematics (ranging from mechanics to navigation, architecture, and perspective). He also ventured into the most general fields, such as cosmology and philosophy (as seen through his criticism of Aristotelian natural conceptions, his favorable opinion on Copernican astronomy and post-Copernican cosmology, and his remarks on "Pythagorean" philosophy of mathematics).

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[12]The figure of the "scientist-engineer" has been introduced into the history of science by Renn 2001, particularly in the contributions by Lefèvre (Lefèvre 2001) and Renn, Damerow, and Rieger (Renn, Damerow, and Rieger 2001). Valleriani discusses it in detail in Valleriani 2010, chap. 6.

[13]Valleriani 2010, 208: "Except for the period of the apprenticeship, an engineer-scientist was almost never personally employed in workshops or building sites, but he was aware of the work procedures followed in these locations and was able therefore either to commission craftsmen or other persons involved with practical activities, to supervise or teach them, or simply be consulted to evaluate their works."

[14]As already discussed in the introduction. The reference work is Elias [1969] 2002.

Actually, he did not hesitate to call his wide and unsystematic work "speculations," an expression that stresses the theoretical character of the endeavor.

From the perspective of a court scientist such as Benedetti, mathematics was the key to practice and theory. It was his specific field of expertise among the Turin courtly elites; through it he acquired a central epistemological status in line with the exaltation of the *certitudo mathematicarum* by many of his contemporaries, among them his correspondent Pietro Catena. At the same time, the practical context surrounding the mathematical approach in many fields such as mechanics led him to emphasize the contingent element of natural phenomena. Thus, the centrality of mathematics in Benedetti's work has a multilayered meaning, including the theoretical, practical, epistemological, and social. The limits of validity and applicability of Benedetti's mathematical science mirror the boundaries of his field of competence in the division of intellectual labor within his courtly environment. Although he used geometry as a sort of universal key, he could not impose his views on other courtiers who were experts in fields such as philosophy and medicine. In this context of enforced openness, Benedetti's criticisms of Aristotelianism appear as a sort of defense of his professional position in the framework of a courtly dialogical pluralism. Such an environment explains the occasional (and fragmentary) character of the *Diversae speculationes*, which brings together occasional materials such as texts for private teaching, letters, short treatises, expert advice, and polemical essays (among others), in which Benedetti made his mathematical expertise manifest and showed its usefulness.

The intellectual distribution of labor in the Renaissance ensured that Benedetti was at the heart of the courtly milieu by virtue of his family's social status and not through his ambition alone. His work exhibits many similarities with the work of other Italian court mathematicians, most eminently that of the aforementioned Del Monte and Galileo, as far as the range of their interests and the overall approach are concerned. Benedetti's most daring passages, which open up unconventional solutions to technical and theoretical problems, and his remarkable disregard for authority qualify him as one of the Italian *novatores*, although he did not make explicit his natural conception as an all-encompassing alternative to the well-established Aristotelian philosophy. His fierce attacks on crucial aspects of the Aristotelian conception—relating to motion, the void, infinite space, time, infinity, and planetary theory—did not result in a systematic new natural philosophy. Rather, he limited himself to collecting results in different areas and to working on the most varied aspects without finding their common denominator. He also made elliptical references to Pythagoreanism and implicitly rehabilitated some aspects of atomist and stoic conceptions, for instance the plurality of worlds and the fluidity of the heavens. Cardano, whom he appreciated, went much further in the inquiry of the common foundations of the sciences (specifically mathematics, practical arts, and medicine) while Benedetti's correspondent Patrizi advanced a systematic natural philosophy inspired by neo-Platonism. In the same years in which Benedetti finished and published his physico-mathematical speculations in Turin, Bruno published philosophical dialogues in London expounding a natural philosophy and an anthropology that led to far more radical consequences for the premises of cosmology, similar to those reached by Benedetti. Another contemporary of Benedetti, Telesio, had offered the first modern attempt to build up a conception of nature on new principles. His *Natura iuxta propria principia* paved the way for the next generations of scholars searching for new foundations in natural science. Among them was his direct follower Campanella, who brought his philosophy to France in the seminal years of the mechanical philosophies of Pierre Gassendi and René Descartes. Benedetti participated in this wide cultural transformation; he contributed to advancing the mathematical and phys-

ical disciplines and discarding consolidated theories—but without offering a systematic alternative.

To summarize the most evident features of Benedetti's endeavor: it was courtly, secular, anti-Academic, unsystematic, occasional, elitist, learned, abstract, pleasant, and useful. It was *secular*, that is, non-theological, as it was linked to the interests of the ruling class and the state. It was a *useful and pleasant* science: on the one hand, it was practice-oriented but not purely empirical; on the other hand, it proved witty and fit for courtly sociability. It was *abstract and disinterested*: superior to the vulgar and tuned to aristocratic values. *Learned*: fit to be exhibited at court alongside the other arts. *Elitist*: Benedetti elevated mathematics from a practical discipline of scientist-engineers to a refined cultural activity. *Occasional*: linked to the variegated political and cultural interests of the court. *Unsystematic*: fragmented, lacking the inner coherence of scholasticism. *Anti-Academic*: free from concerns about respect for university scholarly traditions. All of these characteristics of Benedetti's science were the hallmark of court science: it was technical and abstract without losing contact with practice and experience—a mathematical-empirical science *in nuce*; it was (relatively) free from bookish tradition and theology but not from the contingencies of courtly life.

What is the common denominator of the great variety of subjects dealt with by Benedetti? What is the center around which they all gravitate? Is there one unifying principle behind the apparent disorder and heterogeneity? It should be emphasized that Benedetti first established his fame *as a mathematician*. His early treatment of motion by mathematical means was explicitly directed "against Aristotle and all philosophers" (*contra Aristotilem et omnes philosophos*). In his time "mathematics" had a wide scope. It comprised arithmetic and geometry, astronomy and astrology, as well as music, but also reached far beyond the boundaries of the *quadrivium* by encompassing optics, practical mechanics, architecture, and engineering. The expansion of mathematics into the fields of physics, natural philosophy, meteorology, and even metaphysics and epistemology was a crossing of the disciplinary boundaries. Benedetti's time bears witness to several attempts to expand the boundaries of mathematics. Cardano, for one, claimed that geometry had the function of a universal logic fundamental to rational thought, and that the practical disciplines including statics, mechanics, and architecture were its subordinate fields of inquiry.[15]

Benedetti's intellectual identity, however, proves much more complex than his corporate identity as a mathematician.[16] His pronounced titles vary. In a short biographical note accompanying the birth horoscope published by Gaurico, he was referred to as "Phylosophus, Musicus, atque Mathematicus"; on October 19, 1589, he signed an astrological report cast for Carlo Emanuele I as "Matematico e Astrologiaro";[17] contemporary admirers of his such as the Milan painter and poet Lomazzo and the Danish astronomer Brahe called him "matematico" and "philosophus et mathematicus inprimis excellentem," respectively.[18] Probably, Brahe's designation of Benedetti as both philosopher and mathematician best captures the poles of his intellectual activity. Intriguingly enough, Benedetti generally dropped the title of "mathematician," keeping only that of "philosopher" in his

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[15] Girolamo Cardano, *Encomium geometriae recitatum anno 1535 in Academia Platina Mediolana* in Cardano 1966, vol. 4, 440–445.

[16] By "corporate" we refer here to the *esprit de corps* of a group that considers itself a bounded entity whose interests are marked as separate from other groups. The guild culture of the Middle Ages originated this particular meaning of corporation, which precedes the modern sense of a professional group or legal body.

[17] Roero 1997, 57–58.

[18] Lomazzo 2006, 177: "Del Sig. Gio. Battista Benedetti Matematico" Brahe 1916, 251–253.

publications. On the title page of his *magnum opus* of 1585, the *Diversae speculationes*, he appears as "patritius Venetus philosophus," exactly the same epithet that appears in *De gnomonum umbrarumque solarium usu liber* (1574). In the publications in the vernacular, he correspondingly appears as "filosofo del sereniss. duca di Savoia," e.g., in the *Consideratione… d'intorno al discorso della grandezza terra et dell'acqua* (1579). In the last publication, his self-presentation as court philosopher is interestingly opposed to the designation of his intellectual opponent, Antonio Berga, as "*filosofo nella Università di Torino*," that is, "university philosopher"—which is equivalent to *scholastic* philosopher. These references are telling for Benedetti's self-perception or, to use an in-vogue expression, his *self-fashioning*.[19] In both cases, the image of court philosopher was his intended identity, whether reflected or purposely constructed (or a mixture of both). As was the case with Galileo, the Florentine courtier, the philosopher's social status and reputation was higher than that of the mathematician. This is why, among the conditions for Galileo's appointment as a courtier to the Medicis, he regarded the designation "philosopher" as relevant.[20]

As for the epistemological debates mirroring the disciplinary and social divides and hierarchies of the time, heated controversies began over the "certainty of mathematics." The determination of the degree of certainty of mathematics also concerned the legitimacy of using mathematics in physics. In the case of Benedetti, the tension between his function as court mathematician and his identity as philosopher—and *patrizio*—lies beneath his science. While philosophical legitimacy was essential for the acknowledgment of the intellectual dignity of his endeavor, the practical dimension of mathematics remained fundamental for the social justification of his function as a court expert.

One could single out the social and the political-cultural coordinates of Benedetti's science as two complementary drives. On the one hand, his position as a court mathematician directly determined much of the content of his writings, occasioned by the requests addressed to him as a court *expert* in technical issues pertaining to mathematics. His position also determined formal aspects of his work, in particular its occasional character and fragmentation. On the other hand, Benedetti's identity as a philosopher was directly related to his cultural ambitions and his engagement aimed to affirm *mathematical philosophy* in the intellectual arena against scholastic thinkers and humanistic literati. His political identity as a lay aristocrat made him an organic part of the centralizing project of the court and marked his distance from Counter-Reformist drives which sought to impose Roman universal interests over territorial states' autonomy. His support for a sort of party of the *politiques* resulted in treatises advising on politically relevant technical and cultural issues (e.g., navigation on the occasion of the battle of Lepanto or the calendar reform). His activity as a lay educator, e.g., his arithmetic teaching to the prince, Carlo Emanuele I, is found in his pedagogical writings, some of which were published in his scientific miscellanea. In summary, both content and form, as well as the demarcation of the fields of his scientific competence as a mathematician and philosopher, depended on social settings and cultural engagement.

The fact that Benedetti never established a scientific school around himself can be seen as an indication of the precarity of patronized science, linked to the person of a particular ruler and not institutionalized at the level of an academic body. In the course of the seventeenth century, these limitations of early court society would be solved by securing scientific continuity for patronized science through the foundation of scientific societies.

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[19]Greenblatt 1980.
[20]Biagioli 1993. Also see Biagioli 1989.

These societies constituted an improvement over the volatility of Renaissance patronage, which depended on the humors and interests of a prince, by replacing him with a corporative *persona ficta* deputed to protect, credit, and promote science. This did not imply a diminution of the political relevance of science. As has been argued, the institution of the Académie Royale des Sciences as a means to patronize all of the sciences also meant the conquest of a new kingdom, *la république des lettres tout entière*, for Louis XIV.[21]

Montesquieu was a perspicacious observer of the courtly society in which Benedetti lived and worked. In his opinion, the "courtly air," or the ethos of the ruling elites of a monarchic state, "consists in putting away one's own greatness for a borrowed greatness. This greatness is more flattering to a courtier than is his own."[22] Such *grandeur empruntée*, or borrowed greatness, was a function of a person's distance from the ruler. Benedetti's greatness could have solely consisted in his mathematical acumen, in his mechanical insights and demonstrations, or in his philosophical discernment; these are the virtues that the historian of science is inclined to observe as principal. However, Benedetti saw himself as a court gentlemen, and only valued his capacity as a mathematician as subordinate. He presented himself as a *court* intellectual, more precisely, as a "philosopher *to the Dukes of Savoy*." He "borrowed his greatness" (in Montesquieu's words) from his proximity to the rulers. In the courtly milieu, it was honor and rank, together with their corollary, ambition, rather than skill, diligence, and measure that marked the character of a nobleman who belonged to the hegemonic class of the new state. Greatness is a major motivation for Benedetti's science, which cannot be confined to technical demonstrations or the solution of specific problems. Rather, his treatment of details never departed from concerns about the big picture; in his work, special issues were constantly elevated and received their meaning on the level of a grand overview, natural and epistemological.

Greatness is not the only courtly quality to enter Benedetti's science. As Montesquieu further observed: "At court one finds a delicacy of taste in all things, which comes from continual use of the excesses [*superfluités*] of a great fortune, from the variety, and especially the weariness, of pleasures, from the multiplicity, even the confusion, of fancies, which, when they are pleasing, are always accepted."[23] To be sure, one cannot say that Benedetti's knowledge was superfluous in the sense that it had no concrete application. In the Renaissance, it was evident to anybody how closely mathematics was connected to practical realms ranging from war technology to fortification, navigation, and administration. Benedetti's work and activities related to these realms; even his astrological consultancies can be appreciated for their practical orientation—as astrology notably coincided with the so-called *astronomia practica*, as opposed to mathematical astronomy, or *astronomia theorica*. Still, Benedetti insisted on his lineage as a "philosopher" (connected with his claims about the Pythagorean universality of his method and the fragmentation of its applications) despite the attention given to practice and concreteness in Renaissance mathematics. Such a contention was aimed at confirming his superiority over the immediate application of knowledge or the material origin of arts such as mechanics.

His stress on theory—on "speculation"—is well attuned to the spirit of court society, which was centered on nobility, that is, on disinterest and rank, rather than efficacy. The "superfluity of Benedetti's science" corresponds to the leisure character of knowledge in general, due to fact that its bond with materiality and practice was sublimated. Whereas corporative and merchant societies like those of the Italian Quattrocento (or, more gen-

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[21] Biagioli 1995, 1418 and 1438.
[22] Montesquieu 1989, 33.
[23] Montesquieu 1989, 33.

erally, bourgeois and democratic ones like those emerging in the seventeenth century) would emphasize the practical origin and meaning of science, a court society stresses its symbolic value rather than direct usefulness and economical importance.

Besides the *superfluité* (which applies to Benedetti only if it is not taken too literally), all of the other qualifications Montesquieu attached to the court atmosphere suit his endeavor: good taste (we can add, "wit"), variety, pleasure, multiplicity, even confusion. The main virtue of a court society rested on the sense of honor and ambition: "Honor, meddling in everything, enters into all the modes of thought and all the ways of feeling and even directs the principles."[24] Norbert Elias, who agreed with this assessment, also pointed out the fatal consequences for budgetary issues of a mentality that is so distanced from a modern bourgeois economy. From an economic viewpoint, court society was intrinsically flawed. It was destined to bankruptcy because form, ritual, and etiquette counted more than parsimony. Similarly, courtly science displayed detachment from monetary return. Elias has also emphasized the centrality of etiquette for this detachment. At court, formal etiquette was decisive, as it served to maintain and reinforce distances and hierarchies.[25]

The sense of honor and superiority typical of such social formations appears in Benedetti's intended distance (social, intellectual, moral, and epistemological) from artisanal practice and the erudition of university professors. He appropriated the results and methods of both fields, in particular those of the practical arts, but at a higher level of generalization. He particularly envisaged a reformed natural philosophy as the most cherished fruit of his "mathematical-physical speculation." Such theoretical distance from immediacy is the epistemological parallel of the sense of honor and social distance and, as such, it became an essential ingredient of Benedetti's science and added symbolic value. As a court intellectual, he did not identify himself with traditional forms of higher culture such as Scholastic Aristotelianism or humanistic rhetoric. He proudly affirmed himself as a courtier, free to think and philosophize in the protected space of the court, independent of the most immediate material needs, of academic constraints dictated by tradition, and concerns about systematicity and completeness. Ambition, the companion of aristocratic honor, "meddled in everything" and directed Benedetti's search for the most general principles of a new vision of nature, both mathematical and physical. The court protected and promoted a science and philosophy in which disinterestedness was foremost. In its favorable womb, a daring mind could venture out to explore new realms beyond established disciplinary boundaries. The speculative freedom of the court also determined the specific form of Benedetti's work, its occasional character, and the amazing variety exhibited by his *diversae speculationes mathematicae et physicae*.

The economy of honor in the court society left an enduring epistemological imprint on the social fabric of science. Symbolic capital governed modern science long after it became coupled with economic capital and, in many ways, it still significantly influences science and research. The legacy of courtly ingenuity and leisure has to be acknowledged as a lasting influence upon scientific practice as well. Moreover, the topos of a protected space, so attractive to the emergent category of philosopher-scientists in the sixteenth and seventeenth centuries, contributed to creating the myth of the independence of pure science. Constant claims and controversies about scientists' autonomy have accompanied the modern path to science in its migration from the court to the scientific academy and from the scientific academy to the laboratory. The connections of modern science to the

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[24]Montesquieu 1989, 33.
[25]Elias [1969] 2002, 173.

economy and society at large, politics, and cultural structures can be appreciated by considering the complex historical ties that link knowledge with its material and cultural conditions reaching far beyond the perception of the individual historical actors. The spirit of Benedetti's science can be seen as typical of an age of profound social transformation and political reconstitution, which is reflected in the exceptional re-structuring of knowledge and the transition to novel forms of scientific acquisition, legitimation, and transmission.

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*Diversarum speculationum mathematicarum, et physicarum liber*
**(Turin 1585)**

# IO · BAPTISTAE
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### Patritij Veneti Philosophi.

*DIVERSARVM SPECVLATIONVM*
*Mathematicarum, & Physicarum*

### Liber.

Quarum seriem sequens pagina indicabit.

AD SERENISSIMVM CAROLVM EMANVELEM
ALLOBROGVM, ET SVBALPINORVM
DVCEM INVICTISSIMVM.

TAVRINI, Apud Hæredem Nicolai Beuilaquæ, MDLXXXV.

*Superioribus permissum.*

TRACTATVS QVI IN HOC
*volumine continentur.*

Theoremata Arithmetica.
De rationibus operationum perspectiuæ.
De Mechanicis.
Disputationes de quibusdam placitis Arist.
In quintum Euclidis librum.
Physica.& Mathematica responsa per Epistolas.

# SERENISSIMO
## CAROLO EMANVELI
### Sabaudiæ Duci, &c.

AGITVR *nonusdecimus annus ex quo litte-*
*ris Serenißimi patris tuæ Celſitudinis , ac-*
*cerſitus ex vrbe Parmenſi in hanc me ciui-*
*tatem contuli . Is aduenientem tam humanè*
*excepit , tanta deinde liberalitate fuit com-*
*plexus ego viciſſim ei deſeruiendi, tam vehe-*
*menti cupiditate fui accenſus , vt ſub eius ditione quod ſuper-*
*eſſet vitæ agere conſtituerem . Cuius in me benignitas , mea*
*in illum obſeruantia mirum in modum mutuo vſu, & conſue-*
*tudine eſt adaucta , vt idem Dux me ſecum dum ruſticaretur*
*eſſe vellet, ſæpè etiam ſecum pernoctare ; quo quidem tempo-*
*re de Mathematicis ſcientijs mecum agebat , in quibus perdi-*
*ſcendis mea opera vtebatur, quæſtiones, Arithmeticam , Geo*
*metriam , Opticen , Muſicam , aut Aſtrologiam ſpectantes*
*proponens . Cui vt quod in me eſſet ſatisfacerem , acrius*
*quàm antea in ea ſtudia ( ad quæ tamen ſemper fui propenſiſſi-*
*mus ) incubui . Illiusq̨ imitatione ( vt ferè cæteri Principum*
*ſtudia imitantur ) non pauci aut præſentes, aut per litteras me*
*de his, atque illis Mathematicis quæſtionibus conſuluerunt .*
*Cùmque ego nunquam laborem amicorum cauſa defugerim ,*
*euenit vt poſt tot annorum curricula, mea ſcrinia ſcrutatus,*
*inuenerim tot abſolutas quæſtiones , vt ex eis corpus mediocre*
*effici poſſe videretur . Quas , cùm rationibus in epiſtola ſub-*
*ſequenti allatis edere conſtituiſſem , non ſub cuiuſque alte-*
*rius nomine, & auſpicijs quam tuæ Celſitudinis volui apparere;*
*tum quòd patri debitum libellum filio reddere par erat , tum*

<div align="right">A 2     <em>quòd</em></div>

quòd in tua Celſitudine paternam in me fouendo, & augendo
benignitatem ineſſe ſemper ſum expertus, tum quòd tuæ Celſi-
tudinis interrogationibus excitatus non pauca quæ hoc volu-
mine continentur, elucubraui. Acceſſit, quod ego ſemper in
his dedicationibus ſpectandum putaui, tuam Celſitudinem tan-
tos progreſſus in Mathematicis feciſſe, vt vel idonea æſtima-
trix mearum vigiliarum eſſe poſſit. Quare, & veterum Per-
ſarum Regum gloriam æquauit, & nos veluti in ſpem certam
fælicitatis huius ſæculi induxit, ſi verum eſt Platonis va-
ticinium, beatam eam futuram Rempublicam in qua
Principes Philoſophentur. Tua igitur celſi-
tudo libellum tot ei nominibus debitum,
ea qua ſolet humanitate accipe-
re nè grauetur. Deus tuas
omnes cogitationes,
& conatus ad
fœliciſſi-
mos
ſemper exitus perducat,
teǵ diutiſſimè ſer-
uet incolu-
mem.

# AD LECTOREM.

VM Varijs temporibus permulta in diuerſis diſciplinis contemplatus ſim, partim à præſtantibus viris patronis ac amicis meis excitatus, qui ſuper eis ſententiam meam exquirebant, partim, ab ingenito mihi deſiderio, rationem, & cauſam eorum percipiendi, committendum non putaui, quin qualiacunque mea ſcripta in illis ſcientijs, ſtudioſis impartirer, non dubitans quin illis aliquid commodi atque vtilitatis allatura ſint, prę ſertim cum in eiuſmodi quæſtionibus inueſtigandis atque perpendendis, nemo (quod ſciam) hactenus elaborauerit. Nihil enim his libris à me traditum eſt, quod aut legiſſe, aut ab alijs audiuiſſe meminerim, nam ſi aliena attigi, ea, aut cum aliqua differentia demonſtrationis, aut dilucidius ſcripſi, quod ſi forte alius eadem tradidit, aut eius lucubrationes ad me non peruenerunt, aut earum perlectionis memoria excidit. Vt enim etiam Ariſtoteles ipſe ſenſit facilè fieri poteſt, vt pluribus, eædem opiniones in mentem veniant. Immo multa ſcribenti euenire poteſt, vt cum iamdiu aliquid ſcripſerit, iam oblitus, idem repetat, quod mihi etiam nonnunquam accidit. In his autem libris non ſuſcepi munus integræ ali cuius ſcientiæ tradendæ, ne, quæ ab alijs iam tradita ſunt, ipſe inutiliter re peterem, mihique viderer ex alienis laboribus laudem voluiſſe comparare. Singularum enim ſcientiarum volumina, iam ab alijs collecta, atque in ordinem ſunt digeſta, & ſi pauciſſimi ſint libri quorum omnes ſententiæ, omniaque inuenta vnius ſint authoris, excipio Archimedis volumina. Cumque multi ſint, qui vel vnam rem à ſe inuentam in publicum proferre non dubitent, multo magis mihi qui multa excogitaui, & ſi inter ſe hætereogenea, atque vtcunque expreſſa, idem licere ſum arbjtratus. In his autem meditandis, ex Arithmeticis authoribus quos inſpexi, præcipuus fuit Nicolaus Tartalea, quippe quem ferè omnia ab alijs ſcripta collegiſſe conſtat, nec alios ex præcipuis, quos legere potui omittendos duxi, inter quos ſunt Hieronymus Cardanus, Michael Stifelius, Gemma Friſius, Ioannes Nouiomagus, Cuthebertus Tonſtallus, cæteriq́; huiuſmodi. Quorundam tamen volumina illorum qui à Tartalea citantur, vt Leonardi Piſani, Proſdocimi, Ioannis Infortunati, Fratris Lucæ, Petri Borgi, aliorumque aliquot inſpiciendorum,

<div align="right">facultas</div>

facultas mihi non fuit . Præterea, licet in his libris nonnullę inueniantur
propositiones, quæ disiunctam ab alijs habeant rationem, eæ non sper-
nendæ tamen sunt, viam fortasse alicui aperient vlterius progrediendi.
Quemadmodum enim , exempli gratia, ex sub contraria coni sectione ,
sumpta postea fuit diuina illa Planisferij delineatio,quæ sub Ptolomæi no-
mine legitur, & sicuti ex penultima primi Euclidis , quam Pythagoras
excogitauit propè innumeræ pulchræ consequentiæ in Astronomia , in
Architectura, in multisq; alijs scientijs desumptæ sunt, immo quemad-
modum ex singulis propositionibus à nostris maioribus excogitatis mul-
ta egregia sunt deducta, ita fortasse continget, vt ex mearum inuentio-
num aliqua,nõnihil in posterum vtilitatis desumatur . Si quid verò,hic in-
ueneris, quod tuo genio non arrideat , illa prudentissimi hominis sen-
tentia in mentem veniat. *Quot capita , tot sententiæ* , ac per raro con-
tingere , vt idem omnibus probari, atque placere queat, & perdifficulter
inueniri hominem cui placeant omnia quæ alteri satisfaciunt. Nec te mo
ueat,quod hęc Theoremata siue excogitationes non videas ordine illo di-
spositas , quo collocari debere existimaueris, tum in Arithmeticis, tum in
cæteris. Cum enim in huiusmodi rebus ordo non sit necessarius, vi-
sum est mihi posse me , sine repræhensione, illum negligere , cum spe-
culationi, siue inuentioni pręcipuè adeo mihi incumbendum decreuerim
vt in collocatione operam ponere , & tempus absumere operæpretium
non duxerim , quod idem in epistolarum collocatione feci,in quibus per-
sonarum ad quas scribo nullus ferè graduum ordo seruatus est, nec tem-
poris, quo sunt scriptæ, quæsitorum tantummodo ratione habita . Nec
admirari quenquam velim, quod in speculandis numerorum passioni-
bus, figuris vtar geometricis, ita enim in. 2.libr. fecit Euclides , qui mo-
dus, eo magis mihi arridet, quo minus est abstractus, *quoniam oportet in-*
*telligentem phantasmata speculari*, cum pręterea perspicuum sit, discretum
omne, ex continui diuisione aliquo modo oriri, siue actu, siue potentia.
Deinde si forte meis in demonstrationibus tibi videbor aliquando bre-
uior, illud in causa fuisse scias, quod ibi ad viros scribebam in his discipli-
nis exercitatos,quibus satis fuit rem significare . Libuit autem mihi om-
nes voluminis Arithmetici propositiones potius vocabulo theorema-
tum appellare, quam problematum , quia pars earum speculatiua tan-
tum mea est, & si ex varijs eiusmodi propositionibus etiam operatiuam
adinuenerim . Quoniam verò multis in locis accidit, vt veritatis iudi-
candæ causa necesse mihi fuerit quorundam sententijs aduersari nolim te
                                                                      hoc

hoc mihi vitio tribuere, meq́; hoc nomine carptorem maledicumq́; ha-
bere quod alienos errores aperiam, cum potius habenda sit mihi gratia,
quod in ijs interdum laborans ( quę Antisthenes in disciplinis magis ne-
cessaria esse dixit, *vt mala scilicet prius dediscantur* ) falsas opiniones euel-
lere studeam, veritatemq́; ostendere, quam omnis philosophus, Aristo-
telis exemplo, pluris quam cuiusuis hominis authoritatem, aut gratiam
facere debet. Cumq́ue in hoc volumine aliquid eiusmodi legeris
te oratum volo, vt in iudicando, affectum omnem exuas,
Sallustianum illud præ oculis habens. *Omnes qui de re-*
*bus dubijs consultant, ab odio amicitia, ira, atque*
*misericordia vacuos esse decet.* Hinc fiet, vt
non personæ ( vt multi solent ) sed
veritati, quę summo studio di-
gnissima est, semper po
tius faueas. Vale
nostrisque
labo-
ribus vtere, si quem inde fructum,
sicuti spero tuleris, illi præ-
cipuè habeas gratiam à
quo omnes fluunt
scientiæ.

# IO·BAPTISTAE BENEDICTI

## PATRITII VENETI SERENISS· CAR· EM·
### ALLOBROGVM DVCIS PHILOSOPHI.

### *Theoremata Arithmetica.*

Raeclare' multa veteres mathematici philosophi de numeris eorumque effectibus excogitata posteris tradiderunt, quorum cum vix vllam rationem reddiderint, aut certè per exiguam, occasione diuersorum problematum mihi à Serenissimo Sabaudiæ Duce propositorum præbita, de ijs quæ ab antiquis proposita fuerunt contemplanda nonnulla occurrerunt, quæ posteritati comendare non inutile arbitratus sum, ne hæ meæ cogitationes interciderent, & occasionem præberem quamplurimis abstrusa hęc indagandi, quæ problematibus & thæorematibus inuoluta, vix aliquem qui euolueret nacta sunt.

Inter cætera vero à me quesita, hoc fuit theorema

### THEOREMA PRIMVM.

Interrogavit me Serenissimus Dux Sabaudiæ, qua ratione cognosci posset scientificè & speculatiue ( vt dicitur ) productum ex duobus fractis numeris, quolibet producentium minus esse. Cui respondi, mente & cogitatione concipiendum esse fractos producentes cum fractis productis, non vnius eiusdemque naturæ esse, imò longè diuersæ.

Exempli gratia, fractis numeris propositis . a. i. et . a. c. quorum integri sint . a. b. et. a. d. qui tanquam lineæ cogitentur, apertum sanè esset productum . c. i. superficiale futurum, quod nomen caperet à producto superficiali. d. b. generato ex vno in aliud totorum linearium, nam si constitueretur. a. i. octauum ipsius. a. b. et. a. c. dimidium. a. d. multiplicato. a. i. cum. a. c. produceretur sextumdecimum ipsius. d. b. Quare. d. b. esset totum relatiuũ ipsius. c. i. non aliquod totum producentium. Mirum itaque non est si productum . c. i. minus videatur suis producentibus, cum toto, diuersæ naturæ à primis conferatur, fractum siquidem ab integro eiusdem naturæ, linearis, superficialis, aut corporeæ denominatur.

Quòd si amplioris cognitionis gratia ex scientiæ præceptis speculari voluerit aliquis

A

quis, qua ratione fractus numerus . c. i. minor fit in fuo integro . d. b. fracto. a. i. in fuo integro . a. b. aut fracto . a. c. in fuo integro . a. d. confideret is quo pacto proportio. c. i. ad. d. b. minor fit proportione. a. i. ad. a. b. et. a. c. ad. a. d. hac ratione. Manifeftum eft ex prima fexti de quantitate continua, aut. 18. feptimi Euclidis de difcreta, proportionem ipfius. d. i. ad. d. b. effe ficut. a. i. ad. a. b. & cum. c. i. minor fit. d. i. velut pars fuo toto, proportio, c. i. ad. d. b. minor erit proportione. d. i. ad. d. b. ex. 8. quinti, quare minor erit pariter proportione. a. i. ad. a. b. ex. 12. eiufdé vnà etiam proportio. c. i. ad. d. b. minor erit. a. c. ad. a. d. ex eifdem caufis, medio. c. b. Ex quibus patet ratio, cur fracti diuerfarum denominationum ad vnicam reducantur. Cur etiam numeros integros in partes fractis fimiles frangere liceat, quæ omnia ex fubfequenti figura facilè cognofci poffunt.
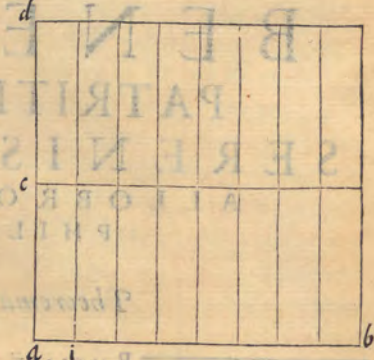
## THEOREMA II.

Q V ae fit ratio, cur hi, qui numeros, fractos diuerfarum denominationum colligere volunt, & in fummam redigere, multiplicent vnum ex numerantibus per denominatorem alterius, & poftmodum denominatores adinuicem, quorum vltimum productum, commune eft denominans duorum priorum productorum, quæ collecta in fummam efficiunt quod quærebatur.

Qua in re fciendum eft, denominantes confiderari tanquam partes vnius eiufdéque magnitudinis quantitatis continuæ, linearum (verbigratia) a. b. et. a. d. æqualiũ in longitudine, quarũ. a. b. in quatuor partes diuidatur, et. a. d. in tres. Quare fi colligere voluerimus duo tertia cum tribus quartis, multiplicabimus. a. c. duo tertia, cum. a. b. diuifa in 4. partes, producetúrque. c. b. octo partium fuperficialium, dehinc multiplicando. a. i. tres quartas cum. a. d. diuifa in. 3. partes producetur. i. d. primis fingulis æqualis, nouem partium fuperficialium, multiplicata deinde a. b. diuifa in. 4. partes per. a. d. in. 3. diuifa, producetur quadratum. d. b. in continuo, in 12. partes diuifum, quod erit totum commune fingulis productis, quorum primum erat. c. b. Quare. c. b. ita fe habet ad totum. d. b. ficut. a. c. ad. a. d. ex prima fexti in continuis, aut. 18. feptimi in difcretis quantitatibus, et. d. i. ad. d. b. ficut. a. i. ad. a. b. ex eifdem propofitionibus. Collectis deinde partibus producti. c. b. cum partibus producti. d. i. manifeftè depræhendetur eiufmodi fummam componi ex partibus vnius totius communis fingulis earum.
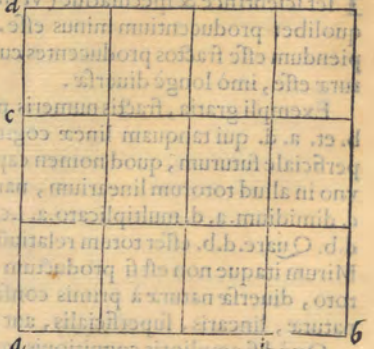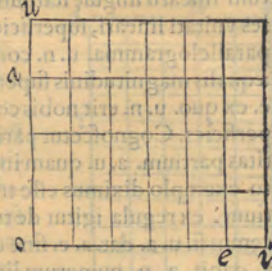
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## THEOREMA III.

CVR reperturi qualis sit fractus aliquis numerus respectu alterius, multiplicare debeant numeratores adinuicem & ita etiam denominatores, ex quo productum ex numeratoribus nomen capiat à producto denominatorum.

Huius si causam nosce vis, sume. o. i. &. o. u. pro totis denominatoribus, tum. o. e. &. o. a. pro numeratoribus ( exempli causa) sit. o. i. senarius. o. u. quaternarius. o. e. quinarius. o. a. ternarius. Si nosce vis quæ sint tres quartę partes quinque sextarum, patet ex regulis practicis oriri quindecim vigesimasquartas. Id quomodo fiat, ex subscripta figura depræhendetur, memores tamen esse oportet, quodlibet productũ considerari tanquã superficiem, producentia autẽ tanquam lineas. In hac igitur figura productum ex totis linearibus est. u. i. aggregatum ex. 24. partibus, &. u. e. productum aggregatum ex. 20. Quod ita se habebit ad productum totale. u. i. sicut. o. e. ad o. i. ex prima sexti aut. 18. septimi, ita. u. e. erunt quinque sextæ partes. u. i. quarum in proposito exemplo, tres quartæ quærũtur. Si itaq; multiplicabitur. o. e. cũ. o. a. orietur productum. a. e. ita proportionatũ ad. u. e. sicut. o. a. ad o. u. reperitur, ex prædictis rationibus. Quòd si statutũ est. o. a. tres quartas partes esse ipsius. u. o. etiã. a. e. tres quartæ partes erũt. u. e. sed. u. e. quinque sextæ sunt ipsius. u. i. ex quo sequitur bonum esse huiusmodi opus.

## THEOREMA IIII.

CVR multiplicaturi fractos cum integris, rectè multiplicent numerantem fracti per numerum integrorũ, partiãturq́ue productum per denominantẽ fracti, ex quo numerus quæsitus colligitur.

Propter quod mẽte concipiamus in subsequenti figura, numerum integrorum tãquam lineam. a. e. qui, verbigratia, sit denarius, quorum vnusquisque sit æqualis a. i. cogiteturq́ue productum ipsius. a. e. in. a. i. sitq́ue. u. e. quod quidem erit denarius superficialis, constituta prius. a. u. æqualis. a. i. &. a. o. sint duæ tertiæ. a. u. quarũ duarum tertiarum productum in numerum. a. e. sit. o. e. pariter. u. i. vnitas sit superficialis prout. a. i. vnitas est linearis, quam. u. i. respicere debet productum. o. e. ex quo integer superficialis. u. i. erit tanquam ternarius, & productum. o. i. tanquam binarius, & quia quælibet pars è viginti ipsius. o. e. æqualis est tertiæ parti. u. i. vnitatis superficialis; si cupiamus scire quot integræ vnitates sint in partibus. o. e. consultum est easdem diuidere per denominantem. u. i. compositum ex tribus partibus superficialibus, & cum tam linea u. a. quam superficies. u. i. diuidatur in 3. partes æquales nosce peroportunũ est eiusmodi partitionem numeri. o. e. fieri per numerum ipsius. u. i. non. u. a. ex prædictis causis.
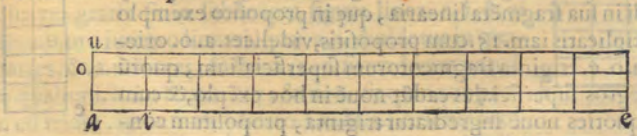
A 2 THEO-

## THEOREMA V.

ALIA quoque via prædicti esse
ctus causa, speculando inno-
tescere potest, cuius rei gratia for-
metur sequens figura. e. o. a. u. n.
eiusmodi, vt. a. e. sit numerus li-
linearis integrorum, &. o. e. produ-
ctum numerantis ipsorum fractorū
in integris, ex quo. a. o. erunt duæ
tertiæ, verbigratia, a. i. aut a. u. qua-
rum linearū singulę statuuntur æqua
les vnitati lineari, superficies autem
parallelogramma. u. n. constituatur
æqualis magnitudinis superficiei. o.
e. ex quo. u. n. erit nobis cognita su-
perficies. Cognoscetur pariter quan
titas partium. a. u. quam in proposi-
to exemplo diximus esse trium par-
tium. ex regula igitur de tribus, di-
cemus si. u. a. dat. a. e. sine dubio. o.
a. dabit. a. n. numerum linearem.
quæ regula ex. 15. sexti in continuis,
& ex 20. septimi in discretis, depro-
mitur. recte igitur multiplicâtur fra-
cti numerantes cum integris, & productum diuiditur per denominantē fractorum.
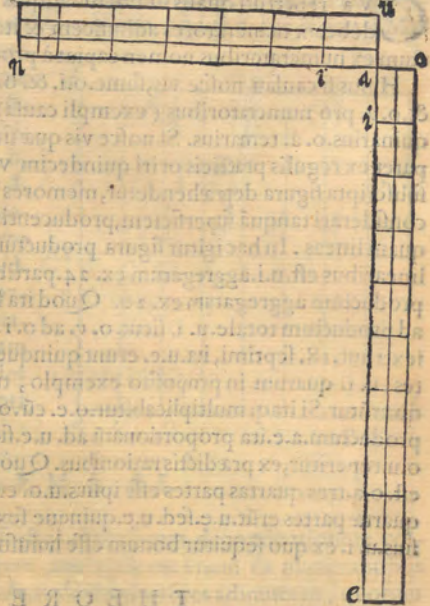
## THEOREMA VI.

ITEM & alia speculatione cognosci potest hoc recte fieri mul-
tiplicantes enim has duas tertias per decem, debemus conside-
rare quantitatem duarum tertiarū decies produci, ex quo oriuntur
20. tertia, quandoquidem singulæ vnitates, tunc pro duobus ter-
tijs sumuntur, sed cum quilibet integer tria fragmenta contineat,
ideo ex ratione partiendi quoties ternarius ingrediatur viginti,
statim cognoscemus quod optabamus.

Id ipsum accideret si integri in eiusmodi specie fractorum diui-
derentur. quo facto hi multiplicandi essent cum numerante propo
sito, & partiendū productum per quadratum denominantis.

Cuius rei hæc est speculatio. Sit linea. a. e. constans ex quinqß
integris numeris, quorum vnusquisqß æqualis sit. a. u. vel. a. i. &. a. o.
sint duo tertia vnitatis integræ linearis. cogitemus nunc hos quinqß
integros diuidi in sua fragmēta linearia, quę in proposito exemplo
erunt. 15. multiplicatis iam. 15. cum propositis, videlicet. a. o. orie-
tur productum. o. e. triginta fragmentorum superficialium, quorū
in singulos integros superficiales cadūt noue in hoc exēplo, & cum
notauerimus quoties noue ingrediatur triginta, propositum con-
sequemur.

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## THEOR. ARITH. 5
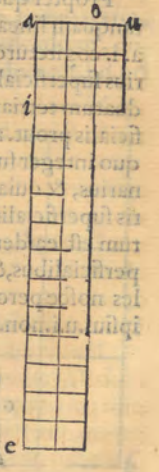
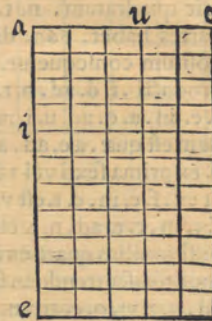### THEOREMA VII.

CVr multiplicaturi integros numeros & fractos, cum integris & fractis, debeant integros reducere ad species fractorum, eos colligendo cum fractis: deinde multiplicare hos vltimos numerantes adinuicem & productum partiri per productum denominantium.

Vt ( exempli causa) si volumus multiplicare vnum & duo tertia, per duo & tria quarta, reducentur omnia in fractos, ex quo vna ex parte essent quinque tertia, multiplicanda cum vndecim quartis ex altera, quo facto oriretur productum quinquagintaquinque fractorum, quod diuisum per productum ternarij in quaternarium, videlicet per duodecim, quatuor integri proferentur cum septem duodecimis fractis vniusintegri.

Detur subsequens figura in qua linea a.i.æqualis sit lineæ.u.a.quarum vnaquæq; cósideretur pro integro numero:cogiteturq;.a.i.valere quatuor in presenti exéplo,&.a. u.tria:detur deinde linea. a. o.æquipollens vni integro cũ duobus tertijs, &.a.e. æquipollens duobus integris & tribus quartis. Iam si hæ duæ lineæ in suos fractos. reducantur,multiplicata(vt in sequenti figura apparet.)a.o. cũ a. e. orietur productum. o. e. fractorum superficialium quinquagintaquinq;, quorum integer superficialis valet duodecim, scilicet. u. i, vt cuique manifestum est, ex quo, quærenti media partitione, quoties duodecim ingrediatur quinquagintaquinque,citra errorem,quæsitum occurret.



### THEOREMA VIII.

IDipsvm accideret si fracti ad vnam eandemque denominationem reducerentur,qui postmodum simul multiplicarentur, productumque partiremur per quadratum denominantis communis.

Exempli causa,sint eadem quinque tertia, & vndecim quarta adinuicem multiplicanda, quæ si reducantur ad vnam & eandem denominationem quinarius numerans vnius, multiplicabitur cum quaternario denominante alterius, & vndenarius secundi cum ternario denominante primi. ex quo vna ex parte essent viginti, ex altera 33. numerantia vnius cómunis denominantis, quod esset productum ternarij in quaternarium, videlicet duodecim, vt ex veteri regula patet. Iam si multiplicentur viginti cum trigintatribus, dabuntur. 660.fracti,quorum integer erit quadratum duodenarij,nempe. 144.quibus quidem.660.diuisis per.144. proferentur quatuor integri & septem duodecimi.



Cuius rei gratia sit in subscripta figura linea. a. i. & ei æqualis.a.u. pro integro lineari, quæ. a. i. diuidatur in quatuor partes, &.a.u.in tres, & linea . a. e. sit vndecim partiũ talium qualium. a. i. est quatũor , &.a. o. sit quinque prout.a.ũ. est trium . nunc multiplicato.a.o.&.a.i.orietur productum.o.i.viginti partium superficialium. tum multiplicato
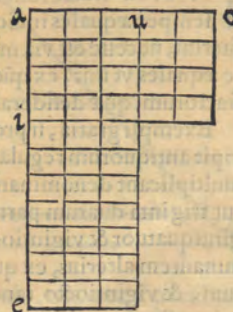
cato. a. e. per. a. u. dabitur productum. u. e. trigintatriũ partium. ad hæc quadratum. u. i. constabit ex duodecim partibus eiusdem rationis cum reliquis duobus productis, quod quadratum. u. i. vnitas est superficialis, & communis denominans duorum productorum. quod si in præsentiarum cogitabimus lineam. c. d. trigintatrium partium æqualium, et. c. t. duodecim similium, et. c. f. viginti. c. n. duodecim, multiplicato. c. d. cum. c. f. dabitur superficies. f. d. 660. fractorum superficialium, quorum vnitas integra superficialis erit quadratum. n. t. 144. partium cuiusmodi. f. d. partes habet. 660. diuiso itaque. f. d. per. n. t. propositum consequetur. eo quòd eadem proportio erit producti. f. d. ad. n. t. quæ producti eius quòd fit ex. a. e. in. a. o. ad. u. i. nam proportio. c. d. ad. c. t. eadem est quæ. a e. ad. a. i. & c. f. ad. c. n. vt. a. o. ad. a. u. ex prima sexti vel 18. septimi, sed vt. f. d. ad id ꝙ fit ex. f. c. in. c. t. est vt. c. d. ad. c. t. & vt eius ꝙ fit ex f. c. in. c. t. ad. n. t. est vt. f. c. ad. c. n. ex dictis propositionibus quare ex æqua proportionalitate, eodem modo discurrendo in figura. o. a. e. ita se habebit. f. d. ad. n. t. vt. o. e. ad. u. i. Porrò ex ijs, quæ hactenus de fractorum multiplicatione considerata fuerunt, apertè ratio deprehenditur, cur productum, singulis producentibus semper minus sit, cum producta sint superficialia producentia verò semper linearia, omissis productis corporeis, quæ omnia ad superficialia reducuntur.
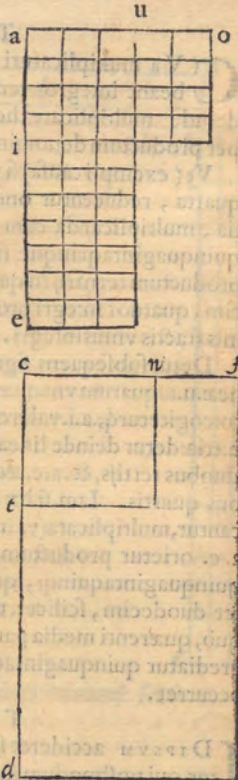
## THEOREMA IX.

IN Ipsa fractorum diuisione, animaduertendum est, denominantes numeros semper æquales inuicem esse debere, vnius scilicet speciei, quòd si æquales non fuerint, necesse est via multiplicationis ipsorum denominantium adinuicem efficere æquales vt sint, ex quo productum oritur eiusmodi, vt aptum sit habere partes fractorum, quæ desiderabantur.

Exempli gratia, si proponerentur diuidenda septem octaua per tria quarta præcipit antiquorum regula, vt ad vnam tantum denominationem reducantur. quare multiplicant denominantes inuicem. ex quo productum in materia proposita oritur triginta duarum partium commune denominans, cuius duo numerantes sunt vigintiquatuor & vigintiocto, producti ex multiplicatione vnius numerantis in denominantem alterius, ex quo dantur vigintiquatuor tamquam tria quarta trigintaduorum, & vigintiocto tanquam septem octaua particularum vniformium, prout ope primæ sexti aut decimæoctauæ septimi in subscripta figura cognosci potest. Sit

Sit itaque linea.a.i. diuisa in partes octo,& ei æqualis in longitudine.a.u. in qua-
tuor, productum verò vnius in alteram
sit. u. i. trigintaduarum particularum
superficialium similium & æqualiū ad-
inuicem. sit deinde . a. e. septem partiū
lineæ. a. i. & . a.o. trium partium. a. u.
tunc productum . a. e. in . a. u. erit. u. e.
particularum superficialium vigintiocto
& productum . a. o. in . a. i. erit. o. i.par-
ticularum superficialiū vigintiquatuor
eiusdem naturæ cum partibus triginta-
duabus totius denominantis communis.
vnde diuiso numerante vigintiocto per-
numerantem vigintiquatuor , dabitur
vnum cum sexta parte illius vnius.
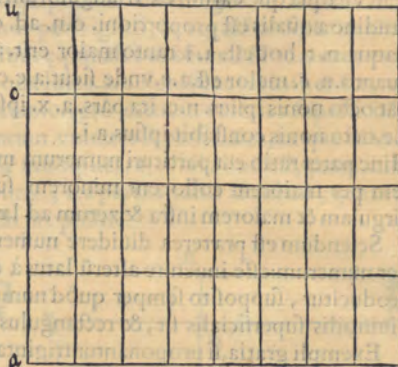
## THEOREMA X.

PARTIRI seu diuidere vno numero alium numerum , est etiam quodammodo
eiusmodi partem numeri diuisibilis inuenire respectu totius numeri diuisibilis,
cuiusmodi est vnitas in diuidente respectu totius diuidentis,partem inquam numeri
diuisibilis sic se habentem ad totum numerum diuisibilem sicut vnitas ad totum di-
uidentem, quod similiter ex regula de tribus præstamus dicentes , si tantus numerus
diuidens dat vnitatē , quid dabit numerus diuisibilis,quemadmodum ex . 15. sexti
seu. 20. septimi licet speculari , Idcircò quotiescunque minorem numerum per
maiorem diuidimus,semper qui prouenit fractus est.

Exempli gratia, si cogitaremus lineam . a. e. diuisam in octo partes æquales,qua-
rum vna scilicet vnitas esset. a. i. & cupere-
mus eam diuidere in nouem partes, ac scire
quanta sit nona illius pars ; manifestum esset ,
nonam partem ipsius . a. e. minorem futuram
ipsa . a. i. cum. a. i. diminui debeat à sua inte-
gritate eadem proportione, qua . a. e. minor
reperitur vna linea nouem partium æqualium
singularum . a. i.

Quod vt dilucidè cuiuis innotescat, hoc
etiam modo licebit videre sit linea . n. c. no-
nupla ad.a. i. & parallela ad . a. e.dubium non
est quin. n. c. maior futura sit ipsa . a. e. iam si
earum extrema congiungantur med js duabus
lineis. n. a. et . c. e. quæ simul concurrant in
puncto. o. ( quod est probatu facillimum ) da-
buntur certè duo trianguli similes. a. o. e. et. n. o. c. Sit deinde . n. t. vna è partibus
ipsius . n. c. quæ. n. t. æqualis erit . a. i. ex præsupposito . ducatur deinde . o. t. quę
intersecet . a. e. in puncto. x. dico. a. x. tanto minorem futuram . a. i. quanto . a. e.
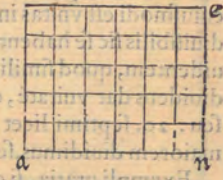minor est. n. c. neque enim dubium esse potest quin proportiones . n. t. ad . a. x. et.
n. c.

## THEOR. ARITH. 9

Hinc manifeſte patet quamlibet diuiſionē aut partitionem oriri ex regula de tribus, quandoquidem ſinguli diuidentes æquipollent vni integro, & loco illius ſumuntur. Perinde enim eſt diuidere centum per viginti, ac regulá obſeruare de tribus dicētes, ſi viginti æquipollent vni, quibus ęquiualebūt cētum? Hoc autem ex ſub ſequenti figura facile deprehendetur, in qua linea.a.b.ſignificat viginti, et.a.o. vnitaté linearé, et.a.c. vnitates lineares centū:o.c. verò centum vnitates ſuperficiales, et.a.d.quinq; vnitates lineares, et.d.b.centum vnitates ſuperficiales, ex quo manifeſtè deprehenditur quòd quemadmodum multiplicare, nihil aliud eſt, quam inueni re productū ex duobus lateribus propoſitis, ita partiri nihil aliud eſt, quàm dato vno latere inuenire aliud latus producti propoſiti.
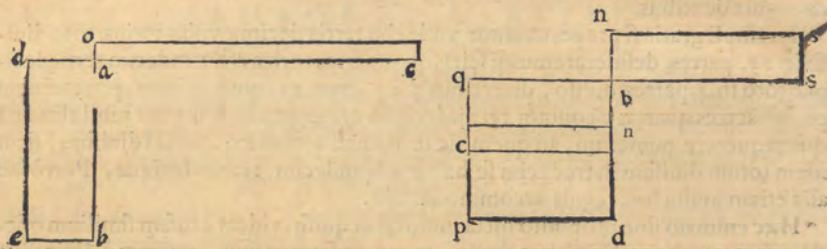


Nam quotiescunq; ratiocinātes dicimus tantundem numeri, immediate produci mus ſuperficiem, mediāte vnitate in huiuſmodi numero, qui numerus antequā producatur in vnitatem, mente concipiendus eſt tanqua m linearis, tanquam linea inquam diuiſa in totidem particulas lineares, ſingulas continuas & æquales vnitati propoſitæ. Cū verò productus fuerit numerus in vnitate ſuperficialis, erit ac ſi tot eſſent vnitates quadratæ, quod ſi ita non eſſet, nulla mentio facienda eſſet quorumuis fractorū. Ex eadē regula de tribus reduci poteſt ad praxim tertiū theorema.

Quare cupientes ſcire quæ ſint illæ partes, quæ ſunt tres quartę, ipſarum quinque ſextarum, dicemus ſi quatuor dant tria, quid dabunt quinq; ſextæ? dabunt. 15. vigeſimas quartas, quæ quindecim ſunt tres quartæ ipſius. 20. viginti autē quinq; ſex tæ vigintiquatuor, quandoquidem nos numerum quęrimus, cui ita proportionentur quinq; ſextæ alterius numeri, ſicut quatuor ad tria, vnde ſic ſe habent. 20. ad. 15. ſicut. 4. ad. 3. ipſe autem. 20. quinq; ſextę partes ſunt vigintiquatuor, vt per ſe notū eſt.

Ex eadem regula de tribus, huiuſmodi quęſito reſponderi poteſt, ſi conſtituamus prædictas quinq; ſextas eſſe numerum, cuius tres quartæ quęrantur, dicentes, ſi vnus integer dat tres quartas, quid dabunt quinq; ſextæ? quare ſequentes regulam de tribus, dabuntur quindecim vigeſimæ quartæ. Valet eadem regula de tribus; vt quis ſcire poſſit, quæ pars aut partes numeri propoſiti ſit aliquis numerus.

Exempli gratia, ſcire cupienti, quæ pars aut partes ipſius vigintiquatuor ſint ſexdecim, conſtituentur. 24. tanquam vnum totum, cuius pars aut partes ſint ſexdecim, dicemus igitur ſi .24. dant ſexdecim, quid dabit vnum? ſexdecim videlicet vigeſimasquartas, quæ cum ad primos numeros reductæ fuerint, erunt duæ tertiæ. Eadem ratione qui ſcire uellet, quæ partes aut pars eſſent tres quartæ, octo nonarum, diceret, ſi octo nonæ dant tres quartas, quid dabit vnum? prouenient. 27. trigeſimęſecundæ.

Subſeruit pariter ad ſciendū naturā partiū numeri propoſiti. Exempli cauſa, ſi quis quærat, cuius numeri, duodecim ſint duæ tertiæ partes. Dicet ſi duo dant tria, quid

B dabunt

dabunt duodecim ? nempe dabunt decemoƈto , numerum quæſitum ſcilicet,
Tunc autem nil aliud preſtamus quam quòd quærimus numerum ad quem ita ſe
habeant duodecim , ſicut duo ad tria. Ita etiam ſi quis quærat, cuius numeri duo
tertiæ ſint tres quintæ, dicet, ſi tria dant quinq;, quid dabunt duo tertia ? nempe da-
bunt integrum cum fraƈto nono. Hoc erit itaq; quęrere numerum ad quem ſic ſe
habeant duo tertia ſicut tria ad quinq;,quod manifeſtum eſt per ſe.

Eadem ratione qui ſcire vellet,cuius numeri duæ ſeptimæ, eſſent oƈto integra-
rum cum duabus quintis,diceret,ſi duo dant ſeptem quid dabunt oƈto integra cum
duabus quintis?nempe dabunt.29.integra cum duabus quintis numerum quæſi-
tum. Sic etiam qui transferre uellet fraƈtum numerum in fraƈtum , id perficeret
ex regula de tribus.

Exempli gratia ſi proponerentur vndecim tertiædecimæ vnius totius, toto diuiſ-
ſo in.13. partes, deſideraremusq; ſcire,quot partes totius eſſét vndecim tertiędeci-
mæ,toto in.4.partes diuiſo , diceremus ſi. 13 .dant. 11. quid dabunt quatuor?nem-
pe dabũt tres quartas cũ quinq; tertijsdecimis unius quartæ,hoc verò nihil aliud eſt
quam querere numerum , ad quem ſic ſe habeat totum in 4. partes diuiſum , ſicut
idem totum diuiſum in tredecim ſe habet ad undecim tertiasdecimas, Porrò ad
alia etiam multa hæc regula accommodata eſt.

Hæc enim nó ſine propoſito diƈta ſunt,ſed ut quiſq; videat cauſam ſimilium ope-
rationum,quæ à praƈticis circa fraƈtos numeros ſcriptæ ſunt , omnem à diuina illa
regula de tribus originem trahere ut etiam in ſequentibus videbimus.

## THEOREMA XI.

CVr produƈtum ex eo quod oritur in diuidente , ſemper æquale eſt numero
diuiſibili ſi queras ita accipe.

Sit numerus diuiſibilis.b.quod oritur ſit.c.diuidens.d.& vnitas diuidentis.t. cum
igitur,vt in præcedenti theoremate diƈtum
fuit, eadem ſit proportio . b.ad.c.quæ eſt.d.
ad.t. manifeſte deprehenditur ex.20. ſepti
mi,produƈtum ex.b.in.t. æquale eſſe pro-
duƈto .c.in d.
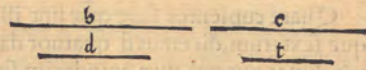
## THEOREMA XII.

ID ipſum alia ratione contemplari licet.
Numerus diuiſibilis ſignificetur per lineam.n.e.diuidens verò per lineam.a.e.
quod oritur linea.u.e vnitas diuidentis.o.e.quá cogitamus eſſe vnitatem linearem;
ad hæc produƈtum ex.u.e.in.a.e.ſit ſuperficies.u.a. Dico ſuperficiem.u.a. componi
ex tot vnitatibus ſuperficialibus quot linearibus conſtat linea.n.e.nam ex ijs quæ
diuidendi ratione notauimus,cóſtituitur
eandem proportionem eſſe .n.e. ad.u.e.
quę eſt.a.e.ad.o.e.At ex prima ſexti aut
18.ſeptimi ſic ſe habet totale produƈtũ.
u.a. ad partiale. u.o. ſicut.a.e.ad.o. e.
quare ſic ſe habebit.u.a.ad.u.o. ſicut.n.
e.ad.u.e.ſed.u.e.et.u.o.numero non differunt,cum ſint vnius & eiuſdem ſpeciei, (ta-
met ſi numerus.u.o.ſit ſuperficialis et.u.e.linearis). Itaq; ex nona quinti numerus.
u.a. æqualis erit numero.n.e.
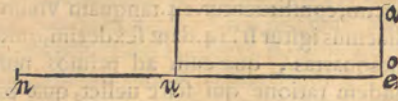
THEO-

## THEOREMA XIII.

CVr diuidentibus numerum diuisibilem per proueniens, oritur numerus diui-
dens?

Sit subscriptus rectangulus. o. e. numerus diuisi
bilis, qui producitur, tam ex. a. o. in. a. e. quàm ex. a.
e. in. a. o. quare si. a. o. diuidens fuerit. a. e. proue-
niens erit, si verò. a. e. diuidens extiterit, a. o. pro-
ueniens erit futurum.
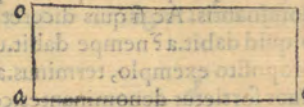
## THEOREMA. XIIII.

HOc ipsum, alia quoq; uia licebit speculari.
Sit linea. a. denotâs numerum diuisibilem, et. o. primi prouenientis linea. e. pri
mi diuidentis .u. secundi prouenientis idest cum. o. pro diuidente sumetur . Iam ex
indicata definitione diuisionis nono theoremate huius libri, dabitur proportio. a.
ad. o. prout datur. e. ad vnitatem significatam li-
nea. i. & permutatim .a. ad. e. sicut .o. ad. i. sed. a.
ad. u. sic se habet prout. o. ad. i. ex eadem definitio-
ne diuisionis , itaq; sic se habebit. a. ad. u. sicut. a. ad.
e. vnde. u. æqualis erit. e. ex. 9. quinti
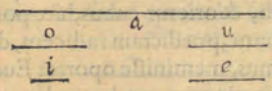
## THEOREMA. XV.

VNde prouenit, vt qui velit cognoscere cuius numeri quatuor quintæ par-
tes, sint , duæ tertie, aut quid simile , cósultissime faciat , si ad unam eandemq;
denominationem reduxerit.

Prout in proposito exemplo, cũ denominâs cómunis sit quindecim, cuius duæ ter
tiæ sunt decẽ, & quatuor quintæ duodecim, cómunis autẽ denominans. 15. multipli
candus sit per quatuor quintas, scilicet duodecim , & productum diuidendum per
duas tertias, hoc est decem, ex quo oriantur decemocto quesitus numerus ?

Quod ad reductionẽ numeratorũ ad vnam & eandem denominationem attinet,
ea de causa fit quo uti possimus regula de tribus, quæ tribus tantummodo notis ter-
minis indiget , quo quartus à prædictis dependens, inueniri possit , quandoquidem
bini illi respectus , tribus terminis comprehendi possũt. At quod ad multiplicatio-
nem spectat denominantis cómunis cũ numerante denominantis incogniti & diui-
sionem producti per numerantem cogniti illẽ nihil aliud sunt, quam quartũ terminũ
inuenire, ita proportionatum tertio, vt secundus primo.

Exempli gratia, sit. a. denotâs nume-
rantem denominantis cogniti, qui signi
ficetur linea. o. et. e. sit denominantis in-
cogniti numerans, denotati linea. u. imò
verò & cogniti. o. nempe quatuor
quintæ, Iam si. o. cum. e. multiplicemus , & productum per .a. diuidemus dabitur. u.
sic se habens ad. e. sicut. o. ad. a. ex. 20. septimi.

B 2 Inue-

12                    IO. BAPT. BENED.
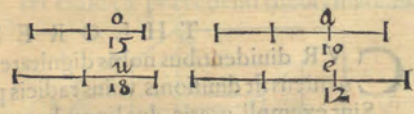
## THEOREMA XVI.

INuenire autem cupienti cuius numeri, duæ tertiæ, sint quatuor quintę partes, mul
tiplicandę essent duæ tertiæ per denominantem communem, & productum diui-
dendum per quatuor quintas ipsius de-
nominantis. Ac si quis diceret si.e.dat.
o.quid dabit.a? nempe dabit.u.nam in
proposito exemplo, terminus.a.loco.e.
duos sortietur denominantes, cognitum
videlicet.o. et.u. incognitum quod po-
stea cognitum oritur ex regula de tribus, vt dictum est.
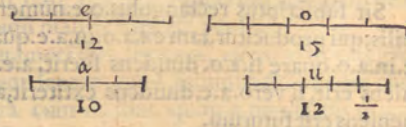
## THEOREMA XVII.

QVA ratione cognosci poterit proportionem quantitatis censicæ censicæ ad
similem quantitatem quadruplam esse ad eam, quæ est suarum radicum; pro-
portionem autē primarum relatarum esse quintuplam, atq; ita deinceps?
   Cuius rei gratia, sciēdus est modus, pductionis harū dignitatū qui oritur ex produ-
ctione primæ radicis in seipsam, prout qui cubū requirit, ducat radicē in suo quadra-
to, & orietur cubus, hæc postea ducta in cubum, quantitatē censicam censicā, et in
hanc, prædictam radicem, dabit quantitatem primam relatam. Quod vbi sciueri-
mus, meminisse oportet Euclidem decimaoctaua sexti aut.11.octaui docere, pro-
portionem quadrati ad quadratū, duplam esse proportioni suarum radicum, &.36.
vndecimi aut.11.octaui, cubi ad cubū triplam esse, ego verò nunc assero, censici cen
sici ad radicum proportionem quadruplam esse, primi verò relati ad primum re-
latum quintuplam atq; ita gradatim.
   Cuius speculationis gratia, detur linea.d.quæ cubum maiorem significet.et. b.
minorem.c. verò sit radix ipsius.d.et.e.ipsius.b. ita ordinate adinuicem, vt in sub-
scripta figura cernitur. Iam.c.cum.d. producatur proueniatq; . q. censicum censi-
cum, tum producatur. e. cum.b. et dabitur. p. alterum censicum censicum. Dico
igitur proportionem. q. ad. p. quadruplam esse proportioni.c. ad. e. hac de
causa quòd proportio. q. ad. p. compo-
natur ex proportione. d.ad.b. et. c.ad. e.
prout facile ex.24. sexti, aut quinta octaui
deprehenditur. Quare cū proportio.d.ad.
b.proportioni.c.ad.e. tripla sit, patet pro-
portionem.q. ad.p.quadruplam esse pro-
portioni.c.ad.e.Idem de cæteris dignitati
bus dico, sumptis semper.d. et.b.pro duo-
bus censibus censuum, aut duobus primis relatis, aut alio quouis axiomate.
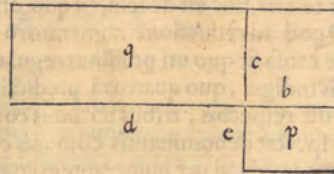
## THEOREMA. XVIII.

CVR diuidentibus nobis dignitatem, per dignitatem, radix prouenientis: pro
ueniens sit diuisionis vnius radicis per alteram?
   Sint exempli gratia duę lineæ.b.q.et. f.g. quæ significent duas radices cuiusuis
dignitatis; demusq; esse radices duorum quadratorum, quadratumq; ipsius b.q.
per quadratum ipsius. f.g. diuidatur; quadrataque radix prouenientis sit.d.q.
vnitas verò linearis sit.i.g. Dico ipsam.d.q. esse proueniens ex diuisione.b.q.
per.f.g. Patet enim ex definitione diuisionis nono theoremate tradita quadra-
                                                                          tum

rum ipſius.d.q.talem eſſe partem quadrati ipſius.b.q.qualis quadratum ipſius.g.i.
eſt quadrati ipſius.f.g. Scimus præterea ex.19.ſexti,aut vndecima octaui, propor-
tioné quadrati ipſius.b.q.ad quadratũ ipſius.d.q. duplam eſſe proportioni. b. q. ad.
d.q. ſuarum radicum (cuborum enim tripla eſſet & cenſuum cenſuum, quadrupla,
atq; ita deinceps ex præcedenti theoremate) Id ipſum dico de dignitatibus ipſius.
f.g.et.i.g.reſpectu radicum.f.g.et.i.g.Vnde
cum proportio dignitatis ipſius.b.q. ad il-
lam.d.q. equalis ſit proportioni dignitatis
ipſius.f.g.ad illam.g.i.ex communi ſcien-
tia aperté cognoſcemus ſimplices propor-
tiones eſſe interſe æquales,nempe eam quę
eſt.b.q.ad.d. q.æqualem eſſe ei, quæ eſt.f.
g.ad.i.g.itaq; ſequitur ex definitione diuiſionis.d.q.eſſe proueniens ex diuiſione.
b.q.per.f.g.
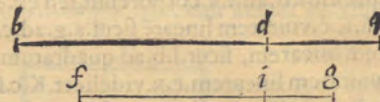
## THEOREMA XVIIII.

CVR productum ex duabus radicibus quadratis, eſt quadrata radix, producti
ſuorum quadratorum ſimul?

In cuius rei gratiam,ſint duo quadrata.d.a.et.n.o. coniuncta ſimul, prout in ſub-
ſcripta figura apparet,ita tamen vt angulus.a.n.u.ſit re-
ctus , quare ex quartadecima primi,duo latera. n.c.et.
n.a.directe coniũgentur adinuicem, prout etiam reli-
qua duo latera.n.u. et. n.d.Cogitato deinde.a.u.pro
ducto ipſius.a.n.in.n. u. duarum videlicet radicum
quadratarum ſimul , dabitur ex prima ſexti, aut de-
cimaottaua ſeptimi,productum.a.u.medium propor-
tionale inter quadratum.a. d. et.u.c. quod ſi cogi-
tem is has tres ſuperficies, tres numeros eſſe , pate-
bit ex vigeſimaprima ſeptimi productum. a. u.in ſe-
ipſum,quadratum ſcilicet. a. u. æquale eſſe producto.
a. d. in.u.c.ex quo propoſiti euidentia conſequetur .
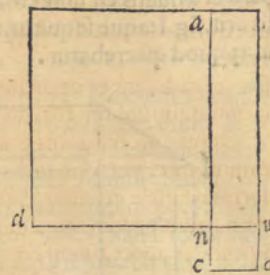
## THEOREMA XX.

QVA ratione id ipſum in cubis cognoſci poterit.

Sit cubus.l. b. & cubus. o.p. quorum productum ſit. u. g. quod aſſero eſſe
cubum , quamuis Eucli. idem probet
in. 4. noni. cuius radicem demonſtra-
bo eſſe numeri æqualis numero.m. q.
qui.m.q. productum eſt ipſius.m.e.in.e.
q.radicum propoſitorum cuborum.Pa-
tet enim ex præcedenti theoremate.m.
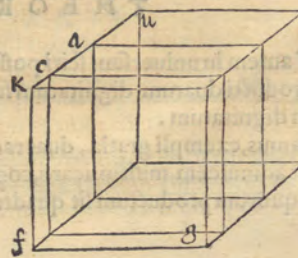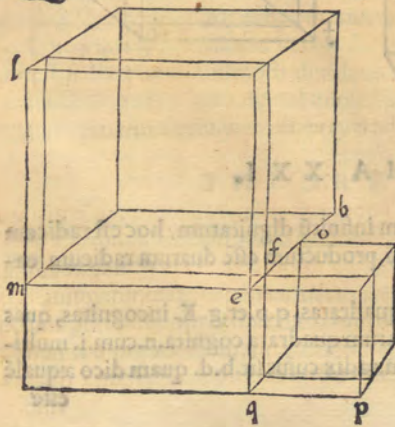
q. radicem esse quadratam producti .l. e. in. e. p. quod productũ sit quadratum corporeum. c. g. cogitemus pariter duo quadrata. l. e. et. e. p. esse pariter corporea, tantę profunditatis, quantam, vnitas linearis radicum. m. e. et. e. q. requirit. Hæc duo corpora producentur à superficie in vnitatem, vocenturq́;.l.x.et. x. p. quo facto, cogitemus corpus. a. g. tamquam productum cubi.l.b. in quadratum. e. p. Vnde ex decimaoctaua, aut decimanona septimi, eadem erit proportio. a. g. ad. c. g. quæ est.l.b.ad.l.x. corporeum, sed ex. 25. vndecimi & prima sexti, ita se habet. a. K. ad. K. c. vnitatem linearé sicut. a. g. ad. c. g. & ex eisdé ita se habebit. b. e. ad. e. x. vnitatem linearem, sicut.l.b. ad quadratum.l.x. corporeum. Itaque sic se habebit. b. e. ad vnitatem linearem. e. x. videlicet. K. c. sicut. a. K. ad ipsam. K. c. Vnde ex nona quinti. a. K. æqualis erit. e. b. & consequenter æqualis. m. e. Iam verò sit.u. g. productum. l. b. cubi, in cubum. o. p. vt supra dictum est, Hinc patebit ex quauis duarum propositionum, decimaoctaua, aut decimanona septimi, eandem futuram proportionem. u. g. ad. a. g. quæ est. o. p. ad. x. p. quadratum corporeum. Quare ex postremis, dictis rationibus, eadem erit proportio. u. K. ad. a. K. quæ est. o. e. ad vnitatem linearem. e. x. at ex dictis decimaoctaua & decimanona septimi, ita se habet numer⁹. m. q. ad numerũ superficialé. m. e. qui ꝓducitur à lineari. m. e. in vnitaté linearé ipsius. e. q. sicut nume rus. q. e. ad suam vnitaté, sed cũ numerus. a. K. æqualis sit numero. m. e. vt probatũ est erit ergo ex vndecima & nona quinti, numerus. u. K. æqualis numero. m. q. At. f. g. pariter æqualis est numero. m. q. ex præcedenti theoremate, vnde. K. u. pariter æqua lis erit. f. g. Itaque sequitur. u. g. cubum esse, & f. g. radicem ipsius, æqualem numero. m. q. quod quærebatur.
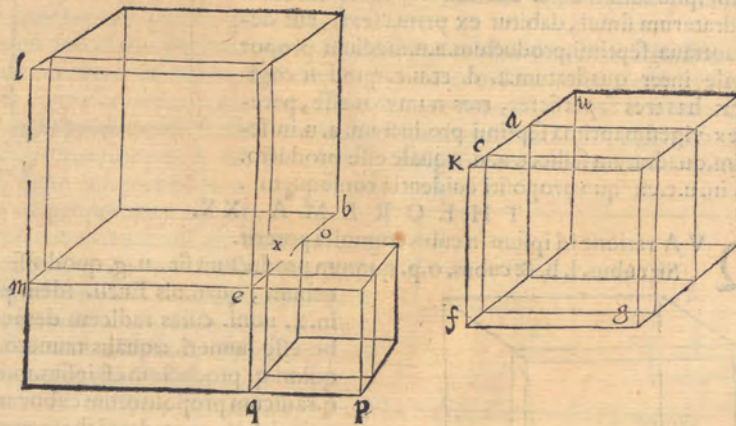


## THEOREMA XXI.

VT autem in vniuersum sciri possit totum infinitũ dignitatum, hoc est radicem producti duarum dignitatum similium, productum esse duarum radicum earundem dignitatum.

Ponamus, exempli gratia, duas radices quadratas. q. p. et. g. K. incognitas, quas qui velit adinuicem multiplicare, cogatur earum quadrata cognita. n. cum. i. multiplicare, quorum productum sit quadratum. m. radix cuius sit. b. d. quam dico æqualé

esse

eſſe ‚pducto.q.p.in.g.κ. qđ autē ſit.o. Patet enim proportionē.o.ad.q.p.eandē eſſe
cum proportione.g.κ.ad ſuam vnitatem linearem, ex decimaoctaua, aut decima-
nona ſeptimi, hæc vero vnitas linearis ſit.t.cuius ſuperficialis ſit.u.vnitas ſcilicet to-
ties in ſeipſam multiplicata quoties propoſita dignitas patitur, tametſi in præſen
ti exemplo quadrata dignitas ſumatur.Itaq; ex eiſdem propoſitionibus decimaocta
ua aut decimanona,ſic ſe habet.m.ad.n.ſicut.i.ad.u. Scimus præterea proportionē.
m.ad.n.(eo quod in propoſito exemplo ſint quadrata ) duplam eſſe proportioni.b.
d.ad.q.p.et ipſius.i.ad.u. pariter duplam proportioni.g.κ.ad.t. iam autem dictum
fuit ſic ſe habere.m.ad.n.ſicut.i.ad.u.Itaq;.

b.d. ſic ſe habebit ad. q.p. ſicut. g.κ.ad.t.
quandoquidem ſic ſe habeat totum ad to-
tū , ſicut pars ad partē,dū ſimiles ſint,proba
tū autē eſt ſuperius ita ſe habere.o.ad.q.p.
ſicut.g.κ.ad.t.itaq;.o.ſic ſe habebit ad.q.p.
ſicut. b.d.ad.q.p. vnde.o. æqualis erit.b.d.
Hoc ipſum cęteris dignitatibus conueniet,
mutatis tantummodo proportionibus. m.
n. ad proportionem.b.d:q.p. ſic propor-
tionibus duarum dignitatum.i.u. ad pro-
portionem ſuarum radicum.g.κ.t.
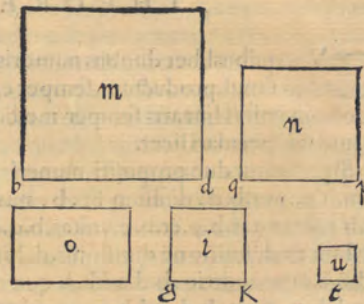
## THEOREMA XXII.

DOCENT veteres, quòd ſi quilibet numerus in duas partes inæquales diuiſus
fuerit,totumq́; diuiſum per vnā partium,& per eandem pars altera diuiſa fue-
rit:differentia prouenientium ſemper vnitas erit. quodquidem veriſſimum eſt.

Detur enim.b.d. propoſitus numerus in duas partes inæquales diuiſus.b.c.et.c.d.
& in primis totū.b.d.per.c.d.diuidatur,ex quo oriatur e.o.vnitas autem.p.i.o.ſigni-
ficetur,tum pars ipſa.b.c.p.eādem. c. d.diuidatur,ſitq́; proueniés.a. Sanè ex defini-
tione diuiſionis,eadem erit proportio.b.d.ad.e.o.quæ eſt.c.d.ad.i.o. et ita.b.c.ad.a.
ſicut.c.d.ad.i.o.Ex.19.autem quinti,ita ſe habet.b. c.ad.e.i.ſicut.b.d.ad.e.o.at.b.d.
ad.e.o.ſic ſe habet ſicut.c.d.ad.i.o.hoc eſt ſicut.b.c.ad. a.Quare ex.11.quinti ſic ſe
habebit.b.c.ad.e.i.ſicut.ad.a.ex quo ex.9.prędi-
cti.a. æqualis erit. e.i. ſed.c.i. minor eſt.e.o.
per.i.o. Quare ſequitur propoſitum verum eſ
ſe. Quod ipſum pauciſſimis verbis ſic definiri
poteſt,ſi dixerimus,eiuſmodi diuidens. in par-
te diuiſibili,quā in toto, ſemel minus ingredi,
quandoquidem altera pars eſt,ex qua totum integrum perficitur.
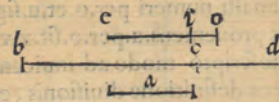
## THEOREMA XXIII.

HOc ipſum alia ratione contemplari po
terimus.

Significetur enim totalis numerus per.a.e.
in duas partes diuiſus.a.u.et.u.e.totius autem diuidens ſit.u. e. & partis alterius.a.u.
totius verò prouenié́s ſit.a.c.partis autē,ſit prouenié́s.a.n.tum differentia ſit.n.c.vni

tas
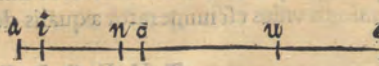
tas vero cui differentiã.n.c.æquari dico,sit.a.i.Patet enim in primis,eandem propor
tionem esse.a.e. ad.a.c. quæ est. u.e.ad.a.i.ex definitione diuisionis, et eandem
esse.a.u.ad.a. n.quæ est.u.e.ad.a.i.vnde ex.
11.quinti sic se habebit.a.e.ad.a.c.sicut.a.
u . ad.a . n. et ex.19.eiusdem sic se habe-
bit. u.e.ad. n.c. sicut. a.e.ad. a.c. sed. sic se

habebat.u.e.ad.a.i.Itaq; ex prædicta.11.quinti,sic se habebit.u.e. ad.n.c. sicut ad.a.
i.Quare ex.9.eiusdem.n.c.æqualis erit.a.i.et idcirco.n.c.pariter vnitas erit.
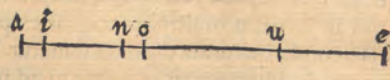
## THEOREMA XXIIII.

CVR quibuslibet duobus numeris diuisis adinuicem,multiplicatisq; prouenien
tibus simul,productum,semper est vnitas superficialis?Nempe ex.20. septimi,
quoniam vnitas linearis semper media proportionalis est inter bina prouenientia.
Quod ita speculari licet.

Significétur duo propositi numeri per.b.p.et.b.d. mutuo diuisi, proueniens au-
tem.b.p. per.b.d . diuisum sit .b . n.tum proueniens.b.d. diuisum per.b.p.sit.b.a.
et.b.t.sit vnitas.b.p.et.b. e.vnitas.b.d.ex quo.b.t.æqualis erit.b.e.

Iam ex definitione diuisionis,dabitur eadem proportio.b.p.ad.b.n.quæ est.b.d.
ad.b.e.et proportio.b.d.ad.b.a. quæ est.b.p.ad.b.t. Sed cum sic se habeat.b.
p.ad. b.n. sicut .b.d. ad.b.e. permutando sic se habebit.b.p.ad.b.d.sicut.b.n.ad.b.
e.hoc est ad.b.t.et cum sic se habeat.b.d.ad.b.a. sicut. b.p.ad.b.t: permutando sic se
habebit. b.d.ad. b.p. sicut.b.a. ad .b.t.
Quare euersim sic se habebit.b. p.ad.
b.d.sicut. b.t.ad.b.a.sed. b.n.ad. b.t. sic
se habebat vt. b.p.ad.b.d. Itaq; ex.11.
quinti sic se habebit.b.n.ad.b.t.sicut.b.
e.ad.b.a.Dictum autem est.b.e.et.b.t.idem omnino esse. Quare ex.20.septimi pro-
positi veritas innotescet.
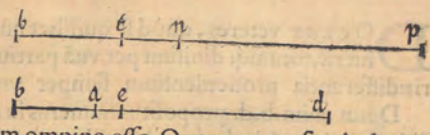
## THEOREMA XXV.

IDipsum & hac altera uia patebit .
Duo illi numeri per.o.et.u.significentur mutuo diuisi,proueniens autẽ.o. per.
u.sit.e.et proueniens.u.per.o.sit.x.vnitas ueró per.i.significetur, quas tamen quanti-
tates subscripto modo ad inuicem disponi-
to.Itaq; ex definitione diuisionis , eadem erit
proportio.o.ad.e.quę est.u.ad.i.et.o.ad.i.quę
est.u.ad.x.Quare ex æqualitate proportionũ.
e.ad.i.sic se habebit sicut.i.ad. x. erit enim.i.
media proportionalis inter.ę.et.x.ex.20. autẽ
septimi propositum concludetur. Huiusmodi rei causa etiam est, quod proueniens
diuisionis vnius est numerator æqualis denominatori diuisionis alterius.
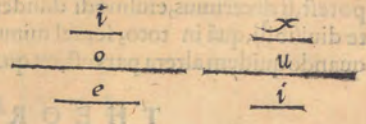
## THEOREMA XXVI.

CVR duobus numeris mutuo diuisis,sũptis deinde prouenientibus simul et adinuí
cem,& per hanc summam,diuisa summa quadratorum dictorum propositorũ
nume-

nu merorum, proueniat numerus æqualis numero producti duorum primorum nu
m erorum fimul.

Sint exempli gratia propofiti numeri.2.et.8.qui mutuo diuifi in primis dent pro
uenientia quatuor integra,tum quartam partem pro altero proueniente,hæc colle-
cta dabunt fummam quatuor integrorum et quartæ partis vnius,fumma autem qua
dratorum binarij & octonarij erit.68. qui quidem numerus per quatuor & quar
tam partem vnius diuifus dabit.16.pro proueniente, quæ.16.æqualia erunt pro
ducto binarii in octonarium.

Cuius rei hæc erit fpeculatio,fint duæ lineæ.o.e.et.o.n. quæ duos numeros pro-
pofitos fignificent, inuicem ad angulum rectum.o.coniunctæ, quarum quadrata
fint.o.a.et.o.p.ipforum productum fit.n.e.tum.o.t.fit proueniens ex diuifione.o.e.
per.o.n.Hęc fingulatim confideremus (nā fi in partibus fimplicibus quod dicimus ac
ciderit,id ipfum in compofitis confequenter eueniet)quamobrem ex definitione di
uifionis dabitur eadem proportio.o.e.ad.o.t.quæ eft.o.n. ad vnitatem, quæ fit.o.
x. Nunc cogitemus fuperficię rectangulā. o.c.æqualę quadrato.o.a.tunc numerus.
c.t.proueniens erit,ut patet, ex diuifione numeri quadrati.o.a.per numerū.o.t.eritq;
eadę proportio.c.t.ad.o.e.quæ eft.o.e.ad.o.t.ex fecunda parte quintædecimæ fexti,
aut.2 o.feptimi.Iā autę dictum eft.o.e.ad.o.t.fic fe habere ficut.o.n.ad.o.x.Itaq; ex.
11.quinti fic fe habebit.c.t.ad.o.e.ficut.o.n.ad.o.x.Sed ex prima fexti,aut.18. vel.
19.feptimi,fic fe habet ,pductum.n.e.ad.e.x.ficut.o.n.ad.o.x.quare denuo fic fe ha-
bebit numerus.c.t. ad numerum.o.e. ficut nume-
rus.n.e.ad numerum.x. e. Sed numerus.o.e. cum
numero.x. e.fpecie idem eft,igitur ex.9.quinti nu
merus.c.t.numero.n.e.æqualis erit.

Id ipfum de quadrato ipfius.o.n.videlicet.p.o.
dico. Nam fi proueniens.o.n.diuifo per.o.e. ideft.
o.i. proportionale refpondens ad.o.t. cum.o.t.
coniunctū fuerit,et per hāc fummam diuifa fumma
quadratorum. o.a.et.o.p.patet per fe proueniens
futurum eiufdem numeri.c.t.ipfumq;.c.t.proue-
niens femper futurum.

Quo autem lucidius res hæc innotefcat. Cogi
temus proueniens quadrati. o. p. diuifi ab.o.i. re-
fpondentisq;.o.t. effe.i.u. quod via prædicta inue-
nitur æqualis effe numero.n.e. ex quo confe-
quenter æquale.c.t: cogitato deinde rectangu-
lo.o.u.æquali.o.p.coniuncto.o.c:totum.t.u. æqua-
le erit compofito duorum quadratorum. o.a. et.o.
p.cum in nullo numerus.c.t.mutetur,tam ex com-
pofito.t.u.quā ex fimplici.o.c.ex quo propofiti fe
fe ueritas profert.
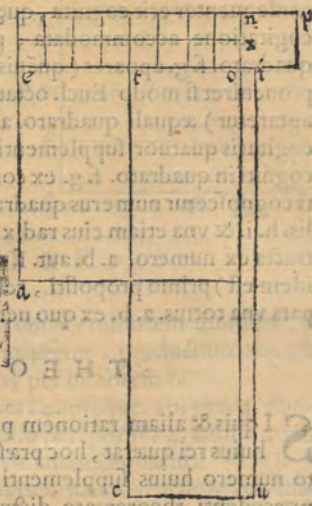
## THEOREMA XXXVII.

PRopofvervnt véteres nobile quidem problema, fed quod tamen citra al-
gebraticam effectionem,aut nefcierunt, aut noluerunt diffoluere, quod nihi-
lominus facillimum eft.

C Propo-

18                    IO. BAPT. BENED.

Proponunt hi numerum in binas eiufmodi partes diuidendum, vt fumma qua-
dratorum dictarum partium, alteri numero pofsibili propofito æqualis fit, pofsi-
bili inquam, etenim fi eiufmodi numerus propofitus, minor effet producto totius
primi in fuum dimidium, effet huiufmodi factum impoffibile. Quod nos exequi
cupientes, fumamus primum numerū propofitum, quem in fe ipfum multiplice-
mus. ab hoc quadrato deducamus fecundum numerum propofitum, tum quod re-
manferit duplicemus, quod duplum denuo iubeo ex eodem primo quadrato detra-
hi, accepta poftea radice quadrata refidui & dempta ex priori numero propofito,
tunc dimidium refidui vna pars erit ex duabus primi numeri quæfita.

Exempli gratia proponantur. 20. diuidenda in duas eiufmodi partes, vt fumma
quadratorum ipfarum partium æqualis fit. 272. qui numerus maior eft. 200. maior
inquam dimidio quadrati. 400. ipforum. 20. hic autem numerus. 272. è quadra-
to. 400. deducatur, remanebūt enim. 128. quod duplicari iubeo, producētur fiquidē.
256. quæ pariter deducta è quadrato totali, remanebunt. 144. cuius radicem fumi
volo, quæ erit. 12. & dempta ex. 20. priori numero dato remanebit. 8. cuius di-
midium erit. 4: pars vna ex quæfitis, quæ ex primo numero propofito. 20. detra-
hetur, remanebitq̑. 16. pro altera parte.

Cuius demouftrationis caufa, in primis cogitemus quadratum. a. c. cognitum nu-
meri. a. b. primò propofiti, qui cogitetur diuifus in duo quadrata. d. e. et. e. b. duo-
que fupplementa. a. e. et. e. c. numerus autem fummæ duorum quadratorum. d. e.
b. pro fecundo propofito datur; ex quo, fumma duorum fupplementorum. a. e. c.
confequenter erit cognita, quę cum duplicata fuerit, & quatuor hæc fupplementa
cogitatione accommodata, prout in
quadrato. f. g. apparet (quāuis idipfum
proueniret fi modo Eucl. octaua fecūdi
aptaretur) æquali quadrato. a. c. ita vt
cogitatis quatuor fupplementis numeri
cogniti in quadrato. f. g. ex confequen-
ti cognofcetur numerus quadrati partia
lis. h. i. & vna etiam eius radix qua de-
tracta ex numero. a. b. aut. f. n. ( quod
idem eft) primo propofiti, relinquetur numerus cognitus duplum. x. n. aut. t. b.
pars vna totius. a. b. ex quo uerum erit hoc meum problema.
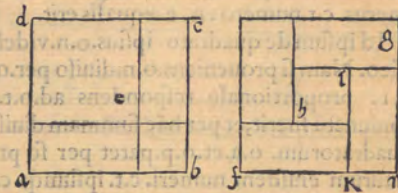
## THEOREMA XXVIII.

SI quis & aliam rationem perficiendæ
huius rei quærat, hoc præftet inuen-
to numero huius fupplementi, cum in
præcedenti theoremate dictum fuerit,
qua ratione manifeftetur duplum fupple-
menti ipfius.
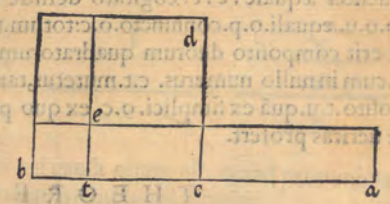
Cogitemus in fubfcripta figura lineam.
a. b. tanquam primum numerum propofi-
tum, & productum. a. e. fupplemento. a. e. primæ præcedentis figuræ æquale fit,
ac deinde ordine ab antiquis tradito procedatur, ad quadratum reducto dimidio.
a. b. videlicet. b. c. quod erit. b. d. ex quo detrahatur deinde. a. e. quare remane-
                                                                                                    bit

bit quadratum. e. d. cognitum , cuius radix æqualis erit. c. t. qua coniuncta dimidio. c. a. ex quinta secundi Eucli. dabit quod propositum erat.

## THEOREMA XXIX.

QVID causæ est , cur subtracto duplo producti duorum numerorum ad inuicem multiplicatorū ex summa suorum quadratorum , semper quod super est duorum numerorum quadratum differentiæ sit?

Exempli gratia si proponerentur duo numeri. 16. et. 4. duplum producti eorum esset. 128. quò detracto ex summa suorum quadratorum, nempè ex. 272. remaneret. 144. cuius quadrati radix esset. 12. tanquam differentia inter. 4. et. 16.

Id vt sciamus, duo numeri propositi, duabus lineis significentur, maiore. q. g. et minore. g. p. directè coniunctis, super quas , totale quadratum extruatur. a. p. in quo cogitetur diameter. a. p. et à puncto. g. ducatur parallela. g. n. c. et à puncto.n. parallela. n.s.r. ex quo duo producta dabūtur.q. n. et.n. u. singula æqualia producto. q. g. in g. p. et. a. n. et. n. p. duo quadrata dictorum numerorum propositorum, quod satis superq́; probatur quarta secundi Eucli. Cogitemus deinde. n. o. æqualem. n. p. et à puncto. o. ducatur. o. m. t. parallela. r. s. et. o. e. ad. n. c. quare ex allatis ab Eucli. octaua secundi , dabitur quantitas. m. n. æqualis. q. n. producto. q. g. in g. p. et quantitas. o. c. minor ipso producto , ex quantitate quadrati. n. p. ex quo quantitas. m. n. e. vnà cùm quadrato. n. p. æqualis erit duplo producti. q. g. in. g. p. sed hæ duæ quantitates, sunt partes duorum quadratorum dictorum, & quæ super est. m. e. quadratum differentiæ vnius numeri propositi ab altero, prout in subscripta figura licebit cui libet considerare. Itaque veritas hæc manifesta erit.
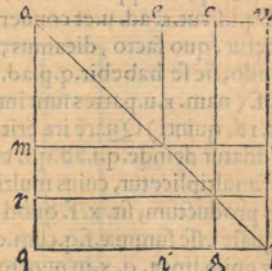


## THEOREMA XXX.

CVR ij qui ex duobus numeris propositis maiorem per minorem diuidunt, si proueniens per maiorem numerum multiplicauerint, productum æquale erit prouenienti ex diuisione quadrati maioris numeri per minorem?

Exempli gratia si proponantur duo numeri. 20. et. 4. ipseque. 20. per. 4. diuidatur, dabit quinque , tum. 400. quadrato. 20. diuiso per priore. 4. dabit. 100. quod proueniens, producto ex. 20. in. 5. primo prouenienti adæquatur.

Cuius speculationis causa, sint duo numeri, qui lineis. x. u. et. x. s. maiore atq; minore significētur, tum. u. x. numerus per. s. x. diuidatur, sitque proueniens. x. n. postmodum quadratum. u. x. sit. x. o. et productum ex. n. x. in. u. x. sit. x. e. quod æquale esse dico prouenienti ex diuisione quadrati. o. x. per. s. x. quod sit. m. Patet enim ex definitione diuisionis, talem futuram proportionem. u. x. ad.n. x. qualis est. s. x. ad vnitatem, & quadratum. o. x. ad rectangulum. e. x. ita se ha-



C 2 bi-

biturum, ficut. u. x. ad.n.x. ex prima fexti aut. 18.vel. 19. feptimi, quare ex 11. quinti ita fe habebit. o. x.ad. e. x. ficut. s. x. ad vnitatem; fed ficut fe habet.s.x. ad vnitatem, ita fe habet pariter. o.x. ad. m. vnde ex. 11. prædicta ita fe habebit. o. x. ad. m. ficut idipfum. o. x.ad.e.x. itaque ex.9.prædicti quinti.m. æqualis erit.o.x.

## T H E O R E M A   XXXI.

C V R propofito aliquo numero in duas partes inæquales diuifo, fi rurfus per quamlibet ipfarum diuidatur, prouenientia tantumdem coniuncta quantum multiplicata efficiant.

Exempli gratia, fit denarius propofitus numerus, per binarium & octonarium diuifus, prouenientia erunt quinque & vnum cum quarta parte, quæ coniuncta erunt. 6. cum quarta parte lineari, quæ fimul multiplicata, pariter erunt. 6. cum quarta parte fuperficiali.

Cuius fpeculationis caufa, totalis numerus, linea. q. p. fignificetur, eius duæ partes, per. κ. maiorem et. u. minorem, ipfa vnitas per. t: proueniens ex diuifione. q.p.per. κ. fit.q.i. proueniens autem ipfius. q.p. per.u. fit.q. f. quare ex definitione diuifionis ita fe habebit. q.p. ad.q. i. ficut. κ. ad.t. et.q.p.ad.q.f.ficut. u.ad.t. hoc eft.q.f.ad.q.p. ficut.t.ad,u.vnde ex æqualitate proportionū fic fe habebit. q. f. ad.q.i.ficut.κ. ad. u.et conuerfim.Ad hæc in linea.q.p.vnitas, per lineam.q.o.fignificetur, quo facto, dicamus, fi.q.p.ad.q.i.fic fe habet vt. κ. ad.q. o. itaque permutando, fic fe habebit.q.p.ad.κ. ficut.q.i ad.q. o. hoc eft. κ. u. ad. κ. ficut. i. q. f. ad. q. f. (nam. κ.u.partes funt integrales totius.q. p. et. κ.u.ad. κ.eft ficut.i. q.f. ad. q. f. ex.18. quinti) Quare ita erit. i.q f.ad.q.f.ficut. q.i. ad vnitatem.q.o. ex. 11.quinti Addatur deinde.q.i.ad.q.f. et.q.i.per.

q.f.multiplicetur, cuius multiplicationis productum, fit. x. f. quod probabo æquale effe fummæ.f.q.cum.q.i. Secetur enim linea. q. x.in puncto. s.ita.vt. q.s. æqualis fit. q. o. figneturque productum. s. f. quare eadē erit proportio quantitatis . x. f.ad.s.f.quæ eft.q.x. ad.q.s,ex prima fexti, aut . 18. vel 19. feptimi, hoc eft, ficut. q.i. ad. q. o. et ex. 11. quinti ( vt dictum eft) ficut.i.q. f. ad.q. f. fed numerus. s. f. fuperficialis tantus eft, quantus linearis. q. f. quare ex. 9. quinti tantus erit ( fuperficialiter) numerus. x. f. quantus ( lineariter ) . f. q.i. quod erat propofitum.
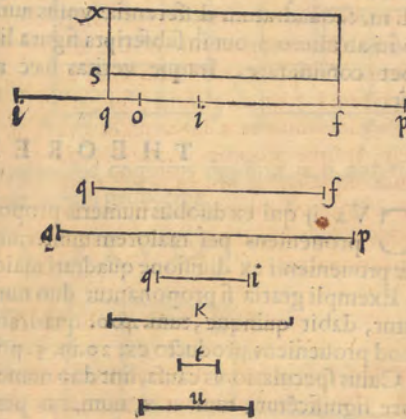


## T H E O R E M A.   X X X I I.

C V R numero aliquo in duas partes inæquales diuifo, fi rurfus diuidatur per fingulas partes, fumma duorum prouenientium per binarium, femper maior fit fumma prouenientium ex diuifione vnius partis per alteram.

Exēpli gratia, fi proponeretur numerus. 24.qui in duas partes inæquales diuide
retur.

retur. 20. scilicet et. 4. certè. 24. per singulas partes diuiso, daretur vnum proue-
niens sex integra, & alterum vnum & quinta pars, quorum summa esset septem in-
tegra cum quinta parte, tum altera parte per alteram diuisa, daretur vnum proue-
niens quinque integrorum & alterum vnius quinti tantum, quorum summa esset
quinque integra, & vna quinta pars, minor prima reliquorum duorum prouenien-
tium per binarium.

Cuius considerationis causa, propositus numerus linea. q. p. significetur, eius duę
partes lineis. q. x. et. x. p. tū. q. f. sit proueniens ex diuisione totius. q. p. per. x. p. et.
q. i. sit proueniens ex diuisione eiusdem. q. p. per. q. x. adhæc. h. m. sit proueniens,
ex diuisione. q. x. per x. p. et. h. κ. proue-
niens ex diuisione. p. x. per. q. x. patet igi-
tur ex. 22. theoremate huius libri proue-
niés. h. m. minus esse proueniente. q. f. per
vnitaté, & proueniens. h. x. minus proue-
niente. q. i. per alteram vnitatem. Itaque.
f. q. i. maior erit. m. h. κ. per numerum binarium, quod erat propositum.
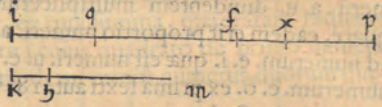
## THEOREMA. XXXII.

QVILIBET numerus, medius est
proportionalis inter numerum
sui quadrati & vnitatem.

Detur enim numerus propositus,
qui linea. a. u. significetur, cuius qua-
dratum sit. u. n. vnitas linearis sit. i. a.
et superficialis. o. patebit ex. 18. sexti
aut 11. octaui proportionem. u. n. ad.
o. futuram duplam proportioni. u. a.
ad. i. a. sed. i. a. et. o. eadem (specie)
res sūt, tanta scilicet. a. i. quanta. o. vni
tas est, Itaque proportio numeri. u. n.
ad. u. a. æqualis erit proportioni. u. a.
ad. i. a. Quare numerus. u. a. inter nu-
merum. u. n. & vnitatem, medius erit
proportionalis.
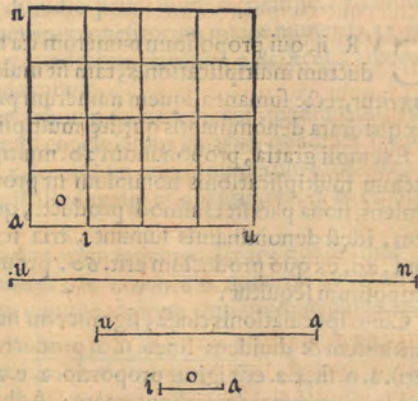
## THEOREMA XXXIII.

HOc ipsum quod diximus & alia ratione speculari licebit.
Propositus numerus, nunc etiam per. a. u. significetur, eius quadratum per.
u. n. vnitas linearis per. a. i. productumq̗;. a. u. in. a. i. terminetur, sitq̗. n. i. quare
n. i. constabit numero superficiali æquali numero lineari. a. u. & ex prima sexti aut.
18. vel. 19. septimi, eadem erit proportio. u. n. ad. i. n. quæ est. a. u. ad. a. i. sed nu-
merus. a. u. cum numero. n. i. idem specie est. Itaque medius est proportiona-
lis inter. u. n. & vnitatem.

THEO-

### THEOREMA XXXV.

Qvivis numerus per alterum multiplicatus, & diuisus, medius est proportionalis inter productum multiplicationis, & proueniens diuisionis.

Exempli gratia, si. 20. multiplicetur per quinque & inde per quinque diuidantur productum erit. 100. proueniens. 4. inter quos numeros. 20. medius est proportionalis.

Hoc vt speculemur, proponatur numerus multiplicandus & diuidendus, qui significetur linea. u. e. multiplicans autem & diuidens linea. a. u. multiplicationis productum sit. e.a. proueniens ex diuisione sit.o. e. Nunc proueniens. e. o. per numerū. a. u. diuidentem multiplicetur, cuius multiplicationis productum sit. e. i. quare, eadem erit proportio numeri. a. e. ad numerum. e. i. quæ est numeri. u. e. ad numerum. e. o. ex prima sexti aut. 18. vel 19. septimi. Sed cum numerus. u. e. ex. 11. theoremate præsentis libri, numero. e. i. æqualis sit. verum esse, quod propositum fuit consequetur.

### THEOREMA XXXVI.

Cvr ij, qui propositum numerum ita multiplicare & diuidere cupiunt, vt productum multiplicationis, tam sit multiplex prouenienti ex diuisione, quam quæritur, rectè sumant aliquem numerum pro multiplicante & diuidente, qui sit radix quadrata denominantis quęsitę multiplicitatis.

Exempli gratia, proponuntur. 20. multiplicanda atque diuidenda, ita vt productum multiplicationis nonuplum sit prouenienti ex diuisione, nempè, vt proueniens, nona pars sit eiusmodi producti, quare quadratam radicem ipsorum nouem, idest denominantis sumunt, tria scilicet, multiplicant igitur & diuidunt data. 20. ex quo productum erit. 60. proueniens autem. 6. cum duabus tertijs. & propositum sequitur.

Cuius speculationis causa, significetur numerus propositus linea. u. e. multiplicans autem & diuidens linea. u. a. productum sit.e.a. proueniens. e. o. quadratum verò. a. u. sit.x.a. erit igitur proportio. a. e. ad.e.o. dupla proportioni. a. e. ad numerum. u. e. ex præcedenti theoremate: Adhæc, cogitemus in linea. u. a. vnitatem. u. i. terminenturģ; duo producta. e. i. et.x. i. quare eadem erit proportio. a. e.ad.e.i. quæ est. a. e. ad.u. e. numerus enim.e.i. (quamuis superficialis) idem est cum numero lineari. u. e. sed. a. e. ad. e.i. sic se habet sicut.a.u.ad. u. i. ex prima sexti aut. 18. vel. 19. septimi, (quod ipsum dico de.a.x.ad.x.i.) quare proportio. a. x. ad.x.i. hoc est.x.u.æqualis erit pportioni.a.e.ad. u.e.at trigesimotertio & trigesimoquarto theoremate probatum est proportionem numeri. a.x. ad vnitatem, duplam esse proportioni eiusdem numeri. a. x. ad. u.x. sequitur igitur cum dimidia sint æqualia, tota etiam æqualia esse: hoc est proportionem numeri. a. e. ad numerum. e.o.æqualem esse proportioni numeri. a. x.ad vnitatem. Itaque rectè sumitur numerus.a. u. eiusmodi vt quadratū ipsius.
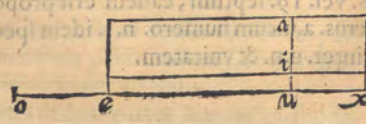
ipſius. a. x. tam ſit multiplex ad vnitatem, quam cupimus numerum. a. e. numero. e. o. multiplicem eſſe.

## THEOREMA XXXVII.

CVR inuenire cupientes duos numeros, quorum quadrata in ſummam colle-ſta, æqualia ſint numero propoſito, & ijſdem numeris multiplicatis ad-inuicem, productum alteri numero propoſito ſit æquale, rectè ſumant dimidium primi numeri propoſiti, cui ſumma quadratorum æquari debet, hocq́; dimidium in ſeipſum multiplicent, vnà etiam alterum numerum propoſitum in ſeipſum multiplicent, quod quadratum detrahunt de primo, & reſidui quadratam radicem, dimidio primi numeri propoſiti coniungunt, ex qua ſumma, quadratam radicem eruũt, quæ duobus quæſitis numeris maior erit, cuius quadrato de primo numero detracto, & ex reliquo eruta radice quadrata, detur minor numerus, duorum quę-ſitorum.

Exempli gratia, ſi proponerentur. 34. pro primo numero cui æquari de-beret ſumma duorum quadratorum, quorum radicum productum æquale eſſe de-beret alteri numero, verbi gratia. 15. iubet antiquorum regula, dimidium primi numeri in ſeipſum multiplicari, cuius dimidij quadratum erit. 289. è quo ſi detra-has quadratum ſecundi numeri, nempe. 225. remanebit. 64. atq; huius ſi quadra-tam radicem ſumas nempe. 8. quam dimidio primi numeri, nempe. 17. coniun-gas, dabitur duorum quadratorum numerorum quęſitorum maior numerus. 25. hac deinde radice è dimidio detracta, minus quadratum dabitur. 9. ſcilicet, quorum radices. 5. et. 3. eſſent ij numeri, qui quæruntur.

Cuius ſpeculationis gratia, cogitemus primum numerum, cui quadratorum ſum-ma æquari debet, ſignificari linea. a. n. tum concipiamus quæſita quadrata ſignifi-cari, coniungiq́; modo ſubſcripto. t. b. ĸ. ſecundum porro numerum propoſitum, ſignificari producto. d. b. Iam nil ſupereſt aliud quam vt quantitates. d. p. et. b. p. quæramus.

Itaque cum in linea. a. n. ſummæ quadratorum numerus detur, quadratum di-midij. o. a. ſit. s. a. quod nobis erit cognitum; ſit etiam. a. u. numerus quadrati ma ioris, et. u. n. minoris, et. a. z. productum vnius in alterum; qui quidem numerus. a. z. æqualis erit quadrato nume-ri. d. b. ex. 19. theoremate hu-ius libri. Itaq; a. z. cognitum erit, cum eius radix. d. b. ſit ſe-cũdus numerus propoſitus, quæ minor erit. a. s. ex quinta ſecundi, aut ſeptima conſequentia poſt. 16. noni Eucli-dis. Iam ſubtracta quantitate. z. a. è quadrato. a. s. cognoſcetur quadratum. t. x. cuius radix æqualis erit. o. u. ex poſtremo adductis, Itaque cognoſcemus. o. u. qui numerus coniunctus dimidio. o. a. cognito, dabit quadratum. a. u. cognitum, at-que ita. u. n. pariter cognoſcetur, & eorum radices conſequenter.

Hoc
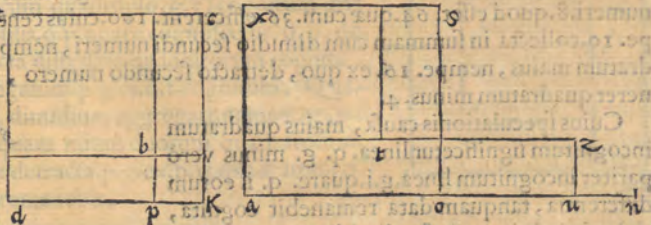
Hoc ipfum & alia ratione perfici poteft, nempe, iuncta fumma. ĸ . b : b . d : et.
b. t. alteri rectangulo æquali.b.d.quod fit. b.c. ex quo totum quadratum lineæ.d.ĸ.
cognitum erit, atq; ita etiam confequenter eius radicem.d. ĸ. cognofcemus, cuius
ope ac producti.d.b.cognofcemus.d.p. et. p.k. prout ex theoremate quadrageſi-
moquinto huius libri patebit.

Michael Stifelius, vndecimo cap. tertij libri, problema eiufmodi proponit,
quod tamen ipfe via algebræ diſſoluit.
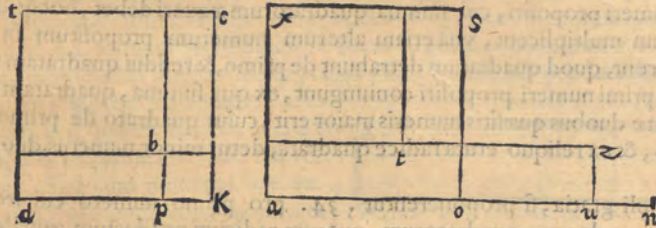


# THEOREMA XXXVIII.

CVR ij, qui duos numeros inuenire volunt, quorum productum alicui nu-
mero propoſito æquetur, & quadratorum eorundem differentia alteri nu-
mero propoſito æqualis ſit. Rectè dimidium fecundi numeri propoſiti in feipfum
multiplicent, cui quidem numero differentia quadratorum æquari debet; porrò
huic quadrato primi propoſiti numeri, cui æquandum eſt productum numerorum
quæſitorum, quadratum adiungant; tum radicem quadratam huius fummæ co-
pulet dimidio fecundi numeri propoſiti, ei inquam, cui differentia quadratorum
æqualis effe debet, ex quo quadratum maius confurgit, à quo, detracto fecundo
numero, fupereſt quadratum minus.

Exempli gratia, ſi proponeretur primo loco numerus. 8. cui æquandum eſt
productum numerorum quæſitorum, tum proponeretur numerus. 12. cui, detra-
cto minore à maiore, differentia quadratorum vtriufque quæſiti numeri æqualis
effe debet, oportet huius vltimi numeri. 12. dimidium in feipfum multiplicare, fiet-
que. 36. quadratum dimidij, vnde in fummam colligeremus quadratum primi
numeri.8.quod effet. 64. quæ cum. 36. efficerent. 100. cuius centenarij radice, nem
pe. 10. collecta in fummam cum dimidio fecundi numeri, nempe. 6. daretur qua-
dratum maius, nempe. 16. ex quo, detracto fecundo numero, nempe. 12. rema-
neret quadratum minus. 4.

Cuius fpeculationis caufa, maius quadratum
incognitum fignificetur linea. q. g. minus verò
pariter incognitum linea.g.i.quare. q. i. eorum
differentia, tanquam data remanebit cognita,
vnà etiam. b.i.et.q.b.fua dimidia; tunc cogite-
tur quadratum. y.g. fuper.b.g. et parallelogrä-
mum rectangulum.g.r.defignatum, & ita etiam
gnomon. u.g.t.prout fexta fecundi Euclidis pro
ponitur, ex quo quadratum. b. i. nempe.u.t.co-
gnitum erit, fed gnomon æqualis eſt rectangulo. g. r.ex prædicta, aut ex.8. poft. 16.
noni,
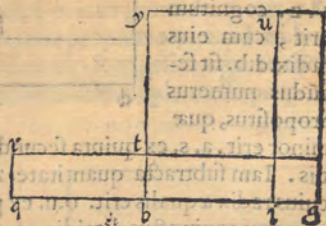
noni, hocq; rectangulum.g.r. quadratum est primi numeri propositi ex.19. theo-
remate huius libri , itaq; cognitum erit : vnà etiam gnomon. u. g. t. cognoscetur;
quare totum quadratum. g. y. eiusq; radix. b. g. manifesta erit ; cui coniuncta. q.b.
data, maius quadratum. q.g. cognoscetur, ex qua. b. g. detracta. b.i. data , cogno-
scetur.i.g. quadratum minus consequenter, etiam eorum radices notæ erunt.

## THEOREMA XXXIX.

ALIA etiam ratione idipsum definiri potest , prætermissa antiquorum via ,
nempe multiplicatis in semetipsis primo & secundo, numeris propositis, qua-
druplicatoq; quadrato primi, qua summa coniuncta cum quadrato secundi nume-
ri, & ex hac altera summa eruta radice quadrata, ex qua detracto secundo nume-
ro, & è reliquo sumpto dimidio, quod erit quadratu minus, quo detracto ex radi-
ce postremo iuncta, supererit quadratum maius.

Exempli gratia, si proponeretur numerus. 8. cui productum duorum numerorum
quæsitorum æquandum est, proponeretur idem . 12. cui differentia quadratorum
duorum numerorum æqualis esse debet. Iubeo primum numerum, nempe. 8. in se
ipsum multiplicari, ex quo exurget. 64. pro numero sui quadrati , quod quadru-
plicari volo, eritq; productum. 256. quod censeo coniûgendum cum quadrato se-
cundi numeri propositi, nempe. 144. eritq; summa. 400. ex qua sumetur radix, sci
licet. 20. & ex hac detrahetur secundus numerus. 12. residuiq; dimidium , nempe.
4. pro quadrato minore, quo in summam collecto cum . 12. dabit quadratum
maius. 16.

Cuius speculationis causa, quadratum maius per lineam.q.g. minus per. g. p. si-
gnificetur : super integram autem.q.p. erigatur quadratum integrum. d.p. diuisum,
vt quadratum. f. g. vigesimiseptimi theorematis huius libri, (idipsum accideret di-
uiso quadrato modo octauæ secundi Euclidis) quæ quidem diuisio, est via quatuor
productorum.q.g. in.g.p. è quibus vnum sit.g. r. quod erit cognitum ex. 19. theore
mate cum sit quadratu primi numeri ppositi, ex quo illa quatuor cognita erût. Iam
verò si cogitemus.q.p. sectam in puncto. t. ita vt.q.t. æqualis sit. p.g. dabitur differen
tia.t.g. cognita, vt radix quadrati. e.o. cum ex præsup-
posito.r.n. æqualis sit.q.g. et.r.e:g.p. ex quo etiam. q. t.
ita pariter.e.n.t.g. æqualis erit. Collecto itaq; quadra
to. e. o. ipsius. t.g. cum quadruplo. g. r : cognitum erit
quadratum. d.p. ipsius.q.p. quare cognoscetur. q. p. de
quo numero detracta differêtia quadratorum cognita .
t.g. supererit aggregatum.p.g. et.q.t. cognitum . Qua-
re ex consequenti, dimidium aggregati, nempe . g . p .
cognoscetur, tanquam minus duorum quadratorum.
cui iuncta. g.t. aut detracta.p.g. ex.p.q. quadratum. q.
g. maius cognitum remanebit.

## THEOREMA XL.

CVR ijs, qui volunt duos eiusmodi numeros inuenire, vt eorum maior mi-
norem, numero proposito superet , & productum vnius in alterum, alteri nu-
mero proposito adæquetur, consultissimum sit dimidium primi numeri propositi ;

                                                                D          numerum

numerum inquam, cui differentia duorum quæsitorum æquanda est, in seipsum multiplicare, atque huic quadrato, secundum numerum propositum iungere, cui, productum numerorum quæsitorum æquale esse debet, & ex hac summa eruere quadratam radicem, quæ coniuncta dimidio primi numeri propositi, dabit maiorem duorum numerorum & ex eadem radice detracto dimidio primi numeri, minorem numerum duorum quæsitorum.

Exempli gratia, si proponeretur. 12. cui differentia vnius numeri ab altero æquari deberet, tum proponeretur. 64. cui productum multiplicationis duorum quæsitorum simul æquadum esset. Dimidium primi numeri in seipsum multiplicaremus, proueniretq; quadratu. 36. cui coniuncto secundo, nempe. 64. totum esset. 100. ex quo detracta quadrata radice. 10. et ipsi coniuncto senario, dimidio primi numeri, & ex eadem detracto eodem dimidio. 6. pro maiore numero proueniret. 16. & pro minore. 4.

Cuius rei speculatio hæc est. Sit.e.o. differentia cognita duorum incognitorum numerorum. a.o. et. a.e. quorum productum datum siue cognitum sit. a. s: consideremus nunc. e.i. dimidium. e.o. datæ differentiæ, & ex composito. a. i. imaginetur quadratum. a. x. in quo protracta sit. t.u. æquidistans lateri. a.i. & tam ab ipsa. a.i. remota, quam. x.i. ab.s.e. vnde. t.e. quadratum erit. e.i. dimidiæ scilicet differentiæ datæ. e.o. et. t.n. rectangulum æquale erit rectangulo. n. c. vt cuilibet licet per se considerare, vnde sequitur gnomonem. e.r. t. æqualem esse producto. a.s. ideo cognitus, qui quidé gnomon, si coniunctus fuerit quadrato. e.t. cognito ex radice. e.i. cognita ( vt dimidia totalis differentiç. e.o. datæ) habebimus quadratum totale. a. x. cognitum, & ita eius radicem. a. i. cognitam & reliqua omnia consequenter quæ quidem speculatio eadem est quæ. 6. secundi seu. x. noni Euclidis.

Poteris tamen ex modo & rationibus præcedenti theoremate allatis, hoc ipsum concludere.

## THEOREMA XLI.

CVR ij, qui aliquo proposito numero, inuenturi sunt duos numeros inter se, differentes, quorum quadratorum summa altero numero proposito æqualis sit, recte primum numerum propositum in seipsum multiplicant, quod quadratum, ex secundo numero detrahut, & dimidium residui sumunt, quod productum erit multiplicationis duorum numerorum inter se, in reliquis præcedentis theorematis ordinem sequuntur.

Exempli gratia, si proponeretur. 12. tanquam numerus, cui differentia duorum numerorum quæsitorum æquanda est, proponerentur præterea. 272. quibus summa quadratorum duorum numerorum quæsitorum æquari deberet, oporteret sane primum numerum, nempe. 12. in seipsum multiplicare, cuius quadratu hoc loco esset. 144. atque hoc detrahere ex secundo numero, superesset. 128. sumpto deinde dimidio huiusce numeri, népe. 64. producto in quam duorum numerorum quæsitoru. Cum hoc. 64. postea et duodenario primo proposito numero, præcedentis theorematis ordinem sequeremur.

Quod

Quod vt speculemus, consideremus subscriptam figuram, vigesiminoni theore-
matis figuræ similem, in qua numeri quæsiti duabus
lineis directè coniunctis. q.g. et.g.p.significentur, ho
rū quadrata erūt.r.c.et.g.s.quorū sūma iterū propo
nitur, quare etiam cognita. Differētia autem duorū
numerorum primo proposita sit. q. i. eius verò qua-
dratum.m. e. quod cognitum est ex sua radice. q. i.
quare gnomon.e.n.m. simul cum quadrato minori.
g. s. cognitus erit, quæ summa æqualis est duplo.g.r.
producto datorum numerorum. Itaque & ipsa. g.
r.cognoscetur, nunc si præcedentis theorematis spe-
culationem in reliquis consuluerimus propositum
consequemur.
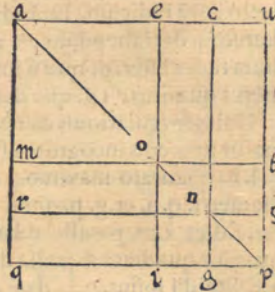
## THEOREMA XLII.

ADhuc etiam & alia ratione idipsum consequi possemus, non consulto qua-
dragesimo theoremate. Nam subtracto quadrato differentiæ, numeri primi
(inquā) propositi, ex sūma duorum quadratorum, nempe ex secundo numero pro-
posito colligendum esset residuum in summam cum prædicto secundo numero, &
ex summa hac desumenda quadrata radix, quæ duorum numerorum summa erit,
de qua detracto primo numero, remanebit duplum minoris numeri quæsiti, cuius
dimidio addito primo numero proposito, aut detracto minore inuento ex radice
postremo inuenta, dabitur numerus maior, qui quæritur.

Exempli gratia, cum superfuerint. 128. hæc si cum secundo numero nēpe.272.
iunxerimus, dabunt. 400. quorum radix erit. 20. de quo numero detracto primo
proposito, nempe. 12. supererunt. 8. quorum dimidiū erit. 4. quo ex.20. detracto
aut coniuncto. 12.maior numerus orietur.

Cuius rei contemplatio, præcedenti figura aperitur. Nam residuum detractionis
quadrati. m. e. ex summa duorū quadratorum. r. c.et.g.s.numerum præbet æqua-
lem duobus supplementis. q. n. et.n.u.ex. 8. secundi Euclidis. qui coniunctus duo-
bus quadratis (quorum summa secundo proposita fuit) cognitionem profert qua-
drati. q.u. & eius radicis.q.p.de qua, detracto primo dato numero, scilicet.q. i.su-
perest. i.p.cuius dimidium nempe.g.p. minor est numerus qui queritur; residuum
verò totius.g.q.maior scilicet.

## THEOREMA XLIII.

CVR ij, qui volunt duos numeros inuenire, quorum summa æqualis propo-
sito alicui numero futura sit, & summa quadratorum maior eorum produ-
cto per quantitatem alterius propositi numeri, rectè dimidium primi dati numeri in
seipsum multiplicant, quod quadratum ex secūdo dato numero detrahunt, sumunt
que tertię partis residui quadratam radicem, quam dimidio primi numeri coniun-
gunt, ex quo maior numerus duorū quæsitorū datur, quo ex toto primo detracto, su-
pererit minor.

Exempli gratia, proposito numero. 20. cui æquanda est summa duorum nume-
rorum quæsitorum, datoq; secundo numero. 208. qui semper maior esse debet

D 2 quadrato

28　　　　　　　I O. B A P T. B E N E D.

quadrato dimidij, prout ex speculatione huiusmodi operis cognoscetur, cui æquáda
est differétia inter summá quadratorũ duorũ qui quærũtur numerorũ, simul cũ pro
ducto eorũ radicum. Dimidium numeri. 20. in seipsum multiplicandum esset, qua
dratumq́; detrahendum ex. 208. vt remanerent. 108. quorum. 108. tertiæ partis qua
drata radix esset. 6. quæ si iuncta fuerit dimidio. 20. nempe. 10. daretur maior nu
merus quæsitus. 16. quo detracto è. 20. darentur. 4.

　　Cuius speculationis causa, datus primus numerus significetur linea. g. h. in qua
maior numerus incognitus sit. g. h. minor verò. b. h. quorum quadrata sint. y. t. et.
b. l. in quadrato maximo. g. p. tum productum. g. b. in. b. h. sit. g. c. cogitenturq́; duo
diametri. q. h. et. g. p. diuisi per medium in puncto. o. per quod duę lineæ ducan-
tur. f. d. et. к. m. parallelæ lateribus maximi quadrati. Hæ dictum quadratum in
quatuor quadrata æqualia diuident, quorum vnumquodq́;, æquale erit quadrato.
g. f. dimidij ipsius. g. h. datę, quare eorum vnumquodq́; cognitum erit. Iterum co
gitemus. s. x. per. e. parallelá. g. к. tantum distan-
tem à. g. к. quantum. y. l. ab. g. h. distare inueni-
tur. Cogitetur pariter. z. i. a. per punctum. i.
parallela. d. p. quare. a. t. æqualis erit. f. c. et. y. x.
æqualis. f. e. et. y. s: b. l. æqualis. Ita subtractis è
duobus quadratis superius dictis. a. t. y. x. et. b. l.
producto. y. b. æqualibus, supererunt. к. d. et. a. c.
x. cognita, tanquam æqualia dato secundo nu-
mero, sed. к. d. quadratum est medietatis. g. f.
cognitæ, cognoscetur igitur residuum. a. c. x. vnà
etiam singulæ tertiæ partes nempe quadrata. o.
i. o. c. et. o. e. & radix. b. f. vel. f. s. singularum,
qua coniuncta dimidio. g. f. rursusq́; ab eodé de-
tracta, propositum consequemur.

# T H E O R E M A  X L I I I I.

**C**VR si quis cupiat numerum propositum in duas eiusmodi partes diuidere, vt
　　quadratum maioris, quadratum minoris superet quantitate alterius numeri
propositi, rectè primum numerum in seipsum multiplicabit, & ab eodem secun-
dum numerum detrahet, residuum verò per duplum primi diuidet, ex quo proue-
niens primi pars minor erit, quæ ex illo primo detracta, partem maiorem
proferet.

　　Exempli gratia, si proponantur. 20. diuisa in duas eiusmodi partes, vt quadratũ
maioris superet quadratum minoris numero æquali ipsi. 240. oportebit primum
numerum, qui quadratus cum fuerit, erit. 400. in seipsum multiplicare, & ex hoc
quadrato secundum numerum nempe. 240. detrahere, tunc remanebunt. 160. quę
diuisa per. 40. numerũ duplũ primo, dabuntur quatuor pro minori numero, à resi-
duo verò. 20. detractis quatuor, erunt. 16. pro maiori numero.

　　Quod vt exactè consideremus, primus numerus propositus significetur linea. q.
h. diuidendus in duas partes. q. p. et. p. h. tales quales quærimus. Postmodum eriga
tur quadratum. q. e. diuisum diametro. f. h. ductisq́;. p. o. t. et. a. o. c. parallelis lateri-
bus quadrati, dabuntur imaginaria quadrata. c. t. et. p. a. duarum partium. q. p. et. p.
h. incognitarum. Ad hæc cogitemus quadratum. u. n. æquale quadrato. p. a. è quadra

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　to

to maiore. c.t. extractum quare refiduum qua-
drati.c.p.cognitum erit,quam quantitatem co-
gnitam , cum fit fecundo loco data, cogitemus
detrahi è toto quadrato cognito. q. e . ex quo
fumma duorum fupplementorum. q. o. et. o. e.
cognofcetur, vnà cum quadratis. u.n.et.p.a.du
plo fcilicet.q. a.quo diuifo per duplum.q.h.aut
fimplex.q.a.per. q. h. fimplicem , dabitur. a. h.
nempe.p.h. minor numerus quæfitus.
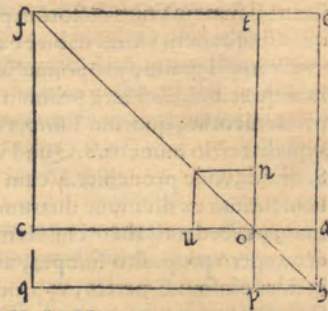
## THEOREMA XLV.

CVR volentes diuidere numerum propofitum in duas eiufmodi partes,vt pro
ductum vnius in alteram, alteri numero propofito æquetur,rectè dimidium
primi dati numeri in feipfum multiplicant, ex quo quadrato fecundum datum nu-
merum detrahunt, refiduiq; radicem fumunt, qua coniuncta vni dimidio primi nu-
meri, pars maior datur,ex altero verò dimidio detracta , minorem manifeftabit.

Exempli gratia, fi numerus partiendus effet. 34.alter verò numerus effet.64. cui
productum vnius partis in alteram æquale effe deberet. Dimidium primi numeri,in
feipfum multiplicaremus, cuius quadratum effet. 289.de quo detracto fecundo nu-
mero nempe. 64.remaneret.225.cuius quadrata radix nempe. 15. coniuncta. 17.
dimidio.34.proferet. 32. maiorem partem , detractoq; ex. 17. fupereffet.2. pars
inquam minor.

Cuius fpeculationis caufa , primus numerus propofitus fignificetur linea. a. d.cu-
ius dimidium.c. d.cognitum erit, vnà etiam eius quadratum.c.f.quo diuifo per dia
metrum. e.d.fupponantur partes ignotæ
ipfius.a.d. effe.a.b.et.b.d. & à puncto.b.
duci lineam.b.h.g.parallelam.d.f.et. m.
h.κ.parallelam.d.a. extructa figura fimi
li figuræ quintæ fecundi Eucli. quare da
bitur gnomó.l. d.g.æqualis producto.b.
κ.& proinde cognitus , quo detracto è
quadrato,c.f.remanebit quadratum.g.l.
cuius radice æquali. c.b. coniuncta. a. c.
& detracta ex.c.d.partes. a.b. et.b.d.quæfitæ dabuntur.
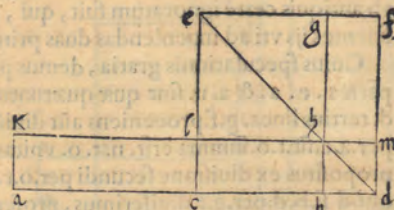
## THEOREMA XLVI.

CVR propofitis tribus numeris, quorum prior in duas eiufmodi partes diui-
dendus fit, ut mutuò diuifæ, & per fummam prouenientium diuifo fecundo
numero, proueniens vltimum fit æquale tertio numerorum propofitorum . Conful
tifsimum fit fecundum numerum per tertium diuidere, ex quo proueniens fit fum-
ma proueniment ium è duabus partibus mutuò diuifis, quam fummam fi quis velit di-
ftinguere,rectè poffit medio operationis precedêtis theorematis fûpta vnitate fuper
ficiali pro fecundo numero diftinctis poftmodum prouenientibus,rectè meo iudi-
cio operabimur per regulâ de tribus ( quod fuit ab antiquis prætermiffum ) Si dixe-
rimus,

rimus, si summa vnius dictorum prouenientium cum vnitate dat primum numerum, quid ipsa eadem vnitas dabit? ex quo propositum oriatur.

Exempli gratia, proponuntur tres numeri, primus. 20. secundus. 34. tertius. 8. Iam quærimus diuidere primum. 2 0. in duas partes quæ mutuò diuisæ præbeant duo prouenientia, quorum summa tanta sit vt per eam diuiso. 34. proueniat numerus æqualis tertio numero. 8. Quod vt præstemus iubet regula secundum. 34. per tertiū. 8. diuidi, vnde proueniet. 4. cum vna quarta parte, quod proueniens erit summa pro uenientium ex diuisione duarum partium quæsitarum, quæ si distinguere voluerimus, præcedentis theorematis methodum sequemur, vnitate superficiali pro secun do numero proposito sumpta, ac si diceremus, diuidatur. 4. cum vna quarta parte in duas eiusmodi partes, vt productum vnius in alteram sit vnitas superficialis, certè fractis integris cum quarta parte coniungendis, darentur vnitatis decemseptem quartæ lineares, verum cum necesse sit, ex præcedenti theoremate, dimidium in seipsum multiplicare, essetq; dimidium. 8. quartarum partium cum octaua, commodius totum constituetur. 34. octauarum, quarum dimidium, nempe decemseptem octauæ, in seipsum multiplicatum erunt. 289. sexagesimæ quartæ vnius integri superficialis, quandoquidem integrū superficiale, cuius vnitas linearis in. 8. partes diuiditur est. 64. vt ex primo theoremate huius libri depræhendi potest. Nunc vnitate hac superficiali, nempe. 64. ex. 289. detracta, supererit. 2 2 5. cuius radix quadrata, scilicet. 1 5. coniuncta dimidio dictorum prouenientium, nempe. 1 7. dabit maius proueniens. 3 2. detractaq; ex altero dimidio, dabit proueniens minus. 2. hoc est pro maiore proueniente. 3 2. octauas, & pro minore duas, quatuor scilicet integros pro maiore, & quartam partem vnius integri pro minore. Nunc si ex regula de tribus dixerimus, si. 4. iuncta vni, nempe. 5. dant. 2 0. primum numerum, quid dabunt. 4. integra (proueniens inquam maius) dabūt certè. 1 6. partem maiorem. Tum si dixerimus, si quarta pars coniuncta vnitati dat. 2 0: quid dabit quarta illa pars(hoc est proueniens minus) dabit prefectò quatuor scilicet minorē partem, quod ab antiquis certè ignoratum fuit, qui, inuentis prouenientibus quieuerunt, nescientes ijs vti ad inueniendas duas primi numeri partes.

Cuius speculationis gratia, demus primum numerum significari linea. e.u. cuius partes. e. a. & a. u. sint quæ quæruntur, alter verò numerus significetur linea. b. d. tertius linea. g. f. proueniens aūt diuisionis. e. a. per. a. u. sit. n. t. diuisionis aūt. a. u. per. a. e. sit. t. o. summa erit. n. t. o. vnitas verò. n. i. et. o. i. Iam si numerus. f. g. tertiò propositus ex diuisione secundi per. o. t. n. proferri debet. Ex. 1 3. theoremate patet, quòd si. b. d. per. g. f. diuiserimus, proferetur. o. t. n. qui cum fuerit inuentus, summā esse oportet duorū prouenientiū, ex diuisione mutua duorū numerorum, nempe. a. e. per. a. u. et. a. u. per. a. e. deinde manifestum est ex. 24. aut. 2 5. theoremate eorū productum (multiplicatis prouenientibus adinuicem) vnitatem superficialem futuram esse. Hactenus igitur, totum. o. n. ex doctrina præcedentis theorematis diuiditur in puncto. t. ita vt productum. o. t. in. t. n. solam vnitatem superficialem cōtineat, quo facto, si, vt antedictum est, cogitauerimus. n. t. proueniēs esse ex diuisione. e. a. per. a. u. et. t. o. proueniens ex diuisione. a. u. per. a. e. patebit ex definitione diuisionis, quod eadem erit proportio. a. e. ad. n. t. quæ est. a. u. ad vnitatem. n. i. et. a. u. ad. o. t. eadem quæ est. e. a.

ad
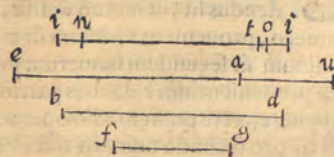
ad vnitatem. o. i. permutandoq; e. a. ad. a. u. ficut. t. n. ad. n. i. & componendo. e. a. u. ad. a. u. ficut. t. n. i. ad. n. i: & euerfim. e. a. u. ad. e. a. vt. t. n. i. ad. t. n. Quare, ex. 20. fepti mi, recte vtimur regula de tribus. Idem & de altera parte dico, quamuis qui vnam teneat, alteram quo que habiturus fit. Non mirum tamen fi huiufmodi problema ab antiquis definitum non fuerit, qui hanc vltimam partem non cognouerunt.

## THEOREMA XLVII.

CVR duobus numeris mutuò diuifis, fi per fummam prouenientium, produ- &um vnius in alterum multiplicetur, vltimum productum, fummæ quadra-tnıom duorum numerorum æquale futurum fit.

Exempli gratia, propofitis. 16. et. 4. mutuò diuifis, fumma prouenientium erit. 4. integrorum cum quarta parte, qua fumma multiplicata cum producto primorū numerorum, nempe. 64. dabuntur. 272. integri fuperficiales, qui fummæ quadra-torum duorum numerorum æquantur.

Hoc vt confideremus, duo numeri partibus. a. e. et. e. i. in linea. a. i. fignificentur, quorum productum fit. e. d. & quadratū ipfius. a. e. fit. e. p: ipfius verò. e. i. fit. e. q. pro-ueniens aūt ex diuifione. e. i. per. a. e. fit. o. u. proueniens aūt. a. e. per. e. i. fit. o. t. quo-rum fumma fit. o. u. t. tum productum. e. d: linea. u. n. fignificetur ad angulum rectū coniuncta in puncto. u. extremo ipfius. o. u. t. productum aūt. u. o. t. in. u. n. fit. n. t. Iam probandum nobis eft. n. t. æqualem effe fummæ duorum quadratorum. q. e. p. Quod fingillatim probo, & affero productum. o. n. æquale effe quadrato. q. e. & productū. s. t. quadrato. e. p. Nam ex. 35. theoremate patet numerum. e. i. medium effe pro-portionalē inter. e. d. et. o. u: cum numerus. e. i. ex præfuppofito ab. e. a. multiplicetur & diuidatur, cuius multiplicationis produ-&um eft. d. e: nempe. u. n. & proueniens ex diuifione eft. o. u: quare ex dicto theorema-te. e. i. media proportionalis eft inter. u. n. et. u. o. Itaq; productum. o. n. æquale eft qua-drato. e. q. ex. 16. fexti vel. 20. feptimi. Idem dico de producto. s. t. népe æquale effe qua-drato. e. p. quandoquidem numerus. a. e. ab e. i. multiplicatur ac diuiditur, cuius multi-plicationis productum eft. d. e nempe o. s. & proueniens ex diuifione. o. t: inter quæ ex. 35. theoremate. a. e. media proportionalis eft. Quare ex allatis propofitionibus productū. s. t. æquale eft quadrato. e. p. fed totū productum. n. t. fumma eft duorum productorum. o. n. et. s. t. ex prima fecundi Eucli. Itaque verum effe quod dictum eft, confequitur.
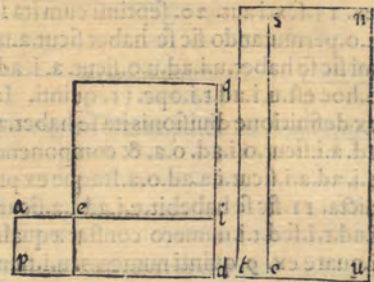
## THEOREMA XLVIII.

CVR fi quis maiorem duorum numerorum fola vnitate inter fe differentium, per minorem diuidat, maioremq; per proueniens multiplicet, productum, fūmæ ipfius maioris cum eodem proueniente æquale erit.

Exempli gratia. 10. per. 9. diuifo, datur vnum cum nona parte, quo multiplica-to per proueniens, ipfo nempe. 10: datur productum. 11. cum nona parte, tantum fci-licet

licet quanta summa est maioris cum proueniente.

Cuius speculationis causa, maior numerus significetur. a. i. et minor linea. a. o. ex quo ex praesuposito. o. i. vnitas erit. Sit autem proueniens ex diuisione. a. i. per. a. o, a. e: quod. e. a. directè coniungatur ipsi. a. i. et productum. a. i. in. a. e. sit. u. i. Probabo numerum superficialem. u. i. aequalem esse lineari. i. a. e. quare meminisse oportet, decimotertio theoremate probatum fuisse, quòd si numerus diuisibilis per proueniens diuidatur, proueniens futurus sit numerus diuidens, quare. a. o. erit proueniens ex diuisione. a. i. per. a. e. & ex definitione diuisionis ita se habebit. e. a. ad. a. i. sicut. o. i. ad. o. a. & componendo ita. e. i. ad. a. i. sicut. i. a. ad. o. a. quare, a. i. erit media proportionalis inter. e. i. et. a. o. sed. a. i. non modò diuisa nūc cogitatur ab. e. a. ex quo sit proueniens. a. o. sed etiam per eandem. e. a. multiplicata, ex quo productum oriatur. u. i. Itaq; ex. 25. theoremate. a. i. media est proportionalis inter. u. i. et. a. o. Quare. ex. 11. quinti. eadem erit proportio. u. i. ad. a. i. sicut. e. i. ad eandem. a. i. Igitur ex. 9. praedicti numerus. u. i. aequalis erit numero. e. i. quod erat propositum.
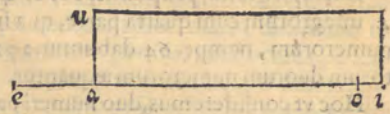
## THEOREMA XLIX.

IDipsum etiam alia ratione considerari potest.

Linea. u. a. secetur in puncto. t. ita vt. a. t. aequalis sit vnitati. o. i. & media paral lela. t. n. terminetur productum. t. i. quod constabit aequali numero, quamuis superficiali, numero. a. i. tametsi lineari. Tum parallela ducatur à puncto. o. ipsi. a. u. termi netur; productum. o. u. ex quo bina producta dabuntur. u. o. et. t. i. inter se aequalia ex. 15. sexti aut. 20. septimi cum ita se habeat. a. i. ad. a. u. sicut. a. o. ad. a. t. sed. a. i. ad. a. o. permutando sic se habet sicut. a. u. ad. a. t. & ex prima sexti aut. 18. vel. 19. septi mi sic se habet. u. i. ad. u. o. sicut. a. i. ad. a. o. hoc est. u. i. ad. t. i. ope. 11. quinti. Iam ex definitione diuisionis ita se habet, a. e. ad. a. i. sicut. o. i. ad. o. a. & componendo. e. i. ad. a. i. sicut. i. a. ad. o. a. Itaque ex prae dicta. 11. sic se habebit. e. i. ad. i. a. sicut. u. i. ad. t. i. sed. t. i. numero constat aequali. a. i. quare ex. 9. quinti numerus. u. i. numero. e. i. aequalis erit.
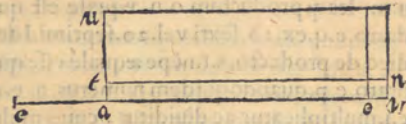
## THEOREMA L.

CVR diuidentes numerum propositum in duas eiusmodi partes, vt productū vnius in alteram cum i psarum differentia in summam collectum, aequale sit alicui alteri numero maiori primo. Rectè primum ex secundo detrahunt, residuum verò conseruant, tum ex primo semper binarium desumunt, dimidiumq; conseruant, alterum verò dimidium in seipso multiplicant, & ex quadrato numerum con seruatum eruunt, residuiq; radicem ex dimidio conseruato, quod vltimum residuum propositi numeri quaesita pars minor est.

Exempli gratia, si proponatur numerus. 20. ita diuidēdus, vt productū vnius partis in alteram, cum partium differentia collectum in summam, aequale sit proposito nume-

numero, verbi gratia. 92. præcepit regula detrahi primum numerum ex secundo, nempe. 20. ex. 92. cuius residuum, scilicet. 72. conseruetur, tum detrahi iubet binarium ex primo, sic in proposito exemplo remanebunt. 18. huius autem. 18. dimidium in seipsum multiplicari iubet, quod cum sit. 9. datur numerus. 81. ex quo. 81. primum numerum conseruatum, nempe. 72. vult regula detrahi, sic remanebit. 9. tum huius. 9. quadrata radix detrahenda est ex dimidio ipsius. 18. quod fuit ante quadratum, sic supererit. 6. hoc est. 9. excepta radice quadrata, qui. 6. erit minor pars quæsita, maior verò. 14. quarum productum. 84. coniunctum cum partium differentia præbet exactè. 92.

Cuius rei hæc est speculatio. Primus numerus minor, qui proponitur diuisibilis significetur linea. q. g. maior verò linea. x. tum cogitemus. q. g. diuisam, cuius maior pars sit. q. o. minor. o. g. differentia. q. p. ex quo. p. o. æqualis erit. o. g. sit autem productum. b. o. Oportet igitur, ut. b. o. simul cum differentia. q. p. æquale sit numero. x. secundò proposito, qui notus est, quare etiam summa producti. b. o. cum differentia q. p. cognita erit, ex qua detracto primo numero. q. g. residuum cognitum erit, nunc igitur quodnam erit hoc residuum? attendamus qua ratione ex summa. b. o. et. q. p. detrahenda sit. q. g. In primis si subtraxerimus ex dicta summa. q. p. quæ pars est. q. g. supererit detrahenda. p. g. ex. b o. pars inquam ipsius. q. g. quod fiet quotiescunque cogitauerimus. q. o. duabus vnitatibus diminutam, et per. o. g. multiplicatam, sit autem productum. b. e. nam cum. o. g. toties. b. o. ingrediatur, quot sunt in. q. o. vnitates ex prima sexti aut. 18. vel. 19. septimi, detrahendaq́; sit. p. g. ex. b. o. quæ. p. g. dupla est. o. g. patebit. o. c. æqualem esse. p. g. supererit itaque. b. e. productum. q. e. in. e. i. cognitum, erutis autem ex. q. g. ijsdem duabus vnitatibus, remanebit. q. i. nobis nota, ex quo. e. i. æqualis erit. e. c. Cum igitur productum. q. e. in. e. i. cognoscamus simul cum. q. i: Si voluerimus partes. q. e. et. e. i. cognoscere, vtemur. 45. theoremate huius libri, & propositum obtinebimus, nam cognoscemus. e. i. & ex consequenti. o. g. eius æqualem.
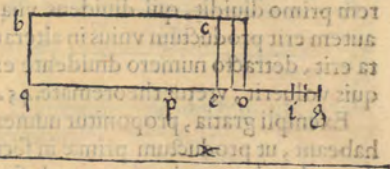
## THEOREMA LI.

DIVIDERE numerum in duas eiusmodi partes, quæ pro medio proportionali alterum numerum propositum recipiant, primi dimidio minorem, aliud nihil est, quàm binas primi numeri partes inuenire, quæ inter se multiplicatæ quadrato secundi numeri numerum æqualem proferant, ex. 16. sexti aut. 20. septimi, quod tamen. 45. theoremate fuit à nobis speculatum.

## THEOREMA LII.

CVR, propositis tribus numeris quibuscunque, si productum primi in secundum per tertium multiplicetur, atque secundum hoc productum corporeũ, per primum numerum diuidatur, proueniens erit numerus æqualis producto secundi in tertium.

Exempli causa, proponantur hi tres numeri. 10. 11. 12. multiplicenturq́; 10. cũ.

E      11.

11, dabuntur. 110. quo producto multiplicato cum. 12. dabuntur. 1320. hoc pro
ueniens per primum nempe. 10. diuisum dabit. 132. numerum æqualem producto
secundi in tertium numerorum propositorum, scilicet. 132.

Hoc vt speculemur, primus numerus significetur linea. o. u. secundus. e. o. tertius.
e. a. productum verò. o. u. in. o. e. sit. o. i. ipsius ve-
rò. o. i. per. e. a. productū corporeū sit. i. e. tum
productū. e. o. in. e. a. sit. e. c. Dico nūc quod di-
uiso numero corporeo. i. c. per primū. o. u. pue-
niens æquale erit numero producti. e. c. Qua-
re in primis cogitandum est, quod cum produ-
ctum. i. c. ortum fuerit ex multiplicatione. o. i.
in. e. a: dictum. o. i. toties ingredietur. i. c. quo-
ties vnitas reperitur in. e. a. eadem ratione, to-
ties. e. c. in. i. c. quot vnitates erunt in. o. u. Itaq;
sequitur quòd diuiso. i. c. per o. u. proueniens sit
e. c. corporeum, æquale nihilominus producto. e. c. superficiali.
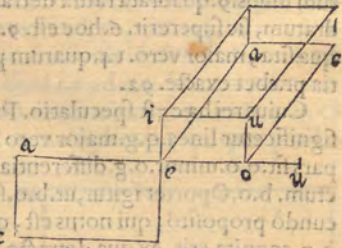
### THEOREMA LIII.

CVR diuidens propositum numerum in tres partes sic se habentes vt produ-
ctum primi in secundam, in tertia multiplicatū, præbeat numerum alteri nu-
mero proposito æqualem. Rectè secundum numerum per quemcunque alium mino
rem primo diuidit, qui diuidens vna erit ex tribus partibus quæsitis, proueniens
autem erit productum vnius in alteram reliquarum duarum, quarum summa cogni
ta erit, detracto numero diuidente ex primo dato, quam quidem si distinguere
quis voluerit, vtetur theoremate. 45.

Exempli gratia, proponitur numerus. 20. in tres partes diuidendus, quæ sic se
habeant, ut productum primæ in secundam in tertia multiplicatum det. 90. itaque
sumenda erit pro prima vna pars ipsius. 20. quæcunque illa sit, verbi gratia. 2. qua
secundus numerus, nempe, 90. diuidatur, dabitur igitur. 45. quod erit productum
cæterarum partium inter se, quarum summa est. 18. quam summam si distinguere
volueris in cęteris duabus partibus separatis, vteris. 45. theoremate, vt quàm citis-
simè quod cupis exequaris, erunt autem partes. 3. et. 15.

In cuius speculationis gratiam nihil aliud occurrit, quàm quod præcedenti theo-
remate, & superiore. 45. allatum est.

### THEOREMA LIIII.

DIVIDERE numerum in. 3. eiusmodi partes, vt quadratum vnius sit æquale
producto reliquarum duarum inter se, idem omnino est cum. 51. theoremate.
Nam qui sumet quamlibet partem propositi numeri, quæ tertia parte maior tamen
non sit, residuumq; in duas tales partes diuiserit, vt prima sumpta, media proportio
nalis sit ex probatione. 51. theoremate allata, propositum consequetur.

### THEOREMA LV.

IDipsum alia ratione ab ea diuersa quā. 51. theoremate adduximus, pfici potest.
                                                                        Suman-

Sumantur enim tres numeri continui proportionales, cuiuscunque denique proportionalitatis, qui in summam colligantur, ac postmodum, regula de trib. dicamus. Si summa hæc primo numero proposito in tres partes diuidendo respondet, cui respondebit vna ex tribus partibus huiusce sūmæ? idem de reliquis duabus partibus dico.

Exempli gratia, si proponatur numerus. 57. diuidendus in tres continuas partes proportionales proportione sesquialtera, tres numeros in eiusmodi proportionalitate distinctos sumemus, vt potè. 4. 6. 9. qui in summam collecti dabunt summā. 19. dicemusq́; si. 19. dant. 4. quid dabūt. 57? vnde proueniens vnius partis erit. 12. Tum si dicamus, si. 19. dat. 6. quid dabit. 57? nempe dabit. 18. Postremò, si. 19. dat. 9. quid dabit. 57? nempe. 26. atque ita dabitur. 18. cuius quadratum æquabitur producto reliquarum duarum partium inter se.

Quod vt sciamus, numerus propositus in tres quaslibet partes diuidendus significetur linea. a. d. tres autem numeri dictæ proportionalitatis, lineis. e. f. f. g. et. g. h. directè inter se coniunctis denotentur. Cogitemus pariter lineam. d. a. in tres partes diuisam. a. b: b. c. et. c. d. eadem cum cæteris proportionalitate, tunc eadem erit proportio. a. d. ad quamlibet suarum partium, quæ est. e. h. ad respondentem ipsius in. a. d: Verbi gratia respondentem. a. b. ipsi. e. f. et. b. c. f. g. et. c. d: g. h. Dico enim quòd ita se habebit. a. d. ad. c. d. sicut. e. h. ad. g. h. Nam cum sic se habeat. a. b. ad. b. c. sicut. e. f. ad. f. g. ex præsupposito, permutando sic se habebit. a. b. ad. e. f. sicut. b. c. ad. f. g. & eadem ratione sic se habebit. c. d. ad. g. h. sicut. b. c. ad. f. g. & cósequenter sicut. a. b. ad. e. f. ex quo ex. 13. quinti sic se habebit tota. a. d. ad totam. e. h. sicut. e. d. ad. g. h. aut. b. c. ad. f. g. aut. a. b. ad. e. f. permutando itaque propositum manifestum erit, ipsum autem productum. a. b. in. c. b. æquale erit quadrato. b. c. ex. 15. sexti aut. 20. septimi.
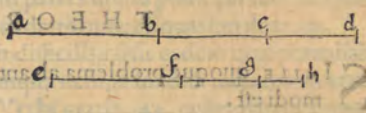


## THEOREMA LVI.

VEteres aliud quoque problema indeterminatum proposuerunt, quod ex more ratione à me definietur, est autem eiusmodi.

Quomodo propositus numerus in tres eiusmodi partes diuidatur, vt quadratū vnius æquale sit summæ quadratorum reliquarum duarum partium.

Hoc vt efficiamus tria quadrata separata sumamus, quorū vnū æquale sit reliquis duobus, eorū autē radices in summam simul colligantur, tum regulam de tribus sequemur, ratione præcedenti theoremate demonstrata, & rectè vt infra docebimus, quod autem dico de quadratis, etiam de cubis, & quibusuis dignitatibus assero.

Exempli gratia, si numerus diuisibilis proponatur. 30. in tres eiusmodi partes diuidendus, vt quadratum vnius æquale sit summæ quadratorum reliquarum duarum partium, in primis radices trium quadratorum sumemus, sic quomodocunque se habentes, vt maius ipsorum æquale sit summæ reliquorum duorum, verbi gratia. 25. 16. et. 9. nempe. 5. 4. et. 3. quæ si colligantur in summam efficiunt. 12. Tum ex regula de tribus dicemus, si. 12. respondet. 30. cui. 5. radix maior respondebit? nempe. 12. cum dimidio.

Deinde si dixerimus si. 12. valet. 30. quid valebit. 4. radix media? nempe valebit. 10. tertia autem minor, 7. cum dimidio. Itaque tota summa erit. 30. & quadra-

E   2                    tum.

tum. 12. cum dimidio erit. 155. quod æquale erit fummæ quadratorum duarum par
tium, nempe. 100. cum. 55.

Hoc vt demóftremus, numerus diuifibilis propofitus fignificetur linea. a. d. & fum
ma radicum, noftro modo fumptarum, linea. e. h. quarum prima & maior fit. e. f. fe-
cunda. f. g. tertia. g. h. cogitemus etiam lineam. a. d. ea ratione diuifam effe qua. e. h.
patebit enim ex modo præcedentis theorematis vnamquanque partium. a. d. ita fe
habituram ad fuum totum ficut fe habent fingulæ. e. h. ad fuum. Quod ideo dico, vt
intelligamus recte nos dicere. Si. e. h. dat. a. d. ergo. e. f. dabit. a. b. atq; ita de cæteris.
Quare permutando fic fe habebit. a. b. ad. b. c. ficut. e. f. ad. f. g. idem dico de reliquis.
Igitur ex. 18. fexti aut. 11. octaui, eadem erit proportio quadrati. a. b. ad quadratū.
b. c. quæ quadrati. e. f. ad quadratum. f. g. tota enim funt æqualia, cum eorum partes
fimiles inter fe funt æquales. Idem dico de proportione quadrati. a. b. nempe ita
fe habere ad. c. d. ficut quadratum. e. f. ad quadratum. g. h, ex quo ex. 24. quinti pro-
portio quadrati. a. b. ad fummam quadratorum duarum partium. b. c. et. c. d. fic fe ha
bebit ut quadrati. e. f. ad fummam quadra-
torum. f. g. et. g. h. At quadratum. e. f. æquale
eft fummæ quadratorum. f. g. et. g. h. igitur
fic etiam fe habebit quadratum. a. b. nempe
æquale quadratis. b. c. et. c. g. Idipfum de cæ
teris dignitatibus dices, vterisq;. 21. theoremate huius libri.

## THEOREMA LVII.

SImile quoque problema ab antiquis indeterminatum proponitur, quod eiuf-
modi eft.

An numerus aliquis in tres eiufmodi partes diuidi poffit, vt quadratum vnius æ-
quale fit fummæ quadratorum cæterarum duarum partium fimul cum producto
vnius in alteram.

Exempli gratia, fi proponatur numerus. 50. vt iam dictum eft diuidendus, repe
riendus erit alius quilibet numerus, qui tamen fumma fit trium radicum fic fe ha-
bentium, vt quadratum vnius æquale fit fummæ quadratorum duarum partium fi-
mul cum producto vnius in alteram, eum autem qui primò occurrit fumamus, utpo
tè. 30. qui fumma eft numerorum. 6. 10. 14. partium fic fe habentium, vt quadratum
ipfius. 14. æquale fit fummæ quadratorum cæterarum partium fimul cum produ-
cto vnius in alteram, agamusque regula de tribus, ac dicamus, fi. 30. valet.
50. quid valebit, 14. nempe. 23. cum tertia parte. Idem efficiemus in cæte-
ris partibus, quarum vna erit. 16. cum duabus tertijs, altera verò. 10. abfque fractis,
ex quo quadratum primæ erit. 544. cum. 4. nonis, fecundæ. 277. cum feptem nonis p
tertiæ. 100. & productum fecundæ in tertiam. 166. cum. 6. nonis, quod productum,
cum quadratis fecundæ & tertiæ collectum erit. 544. cum. 4. nonis.

Huius rei fpeculatio eadem eft, que fuit præcedentis theorematis. vfque quo no-
ueris eandem proportionem effe quadrati. a. b. ad fummam quadratorum. b. c. et. c.
d. quæ quadrati. e. f. ad fummam quadratorum. f. g. et. g. h. Sed cum hic non demus
quadratum. e. f. æquale fummæ quadratorum. f. g. et. g. h. fed maius ex producto. g. h.
in. f. g. aut quod idem eft, è contrario, fubfequentes figuræ cogitandæ erunt, qua-
rum. i. fit quadratum. a. b: l. fit quadratum. c. f: x. quadratum. h. c: y. quadratum. f. g: p.
quadratum. c. d: q. quadratum. g. h: k. fit productum. b. c. in. c. d: m. fit productum. f.

g. in

g.in.g.h.Nunc ex ſpeculatione præcedentis theorematis, eadem erit proportio. n.
t.ad.o.u.quæ eſt.n.s.ad.o. n. quare pro-
ductum. κ. ex definitione ſimile erit
producto. m. cum vtraque ſint rectan-
gula, vnde proportio. κ. ad. m. ad pro-
portionem.n.t.ad.o.u. ex. 18. ſexti du-
pla erit. Igitur proportio. κ. ad. m. æ-
qualis erit proportioni. x. ad. y. et. p.
ad.q. et.i.ad.l.& permutando ſic ſe ha-
bebit. κ.ad.i.ſicut.m.ad.l.ſed.x. p.ad.i.
ſic ſe habere probatum eſt vt.y.q. ad.l.
Quare ex eadem. 24.quinti ſic ſe habe
bit.x.p.κ.ad.i.ſicut.y.q.m.ad.l.ſed.y.q.
m.æqualis eſt.l. Itaque.x.p.κ. pariter.i.
æqualis erit.
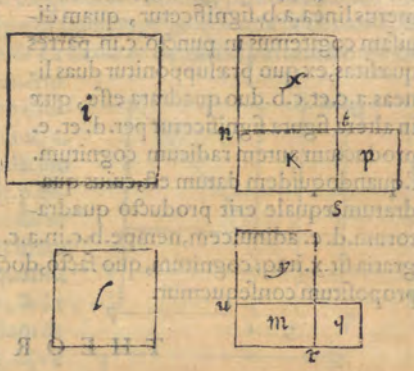
## THEOREMA LVIII.

ALIVD quoque problema, nec tamen definitum, veteres propoſuerunt,
nempe an aliquis numerus in.4.eiuſmodi partes diuidi poſſit, vt ſumma qua-
dratorum duarum partium dupla ſit ſummæ quadratorum reliquarum duarum.

Verum huius effectio & ſpeculatio non erit difficilis, cũ ſit eadem quæ præmiſsis
proximè duobus theorematibus allata fuit, ſumpta nempe ſumma radicum quarun
cunque ſic ſe habentium, prout dictum fuit. Verbi gratia. 44. cuius partes erunt.
16 12. 14. 2. tũc progrediemur regula de tribus dicentes.Si 44 numerum propoſi-
tum valet, quid. 16.pars maior? nempe valebit partem maiorem numeri propoſi-
ti reſpondentem. 16. idem de cæteris dico.

Porrò ſpeculatio eadem eſt cum ſuperioribus.

## THEOREMA LIX.

CVR diuidens propoſitum numerum in duas eiuſmodi partes, vt productum
radicum quadratarum ipſarum partium æquale ſit alteri numero propoſito,
cuius tamē quadratum maius nõ ſit quadrato dimidij primi numeri propoſiti. Rectè
ſecundum numerum propoſitum in ſeipſum multiplicat, & eundē ex quadrato di-
midij primi detrahit, reſidniq̃ quadratam radicem ſubtrahit ex dimidio ipſius pri-
mi, ex quo datur minor pars quæſita, qua ipſi dimidio coniuncta, maior pars ha-
betur.

Exempli gratia, ſi proponatur numerus. 20. propoſito modo, in duas partes
eiuſmodi diuidendus, vt productum radicum æquale ſit ( verbi gratia ) 8. Dimi-
dium primi numeri in ſeipſum multiplicabimus, cuius quadratum erit. 100. ex
quo quadratum ſecundi numeri, nempe.64.detrahemus,remanebitq̃; 36.cuius radi
ce quadrata coniuncta. 10. dimidio inquam primi numeri propoſiti,dabitur nume-
rus. 16. pars maior, & ſubtracta a dimidio, dabitur minor pars, nempe. 4.

Hoc

Hoc vt demonftremus, primus nu-
merus linea.a.b.fignificetur, quam di-
uifam cogitemus in puncto.c.in partes
quæfitas,ex quo præfupponitur duas li-
neas.a.c.et.c.b.duo quadrata efle, quæ
in altera figura fignificetur per.d. et. e.
productum autem radicum cognitum.
f.quandoquidem datum eft,cuius qua-
dratum æquale erit producto quadra-
torum.d. e. adinuicem,nempe.b.c.in.a.c. ex.19. theoremate huius. Quod verbi
gratia fit.x.itaq; cognitum, quo facto,doctrinam.45.theorematis libri huius fecuti,
propofitum confequemur.
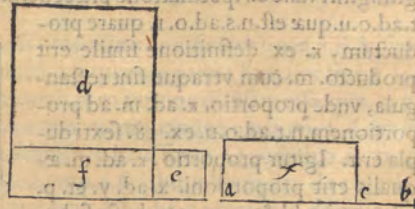
## THEOREMA LX.

CVR productum differentiæ duarum radicum in fummam ipfarum , femper
differentia fit quadratorum ipfarum radicum.

Exépli gratia, quoslibet duos numeros pro radicibus fumpferimus,vt potè. 3.et.
5. quorum differentia eft.2. certè fi differentiam hanc per fummam radicum fcili-
cet. 8.multiplicauerimus , dabitur numerus. 16. quod productum differentia eft
fuorum quadratorum, nempe inter.9. et.25.

Hoc vt fpeculemur,duæ radices in linea.n.i.fignificentur, quarum vna fit.n. c. &
altera.c.i. ipfarum autem differentia.n.t. ex quo.t.
c.æqualis erit.c.i. Tum cogitato toto quadrato.d.i.
cum diametro. d. i. ductaq; parallela lateri. n. d. à
puncto.c.& altera à puncto.t. & à puncto. o. tertia
ipfi.n.i. & à puncto. a.quarta. x. a. e. parallela ipfi.
o. inueniemus. b.n.productum effe differentiæ. n.
t. in fumma radicum.n. i. & cum. d.o. et . a. o. fint
quadrata radicum prædictarum : b. e. æquale erit .
n.u.cum vtrunque horum productorum æquale fit.
x.u.ex quo gnomon.e. d. u. æqualis erit producto.
b. n.quod fcire cupiebamus.
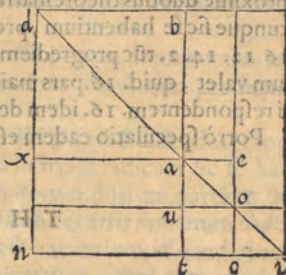
## THEOREMA LXI.

CVR propofitum aliquem numerum diuifuri in duas eiufmodi partes, vt diffe-
rentia radicum quadratarum æqualis fit alteri numero propofito , cuius ta-
men quadratum dimidij primi quadratum non excedat.Rectè fecundum numerum
in feipfum multiplicant , productum verò ex primo numero detrahunt , rursusq; di
midium refidui quadrant, & quadratum hoc ex quadrato dimidij primi fubtrahunt,
atque ita radice quadrata refidui, dimidio primi coniuncta, pars maior datur, qua
ex ipfo dimidio detracta , pars minor relinquitur.

Exempli gratia, propofito numero. 20. ita ut propofitum eft, diuidendo, nem-
pe vt differentia radicum quadratarum dictarum partium æqualis fit binario, bina-
rium hoc in feipfum multiplicabimus , cuius quadratum.4. è primo numero.20.de-
trahemus,

trahemus, supererítq; numerus. 16. cuius dimidium scilicet. 8. in seipsum multipli-
cabimus, dabiturq; numerus. 64. qui cum ex quadrato dimidij primi detractus fue-
rit, nempe ex. 100. & residuo. 36. radix quadrata nempe. 6. coniuncta denario, di-
midio primi, dabit. 16. partem maiorem, & ex denario detracta, partem minorem.

Cuius speculationis causa, primus numerus
propositus significetur linea. x. y. pro voto diui-
sa in puncto. c. et. x. t. productum sit ipsius. x.
c. in. c. y. pariter etiam. q. p. sit summa radicum
quadratarum, nempe. q. g. ipsius. t. c. et. g. p. ip-
sius. c. y. Tum super. q. p. extruatur & diuidatur
quadratum. q. u. ea ratione qua. 41. theoremate
aut. 29. diuisimus, in quo sane quadrato, quadra
tum ipsius. q. i. cernemus datæ differentiæ, & in
eo collocata quadrata. x. c. et. c. y. ita etiam &
rationem, qua cognoscimus productum. g. r. (vsi
modo. 29. theorematis) cuius quidem. g. r. qua-
dratum, ex. 19. theoremate æquale erit produ-
cto. x. t. ideo etiam cognitu, ac proinde cum no
uerimus. x. y. si rationem sequemur. 45. theore
mate cognoscemus non solum ratione. 41. theoremate allata hoc recte perfici, sed
hac etiam alia ratione.
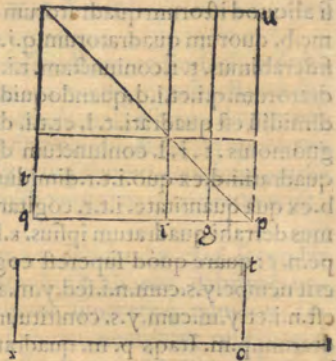
## THEOREMA LXII.

CVR propositum numerum diuisuri in duas eiusmodi partes, vt differentia
suarū radicū quadratarū æqualis sit alteri numero proposito. Cuius tamē qua-
dratū maius non sit quadrato medietatis ipsius primi propositi numeri. Recte etiā
quadratū dimidij secundi numeri ex dimidio primi detrahūt, residuiq; radicem per
secundum multiplicant, & productum ex dimidio primi detrahunt, vt residuum
pars quæsita minor sit, & illud alterum totius residuum, pars maior.

Exempli gratia, si numerus. 50. in
prædictas duas partes diuidendus pro-
poneretur, & alter etiam. 6. quadratum
dimidij secundi numeri esset. 9. eo detra
cto ex dimidio primi, remaneret. 16. cu
ius radix. 4. scilicet per totum secundum
nempe. 6. multiplicata, proferet. 24.
quo producto ex dimidio primi detra-
cto, nempe. 25. dabitur. 1. pars minor,
maior autê erit residuū. 50. hoc est. 49.
radices autem erunt. 1. et. 7. differentes
inter se, numero senario.
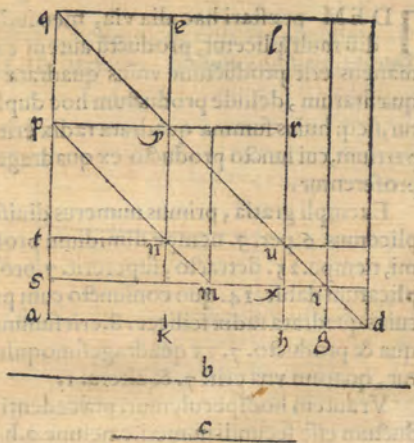
Hoc vt sciamus, duo numeri lineis si-
gnificétur, primus linea. b: secūdus linea.
c. duæ autem partes. b. duobus quadra-
tis. q. i. et. i. d. notentur, eorum verò radi-
ces lineis. a. g. et. g. d. differentia porrò ip
si. c. æqualis & cognita sit. a. h. ex quo. h.
g. æqua-

## IO. BAPT. BENED.

g. æqualis erit.g.d.tum productum.a.g. in.g.d.sit.a.i.et.t.i.æqualis. a.i.et.l. i.páriter
secetur æqualis.t.i.quæ omnia ex diametro.q. d. cogitari possunt:erit igitur. u. i. æ-
qualis. i. d. supereritq́; quadratum. q.u.differentiæ.a. h.cognitum , hoc verò cogi-
temus diuisum esse in.4. partes æquales medijs diametris. p.r.et.n.e.quare vnaqueq;
partium cognoscetur , & quadratū erit ipsius. a. κ.aut ipsius.κ. h.dimidij.a.h. Quòd
si aliquod istorum quadratorum detrahere voluerimus, nempe.n.r.ex dimidio sum
mẹ.b. duorum quadratorum.q.i.et. i. d.cognitæ, hac via procedemus, primum còn
siderabimus. t. r.coniunctam. t.i. quæ quantitates erunt summa dimidij duorū qua-
dratorum.q.i.et.i.d.quandoquidem.t.r.
dimidiū est quadrati. t. l. et.t.i. dimidiū
gnomonis . t . i . l. coniunctum dimidio
quadrati.i.d.ex quo.i.t.r.dimidium erit.
b.ex qua quantitate. i.t.r. cogitare debe
mus detrahi quadratum ipsius. κ.h. nem
pe.n. r : quare quod superest cognitum
erit nempe.y.s.cum.n.i.sed.y.m.æqualis
est.n.i.et.y.m.cum.y.s. constituunt qua-
dratum.p.m. Itaq;. p. m. quadratum &
consequenter.p.s.eius radix cognosce-
tur, ita etiam & productum huius.p.s.in.
s.x. æqualis.c. nempe. p. x: estq; produ-
ctum huiusmodi semper minus quantita
te.r.t.i:per.u.i.æquale quadrato minori.
i. d. quare.i.d.cognoscetur, consequen-
ter.i.q. tanquam residuum ex.b. & eo-
rum radices quadratæ cognoscentur. a.
g. et. g. d.
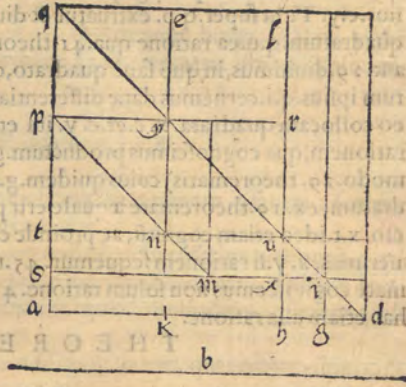


## THEOREMA LXIII.

IDEM præstari hac alia via, meo iudicio potest. Secundus numerus in suū dimi
diū multiplicetur, productū autem ex dimidio primi detrahatur , ex quo re-
manens erit productum vnius quadratæ radicis in alteram partium primi numeri
quæsitarum , deinde productum hoc duplicetur, & primo numero dato coniúnga-
tur,sicq; huius summæ quadrata radix erit summa radicum quadratarum dictarum
partium,cui iuncto producto ex quadragesimoquinto theoremate singulæ radices
proferentur.

Exempli gratia , primus numerus diuisibilis erat. 50. alter verò. 6. Iam si multi-
plicemus. 6.per. 3. nempe dimidium proferetur numerus . 18. quo ex dimidio pri-
mi, nempe.25. detracto , supererit. 7.productum vnius radicis in alteram, quod du
plicatum dabit. 14. quo coniuncto cum primo numero. 50. dabitur numerus. 64.
cuius quadrata radix scilicet. 8.erit summa radicum duarum partium quæsitarum ,
qua & producto. 7. ex quadragesimoquinto theoremate dictæ radices distinguen,
tur, quarum vna erit. 7. & altera. 1.

Vt autem hoc speculemur, præcedenti figura vti poterimus,in qua patet.t.r. pro
ductum esse secundi numeri.c.nempe.a.h.hoc est. t.u.in dimidio.a.e.scilicet.p.t.re-
siduum autem dimidij primi.b. esse.t.i.nempe.a.i.productum radicum,quod supple

men-

mentum est quadrati. q.d. totalis. Quaré duplicato.a.i. & coniuncto. b. cognosci-
mus totum. q.d. & consequenter.a.d. suam radicem, hoc est summam duarum radi
cum.a.g.et.g.d. quæ medio.a.i. cognito, & quadragesimoquinto theoremate singu-
læ cognoscuntur.

## THEOREMA LXIIII.

CVR propositum aliquem numerum in duas eiusmodi partes diuisuri, vt sum-
ma radicum dictarum partium æqualis sit alteri numero proposito. Recté se-
cundum numerum in seipsum multiplicant, ex quo quadrato, primum datum nu-
merum detrahunt, rursusq; residuum in seipsum multiplicant, & ex eo quadrato
quartam partem desumunt, quá ex quadrato dimidij primi numeri detrahunt, radi-
cemqué quadratam residui cum iunxerint,& ex dimidio primi numeri detraxerint,
partes quæsitæ proferuntur.

Exempli gratia, si proponeretur primus numerus.20. diuidendus et.6. secundus
pro summa radicum, hunc secundum.6. in seipsum multiplicabimus, dabiturq; nu-
merus.36. ex quo quadrato primus numerus detrahetur, supereritq; numerus.16.
qui quadratus dabit.256.cuius numeri quarta pars sumetur, nempe.64.quæ ex qua
drato dimidij primi numeri detrahetur, nempe.100.supereritq;.36.cuius radix qua
drata.6. coniuncta & detracta ex.10.dabit.16. partem maiorem et.4.minorem.

Cuius rei hæc speculatio, primus numerus diuisibilis significetur linea.a.b. diui-
sa in puncto. e. in partes adhuc incognitas, et.a.c. sit productum.a.e.in.e.b. item.q;
p. secundum numerum significet, æqualem summæ radicum, quæ puncto. n. distin-
guantur. Postmodum totum quadratum.p.d. erigatur ( quod nobis est cognitum )
in duo quadrata diuisum.o.p.et.o.d. quorum summa.a.b. cum detur, cognita rema-
net summa duorū supplementorū.o.u.et.o.q. qua quadrata cū fuerit dabit quadru
plū quadrati supplemēti.o.q. nēpe quadruplū producti.a.c.etenim.a.c.ex.19. theo
remate huius libri quadratum est ipsius.q.o. sicq; poterant etiam veteres quadrare
dimidium differentiæ. a.b.ab.p.d. nempe quadrato tantummodo supplemento. q.
o. Tunc habito.a.c. eius ope tanquam producti.a.e.in. e.b.ex.45.theoremate singu
læ partes cognoscentur.

Quod alia etiam ratione præstari poterat, nempe cognito supplemento .
q.o. distinguendæ radices q.n.et.n.p.ex. 45. theoremate, quibus cognitis, eorum
etiam quadrata cognoscuntur.
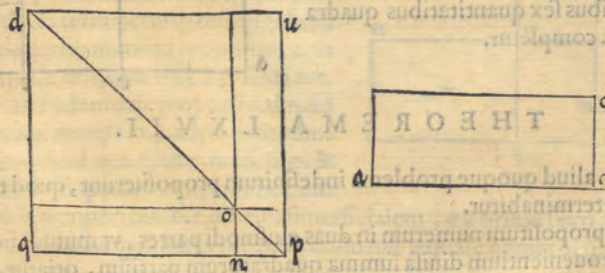
## THEOREMA LXV.

CVR proposito numero in tres qualescunque partes diuiso, si prima in tertiam multiplicetur, & huic producto, secundæ in primam productum coniungatur, itemq; secundæ in tertiam, hæc summa duplicata æqualis sit summæ productorum singularum in cæteras duas.

Exempli gratia, si proponatur. 20. diuisus in tres partes nempe. 12.5.3. multiplicato primo. 12. per. 3. tertiam partem dabitur. 36. secunda verò multiplicata per reliquas duas, hoc est. 5. per. 12. et. 3. in primis dabitur. 60. postea. 15. quorũ triũ productorum summa erit. 111. quæ duplicata dabit. 222. qui numerus æqualis esse dicitur summæ productorum singularum partium in reliquas duas, nempe summæ. 60. 36. 60. 15. 36. 15. hoc est ipsis. 222.

Cuius rei per se patet speculatio, cum in his sex vltimis productis, singula tria prima duplicentur.

## THEOREMA LXVI.

CVR proposito numero in. 3. qualescunque partes diuiso, si in reliquas duas singulæ multiplicentur, & hæc producta cũ summa suorum quadratorum coniungantur, tota summa hæc vltima æqualis erit quadrato totali propositi numeri.

Exempli gratia, si fuerit idem numerus. 20. in. 3. partes diuisus. 12.5.3. Si. 12. in 5. et. 3. producatur, summa productorum erit. 96. at. 5. in. 12. et. 3. erit. 75. postmodum. 3. in. 12. et. 5. erit. 51. nempe in vniuersum. 222. quadratorum porrò summa erit. 178. quæ coniuncta. 222. dabit. 400. quadratum ipsius. 20.

Erit autem huiusce rei facillima speculatio, si sequentem figuram mente conceperimus, in qua. a. b. propositum numerum significet, cuius partes distinctæ sint medio. e. et. c. Ipsum autem. q. b. sit quadratum totale parallelis. e. s. et. c. x. diuisum, quæ quadratum in tria rectangula diuident, quorum primum erit. q. e. compositum ex producto. a. e. in semetipsam, nempe quadratum. o. e. & ex producto eiusdem. a. e. in. c. b. quod erit rectangulum. o. s. ex quo tria rectangula. o. s. et. n. x. et. t. u. tria producta erunt singularum partium in cæteras duas, et. e. o: c. n: b. t. tria quadrata erunt: quibus sex quantitatibus quadratum totale. q. b. completur.



## THEOREMA LXVII.

VETERES aliud quoque problema indefinitum proposuerunt, quod tamen à nobis determinabitur.

Cur diuisuri propositum numerum in duas eiusmodi partes, vt mutuò diuisis, & per summam prouenientium diuisa summa quadratorum partium, oriatur proueniens alter numerus propositus.

Proposito deinde tertio quolibet numero diuidendo per singulas partes primi,

ita

ita vt simul prouenientibus in summam collectis huius summæ ad primum nume-
rum propositum proportio futura sit ea quæ est tertij ad secundum. Recte dimidium
primi numeri in seipsum multiplicant, ex quo quadrato secundum numerum detra
hunt, tum residui radicem sumunt, quam iungentes, & detrahentes ex dimidio
primi, partes quæsitas habent, cætera ex necessitate subsequuntur, prout nunc à
me docebitur.

Exempli gratia, proponitur numerus. 20. in duas partes diuidendus, quibus pò
stea mutuò diuisis, & per summam prouenientium diuisa summa quadratorum,
dent secundũ numerum propositum. 36. nam reliqua consequuntur. Itaque. 10.
dimidium primi in seipsum multiplicatur, & ex quadrato. 100. eruitur numerus. 36.
nempe secundus propositus residui porrò. 64. quadrata radix. 8. sumitur, quàm con
iungimus & detrahimus ex dimidio primi scilicet. 10. ex quo partes quæsitæ dabun
tur. 18. et. 2. quæ mutuo diuisæ dabunt suorum prouenientium summam. 9. cum no-
na parte, per quam diuidentes. 328. summam quadratorum ipsarum partium,
exactè dabitur numerus. 36. qui fuit secundò propositus. Tum si per singu-
las iam inuentas partes quilibet numerus diuisus fuerit, verbi gratia. 72. summa pro
uenientium erit. 40. qui numerus eandem proportionem cum primo nempe. 20. ser
uabit, quam tertius propositus. 72. cum secundo. 36.

Quod vt speculemur, primus numerus significetur linea. n. e. ita diuidendus à
puncto. o. vt diuisa parte. n.o. per. o.e. et. o.e. per. n.o. & per summam prouenien-
tium diuisa summa quadratorum. n.o. et. o.e. detur secundus numerus notatus linea.
q. к. Porrò meminisse oportet quòd. 26. theoremate probatum fuit vltimum hoc
proueniens æquale producto partium inter se futurum, nempe producto. n.o. in. o.
e. quod significetur rectangulo. n.e. Itaque datis. n.e. et. q.к. si. 45. theorema consu-
luerimus, partes. n.o. et. o.e. cognoscemus.

Proponitur deinde tertius quilibet numerus, verbi gratia. x. diuidendus per. o. e.
et. o. n. qui si diuidatur per. o. e. dabit pro
ueniens. b. o. Si verò per. n.o. proueniens
erit. d. n. nunc asserimus summã duorum
horum prouenientium, sic primo nume-
ro. n. e. dato proportionatam esse, sicut
tertius. x. secũdo. q.к. Producatur enim li-
nea. d. n. donec. n. q. æqualis sit. o. b. ex
quo. q. d. erit summa vltimò proueinen-
tium: item producatur. e.n. donec. n. u. æ-
qualis sit. o.e. termineturq́; rectangulum.
q. u. quod tertio numero proposito. x. vt
patet, æquale erit, quare ex. 15. sexti aut.
20. septimi eadem erit proportio. d. n. ad
n. q. quæ. u.n. nempe. o.e. ad. o.n. & com-
ponendo. d. q. ad. q. n. sicut. e. n. ad. n. o. &
permutando. d. q. ad. e. n. quę. q. n. hoc est.
b. o. ad. o. n. nempe sicut. b. e. ad. e. n. superficialem, ex prima sexti aut. 18. vel. 19.
septimi, sed rectangulum. e. n. constitutum fuit æquale numero. q.к. itaque verum
est propositum.

## THEOREMA LXVIII.

CVR numero per numerum diuiso, productoq; duorum numerorum per pro-
ueniens multiplicato, quod vltimò productum est, diuisi numeri semper qua
dratum existat.

Exempli gratia, si diuidamus. 10. per. 2. proueniens erit. 5. quo producto ex duo
bus numeris multiplicato, nempe. 20. habe
bimus. 100. quadratum numeri diuisi.

Cuius gratia duo numeri sint. a. et. e. por
rò. a. per. e. diuiso detur. u. tum. o. produ-
ctum. a. in. e. esse constituatur, quo per. u.
multiplicato dabitur. x. quadratum. a. pro-
pterea quòd. a. medium est proportionale
inter. o. et. u. ex. 35. theoremate. itaque
ex. 16. sexti aut. 20. septimi, propositi veri-
tas elucescet.
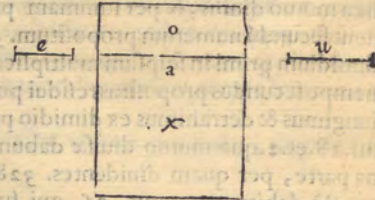
## THEOREMA LXIX.

CVR numero aliquo per duos alios multiplicato & diuiso, si per horum duo-
rum productum, summa duorum primorum productorum diuisa fuerit, vl-
timum proueniens, summæ duorum primorum prouenientium æquale sit.

Exempli gratia, proponitur numerus. 24. per. 8. et. 6. multiplicandus & diuiden
dus summa productorum erit. 336. prouenientium autem. 7. si igitur summam. 336.
productorum per productum duorum secundorum numerorum nempe. 48. diuise-
rimus, proueniens pariter erit. 7.

In cuius gratiâ primus numerus significetur lineâ. q. b. multiplicandus & diuiden-
dus numeris designatis per. κ. m. et. y. m. productorum summa sit. κ. z. prouenien-
tium autem. a. e: et. a. o. ex. κ. m. et. o. e. ex. y. m: tum producto. κ. m. in. m. y. sit. f.
m. Dico quòd si. κ. z. per. f. m. diuiserimus proueni et. a. e. Quod cum sic fuerit, erit
quoque verum quòd diuisa. κ. z. per. a. e. proueniet. f. m. numerus scilicet æqualis
numero. f. m. ex. 13. theoremate huius. Itaque quotiescunque probauero quòd di-
uisa. κ. z. per. a. e. proueniat numerus æqualis ipsi. f. m. propositum verum esse con
sequetur. ex. 13. theoremate. Quòd si proueniens ex diuisione. κ. z. per. a. e. æqua
le fuerit. f. m. patet ex. 7. quinti quòd eadé erit proportio numeri. κ. m. y. ad ipsum
proueniens, quæ ad numerum. f. m. Cogitemus itaq;. κ. u. æqualem. a. e. super quam
mente concipiamus rectangulum. u. p. æqualem. κ. z. ex quo eadem erit proportio.
κ. p. ad. κ. y. quæ. g. κ. ad. κ. u. ex. 15. sexti, aut. 20. septimi, numerus autem. κ. p. erit
proueniens, quod probandum est æquale esse. f. m.

Probabitur autem sic, ex. 9. quinti, nempe demonstrato quòd numerus. κ. p. ean
dem proportionem habeat ad numerum. κ. y. quam habet numerus. f. m. ad eundem
κ. y. Sed probatum est sic se habere. κ. g. ad. κ. u. sicut. κ. p. ad. κ. y. sufficiet igitur pro-
bare sic se habere. κ. g. ad. κ. u. sicut. f. m. ad. κ. y. Sed. κ. g. dicitur æqualis esse. q. b: et. κ.
u: a. e. satis erit igitur probare ita se habere. q. b. ad. a. e. sicut. f. m. ad. κ. y. Scimus au-
tem quòd eadem est proportio. q. b. ad. a. o. quæ. m. κ. ad vnitatem, quæ sit. x. & quod
proportio. o. e. ad. q. b. eadem est, quæ. x. ad. m. y. ex definitione diuisionis. Quare
ex æqualitate proportionum eadem erit proportio. κ. m. ad. m. y. quæ. e. o. ad. o. a. &

componen-

componendo sic se habebit. κ.y.ad.m.y.sicut.e.a.ad.o.a.& permutando.κ.y. ad. e.
a.sicut.m.y.ad.o.a.& ex.19.quinti ita.κ.m.ad.e.o.sicut.κ.y.ad.e.a.& permutando.
κ.m.ad.κ.y.sicut.e.o.ad.e.a. Nunc producatur.f.t.donec.t.i.æqualis sit. κ.y.produ-
ctaq;.m.t.donec.t.s.æqualis sit vnitati.x.terminéturq; rectangulum. s. i.ex quo da-
bitur proportio numeri. f. m. ad numerum.s.i.composita ex.m.t.ad.t.s.et.f.t.ad.t.i.
ex.24.sexti,aut quinta octaui,sed ita etiam proportio.q.b.ad. a. e. componitur ex
eisdem proportionibus,nempe ex.q.b.ad.o.e.æquali.m.t.ad.t. s.& ex proportione.
o.e.ad.a.e.æquali. f.t.ad.t.i.itaque proportio numeri.f.m.ad.s. i. hoc est ad numerū
ipsius.κ.y.equalis est proportioni numeri.q.b.ad.a.e.nepe.κ.g.ad.κ.u. hoc est.κ.p.ad
κ.y.ex quo sequitur.κ.p.constare numero equali.f.m.proueniens igitur ex diuisione
numeri.κ.z.per.f.m.æquale est numero ipsius.a.e.
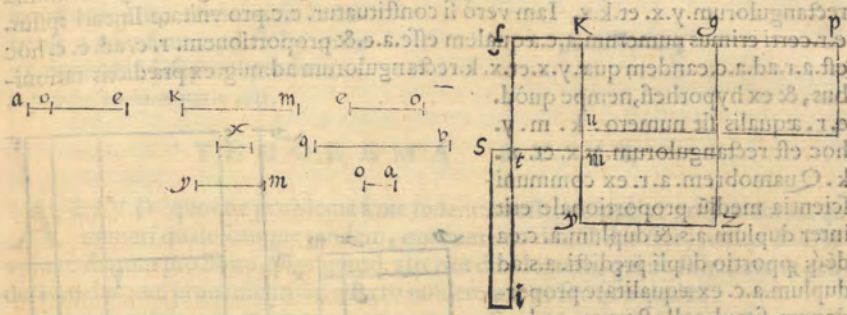
## THEOREMA LXX.

HAEC porrò conclusio alia etiam via demonstrari potest.
Significetur numerus diuidendus atque multiplicandus linea. b. a. Deinde
diuidentes & multiplicátes sint. κ.m.et.m.y.prouenientia ex diuisione sint.a.o.et.o.
e.atque.a.o.ex.m.y:o.e.verò ex.κ.m.proueniat, quorum summa sit.a.e:productum
autem.b.a.in.κ.m.sit.b.p.et.p.s.productum.b.a.in.m.y.ad hæc rectangulum. κ.y.sit
productum.κ.m.in. m. y: quo to-
tum productum.a.s.diuidatur, pro
ueniensq; sit.a.c.cui,a. c: productū.
a.s.eádé proportioné seruabit,quā
k. y. rectangulum ad vnitatem ex
definitione diuisionis , hoc autem
proueniens.a.c.cóstare numero æ-
quali assero summæ. a. e .Primum
enim ex dicta definitione diuisio-
nis habemus eandem esse propor-
tionem.b.a.ad. a. o, quæ. m. y. ad
vnitatem, & quod sic se habet.b.a.
ad.o.e.sicut. κ.m.ad eandem vnita
tem. Itaque vnitas hæc linearis si-
gnificetur per.m.x.in singulis late-
ribus. κ.m.et. m.y.producentibus rectangulum. k.y: superficialis autem vnitas sit.
g.m.
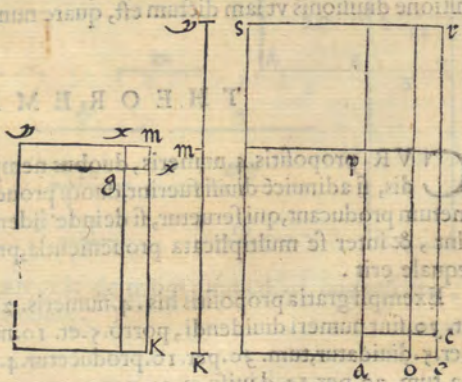
g.m.cogiteturq́; rectangulum.y.x.& rectangulum.k. x. Itaque dabitur eadem pro
portio.k.m.ad.m.x.nempe.k.x.rectanguli ad.m.g.quæ eſt.b.a.ad.o.e.et.y.x.ad. m.
g.quæ.b.a.ad.a.o.ſed ex prima ſexti aut.18.vel.19.ſeptimi , ſic ſe habet rectangu-
lum. k.y.ad.x.y.ſicut.k.m,ad.m. x quare ſicut.b.a.ad.o.e.ex. 11.quinti,& eiuſdem
rectanguli. k. y.ad rectangulum. k.x.ſicut.y.m ad.x.m.nempe.b.a. ad. a. o. Quare
ex communi ſcientia , ſic ſe habebit duplum rectanguli. k.y.ad ſummam. y.x. cum.
k.x.rectangulorum , ſicut duplum. b.a.ad ſummam.a.o.e. et proportio ſummæ re-
ctangulorum. y.x.et.k.x.duplo.g.m.ſicut duplum.b.a.ad.a o.e . Igitur ſumma duo-
rum rectangulorum.y.x. et.x.k.media proportionalis erit inter duplum rectanguli.
k.y.& duplum vnitatis ſuperficialis. g.m. Nunc terminetur rectangulum.a.r.ex quo
dabitur eadem proportio dupli.a. s.ad.a.r.ſicut dupli.b. a.ad. a. e. ex propoſitioni-
bus notatis , ſexti aut ſeptimi . Quare etiam ſicut dupli rectanguli. k. y. ad ſummā
rectangulorum.y.x. et.k.x, Iam verò ſi conſtituatur. e.c.pro vnitate lineari ipſius.
e.r.certi erimus numerum.a.c.æqualem eſſe.a.e.& proportionem. r. e. ad. e. c. hoc
eſt.a.r.ad.a.c.eandem quæ.y.x.et.x. k.rectangulorum ad.m.g.ex prædictis rationi-
bus, & ex hypotheſi,nempe quòd.
e. r. æqualis ſit numero. k. m. y.
hoc eſt rectangulorum. y.x. et. x.
k. Quamobrem. a. r. ex communi
ſcientia mediū proportionale erit
inter duplum.a.s.& duplum. a. c.ea
déq́; pportio dupli prędicti. a.s.ad
duplum.a.c. ex æqualitate propor-
tionum ſimul collectarum , eadem
erit quæ proportio dupli rectangu-
li.k.y.ad duplum.m. g. hoc eſt. a. s.
ſimplicis ad ſimplicem.a.c. quæ ſim
plicis rectanguli.k.y.ad ſimplicem
vnitatem.g.m.ſic enim ſe habet ſim
plex ad ſimplex, ſicut duplum ad
duplum. Sed pariter ita ſe habet.a.s.ad.a.c. cogitato. a.c. tamquam proueniente
ex diuiſione . a.s.per rectangulum.k.y.vt conſtitutum eſt, ſicut.k.y.ad.m.g.ex defi-
nitione diuiſionis vt iam dictum eſt, quare numerus.a.c.æqualis erit numero. a.o.e.
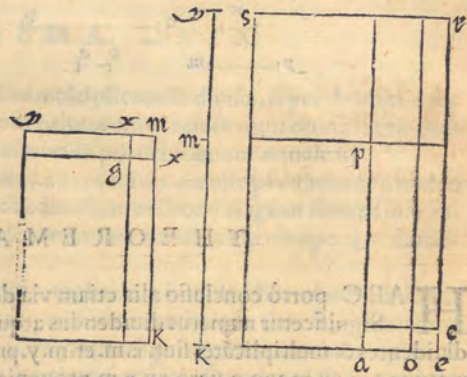
## THEOREMA LXXI.

CVR propoſitis.4.numeris, duobus nempe diuidentibus ac duobus diuiden-
dis, ſi adinuicē diuiſi fuerint,duoq́; proueniētia inuicē multiplicata quéuis nu
merum producant,qui ſeruetur, ſi deinde ijdem numeri verſa vice mutuo diuiſi fue
rint , & inter ſe multiplicata prouenientia,productū hóc , primo ſeruato numero
æquale erit .

Exempli gratia propoſitis his .4.numeris. 20.30.5. 10. duo autem. 20. ſcilicet
et. 30.ſint numeri diuidendi , porrò.5.et. 10.numeri diuidentes,népe vt primo.20
per.5.diuidatur,tum. 30.per.10.producetur.4.et.3.qui ſimul multiplicati proferét.
12.tum. 20.per.10.d iuiſo et.30.per.5. prouenientia erunt.2.6.quæ inter ſe multi-
plicata producent etiam.12.

Cuius

Cuius rationem si quæris, significentur.4.numeri lineis. a.e. o. u. diuidaturq;. a. per.o. & oriat.s.& per.u.oriat.y. et. e. diuiso per. o. oriatur. z. & per. u. proueniat.f.tum.n.sit productum.z. in.y. et.m. productum. s. in. f. Dico n. futurum æquale . m. Sit deinde. x.vnitas, quare ex definitione diui- sionis eadem erit proportio.s.ad.a. et.z.ad.e.quæ.x.ad.o. Sed ita se ha- bet.a.ad.y.et.e.ad.f.sicut.u.ad.x. ex quo sic se habebit.s. ad.a.sicut. z. ad e. et.a.ad.y,sicut.e.ad. f. Itaque ex æqualitate proportionum sic se ha- bebit s.ad.y.sicut.z.ad.f. Igitur ex 15.sexti aut.20.septimi productum, n.producto.m.æquale erit.

## THEOREMA LXXII.

ALIVD quoque problema à me inuentum est, nempe vt proponantur.4. numeri qualescunque tandem, quorum duo diuisibiles sint, tertius diuisor vnius è duobus pro libito,quæramusq; alterius diuidentem, qui sic se habeat vt pro ductum duorum prouenientium quarto numero proposito sit æquale.

Exempli gratia, proponuntur.4.numeri.20.48. 5. 12. porrò. 20. et. 48. numeri sint diuisibiles et. 5. diuidés vnius,ut potè.20.Quærédus nunc erit diuidens alterius nempe.48. eiusmodi vt productum prouenientium æquale sit.12.Diuidam itaque. 20.per.5.prouenietq;. 4.quem per.48. multiplicabo, nempe per alterum diuisibi- lem,sicq; proueniet.192.quod productum per quartum numerum nempe.12. diui- sum dabit.16. qui erit diuidens quæsitus,quo diuiso. 48.proueniet.3. secundum sci licet proueniens, quo per alterum hoc est.4.multiplicato producetur quartus nu- merus. 12.

Quod vt sciamus, primus nume- rus diuisibilis significetur rectangulo. a. i.secundus rectangulo.o.u.primus diuidens latere.a.e. quartum nume- rum rectangulo.i.o. primum proue- niens latere.e.i.secundus diuidens la tere.e.u.(hic autem est quem quæri- mus ) tum alterum proueniens signi ficetur latere.e.o.Iam eadé erit pro- portio. e.i.ad.e.u.quæ.o.i. ad. o. u. Sed cum cognitæ sint tres quantita- tes.e.i:i.o:et.o.u.quarta quoque.e.u. ex regula de tribus immediatè cognoscetur, cætera in subscripta figura facillimè patebunt.

THEO.

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## THEOREMA LXXIII.

HOC etiam problema à me inuentum eft, nempe fi duæ radices quadratæ in fummam collectæ fuerint, & ex dimidio eiufmodi fummæ detracta fuerit mi nor radix, refiduiq́; quadratum duplicatum eiq́; fummæ coniungatur du plum producti ipfius refidui in dimidium fummæ radicum, atque huic fummæ duplum producti eiufdem refidui in radicem minorem coniunctum fuerit; vltima hæc fumma differentia erit duorum quadratorum propofitorum.

Exempli gratia duæ radices quadraræ fint. 5. et. 11. harum fumma erit. 16. & dimi dium. 8. differentia minoris ab ipfo dimidio erit. 3: duplum quadrati huius differen tiæ erit. 18: duplum producti huius differentię in dimidium fummę radicum erit. 48. item & huius differentiæ duplum in minorem radicem erit. 30. quarum omnium fummæ erit. 96. tantaque erit differentia fuorum quadratorum, quorum vnum erit. 25. alterum verò. 121.

Pro cuius rei fcientia, duæ quadratæ radices fint. h. o. et. o. d. directæ inter fe coniunctæ, quæ fumma per medium in puncto. e. diuidatur, tum cogitetur. e. b. æqualis o. e. perpendicularis. h. d. ducanturq́; lineæ. b. h: b. o. et. b. d. Iam ex. 4. primi. b. h. æqua lis erit. b. d. & quadratum. b. h. æquale quadrato. h. o. & quadratum. o. b. fimul cum du plo producti. o. e. in. o. h. ex. 12. fecundi Eucli. Sed ex. 13. eiufdé quadratum. b. d. minus eft quadrato. o. d. cum quadrato. o. b. ex duplo producti. o. e. in. o. d, at duplum eiufmodi producti æquale eft duplo quadrati. o. e. & duplo producti. o. e. in. e. d. ex tertia eiufdem, itaque duo quadrata fcilicet. o. b. et. o. d. maiora erunt duobus quadratis, nempe. o. b. et. o. h. collectis cum du plo producti. o. e. in. o. h. ex duplo quadrati o. e. vnà cũ duplo producti. o. e. in. e. d. Qua re differétia fummæ duorum quadratorum o. b. et. o. d. à fumma duorum. o. b. et. o. h. du plum erit quadrati. o. e. cum duplo producti. o. e. in. e. d. & duplo producti. o. e. in. o. h.

Quòd fi ex fingulis duabus fummis quadratorum demptum fuerit quadratum. o. b. eadem producta & quadrata ipfius. o. e. remanebunt, tanquam differentia duorum quadratorum. o. u. et. h. c.

## THEOREMA LXXIIII.

CVR fumma duorum extremorũ quatuor terminorum proportionaliũ arithmeticè, æqualis eft fummæ duorum mediorum, vbi nota hac in re neceffarium non effe proportionalitatem continuam exiftere.

Exempli gratia, fi darentur hi quatuor termini. 20. 17. 9. 6. quorum proportio ea dem effet primi ad fecundum quæ tertij ad quartum, fumma primi cum quarto effet 26. tantaq́; fecundi cum tertio.

Cuius fpeculationis caufa, primus maiorq́; numerus fignificetur linea. e. o. fecundus. s. q. tertius. u. c. quartus. g. t. differentia porrò inter. e. o. et. s. q. fit. i. o. quæ æqualis erit differentiæ. r. c. qua quartus à tertio fuperatur ex hypothefi. Itaque affero fummam. e. o. cum. g. t. nempe. a. o. æqualem effe fummę. q. s. et. u. c. fitq́;. q. p. Nam in. a. o.

Secundus

Secundus tertiusque terminus reperiuntur, est
enim secundus. e. i. tertius. i. o. et. e. a. quando-
quidem ex præsuppofito. e. i. æqualis est. s. q. et
i. o. æqualis. r. c. et. a. e. cum fit æqualis. g. t. cui
pariter æqualis est. r. u. ex quo. a. e. æqualis
est. u. r. Itaque illud sequitur. a. o. ipsi. q. p.
æqualem esse.

## THEOREMA LXXV.

CVR summa duorum terminorum extremorum imparium arithmeticæ pro-
portionalitatis semper duplo medij termini æqualis est.

Exempli gratia, sunt hi tres termini proportionalitatis arithmeticæ. 20. 15. 10,
summa duorum extremorum erit. 30. quæ duplum est medij termini. 15.

Quod vt speculemur, tres termini, tribus lineis. b. d: n. u. et. q. p. significétur. Di-
co nunc quòd summa. b. d. cum. q. p. nempe.
h. d. semper duplo. n. u. scilicet. g. u. æqualis
erit. Tum differentia. b. d. ad. n. u. sit. c. d. quæ
æqualis erit. e. u. differentiæ inter n. u. et. q. p.
patet enim in linea. h. d: b. c. æqualem esse. n.
u. sed. n. u. ex. n. e. componitur æquali. q. p. et
ex. e. u. æquali. c. d. cum itaq; in. h. d. partem.
h. b. reperiamus æqualem. n. e. gratia. q. p. &
partem. c. d. æquale m. e. u. manifestum erit
h. d. æqualem esse. g. u.
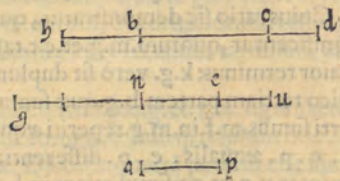
## BINA PROBLEMATA EX DVOBVS PRAEDICTIS
### THEOREMATIBVS DEPENDENTIA.

EX duobus prædictis theorematibus duo problemata oriuntur, quorũ primum
est. Datis tribus quantitatibus cognitis, si quis quartam inuenire voluerit,
quæ eiufmodi sit respectu tertiæ, qualis est secunda respectu primæ, secunda cum
tertia in summam colligenda erit, ex qua detracta prima, supererit quarta.

Exempli gratia, cognitis tribus quantitatibus. 20. 17. 9. si quartam inuenire vo
luerimus eiufmodi proportionem cum tertia arithmetice seruantem, quam secunda
cum prima, secundam cum tertia in summam colligemus, dabiturq; summa. 26. ex
qua detracta prima quantitate, quarta relinquetur nempe. 6. quod ex. 74. theore-
mate dependet.

Idipsum tamen proueniret si quis ex tertio termino differentiam primi atque se-
cundi detraheret; hæc tamen via non tam vniuerfalis estqu àm illa. N si quartus ter
minus incognitus tertio maior esse deberet, dictam differentiam cum tertio termi-
mino in summam colligere oporteret.

Alterum problema est, quòd inuentis duobus terminis, si tertius requiratur, se-
cundus duplicandus erit, ex qua summa detracto primo, statim tertius proferetur,
quod problema ex præcedenti theoremate dependet.

G        Progre-

50      IO. BAPT. BENED.

Progredi nihilominus etiam hac in re poſſemus per differentiam primi & ſecun-
di termini, eam detrahendo aut in ſummam cum ſecunda colligendo, attamen prior
ratio magis latè patet, ideſt vniuerſalior eſt.

## THEOREMA LXXVI.

CVR ſi quis cupiat ſecundum terminum inuenire, quatuor terminorum arith-
meticè proportionalis continuæ, quorum nobis duo extrema proponantur.
Rectè primum duplicabit coniungetq́; vltimo termino, nempe quarto, ex qua ſum-
ma tertiam partem deſumet, quæ erit ſecundus terminus quæſitus.

Exempli gratia, ſi horum quatuor terminorum. 12. 9. 6. 3. duo nobis extrema
proponantur. nempe. 12. et. 3. quorum ſecundus inueniendus ſit, ſumpto quolibet
pro primo, ſit autem. 3. primus numerus, quartus verò. 12. quare duplicato. 3. vt͜po
tè primo, & coniuncto. 12. quarto, ſumma erit. 18. cuius eſt tertia pars. 6. ſecundus
numerus ſcilicet ſumpto principio à minimo. Idipſum euenit ſumpto principio à
maximo. Nam ſi datur ſecundus à minimo aut à maximo, illico tertius datur diffe-
rentia inter hunc & primum, ſecundo coniuncta, aut ex eodem detracta.

Cuius ratio ſic demonſtratur, quatuor termini quatuor lineis. m. g: q. p: u. n: c. t.
ſignificentur, quorum. m. g. et. c. t. tantummodo cognoſcantur. ſitq́;. m. g. primus ac
maior terminus: k. g. verò ſit duplum primi. m. g: cui coniungatur. b. k. æqualis. c. t.
Dico tertiam partem. b. g. quæ ſumma totalis eſt, æqualem eſſe. q. p. In primis enim
certi ſumus. m. f. in. m. g. reperiri æqualem. q. p. ſupereſtq́;. f. g. differentia inter. m. g.
et. q. p. æqualis. e. p. differentiæ inter. q. p. et. u. n. & æqualis. o. n. differen-
tiæ inter. u. n. et. c. t: ſimul etiam in. k. m. habemus. d. m. æqualem. m. f. quare etiam. q.
p. et. k. d. æqualem. f. g. nempe. e. p. aut o. n: Hactenus in. k. g. reperimus duplum. q.
p. ſimul cum. f. g. et. k. d. æqualibus. e. p. et. o. n. & quia. b. k. æqualis. c. t. fuit coniuncta.
conſiderandum eſt an hæ tres quantitates. f. g: k. d. et. b. k. ſimul æquales ſint. q. p.
quod tamen per ſe manifeſtum eſt. nam. q. p. ſuperat. u. n. per. e. p. et. u. n. ex-
cedit. c. t. per. o. n. æqualem. e. p. quare. q. p. per duplum differentię. f. g. ſuperat. c. t. ita
que. f. g: k. d. et. k. b. ipſi. q. p. ſunt ę-
quales, ex quo ſequitur. q. p. tertiã
partem eſſe. b. g. Hæc quæ hacte-
nus dicta fuerunt, in genere maio-
ris inæqualitatis probata fuerunt.
At in genere minoris, ſumpto or-
dinis principio à minimo termino
rum, duplicetur. c. t. ſitq́; duplum
hoc. k. t. cui. k. b. æqualis. m. g. con-
iungatur, quæ ſumma ſit. b. t. Di-
co. u. n. tertiam eſſe partem ipſius.
Nam in primis in. b. t. datur termi
nus. b. k. æqualis vltimo. m. g. in
quo ſemel reperitur. u. n. vnà cum
duabus differentijs, nempe. i. g. in
ipſa autem. b. t: u. n. ſignificetur pri

mo loco per. r. k. ex quo ſupererit. b. r. duabus differentijs prædictis æqualis, ſed ex
præſuppoſito. u. n. componitur ex. o. u. æquali. c. t. et. o. n. æquali vni differentiæ. Itaq;
                                                                     cum
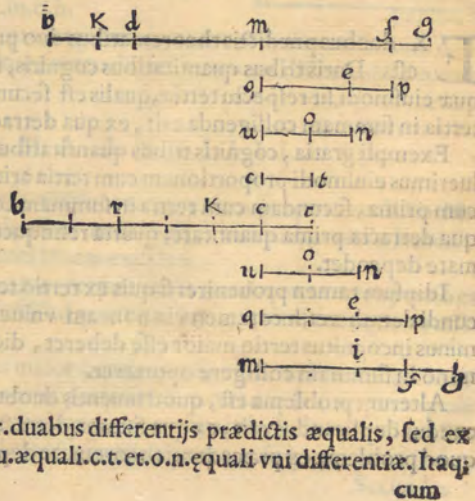
cum in.b.t.præter.r.x.bis detur.c.t. nempe.x.t.et. b. r. duabus differentijs æquipol-
lens, illud efficitur . u.n.pariter ipsius.b.t. esse tertiam partem , quod erat propositū.

## THEOREMA LXXVII.

CVR si quis velit secundum quinque continuorum proportionalium termi-
num inuenire, solis extremis cognitis . Rectè vltimū triplo primi coniunget,
ex qua summa quartam partem detrahet, quæ erit secundus terminus quæsitus .
Quod ipsum faciet qui inuenire vult secundum terminum senarij septenarij, octo-
narij aut alterius cuiuscunque, crescente tamen multiplicatione primi , vltimoq́;
cōniuncto.

Exempli gratia , dantur duo extremi termini , horum quinque numerorum . 18.
16.14.12. 10. nempe.18.et. 10.si.18.primus erit, hoc est , si à genere maioris inæ-
qualitatis progrediemur, triplicabimus terminum.18.dabunturq́;. 54.cui numero
coniuncto quinto termino. 10. dabitur numerus.64.cuius quarta pars erit.16.vtpo-
tè secundus terminus gratia, aut secundi sex terminorum, quadruplicandus esset pri-
mus.18.deinde adiuncto vltimo , quinta pars summæ esset secundus terminus, atq́;
ita deinceps.

Cuius speculationis gratia, dicti termini lineis.z.h:f.s:u.p:e.g.et. r. x. significētur.
In primis ex genere maioris inæqualitatis, triplicabimus. z.h.sitq́; triplum hoc . x.
h.cui cōiungatur.b.x.equalis vltimo termino.r.x. Dico.f.s.quartā partem esse sum-
mę.b.h. Nam in.x. h.secundus terminus.f.s.ter cum tribus differentijs æqualibus.n.h.
reperitur . Probandum nunc est tres has differentias. n.h:a.c.et.d. x. simul cum . b.
x.equales esse.f.s.
quod in dubiū re
uocari nō potest,
cum. f. s. superet.
r.x.per.o.s:t.p.et.
i. g . At in genere
minoris inæquali
tatis,triplum. r.x.
sit. x. a. et. a. b.sit
æqualis.z.h. & cū
z.h. tribus differē
tijs.n.h:o.s:t.p.su-
peret.e.g. quæ in.
a. b. sint. b. x: x. d:
d.c. ex quo. a.c.
æqualis erit. e. g.
et. a.x.cum.b. c.tripla.e. g. Itaque tota summa.b.x.quadrupla erit.e.g.

## THEOREMA LXXVIII.

QVantitates quæ fuerint inuicem in proportionalitate arithmetica proportio-
nales , permutan do quoque proportionales erunt.

G 2 Sint

Sint exempli gratia.4.quantitates.a.b:c.d:e.f.et.g.h: inuicem proportionáles in proportionalitate arithmetica. Hoc eſt vt quæ proportio (licet impropriè dicta ) eſt ipſius.a.b.ad.c.d.eadē ſit ipſius.e.f.ad.g.h. Tunc permutando dico eandem proportionem fore ipſius.a.b.ad.e.f.quæ ipſius. c.d.ad.g.h.

Nam, ex hypotheſi, differentia qua.a.b. ſuperat. c.d. (quæ ſit.m.b. ) æqualis eſt differentiæ qua.e.f.ſuperat.g.h.(quæ ſit.i.f.) vnde.a.m.reſiduum ex. a.b.æquale erit c.d.& reſiduum.e.i.æquale.g.h. Sit igitur exempli gratia. c. d. maior.g.h.per. c.n. vnde.n.d.æqualis erit.g.h.quare.à.m. maior erit.e.i.per. a. к.æqualem.c. n. ex communi ſcientia. Vñde.к.m.æqualis erit.n.d.hoc eſt ipſi.g.h.hoc eſt ipſi.e.i. Quare ex communi conceptu.b. к.æqualis erit ipſi.f.e.ſed.n.d.æqualis eſt.g. h. vt dictum eſt. Cum ergo.b.к.æqualis ſit.e.f.et.d.n.ipſiig.h.et.a.b.maior ſit ipſa. к.b.per.a.к.æqualem ipſi.c.n.per quam.c.n.d.e.maior eſt ipſa.d.n.ſequitur verum eſſe propoſitū hoc eſt, quod eadem proportio ſit ipſius.a.b.ad. e.f.quæ.c.d.ad.g.h.arithmetice ſcilicet.
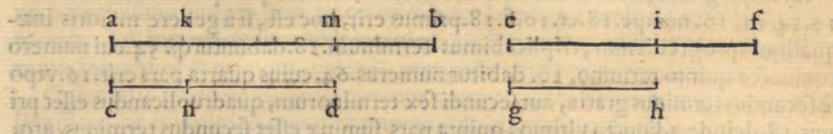


### T H E O R E M A   L X X I X.

CVR prouenientia duorum numerorum diuidentium eiuſdem numeri diuiſibilis, geometricè eandē inter ſe proportionē ſeruant, quā ipſimet diuidētes. Exempli gratia ſi per ſenarium & octonarium numerus vigintiquatuor diuidatur, prouenientia erunt.4.et. 3.eadem proportione, qua diuidentes.

Cuius eſt ratio numerus diuiſibilis ſighificetur rectangulis.u.x. et.n.e.diuidentes autem ſint.u.o.et.e.o.quare ex ijs, quæ. 10. theoremate dicta fuerunt.u.x.per.u.o.diuiſo dabit. x.o.& diuiſo.n.e.per.e.o.dabit.o. n. Dicimus itaque eandē eſſe proportionē o. x.ad.o.n.quæ.e.o.ad.o.u.quod patet ſub ſcriptam figuram conſiderantibus, in qua, ex.15.ſexti aut.20.ſeptimi, eadem proportio cernitur. o. x. ad.o.n.quæ. o. e. ad.o.u.
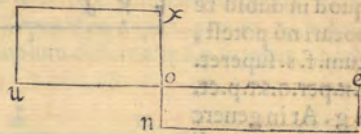


### T H E O R E M A   L X X X.

CVR quauis quantitate, tribus aut quatuor aut etiam pro libito pluribus diuidentibus numeris diuiſa, prouenientia eandem prorſus inter ſe proportionem ſeruabunt, quam ipſi diuidentes habere compe riuntur.

Exempli gratia, proponitur numerus.60.quinque numeris diuiden dis, vtpote.30.20.15.12.10. prouenientia erunt.2. 3. 4. 5. 6. eadem pro-

proportione diuidentium , quamuis ex aduerſo.
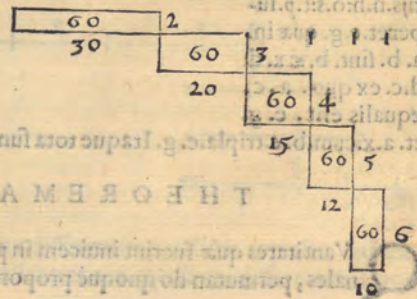
Cuius ratio ex. 15. ſexti aut. 20. ſeptimi dependet. prout in ſubſcripto ordine fa-
cillimè deprehendi poteſt.

### THEOREMA LXXXI.

C VR quantitate in tres continuas partes proportionales ſecta , & per ſingulas
ipſarum diuiſa, ſumma trium prouenientium quadrato medij prouenientis
æqualis eſt .

Exempli gratia , proponitur. 14.diuidendus in tres continuas partes proportio-
nales , nempe.8.4.2.ipſeq́; numerus.14.per ſingulas diuiditur, ex quo tria proue-
nientia oriuntur, nempe ex prima parte. 8.proueniés erit. 1.cum tribus quartis par
tibus ex ſecunda.4. datur proueniens. 3.cum dimidio vnius , & ex tertia. 2. proue-
nient. 7. integri, qui in ſummam collecti dant. 12. integros & vnam quartam par-
tem tantumdem , videlicet quantum quadratum prouenientis medij, nempe . 3 .
cum dimidio .

Cuius ſpeculationis gratia , totalis numerus ſignificetur linea.n.c.qui in tres par-
tes diuidatur.n.a: a.e.et. e.c.quæ ſint continuæ proportionales , quarum ſingulis ,
numerum.n.c. diuiſum eſſe cogitemus , proueniens autem ex diuiſione.n.c. per. n.
a.ſit.i. d.quod verò prouenit ex diuiſione.n.c.per.a.e.ſit.d.u.proueniens quoque ex
diuiſione.n.c.per.e.c.ſit.u.o.quorum ſumma ſit.i.o.quæ aſſeritur eſſe numeri æqua-
lis numero quadrati.d. u.Quod hac ratione probabo,producatur linea.i. o. doneè.
o.p.æqualis ſit.o.u. erigaturq́;.o.m.æqualis.d.i. perpendiculariter.o.p.in puncto. o.
quæ producatur donec.o.q.vnitati ſit æqualis , terminenturq́; duo rectangula. m. p.
et.q.i.ex quo habebimus rectangulum , aut productum.m.p.æquale quadrato. d. u.
ex. 16 ſexti aut. 20. ſeptimi, quandoquidem tria prouenientia.o.u:u. d. et. d. i. ex
præcedenti theoremate ſunt inter ſe continua proportionalia,proportionalitate qua
partes. n.c.Iam verò ſi probauero.q.i.productum , producto.m.p.æquale eſſe, pro-
poſitum quoque probatum erit.Numerus enim producti.q. i. æqualis eſt numero .
ſummæ.i.o.Habemus autem ex definitione diuiſionis ita ſe habere.n.c.ad.i.d. ſicut.
n.a.ad.o.q. Itaque permutando ſic ſe habebit.m.c.ad.n. a.ſicut.d.i.hoc eſt.m.o.ad.
o.q.ſed ſicut ſe habet.n.c.ad.n. a. ita pariter ſe habet.i.o.ad. o.u.hoc eſt ad.o.p.Ita-
que.i.o.ad.o.p.ſic ſe habebit ſicut.m.o.ad.o. q.ex quo ex. 15. ſexti aut. 20. ſeptimi.
q. i.æqualis erit.m.p.& conſequenter quadrato.d.u. Vt autem lector minori labo-
re cognoſcere queat.i.o.ad.o.u.ſic ſe habere,vt.n.c.ad.n.a. ſciendum eſt quòd, ſic
ſe habet. i. d.ad.d.u.ut.c.e.ad.e.a.ex quo componendo ſic ſe habebit.i.u.ad.d.u. ſi-
cut.c.a. ad.a.e.& permutando ita.i.u.
ad.c.a.vt. d.u.ad.e.a.ſed cum ex præ-
cedeti theoremate ſic ſe habeat. d. u.
ad.u.o.ſicut.e.a.ad.a. n. permutando
ſic ſe habebit.d.u.ad.a.e.ſicut.u.o. ad
a.n.ex quo ex. 11.quinti ſic ſe habe-
bit.i.u.ad.c.a.prout.d.u.ad.a. n. per-
mutandoque. i. u. ad.u.o. vt. c. a.ad.a. n. & componendo , ita.i. o.ad.u.o. ſicut.c. n.
ad.a. n.
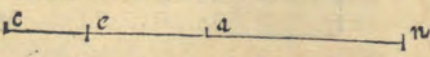
## THEOREMA LXXXII.

CVR quantitate aliqua in quatuor partes côtinuas proportionales secta per-
que singulas diuisa, summa quatuor prouenientium æqualis sit producto se-
cundi in tertium.

Exempli gratia, si triginta in quatuor partes proportionales secetur, hoc est.
16.8.4.2.perq; harum singulas idem numerus. 30. diuidatur, primum proueniens
erit. 1.cum septem octauis partibus. Secundum.3. cum tribus quartis, tertium. 7.
cum dimidio, quartum. 15. integri, quorum summa erit.28.cum octaua parte, tan
tumq; erit productum secundi prouenientis in tertium.

Quod vt sciamus, quantitas.n.c. in partes continuas proportionales quatuor se-
cetur. n. a : a. t: t. e. et. e.c.rursusq; per singulas partes illa ipsa diuisa, prouenientia
sint. i.d:d.x:x.u:u.o.quorũ summa sit.i.o.hanc summã dicimus æqualem esse nume-
ro producti.d.x.in.x.u.

Quod hac ratione probo, cogito productam esse lineam.i.o.quousq;.o.p.æqua
lis sit.o.u. erectamq;.m.o.æqualem.i.d.perpendiculariter.o.p.& productam donec.
o.q.vnitati sit æqualis. Iam terminatis rectangulis.m.p.et.i.q. patebit ex. 15. sexti
aut.20.septimi, productum.m.p.producto.d.x.in.x.u. æquale esse. Ita quòd si pro-
bauero productum.i.q.producto.m.p.æquale esse, facile patebit propositum.Cuius
gratia,sequuti præcedentis theorematis ordinem,primum ex definitiône diuisionis,
eadem proportio erit.n.c.ad.i.d.quæ.n.a.ad.o.q.ex quo permutando.n.c.ad.n.a.sic
se habebit vt. i.d. hoc est.m.o.ad.o.q.& si progrediamur eodem ordine, quo præ-
cedenti theoremate,sumpto principio ab.i.d.et.e.c.versus.d.x.et.e. t. gradatimque
permutando ac coniungendo,inue-
niemus eandem proportionem esse
c. n.ad.n.a.quæ.i.o.ad.o. u. nempe.
o.p.ex quo ex.11 quinti,ita se habe
bit.i.o.ad.o.p.vt. m.o.ad.o.q.quare
ex.15.sexti aut. 20. septimi produ-
ctũ.i.q.erit producto.m. p. æquale,
ex quo etiam æquale erit producto.
d.x.in. x. u. Idem ordo in qualibet
quantitate in quantasuis partes diuisa seruari poterit, cum huiusmodi sciêtia in vni
uersum pateat.
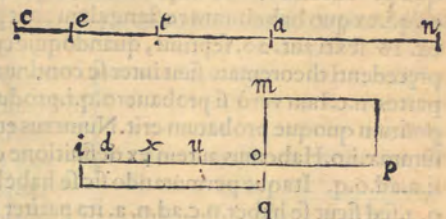
## THEOREMA LXXXIII.

CVR termini medij cubus,trium continuè proportionalium,semper producto
rectanguli compræhensi à maximo & medio in minimo termino æqualis sit.

Exempli gratia, datis his tribus terminis continuis proportionalibus. 9.6.4.si
sumpserimus productum maximi in medium nempe.54.quod per minimũ.4.multi-
plicemus, dabitur numerus.216. cubo medij.6.æqualis.

In cuius gratiam tres numeri côntinui proportionales tribus lineis. a.e.i. signifi-
cétur, cubus autem.e. significetur figura. d. n. productumq;.a.in. e.sit.b.n. ipsius au-
témet in.i.sit.p.o.ita quod.q.p.aut.b.o.cum sint eiusdê speciei, æqualis erit.a:et.o.n.

æqua-

æqualis.e:et.q.n. æqualis. i. Nunc co-
gitemus abſolui corpus. n.h. ita ut. b.
o.c.ſit vnica recta linea,ex quo ex.25.
vndecimi proportio. n.h.ad. n. k. ea-
dem eſt quæ. o. h.ad o.k.ſed ſic ſe ha-
bet. o. h. ad. o. k. vt. h. b. ad. b. k.
ex prima ſexti aut. 18. vel. 19.ſe-
ptimi itaque. n.h. ad. n. k. ex. 11.
quinti ſic ſe habebit. vt. h. b. ad. b. k.
ſed.n.h.ad.n.d.ex eiſdem ſic ſe habet
ut.h.u.ad.d.u. et.h.u.ad.u. d. ita ut. h.
b.ad.b.k. ex præſuppoſito. Itaque ex
11. prædicta. n.h.ad. n. k. eadem erit
proportio quæ. n. h. ad. n. d. Quare
ex. 9. quinti. n. k. æqualis erit. n. d.
Quod erat propoſitum.
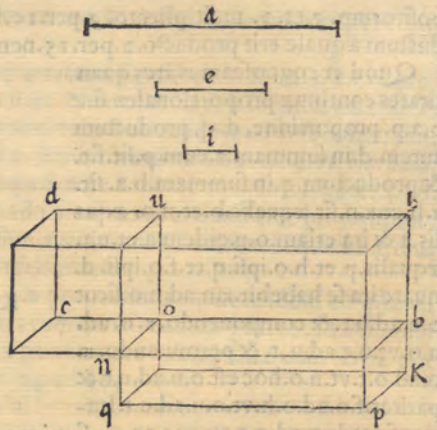
## THEOREMA LXXXIIII.

CVR quadrato vnius quantitatis radice proportionalis,per ſingulos tres termi
nos diuiſo, prouenientia, ſingulis dictis terminis ſint æqualia.

Exēpli gratia, datis tribus terminis continuis proportionalibus.9.6. 4. qua
dratum medij erit. 36. quod per. 9. diuiſum dabit. 4: per.6:6.per.4:9.

Cuius gratia,ſint tres termini cōtinui ρportionales.a.o:o.c.et.c.q. quadratū autē
medij ſit.e.c. Iam ſi applicetur rectangulū. a. d. æquale quadrato.e.c.ipſi.a.o. & re-
ctangulum.q.p. æquale eidem quadrato.e.c.ipſi.c.q. ſi quadratum.e.c.per.a. o.diui
ſerimus, proueniens erit.o.d.diuiſoq; per.c.q.proueniens erit. c.p.quod ſi per ſuam
radicem.o.c.diuidatur, proueniens erit. o.
e.quod ſine dubio æquale eſt. o.c. ſed dico.
o.d.æqualem eſſe.c.q. Nam ex.16.ſexti aut
20.ſeptimi eadem eſt proportio.a. o. ad. o.
c. quę.o.e.ad.o.d. nempe.o.c.ad.o.d.itaque
o.d.ex.9. quinti æqualis eſt.c.q. quandoqui
dem ex.11.ſic ſe habet.o.c.ad. o. d. ſicut.o.
c.ad.c.q. Applicatis ijſdem rationibus ipſi.
p.c.probabimus.c.p.æqualem eſſe.a.o. cum
o.c.media ſit proportionalis,tā inter. c.p. et
c.q.quam inter.a.o.et.c.q.itaque.c.p.æqua-
lis eſt. a.o.
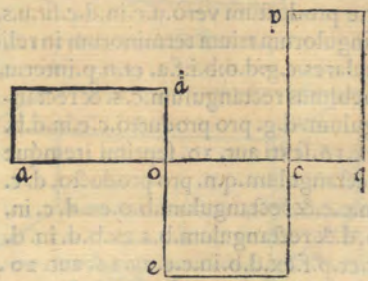
## THEOREMA LXXXV.

CVR propoſitis tribus quantitatibus continuis proportionalibus proportione
aliarum duarum nobis datarum, multiplicata maiori poſtremarum dua-
rum in ſummam mediæ cum minima trium primarum,productum æqua-
le ſit producto minoris duarum in ſummam maximæ cum media trium.

Exempli gratia proponuntur quantitates.9.6. 4. proportione numerorum pro-
poſi-

positorum. 3. et. 2. multiplicato. 3. per. 10. summã. 6. cum. 4. dantur. 30. quod productum æquale erit producto. 2. per. 15. nempe per summam. 9. et. 6.

Quod vt cognoscamus, tres quantitates continuæ proportionales sint b. a. p. proportione. d. q. productum autem. d. in summam. a. cum. p. sit. f. t. & productum. q. in summam. b. a. sit. κ. h. et. κ. n. sit æqualis. b. et. n. o. æqualis. a. & ita etiam. o. u. eidem. a. et. u. t. æqualis. p. et. h. o. ipsi. q. et. f. o. ipsi. d. quare ita se habebit. κ. n. ad. n. o. sicut o. u. ad. u. t. & componendo. κ. o. ad. n. o. vt. o. t. ad. u. t. & permutando. κ. o. ad. o. t. vt. n. o. hoc est. o. u. ad. u. t. & pariter. f. o. ad. o. h. vt. o. u. ad. u. t. Itaque sicut. k. o. ad. o. t. ex quo ex. 15. sexti aut. 20. septimi. κ. h. æqualis erit. f. t.
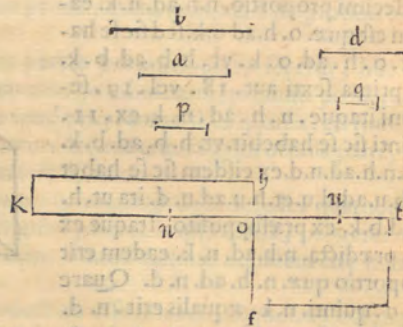
### THEOREMA LXXXVI.

CVR multiplicatis singulis tribus quantitatibus continuis proportionalibus in reliquas duas, sex producta æqualia sint producto dupli summæ ipsarum trium in mediam proportionalem.

Exempli gratia, proponuntur hi tres termini continui proportionales. 9. 6. 4. productum. 9. in. 6. erit. 54. at. 9. in. 4. erit. 36. et. 6. in. 9: 54. et. 6. in. 4: 24. et. 4. in. 9: 36. et. 4. in. 6: 24. quæ producta simul collecta efficiunt numerum. 228 sed tantũ est productum dupli summæ trium terminorum in secundum nempe. 38. in. 6.

Cuius intelligētiæ causa, tres termini cõtinui proportionales significentur linea. b. e. nempe. b. d: d. c: c. e. cuius duplum sit. u. e. et. b. f. æqualis sit. b. d. et. f. n: d. c. et. n. u: c. e productum verò. u. e. in. d. c. sit. u. s. cui dico æqualem esse summam productorum singulorum trium terminorum in reliquos duos. Quamobrem ducantur perpendiculares. c. g: d. o: b. i: f. a. et. n. p. inter. u. e. et. q. s. ex quo pro producto. c. e. in. c. d. habebimus rectangulum. c. s. & rectangulum. d. g. pro producto. c. e. in. d. b. ex. 16. sexti aut. 20. septimi itemque rectangulum. q. n. pro producto. d. c. in. c. e. & rectangulum. b. o. ex. d. c. in. b. d. & rectangulum. b. a. ex. b. d. in. d. c. et. p. f. ex. d. b. in. c. e. ex. 16. aut. 20. prędictas. Quare sex producta æquantur inter se, replentq; productum. u. s. ex quo verum est propositum.
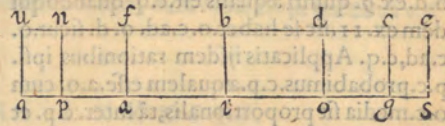
### THEOREMA LXXXVII.

QVA ratione cognosci possit verũ esse proportionem summæ quatuor quantitatum continuarum proportionalium ad summam secundæ & tertiæ, eandem esse, quæ summæ primæ & tertiæ ad secundam simplicem.

Exempli gratia, si inuenirentur hæ quatuor quantitates continuæ proportionales. 16. 8. 4. 2. earum summa erit. 30. summa verò secundæ & tertiæ. 12. tum summa

primæ

primæ cum tertia. 20. ex quo fic fe habet. 20. ad. 8. nempe ad fecundam , vt. 30. ad. 12.

Quod vt fciamus, quatuor prædictæ quantitates fignificentur linea. a. e. i. o. probabo ita fe habere. a. e. i. o. ad. e. i. vt. a. i. ad. e. Nam cum fic fe habeat. a. ad. e. ut. e. ad. i. & vt. i. ad. o: ex æqualitate proportionum vel permutando ita fe habebit. a. ad. i. vt. e. ad. o. & è converfo ita. o. ad. e. vt. i. ad. a. & cóponendo ita. o. e. ad. e. vt. i. a. ad. a. permutandoq́;. o. e. ad. i. a. vt. e. ad. a. nempe. i. ad. e. & componendo ita. o. i. e. a. ad. i. a. vt. i. e. ad. e. & permutando ita. o. i. e. a. ad. i. e. vt. i. a. ad. e. quod erat propofitum. Ex quo patet error antiquorum qui idipfum, accidere arbitrati funt in quantitatibus difcretæ proportionalitatis, quod tamen falfum eft.

Exempli gratia, fi proponantur. 12. 6. 4. 2. proportio. 12. ad. 6. eadem eft quæ. 4. ad. 2. Sed à proportione. 6. ad. 4. frangitur, cum non fit eadem quæ. 12. ad. 6. harum autem fumma erit. 24. & fumma fecundæ cum tertia. 10. fed primæ cum tertia erit 16. ex quo. 16. ad. 6. non fic fe habebit vt. 24. ad. 10.

At in fpeculatione quatuor quantitatum. a. e. i. o. fi proportio. e. ad. i. non effet eadem quæ. a. ad. e. minimè licuiffet dicere ita fe habere. i. ad. e. vt. e. ad. a.
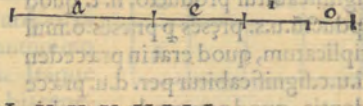
## THEOREMA LXXXVIII.

CVR ex tribus quantitatibus quibuflibet, productum duarum in tertiam, vna femper eademq́; fit quantitas.

Exempli gratia, proponuntur. 15. 8. 2. fi multiplicauerimus. 15. per. 8. tum produ ctum per. 2. tantum erit quantum fi quis multiplicaret. 8. per. 2. & hoc per. 15. et. 15. per. 2. rurfusq́; per. 8.

Quod ut pateat, tres quantitates tribus lineis fignificentur. m. f. a. et. o. Dico productum. m. f. in. a. multiplicatum. per. o. æquale effe producto. a. in. o. multiplicato per. m. f. aut producto. m. f. in. o. multiplicato per. a. Sit enim corpus. d. u. rectagulum, cuius latus. n. u. fit æquale m. f. et. u. t: a: et. u. c: o. patebit manifeftè n. t. effe productum. m. f. in. a. quod. n. t. multiplicatum in. u. c. æquali. o. producit corpus. d. u. fed idipfum corpus. d. u. ex multiplicatione producti. c. t. in latus. n. u. æquale. m. f. oritur, & idipfum. d. u. ex multiplicatione. n. c. in latus. u. t. æquale. a. profertur.
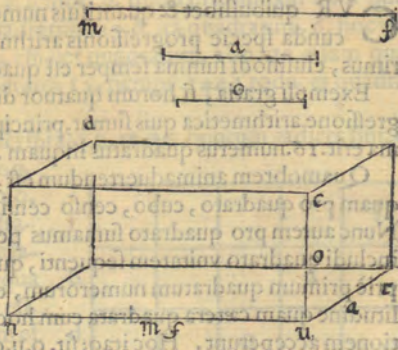
## THEOREMA LXXXIX.

CVR quarumcunque quatuor quantitatum, fi prima in fecundam multiplicetur & hoc productum in tertiam, rurfusq́; hoc alterum in quartam, vltimum productum æquale fit producto producti fecundæ in tertiam, in productum primæ in quartam.

H     Exempli

Exempli gratia, casu sese offerunt hi quatuor numeri.8.5. 3. 2. multiplicato. 8. per.5. & hoc. 40.per.3.rursus hoc.120. per.2.vltimum productum esset.240.æqua le producto.15.( quod ex.5.in. 3.oritur ) in productum. 16 . quod ex. 8. in. 2. profertur.

Cuius speculationis gratia, cogitemus quatuor numeros quatuor lineis. a. e. i.o. signi ficari, productum autem.e.in.i.esse.m.f.et.r.s.similiter & productum.a.in.o.es se. m.z:et.z. f.productum esse.m. f.in.m. z.cui productum.a.in.e. multiplicatum per i. & hoc tandem per.o.æquari debet .

Sit itaque.u. y.productum. a.in. e.quod.u. y. per. i.multiplicatum proferat. u. s. hocque. u.s. multiplicatum per.o.Dico quod dabit numerum æqualem numero.f.z. Quamobrem. r.s.aut. m. f.quod idem est , in figura præcedentis theore matis signi ficetur linea.n.u. & linea. r. u . hu ius, nempe. a. significetur per. u. t. præcedentis, ex quo numerus pro ducti. u.s. præsentis,in præcedenti significabitur producto. n. t. quod pductũ.u.s. presẽs p præses.o.mul tiplicatum, quod erat in præceden ti.u.c.significabitur per. d.u. præce dentis, quod non modo ex multi-

plicatione.n.t.præcedentis, nempe.u.s.præsentis.in.u.c.præcedentis æquali. o. præ sentis oritur , sed etiam ex.c.t.præcedentis æquali. m. z.præsentis in. n.u. præceden tis æquali.m.f.præsentis. Itaque verum est propositum.
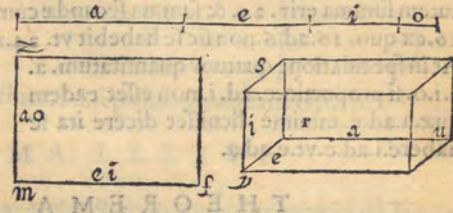
## THEOREMA XC.

CVR quibuslibet & quantisuis numeris in summam collectis,si ab vnitate in se cunda specie progressionis arithmeticę imparium numerorum progressi fue rimus , eiusmodi summa semper est quadratus numerus.

Exempli gratia , si horum quatuor disparium numerorum summã, in dicta pro gressione arithmetica quis sumat,principio ab vnitate sumpto,nempe.1.3.5.7.sum ma erit.16.numerus quadratus inquam . Idem de cæteris.

Quamobrem animaduertendum est , vnitatem,tam sumi pro sui ipsius radicem, quam pro quadrato , cubo , censo censi, primo relato , & alia quauis dignitate . Nunc autem pro quadrato sumamus per. o.significato, cogitemusq; quadratum.o. includi quadrato vnitatem sequenti , quod, vt patet , est quatuor vnitatum, ac pro priè primum quadratum numerorum, ex quo etiam nomen accepit , vnde ex simi litudine quam cætera quadrata cum hoc primo retinent ,ex quaternario denomina tionem acceperunt . Hoc itaq; sit. o.u.c.e.ita ex communi scientia quadrato.o.iun gitur gnomon.e.c.u.constans tribus vnitatibus, quare primus gnomon,numero im pari constat.Scimus etiam ex additione numeri binarij ad imparem, numeris di sparibus summam excrescere,cum propiùs accedere quã binario nequeant, ex quo medio binario , sibi inuicem succedunt . Dico igitur quòd quinario ternarium sub sequente , coniuncto quadrato.o.u.c.e.profertur quadratum , quod in numeris, bi narij quadratum sequitur,eritq; ternarij,quodq; significetur per.o. f. patet enim pri mo non differre ab.o.c.præter quam gnomone.b.f.d. qui coniungitur quadrato . o. c.quique duabus vnitatibus maior est.e. c.u. Iã scimus gnomonem.e. o. u. æqualem

H esse

esse gnomoni. e. c. u. itemq́; gnomonem. b. f. d. æqualem gnomoni. b. o. d. at hic gno-
mon. b. o. d. ex præsupposito, maior est gnomone. e. o. u. duabus vnitatibus. b. et. d.
Itaque etiam gnomon. b. f. d. duabus vnitatibus gnomonem. e. c. u. superabit. Qua-
re. b. f. d. erit impar immediatè sequens ternarium, qui coniunctus quadrato. o. c.
quadratum subsequens componet. Eadem ratione probabitur de quadrato. o. n. se
quenti. o. f. & gnomone. i. n. a. cum hic ordo speculationis sit vniuersalis. In
quo cernitur quemlibet gnomonem sibi contiguũ inferiorem semper duabus vni-
tatibus excedere, cumque quadrata non nisi gnomonibus sibi inuicem succedant.
Sed cũ primus. e. c. u. dispar fuerit, pculdubio etiã necessarioq́; cæteri dispares erũt.
Ex qua speculatione, oritur regula ab antiquis tradita
inueniendi vltimi numeri disparis cõcurrentis ad cõpo
sitionem alicuius quadrati. Vt si quis scire desideret nu-
merum vltimum disparem, quo mediante quadratum.
o. n. constitutum fuit, quod aliud non est quam scire
quantus sit numerus vltimi gnomonis. i. n. a. æqualis gno
moni. i. o. a. Itaque vt sciamus hunc gnomonem. i. o. a.
patet duplicandam esse radicem. o. e. b. i. dabiturq́;. o. e.
b. i. et. o. u. d. a. vbi bis reperitur. o. nos autem tantummo
do quærimus scire gnomonem. i. b. e. o. u. d. a. Itaque
minor est vnitate duplo radicis, cum unitas. o. bis repe-
tatur, quæ tamen in gnomone semel tantum sumebatur.

| o | e | o | i |
|---|---|---|---|
| u | c |   |   |
| d |   | f |   |
| a |   |   | n |

## THEOREMA XCI.

CVR summa quadratorum, quorum radices sunt in proportione sesquitertia
nempe. 4. ad. 3. quadrata sit.

Exempli gratia, sumemus quadratum. 3. scilicet 9. quod in summam cum qua-
drato. 4. colligemus, nempè. 16. eritq́; quadratum. 25. & ita quadratum. 6. hoc est.
36. collectum cum quadrato. 8. nempè. 64. efficiet quadratum. 100. ita etiam qua-
dratum. 9. hoc est. 81. coniunctum quadrato. 12. nempè. 144. producet quadra-
tum. 225.

In cuius gratiam sint duo quadrata subscripta. q. o. et. q. a. quorum radices sint. q.
g. et. q. p. hoc est. q. g. quatuor vnitatum, et. q.
p. trium, ex quo. q. a. erit. 16. vnitatum et. q. o.
nouem. Ad hæc cogitemus applicari quadra-
to. q. a. gnomonem. f. s. h. tam amplum siue la-
tum quã gnomon. b. a. g. nempè vt. h. sit æqua
lis. g. g. verò differentia sit qua. q. g. maior est.
q. p. huncq́; gnomonem. f. s. h. dico æqualem es
se quadrato. q. o. nam ex præsupposito. g. ter ra
dicem. q. p. ingreditur, & quater. q. g. ex quo,
tres partes. q k. p. inter se æquales sunt vnde
etiam quadratum. q. o. nouem partibus super-
ficialibus quadratis constabit, quarum singula
rum radix æqualis erit. g. cumque præcedenti
theoremate didicerimus quemlibet gnomo-
nem quadrati immediatè sequentis æquæ amplitudinis cum gnomone præcedentis,

per duab.vnitatibus superficialibus crescere, quarū singularū radix æqualis est. g. ne
cessariò sequitur gnomonem. b.a.g. duabus partibus aut vnitatibus gnomonem. d.
o.p. superare, ita vt gnomon. b.a.g. septem vnitatibus, aut partibus superficialibus
quadratis constet. Quare eadem ratione gnomon.f.s.h.constabit nouem similibus.
Itaque æqualis erit quadrato.q.o. Quamobrem verum est, quòd quadrato. q. o.
coniuncto quadrato.q.a.proueniet quadratum.q.s.cuius radix ita differet à. q. g.vt.
q.g.à.q.p: ex quo tres radices arithmeticè inter se continuæ proportionales erunt.
Idipsum dico si.q.p.fuerit. 6.et.q.g:8:tunc enim singulæ partes.q. k.p.g.h. æquipol
lebunt duabus vnitatibus , quæ cogitabuntur
in summam collectæ, ut cum patribus.q. κ. p.
g.h. integris contemplari liceat. Idem acci-
det si.q.p.erit.9.et.q.g.12.singulæ enim par-
tes.q.κ.p.g.h. tripartitæ erunt. Idcircò dixi
gnomonem. f. s.h. tam amplum cogitari de-
bere, quam gnomon.b.a.g.nempè ut.h.æqua
lis sit.g. Idem occurret si.q.g.erit. 12.et. q. p.
quinque, quod cum fuerit patebit ex præce-
dentis theorematis speculatione, gnomonem
f.s.h:25. vnitatibus constare , cogitatum am-
plitudinis simplicis vnitatis denominatæ in.q.
p.aut.q.g.non amplitudinis gnomonis. b.a. g.
qui septem vnitatibus latus esset. Cum igitur.

q.p.quinque vnitatibus linearibus constet scinius.q.o:25.superficialibus constare ,
collecto itaque in summam quadrato.q.o.cum quadrato.q.a. cognoscetur quadra-
tum.q.s.vnà etiam eius radix. Eadem ratione , alia multa quadrata similia contem-
plari licebit.

# THEOREMA XCII.

CVR proposito numero pari maiori binario , qui detrahi & in summam colli-
gi debeat ex altero numero quærendo , vt tam residuum quam summa sint
quadrata numerorum integrornm. Rectè dimidium propositi numeri in seipsum
multiplicamus , & quadrato huic addimus vnitatem, eritq; numerus quæsitus.

Exempli gratia proponitur. 12.numerus detrahendus, & coniungendus nume-
ro inuestigando, ut residuum detractionis, & summa sint quadrati numeri. Addi-
ta vnitate ipsi.36.quadrato dimidij, dabitur. 37. numerus quæsitus.

Cuius speculationis gratia , subscripta quatuor quadrata cogitemus.g.p:u.i:t.c:n.
κ.cogitemusq; quadratum.g.p.esse quadratum summæ, κ. n.verò residui subtractio-
nis:u.i. aut numerum inuestigādū,ex quo gnomon.u.d.i.cognoscetur ita etiam et.n.
o. κ.qui inter se sunt æquales. Iam certi erimus.e.i.esse plus quam dimidium gno-
monis.n.o.κ. Itaque cogitemus rectangulum.r.c. exactum dimidiū esse gnomonis.
n.o.κ.ex unitatibus superficialibus quarum una erit.m.a.

Cuius numeri quadratum sit.t.c.vnde etiam cognitum & cum. κ. c. ex communi
scientia sit vnitas linearis, propterea quod. m. a . est superficialis hoc est quadrata,
quæ detracta ex. q.c.dimidio gnomonis.n.o.κ. (quamuis lineari) supererit. κ.q.co
gnita,numerorum integrorum ( nota φ. κ. i.semper minor erit duabus vnitatibus li-
nearibus & maior vna ex dictis vnitatibus, ut ex te ipso contemplari potes ) quare.

n. κ.

n.k.ipſius quadratum numerorum integrorum cognoſcetur, cui addito gnomone.
n.o.k.cognoſcemus numerum.u.i.quæſitum.

Sed cum nobis hæc via, tenenda propoſitum non fuit, hoc eſt primo loco inue
niendi quadrati minoris.n.k. ideo ſupereſt probandum gnomonem.t.o.c. vnitati ę-
qualem eſſe, nempe quadratulo.m.a.quod patebit, ſi conſideremus nos ſumpſiſſe
rectangulum.r.c.pro dimidio gnomonis.n.o.k.etenim ſi ſupplemento etiam.n.r.qua
dratulum æquale. m.a.adderetur, pateret gnomonem.n.a.k.cum dicto quadratulo
collectum, æqualem eſſe gnomoni.n.o.k:cum duo ſupplementa.m.t.et.m.c.inter ſe
ſint æqualia. Quamobrem inuento quadrato.t.c. ex dimidio gnomonis cognito,
additur vnitas, gnomon ſcilicet.t.o.c.ex quo cognoſcitur numerus.u.i. quæſitus.
Quod autem quadratum.g.p.numeris integris conſtet, hac ratione probatur viſum
enim fuit ſupra quadratum.n. k. verè quadratum eſſe, & numeris integris conſtare,
pariter etiam.t.c.ſeq́ mutuò conſequi (nam.k.c.eſt vnitas linearis) ex quo gnomon
n. a.k. numero diſpari conſtabit, ex ijs quæ.90. theoremate probata fuerunt. Itaq;
ex eodem theoremate neceſſe eſt gnomonem.t.d.c.etiam numero diſpari conſtare,
ita vt à numero.n.a.k.non niſi duabus vnitatibus differat, nempe vt.c.p.ſit vnitas li-
nearis, ſed ita reuera eſt, numerus enim.u.d.i. ex præſuppoſito par eſt,quare nume
rus. t.d. c. diſpar erit, cum alterum vnitate ſuperet, videlicet gnomone.t.o. c. vnita
ti æquali, tum.n.a. k.minor eſt.n.o.k. ex eodem gnomone.t.o.c.unitati æquali. Ita
que.n.a.k.minor erit.u.d.i.per vnitatem, & minor.t.d.c.per duas unitates,ex quo ſe-
quitur.g.p.eſſe quadratum integrorũ ex dicto theoremate ac conſequens quadrato
t.c.quare.c.p.vnitas erit, & radices. q.k.et.q.p.horum quadratorum numero binario
rio inter ſe different. Vnà etiam ſcienda eſt cauſa, cur numerus propoſitus neceſſa
rió binario maior eſſe debeat. Etenim cũ ipſe
ſit futurus gnomon.n.o. k. nec poſſit minor eſſe
numero ternario, vt patet ex. 90. theoremate,
idcirco ſequitur neceſſarió maiorem eſſe bina-
rio debere. Quòd ſi diſpar numerus propone-
retur,nec forma operis nec ſpeculationis mutã-
da eſſet. Non erit tamen neceſſarium vt ipſa
quadrata.n.k.et. g. p. numeris integris conſta-
rent. Sæpius enim fractis cóponerentur, quòd
ex. 90. theoremate facile erit ſpeculari nihilo-
minus fractis integris, ipſiſq; collectis cum ſuis
fractis ſummæ eſſent quadratæ.
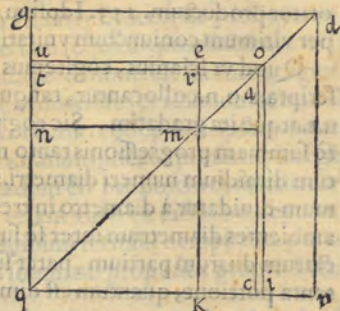
## THEOREMA XCIII.

CVR propoſitis duobus numeris altero pari, altero verò diſpari, duplo primi
minore per vnitatem,ſi alium inuenire numerum voluerimus,cui alterum iſto
rum coniunctum proferat quadratum, & altero detracto, quadratum ſuperſit. Re-
ctè datos numeros in ſummam colligemus, quam ſummam in duas quam maximas
poterimus partes diuidemus, quarum vna pari, altera diſpari conſtet, tum vtran-
que in ſeipſam multiplicabimus, & quadrato minori,duorum numerorum propo-
ſitorum quemuis ademus, ex quo cupimus nobis quadratum minus ſupereſſe,& pro
ueniet nobis numerum quæſitum.

Exempli gratia, proponuntur numeri.11.et. 6.quorum alter alicui numero ad-
den-

dendus , alter ex eodem detrahendus fit , ex quo proferri debeant bina qua-
drata. Itaq; numeri illi in fummam collecti dabunt. 17. differentiam minoris quadra
ti & maioris . Iam fi ex hoc. 17. binas partes fecerimus, altera erit . 8. altera. 9. qui
bus in feipfis multiplicatis alterum quadratum erit. 64. alterum. 81. addito itaq; ipfi.
64. 11. aut. 6. pro libito , propofitum numerum confequemur. cui addito. 6. vel. 11.
dabit nobis. 81. vel ex ipfo detracto. 11. vel. 6. relinquet nobis 64. in prefenti autem
exemplo talis numerus erit, aut. 70. vel. 75. Huius autem theorematis fpeculatio
ex. 90. dependet , quo demonftratum fuit gnomonem proximè quadratum fequen
tem , vnitate duplo radicis minorem effe .

## T H E O R E M A   X C I I I I.

C V R  fi quis cupiat fummam progreffionis arithmeticæ quam citiffimè cogno
fcere. Rectè coniunget vltimo termino vnitatem primum terminum , huius
poftea vltimi termini dimidium cum numero terminorum multiplicabit, ex
quo multiplicationis productum , erit omnium propofitorum terminorum fumma ,
aut eundem vltimum terminum iunctum primo, per dimidium numeri terminorum
multiplicabit. Nam idipfum eueniet.

Exempli gratia , fi proponerentur. 17. termini in prima progreffione arithmeti-
ca naturali , vltimus effet. 17. cui coniuncta vnitate primo termino fumma erit. 18.
cuius dimidium cum numero terminorum, nempe. 17. multiplicatum cum fuerit ,
oritur productum. 153. Idpfum eueniet, multiplicato dimidio numeri terminorū
per vltimum coniunctum vnitati primo termino .

Quod vt fciamus, cogitemus terminos progreffionis collocari, vt in figura fub-
fcripta. a. o. n. collocantur, tanquā per gradus, fumpto principio ab vnitate. n. tum.
u. t. atque ita gradatim . Sic cogitato abfoluto parallelogrammo. q. o. fciemus aper-
tè fummam progreffionis tanto maiorem effe dimidio totius parallelográmi , quan
tum dimidium numeri diametri. a. e. i. c. u. n. requirit . Nam cum parallelogram-
mum diuidatur à diametro in tres partes, diameter vnam occupat, reliquæ verò duę
ambientes diametrum inter fe funt æquales . Sumpto itaq; diametro cum altera di
ctarum duarum partium , patet fumi plufquam dimidiū totius parallelográmi. pro
tanta portione, quantum eft dimidiam occupatam à diametro , qui cū ex difcretis
refpondentibus numero terminorum componatur, conftat numero æquali effe di-
cto numero terminorum. o. n. Iam fi quis multiplicet. a. o. per dimidium. o. n. procul
dubio, ex prima fexti aut. 18. feptimi, oriètur dimidiū numeri parallelográmi. q. o.
quod minus erit fumma progreffionis dimidio numeri diametri , aut quod idem eft
dimidio. o. n. fed hoc dimidium. o. n. æquale eft producto dimidij vnitatis. n. in. o. n.
ex. 20. feptimi, cum dimidium. o. n. fit eius productum in vnitatè . Itaque multipli-
cato. n. o. per dimidium. o. a. coniunctum dimidio vnitatis. n. oritur fumma quæfita
propofitæ progreffionis. Idipfum accidit multiplicata fumma. o. a. & vnitate. n. p
dimidium. o. n. ex. 20. feptimi, cum proportio totius ad totum eadem fit, quæ dimi
dij ad dimidium, ex caufa permutationalitatis . Patet etiam in progreffionibus ,
quæ ab vnitate initium ducunt, fi fiat afcenfus per binarium fumma vltimi termini
cum primo femper duplam futuram effe numero terminorum, quod fequentes figu

ras conſideranti ſpeculari licebit , Diametros harum figurarum notaui literis ſiue
characteribus.a.e.i.c.u.n.
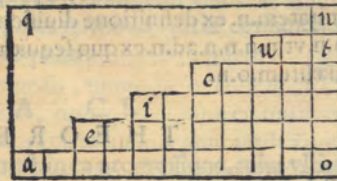


## THEOREMA XCV.

IN progreſſionibus, quæ ab alio termino quam vnitate incohantur , idipſum vt
monuimus accidit , hoc tamen notato , quòd ex conſequenti quælibet pars dia-
metri parallelogrāmi , minimo termino æqualis erit, prout in progreſſionibus quæ
ab vnitate originem ducunt , ſingulæ pàrtes diametri , vnitati ſui primi termini æ-
quales ſunt . At in reliquis progreſſionibus, vt in figura patet , eadem eſt propor-
tio totius diametri ad.o.n.quæ minimi termini ad vnitatem ex. 13 . quinti , nempe.
a.o.ad.o.n.vt.n.n.n.n.ad.n. In eiuſmodi progreſſionibus accidit quoque parallelo-
grāmum à diametro in tres partes diuidi , quarum vnam ipſe occupat , reliquæ ve-
ro inter ſe æquales ipſum ambiunt. Ex quo illud etiam ſequitur,productum.a. o. in
dimidium.o.n.æquale eſſe dimidio parallelogrāmi,quod minus eſt ſumma progreſ-
ſionis dimidio diametri,quod dimidum ſi inuenire voluerimus , minimum terminū.
n.n.n.n.per dimidium.o.n.multiplicabimus, & ex. 18.aut.19.ſeptimi ipſum habe-
bimus,quandoquidé minimo termino per totum.o.n.multiplicato profertur integer
diameter ex.2o.prædicti . Etenim vt diximus , eadem eſt proportio totius diame-
tri ad.o.n.quæ minimi termini ad vnitatem . Ita etiam dico ex dicta. 2o. ſeptimi .
idem dimidium diametri oriri , ſi quis dimidium minimi termini nempè.n.n. per to-
tum.o.n.multiplicauerit. Quamobrem qui ſtatim ſummam propoſitæ progreſſionis
cognoſcere voluerit ,
ſemper primum termi
num.n.n.n.n.cum.a. o.
coniunget,qua ſumma
per dimidiū.o. n.mul-
tiplicata , aut.o. n. per
dimidium dictæ ſum-
mæ , ex prædictis rationibus propoſitum conſequemur .
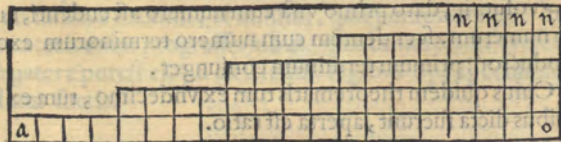


## THEOREMA XCVI.

CVR ſi quis numerum terminorum inuenire velit, cognitis tantummodo prì
mo atque vltimo, rectè vltimum per primum diuidet , ex quo proueniens
nume-

numerus quæsitus erit.

Quod intelligendum est tamen quoties primus terminus differentia terminorũ est, nempe ascendens ipsorum terminorum.

Cuius ratio manifestè speculari potest in figura præcedentis theorematis. Nam diuisa. a . o. per . n . n . n.n. eadem proportio erit. a.o.ad proueniens, quæ . n . n.n. n. ad vnitatem.n. ex definitione diuisionis. At superius dictum fuit ita se habere.a. o.ad.o.n. vt.n.n.n.n. ad.n. ex quo sequitur ex. 11. et. 9.quinti proueniens esse numerum quæsitum.o.n.

## THEOREMA XCVII.

VBI verò primus terminus, reliquorum non erit differentia. Hac de causa necesse est detrahere primum ex vltimo, residuumq́; per numerum ascendentem differentiam scilicet, partiri, proueniensq́; vnitati coniungere, quò numerum terminorum habere possimus. Scimus etenim tam multas vnitates esse in vltimo terminorum quot in omnibus interuallis, aut differentijs in summam collectis simul cum vnitatibus primi termini, totq́; sunt termini, quot interualla simul cum primo termino. Quare si minimus terminus interuallo æqualis fuerit. Vltimo per primum diuiso, ex a deductis præcedenti theoremate propositum consequemur. Itaq́; primo termino ex vltimo detracto residuoq́; per interuallum, hoc est numerum differentiæ diuiso, proueniens erit numerus terminorum absque primo qui vnus est, coniuncto quoque dicto prouenienti propositum consequemur.

## THEOREMA XCVIII.

CVR si quis arithmeticæ progressionis dato primo & vltimo simul cum numero terminorum, ascendentem numerum cognoscere voluerit. Rectè primum ex vltimo detrahet, residuumq́; per numerum terminorum excepto vno diuidet. Huius theorematis speculatio ex. 13. theoremate manifesta erit, nam in præcedenti cap. numerus terminorum erat proueniens diuisionis residui subtractionis primi termini ex vltimo.

## THEOREMA XCIX.

CVR si quis maximum omnium terminorum dictæ progressionis cognoscere voluerit, dato primo vnà cum numero ascendenti, numeroq́; terminorum. Rectè numerum ascendentem cum numero terminorum excepto vno multiplicabit, productoq́; primum terminum coniunget.

Cuius quidem theorematis tum ex vndecimo, tum ex ijs quæ præcedentibus capitibus dicta fuerunt, aperta est ratio.

## THEOREMA C.

CVR veteres cupientes obtinere summam progressionis continuæ naturalis, quæ ab vnitate initium ducit, dato vltimo termino tantummodo. Dimidium vltimi termini cũ toto sequenti multiplicabant, productumq́; summa quæsita erat. Exempli gratia, si vltimus terminus eiusmodi progressionis fuerit. 7. multiplica-

to

to dimidio ipsius nempe. 3. & dimidio, cum numero ipsum terminum sequéti, nem
pè. 8. summa dictorum terminorum erit. 28.

Huius autem speculatio ex. 94. theoremate dependet, in quo facilè depræhen-
dere dicet ex figura continuæ progressionis naturalis, numerum terminorum maxi-
mo termino semper æqualem esse; ex quo tatum est dimidium numeri terminorum,
quantum maximi dimidium, tantusq; est vltimus terminus vnitati coniunctus, quan
tus numerus is, qui vltimum terminum consequitur.

### THEOREMA CI.

CVR antiqui idipsum, quod iam dictum est, in ea progressione, cuius vltimus ter-
minus dispar est scire cupientes, numerum integrorum proximè dimidium
maximi sequentem sumebant, quem per maximum multiplicabant, ex quo
summa quæsita oriebatur.

Exempli gratia, si dimidium maximi fuisset. 3. cum dimidio, sumebant quatuor,
& per maximum. 7. multiplicabant, ex quo pariter proferebatur summa. 28.

Cuius ratio ex. 20. septimi Euclidis oritur, cum eadem sit proportio numeri se-
quentis maximum ad numerum dimidium maximi sequentem, quæ maximi ad suũ
dimidium, est enim dupla.

### THEOREMA CII.

TRaditum est à nonnullis, à veteribus obseruatam fuisse hanc regulam, qua sci-
re possent summam alicuius progressionis arithmeticæ discontinuæ aut inter-
cisæ, quæ numero pari terminetur. Multiplicabãt enim dimidiũ vltimi termini per
proximum numerum dimidio dicto maiorem, ex quo inquiebãt semper productum
summæ quæsitæ æquale esse, subijciuntq; exemplum progressionis, quæ à binario in-
choata crescit per binarium. In qua quidem progressione non per se, sed per acci-
dens regula vera est. Hoc est, non quia ex se vnus ex producentibus numeris, dimi-
dium termini maioris futurus sit, alter uerò proximè sequens dimidium, sed quia
vt dictum est. 95. theoremate, eadem est proportio maximi termini ad numerum
terminorum, quæ minimi ad vnitatem. Cumque in præsenti exemplo minimum
sit duplum vnitati in eiusmodi casu, numerus terminorum, dimidio maximi termini
æqualis est, qui terminorum numerus ex se, vt patet, vnus est ex producentibus, al-
ter verò producens numerus, est proximè dimidium sequens, non ex se, sed quia nu
merus sequens, dimidium est summæ maximi, & minimi, quæ per se alter esse de-
bet producens numerus. In cæteris enim progressionibus, quæ binario non crescũt
regula falsa est, prout facilè patere potest ei, qui ex scientiæ legibus ope speculatio-
nis. 95. theorematis speculatus fuerit.

### THEOREMA CIII.

ALIAM quoque tradunt regulam, qua veteres vsos fuisse dicunt, quo sum-
mam scire possent progressionis discontinuæ, quæ numero dispari absolui-
tur. Ea autem est eiusmodi. Vltimum terminum in duas quam maximè poterant ma-
ximas partes diuidebant, quarum vna semper altera maior erat, hanc autem maio-
rem in seipsam multiplicabant, atque quadratum hoc, summam progressionis esse

I affir-

affirmabant, Quæ fanè regula, non femper, etfi interdum vera fit.

Sumebant hi exemplum progreffionis, quæ ab vnitate incohata crefcit per binarium, in qua per accidens euenit vt numerus dimidium vltimi termini proximè fequens, nempe è duabus partibus vltimi termini maior, æqualis fit numero terminorum, qui per fe vnus è producentibus, ex ijs quę. 94. theoremate diximus, effe debet; alter vero producens, qui per fe dimidium fummæ primi & vltimi effe debet, per accidens pars maior eft duarum vltimi termini, & alteri producenti æqualis.

Aut alio modo ratiocinemur, dicentes, in huiufmodi progreffione dimidium fummæ vltimi termini cum primo, femper medium proportionale eft inter eam fummam & dimidium numeri terminorum, etenim huiufmodi fumma numero terminorum femper dupla eft, prout. 94. theoremate tradimus. Itaque ex. 20. feptimi, quadratum partis maioris, producto fummæ dictæ in numerum dimidij terminorū æquale erit, quod productum per fe fummæ progreffionis eft æquale. At in cæteris eiufmodi progreffionibus fallit regula, vt ex fupradictis facilè demonftratur.

## THEOREMA  CIIII.

PErmultis terminis ad libitum propofitis, difpofitis nihilominus progreffione, aut proportionalitate geometrica continua, fi minimus ex maximo & ex fequenti minimum detrahatur, refiduum maximi, eam proportionem ad fummam reliquorum omnium terminorum retinebit, quam refiduum fecundi ad primum.

Proponuntur, exempli gratia, quatuor termini. 3. 12. 48. 192. continui geometricè proportionales, fi primum, hoc eft minimum, ex fecundo, & maximo detrahas, ex fecundo fupererit. 9. ex maximo. 189. quod fi minimum per refiduum maximi multiplicaueris, hoc eft. 189. orietur. 567. tum fi huiufmodi productum per. 9. (refiduum fecundi) diuiferis, proueniet. 63. quod proueniens æquale erit fummæ reliquorum omnium terminorum, maximo excepto. Ex quo inferre licet ex. 20. feptimi eandem proportionem effe. 189. ad. 63. quæ. 9. ad. 3. aut fi refiduum fecundi per fummam dictorum terminorum multiplicaueris producetur idem. 567. quare ex. 20. feptimi & cætera.

Quod vt fciêtificè poffimus, & in vniuerfum fpeculari. Quatuor termini propofiti, quatuor fubfcriptis lineis fignificêtur. b. i. c. a. f. r. m. s. (quod aūt de his quatuor dico de centūmillibus, & eo amplius dicere poffum.) Nunc minimus terminus. m. s. ex maximo. b. i. detrahatur, fuperfitq;. n. i. idemq;. m. s. ex fecundo termino. f. r. fubtrahatur, fuperfitq;. o. r. Dico proportionem. n. i. ad fummam reliquorum omnium terminorum. c. a. f. r. m. s. eandem effe, quæ. o. r. ad. m. s. Quamobrem ex tertio & quarto fecundus. f. r. detrahat, ex tertioq; fuperfit. t. a. & ex quarto. e. i. ita etiam tertius. c. a. ex quarto. b. i. fuperfitq;. d. i. fanè sic se habebit. c. a. ad. f. r. vt. c. t. ad. f. o. vt quifq; per fe fcire poteft. Quare ex 19. quinti fic fe habebit. a. t. ad. r. o. vt. c. a. ad. f. r. & permutando ita. a. t. ad. a. c. vt. o. r. ad. r. f, & feparando fic. a. t. ad. a. c. (hoc eft. f. r.) vt. r. o. ad. o. f. videlicet. m. s. Idē dico de. d. i. ad. a. c. nempe fic fe habebit. d. i. ad. a. c. vt. a. t. ad. r. f.
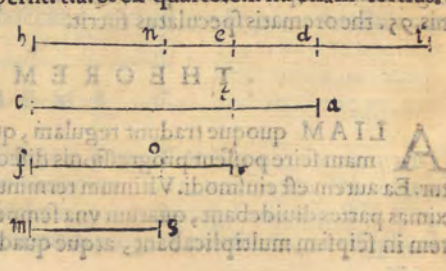
r̃. f. hoc eſt. o. r. ad. m. s. ex. 11. quinti. Itaque ex communi ſcientia ſic ſe habe-
bit. d. i. ad. d. b. vt. e. d. ad. e. b: cum. e. d. æqualis ſit. t. a. Ita etiam vt. e. n. ad. n. b: cum. n.
e. æqualis ſit. o. r. Iam ſi ſic ſe habeat. d. i. ad. d. b. vt. d. e. ad. e. b. permutando quoq; ſic
ſe habebit. d. i. ad. d. e. vt. d. b. ad. b. e. & componendo ita. i. d. e. ad. e. d. vt. d. b. e. ad. e.
b. & permutando ſic. i. d. e. ad. d. b. e. vt. de. a. d. e. b. nempe vt. e. n. ad. n. b. & permutan
do ita. i. d. e. ad. e. n. vt. d. b. e. ad. b. n. & componendo ita. i. d. e. n. ad. n. e. vt. d. b. e. et. b.
n. ad. b. n. & permutando ſic. i. d. e. n. ad. d. b. ẹ. et. b. n. nempe ad. a. c: f. r: m. s: vt. e. n. ad.
n. b. hoc eſt. ut. o. r. ad. m. s. quod erat propoſitum.

## THEOREMA CV.

CVR deſideranti ſummam quorumcunque terminorum progreſſionis conti-
nuæ geometricæ cognoſcere. Rectè minimus terminus ex maximo detrahen
dus eſt, reſiduumq́; per denominantem progreſſionis dempta vnitate diuidendum,
prouenientiq́; maximum terminum addendum, ex quo oritur ſumma quæſita.

Exempli gratia, ſi darentur quatuor termini continui proportionales. 8. 12. 18.
27. primum hoc eſt minimum. 8. ex vltimo. 27. detraheremus: remaneretq́;. 19. quî
per denominantem progreſſionis, dempta vnitate, diuideretur. Quo loco animad
uertendum eſt, quamlibet denominationé cuiuſcunque proportionis numerorum
ſupra vnitatem fieri, nam de proportionibus multiplicibus dubitandum non eſt, &
idipſum de ſuperparticularibus, & ſuperpartientibus eſt intelligendum, vt in præ-
ſenti proportio ſeſquialtera inter duos terminos cogitanda eſt, nempe inter vnum
& dimidium, atque vnum. Seſquitertia autem inter vnum & tertiam partem,
& vnum. Seſquiquinta inter vnum cum quintæ parte, & vnum. De ſuperpartien
tibus idem aſſero quod de proportione ſuperbipartiéte tertias appellata, vt. 5.
ad. 3. quæ cogitanda eſſet inter vnum duas tertias, & vnum, ſuperbipartiens quar-
tas inter vnum tres quartas, & vnum, ita vt minor terminus, numerans ſcilicet, ſem
per ſit vnitas, alter verò denominans. Idem de cæteris. Quare in præſenti exem
plo, detracta vnitate ex denominante progreſſionis, ſupererit tantummodo dimi-
dium, quo diuiſo. 19. proueniet. 38. qui numerus æqualis erit ſummæ reliquorũ
omnium terminorum, cui coniuncto vltimo termino. 27. dabitur ſumma quæſita. 65

Pro cuius ſpeculatione, quatuor termini ſignificentur, quatuor lineis. m. s: f. r: e. a.
b. i. primus autem terminus. m. s. ex vltimo. b. i. detrahatur, reſiduumq́; ſit. n. i. & ex
ſecundo. f. r. cuius reſiduum ſit. o. r. proportio verò progreſſionis ea ſit, quæ. g. h. ad.
y. quo vnitas repræſentatur ( ex quo ſic ſe habebit. g. h. ad. y. vt. f. r. ad. m. s.) qua. y. de
tracta ex. g. h. ſuperſit. h. Tum erecta
cogitetur linea. n. u. x. indefinita per
pendicularis. b. i. à puncto. n. quẹ diui
datur in puncto. x. ita vt. n. x. æqualis
ſit vnitati. y. & in puncto. u. ita. vt. n.
u. æqualis ſit. h. ex quo eadem erit
proportio. n. u ad. n. x. vt. h. ad. y. né-
pe. o. r. ad. m. s. Nam cũ ſic ſe habeat.
f. r. ad. m. s. hoc eſt ad. f. o. vt. g. h. ad. y
hoc eſt ad. g. permutando quoq; ſic
ſe habebit. f. r. ad. g. h. vt. f. o. ad. g. Ita
que ex. 19. quinti. o. r. ad. h. vt. f. r. ad. g. h. ex quo ex. 11. eiuſdem. o. r. ad. h. vt. f. o. ad

I 2 g. &
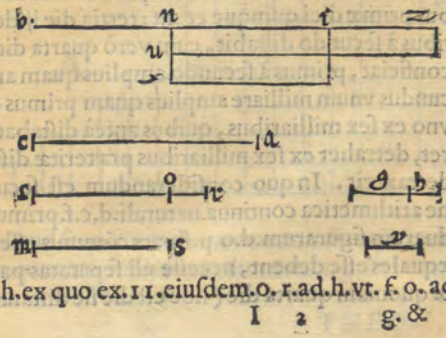
g.& permutando.o.r.ad.f.o.hoc est ad.m.s.vt.h.ad.g.hoc est . y. Quamobrem ea-
dem erit proportio.o.r.ad. m.s.quæ.n.u.ad.n.x. Absoluantur itaque duo rectangu
la.x.i.et.u.z.ita tamen vt rectangulū. u. z.cogitetur æquale rectangulo.x.i.cuius.x.i.
superficialis numerus ex communi conceptione lineari.n.i.æqualis erit , quare ex
eadé communi conceptione , numerus superficialis.u.z.lineari.n.i.æqualis erit,qui
quidem numerus in figura rectangu-
la superficialis cogitandus erit , cum
diuidendus sit per.h.hoc est per.n.u.
ex quo proueniens ex huiusmodi di
uisione erit numerus . n . z . ex ijs
quæ. 10.theoremate dicta fuerunt .
Sed ex. 15. sexti aut. 20. septi-
mi eadem est proportio.n.i.ad. n. z.
quæ.n.u.ad.n.x.hoc est.o.r. ad. m. s.
videlicet vt.n.i.ad aggregatum reli-
quorum omnium terminorum.c.a: f.
r : m.s. ex præcedenti theoremate , & ex. 11. quinti Euclidis.Itaque ex.9. eiuſ-
dem numerus.n.z.æqualis erit summæ trium terminorum.c.a:f.r:m.s.cui coniuncto
quarto termino.b.i.propositum obtinetur.
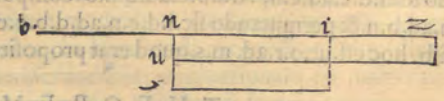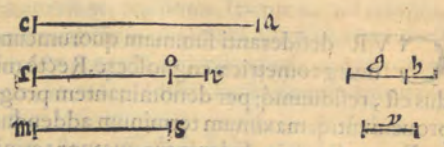
## THEOREMA CVI.

PRoposuere veteres quæsita nonnulla de itineribus interq; hoc vnum fuit . Po-
namus duos iter agere per eandem viam quorum alter quatuor milliaria sin-
gulis diebus conficiat,alter verò prima die milliare vnum,secunda duo , tertia tria,
atque ita singulis diebus milliare addit ; quærimus quot dierum spacio socium con
sequetur.

Quamobrem numerus milliarium primi viatoris duplicatur, sic sunt.8.milliaria,
ex quo semper vnitas detrahitur, quæ in præsenti exemplo erit . 7. totq; dies erunt
quibus socius socium consequetur, & milliarium numerum æqualem absoluerit.

Cuius rei facilis erit speculatio , si subscripta figura diligenter consideretur , in
qua primus viator, die prima, quatuor milliaria linea.q.d.significata conficit, at-
que illa ipsa die alter vnum tantum designatum per.d.perficit, ita vt primus via-
tor tribus milliaribus socium antecesserit, altera verò die secundus uiator cum duo
milliaria côficiat,excedetur à primo duobus milliaribus tantummodo, quę cum tri-
bus primæ diei quinque erunt ; tertia die ijsdem de causis primus sex tantum millia-
ribus à secundo distabit, cum verò quarta die tot secundus quòt primus milliaria
conficiat, primus à secundo amplius quam antea non distabit ; quinta verò cum se
cundus vnum milliare amplius quam primus conficiat. propius accedit ad primum
vno ex sex milliaribus, quibus antea distabat, tum sexta cum duobus primum supe-
ret, detrahet ex sex milliaribus præteritæ distantiæ tria , septima tandem illa sex
detraxerit. In quo considerandum est secundum viatorem iter agere progressio-
ne arithmetica continua naturali.d.c.f.primum autem per rectangulum. q. f. quarū
duarum figurarum.d.o.p.f.pars cômunis esse reperitur, quæ quantitates si inuicém
æquales esse debent, necesse est separatas partes.u.q.n.et.t.i.c.inter se æquales esse,
& quoniam quarta die ( hoc est die sic distante à primo , nempè numero milliarium
primi

primi viatoris ) tot milliaria abfoluat vnus
quot alter abfque vlla differentia, quæ figni-
ficetur per.o.s. neceffe eft itaque ex communi
conceptione tot dies effe poft.o. s. quot ante-
cefferant, vt exceffus æqualis fit defectui, qui
fimul collecti, iuncta etiam.o.s. duplum erunt
d.s. dempta vnitate, prout facilè in fubfcripta
figura qui fque per fe fcientificè poterit fpecu
lari. Quamobrem confultum erit duplicare
numerum.o. s. & ex duplo vnitatem detrahe-
re, quandoquidem dies fupra infraq;.o.s. cum
die. o.s. minores funt duplo numeri.d.s. aut.o.
s. ( quod idem eft) vnitate.
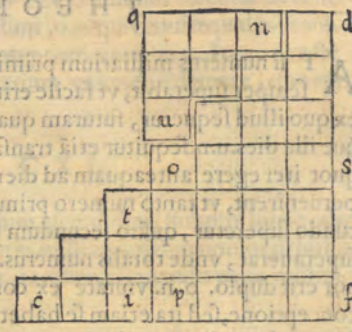
## THEOREMA CVII.

QVOD fi fecūdus viator ordinē fecūdæ progreffionis arithmeticæ feruās iter
agat, nempe ea quæ ab vno per binarium afcendit, femper numerus dierum
æqualis erit numero milliarium diurnorum primi viatoris.

In cuius gratiam animaduertendum eft numerus ne milliarium diurnorum pri-
mi viatoris par an impar fit. Etenim fi par eft, primus viator in fine fingulorum die-
rum primæ medietatis numeri omnium dierum fecundum antecedet numero difpa
ri milliarium; altero verò dimidio numero dierum, à fecundo numero etiam difpa
ri præteribitur, vt in fequenti figura patet. Nam prima die, fecundus ex primo
milliare vnum ex numero pari, qui à primo conficitur detrahit; fecunda verò die
idem fecundus, duo fubtrahit milliaria ex difpari, qui primo reliquus fuerat, ficq;
perpetuò difpar remanet vfque ad vnitatem, ad quam cum peruenerint, nempe ad
illius diei exitum, quo primus fecundum vnitate tantummodò fuperat, manifeftè
depræhendetur fubfequente die fecundum vnitate primum fuperaturum, altera ve
rò tribus vnitatibus, prout penultima die fecundus à primo tribus vnitatibus fupera
batur. Quare neceffe erit, tot diebus fecundum cum primo iter agere, inchoan-
do ab ea die, qua fecundus primam fuperabit, quot egerat dum à primo fuperare-
tur, vt ex communi conceptione, media figura. A. depræhendi poteft. Quod au-
tem fingula dimidia dierum, dimi-
dia fint numeri milliarium diurno-
rum primi, patebit ex fequenti fi-
gura, cogitato termino. u. n. vlti-
mo progreffionis fuperatę à primo
vfque ad vnitatem. e. qui terminus
u. n. coniunctus primo. o. nempe. e.
femper duplę eft numeri termino-
rum. o. n. vt. 94. theoremate circa
finem dictum fuit. Sed. u. n. cum. e.
numero æquali conftat numero
milliarium diurnorum primi viatoris, ex quo fequitur totum numerum dierum, quo
rum. o. n. dimidium eft, æqualem effe numero milliarium diuruorum primi via-
toris.
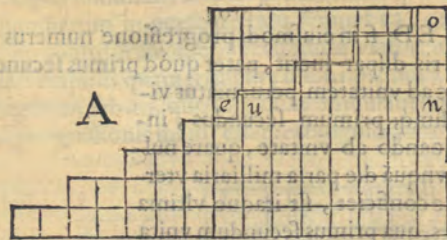
         THEO-

## THEOREMA CVIII.

AT si numerus milliarium primi viatoris dispar fuerit, secundum numero pari semper superabit, vt facile erit sequentem figuram consideranti intelligere, ex quo illud sequetur, futuram quandam diem, qua paria milliaria conficient. Sitque illa dies.u.n.sequitur etiã transacta ea die, tot diebus vtrique ambulandum esse quot iter egere anteaquam ad diem. u. n. peruenirent, vt tanto numero primus à secundo superetur, quãto secundum primus superauerat, vnde totalis numerus.o.s. minor erit duplo. o, n. vnitate ex communi conceptione, sed ita etiam se habet terminus.u.n.hoc est minor duplo.o.n.per. o. vt 94.theoremate dictum fuit, itaque.o.s. æqualis erit.u.n.quod erat propositum.
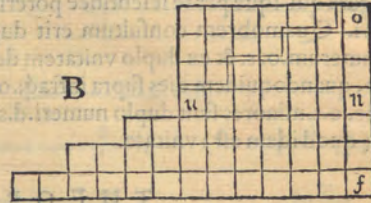
## THEOREMA CIX.

SIN verò progressio secundi viatoris, non ab vnitate sed à binario inchoata, per binarium quoque ascenderet, numerusq; milliarium diurnorum primi viatoris par esset, absque dubio quadam die paria milliaria vterq; conficeret, quę significetur.u.n.qua transacta, tot diebus vtrique ambulãdum erit, quot fuerũt dũ primus secundum superaret, vt totidem alijs primus à secundo superetur, in qua tãmen progressione terminus.u.n.semper duplus est numero terminorum.o.n.ex.95.theoremate, totq; sunt infra.u.n.termini vsque ad.f.quot supra. ex quo illud sequitur omnes terminos aut dies.o.n.f.pauciores esse u.n.vnitate, atque ita præcipit regula detrahendam esse vnitatem ex numero milliarium diurnorum primi viatoris, si dierum numerum habere voluerimus.
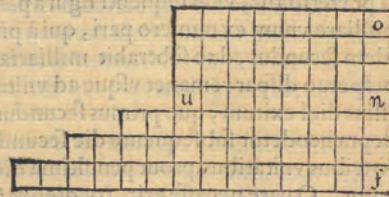
## THEOREMA CX.

SED si in eiusmodi progressione numerus milliarium diurnorum primi viatoris dispar fuerit, patet quòd primus secundum numero dispari superabit, donec ad vnitatem perueniatur vicissimq; primum secundus, inchoando ab vnitate, quare nulla vnquã die paria milliaria vterque conficiet, sit itaque vltima dies, qua primus secundum vnitatem antecedit.u. n. qui terminus duplus est numero terminorum o.n. & cum illa die primus secundum milliario antecedat, sequente
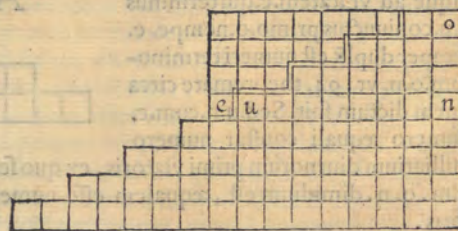
te verò à secundo milliario vno primus antecedatur , ex communi scientia necesse
est secundum tot diebus cũ primo iter agere quot sunt .o.n.qui simul æquales erunt.
u.n.sed.u.n.minor est numero milliarium diurnorum primi vnitate. e. Itaque rectè
sequemur regulam , quæ iubet ex numero milliarium vnitatem demere, quo nu
merum dierum habere possimus.

## THEOREMA CXI.

SI verò secundi viatoris progressio per ternarium ascenderet, sumpto initio ab
ipso ternario, animaduertendum est an numerus milliarium diurnorum primi,
ternario mensuretur nec né, etenim si mensuretur , tandem aliquando paria millia-
ria conficient,quæ dies sit.u.n.quare sub
u. n. totidem quot supra termini erũt,
& cũ. o.n.tertia sit pars.u.n.ex.95.theo-
remate . Itaque tota. o. f. minor erit
duabus tertijs. u. n . vnitate, vt iam re-
ctè sumendæ sint duæ tertiæ partes.u.n.
ex quibus vnitas detrahatur supersitque
numerus. o.f. dierum quæsitorum.
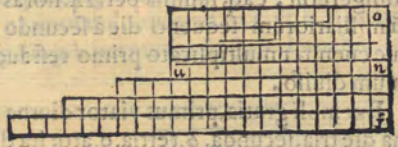
## THEOREMA CXII.

CVM verò milliarium numerus p rimi viatoris metiri non poterit à numero
ascendente secundi,patet nullam futuram diem qua pari milliaria conficient,
quare illa vltima qua primus secundum antecedet, vno aut duobus milliaribus an-
tecedet in præsenti casu. Antecedat itaque duobus milliaribus, sitq; dies.u.n.& alte
ra.t.i.secundus primum vno milliari superabit , ita quod sub.t.i. non poterunt plu-
res integros dies iter agere, quam ambulauerunt ante diem. u.n. hoc est vsquequò
secundus iunctus sit primo, qui numerus dierum , tertia parte.o.n.ipsius.u.n.vnitate
minor erit,cum ex. 95.theoremate.o.n.sit tertia pars. u.n.ex quo numerus. o. f. ter-
minorum aut dierum intergrorum cognitus erit , qui si cum numero ascendente
cognoscetur, statim ex. 99. theoremate deueniemus in cognitionem vltimi diei in
tegri.s.f.atque ita etiam totius summæ progressionis ex. 95.theoremate. Iam verò
cognito numero milliarium diurnorum primi,simul cum numero terminorum, aut
dierum consequenter nouerimus rectanguli summam, hoc est productum à primo
viatore formatum, quarum duarum summarum in præsenti casu semper ea , quæ
huiusmodi producti est, maior erit, cum constitutum fuerit secundum viatorem à
primo superari ipsa die.u.n.vno milliari amplius quam sequente die.t.i.primus à se
cundo superatur, tum pari gradu iter egerunt sub.t.i. quo supra.u.n. ambulauerant.
Hoc animaduertendo, quòd si summa progressionis maior esset rectangulo, ex ea
summa necesse esset numerũ mil
liarium vltimi termini in summa
inclusi detrahere, & residuo ope-
rari. Nunc verò summam pro-
gressionis ex summa rectanguli à
primo viatore facti subtrahi de-
bet,residuumq; seruari voceturq;
pri .

     **IO. BAPT. BENED.**
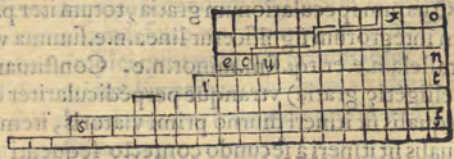
primũ refiduũ. Ad hæc numerũ milliariorũ, quæ fecũdus viator die feqẽti.s.f.confi
ciet fumaf, ex quo numerus milliariũ diurnorũ primi detrahatur, refiduũq́; pariter re
feruetur, voceturq́; fecundum refiduum, poftmodum numerum milliarium primi
vnius diei multiplicetur per primum refiduum feruatum, productumq́; per fecundũ
refiduum diuidatur.a.c.proueniés (q́ erit iter primi in fequenti die) iungatur refi-
fiduo primo, tot enim erunt milliaria conficienda a fecundo fequenti die, vt
fefe confequantur.     THEOREMA CXI.

Vt autem fciamus quantam partem diei feqẽtis, fingulos itinere agere oporteat,
proueniés per. 24. horas multiplicetur (fuppofito quod ambulãtes nullã requiẽ nec
die nec nocte capiãt), pductũq́; p numerũ milliariorum vuius diei primi viatoris di
uidatur, ex quo dabitur quãtitas horarũ, & pars horæ, qua cuiq; illa die ambulandũ
eft. Idem accideret fi primum refiduum referuatum cum proueniente in fummam
colligeretur, eaq́; fumma per. 24. horas multiplicaretur, productũq́; per nume-
rũ milliariorum fequenti die à fecundo conficiendorum diuideretur. Idipfum quo-
que eueniret multiplicato primo refiduo per. 24. & producto per fecundum refi-
duum diuifo.

Exempli gratia, primus viator diurna milliaria vndecim conficit, fecundus, pri-
ma die tria, fecunda. 6. tertia. 9. atq; ita deinceps, diuidatur ergo. 11. per. 3. vnde
pro numero.o.n.dabitur. 3. fupereritq́;. 2. quare:u.n.ab.e. n. duobus milliaribus fu-
perabitur, et.i.t.dictum. e.n. vno milliario, ex quo ante diem.e.u.n. duobus diebus
iter egerunt, totq́; diebus ambulandum erit poft.t.i. hoc eft. 6. in vniuerfum inte-
gris. Ad hęc multiplicato.o.f.hoc eft. 6. per.x.o.hoc eft. 3. habebimus.s.f. milliariorũ
18. tũ cõiũcto.x.o. primo termino hoc eft. 3. cũ.s.f. hoc eft. 18. vltimo termino, habe
bimus. 21. quo multiplicato cũ dimidio.o.f. hoc eft. 3. habebimus totam fummam
progreffionis. 63. fex dierum integrorum ex. 94. theoremate, tum multiplicato. 11.
nempe numero milliariorũ diurnorum primi cum. 6. hoc eft cum.o.f. habebimus pa
rallelogrammum à primo fex diebus integris confectum milliariorum. 66. ex quo
detracta. 63. fumma inquam progreffionis, fupererit pro primo refiduo. 3. fumpris
poftea milliaribus. 21. pro itinere, quod fecundus die fequenti.s. f. conficeret, & ex
ijs detracto numero milliariorũ diurnorum primi, nempe. 11. fecundum refiduum
erit. 10. quod pro diuidenti feruabitur. Iam multiplicato. 11. cum primo refiduo.
3. dabitur. 33. qui diuifus per. 10. fecundum refiduum profert. 3. cum tribus decimis,
eritq́; iter à primo viatore fequenti die conficiendum, hoc etiam ipfum proueniens
cum primo refiduo. 3. coniunctum, dat. 6. cum tribus decimis, quod eft iter fecundi
viatoris illa fequenti die. Ad inueniendam autem quantitatem diei, qua vtriqe
ambulandum eft, perinde erit multiplicare proueniens. 3. & tres decimas per. 24. ho
ras, & productum per. 11. dimidium iter primi viatoris partiri, ac multiplicare fum
mam. 6. & tres decimas cum. 24. horis, productumq́; diuidere per. 24. hoc eft per iter
fecundi viatoris fequentis diei, vtrinque enim femper feptem horæ cum. 12. m. n1
tis prouenient. Idipfum accidet multiplicato per. 24. horas primo refiduo. 3. pro-
ductoq́; diuifo per fecundum refiduum. 10.      fumma neceffe effe numerum mil

Quarum fpeculationum gratia, totum iter parallelogrammi primi viatoris die-
rum integrorum fignificetur linea.n.e. fumma verò progreffionis fecundi linea.f.m.
parallela.n.e. eritq́;.f.m.minor.n.e. Conftituamus deinde à termino, f. n. (maioris
intelligẽtię gratia) vtranque perpẽdiculariter duci, pducatur deinde.n.e. donec. e.
d. æqualis fit itineri diurno primi viatoris, item etiam producatur.f.m, donec. m. x.
æqualis fit itineri à fecundo confecto fequenti die vltimum integrum progreffio-
                          nis,

nis, ex quo.m.k. prolixior erit.e.d. ex præfuppofito.Poftmodum.m.e.et. k.d.dua-
bus lineis rectis coniungantur, quæ productæ concurrent in puncto.b. ducatur pari-
ter.e.g.à puncto.e.parallela.b. k.et.m.a:e. h.et.b.q. parallelæ.f.n.ex quo.f.m. æqua-
lis erit.n.a.et.m.h:a.e. et.h.q:e.o. et.g.k: e.d. et.f.q:n.o. ex.34.primi Eucli. vnde pro
portio.m.h.ad.h.q. erit vt.m.g.ad.g.k. quandoquidem vtraque æqualis eft propor-
tioni.m.e.ad.e.b. ex. 2 fexti, fed cum.m. k. et.g.k. notæ fint, pariter cognofcetur. m.
g. fecundum refiduum, cum etiam notæ fint.n.e.et. n.a. Itaque cognofcemus.a.e. hoc
eft.m.h. cognitis verò, m.g:g.k. et.m.h.ex.15. fexti aut.20. feptimi cognofcetur. h.
q. erit igitur.a. e.aut quod idem eft.m.h primum refiduum,et. m.g. fecundum, et. h.
q. aut.e.o. proueniens, et.n.o. et.f.q. itinera vtriufque viatoris inter fe æqualia.
Nec verò prætermittenda eft fpeculatio vltimæ rationis inueniendæ quantitatis
diei, quæ conftat ope diuifionis producti.m.h.in.24. per. m. g. Ea autem eiufmodi
eft. Probatum fuit fic fe habere.m.h.ad. h. q.ut.m.g.ad.g.k. Itaque componendo
fic fe habebit.m.q.ad.h.q.vt.m.k.ad.g.k.& permutando.m.q.ad.m.k.vt.h. q. ad. g.
k. Sed cum fic fe habeat.m.h.ad.h.q.vt.m.g.ad.g.k. permutando fic fe habebit. m.
h.ad.m.g.vt.h. q. ad. g. k. itaque
ex.11.quinti ita.m.h.ad.m.g.vt.
m.q.ad.m.k. ex quo permutando
m.h.ad.m.q.vt.m.g.ad. m. k. fed
cū.m.k. fit motus toti diei refpon
dens, fecurè dicere poterimus, fi
m. g. talis eft refpectu. horarum.
24.fignificatarum per.m.k. qualis
erit.m. h. & quo
tæ parti diei re-
fpondens : quæ
poftmodũ erit.
m. q. quæ,vt di-
ctū fuit,talis eft
refpectu. m . k.
qualis. m. h. re-
fpectu.m.g.Reli
quę duæ fpecula
tiones priorum
modorũ, vna &
eadem eft,facilisq; per fe mediocriter intelligenti. Eodem modo reliquæ omnes
progreffiones fecundi viatoris cū rectangulo, primi conferri ex hoc theoremate
poterunt.

## THEOREMA CXIII.

PRoponitur & aliud, primum fcilicet viatorem iter incipere diebus aliquot an-
tè fecundum, primum tamen lentius,quàm fecundum ambulare, & vtrunque
eorum certa quædam milliaria conficere. Iam fi fcire voluerimus in quot diebus
fefe confequentur, vulgaris regula iubet, infpici quot milliaria primus folus iter a-
gens confecerit, tum animaduerti differentiam diurnam motus vnius ab altero,atq;
milliarium numerum primi viatoris foli abundantis per hanc differentiā diuidi,pro
ueniens autem erit numerus dierum quæfitus.
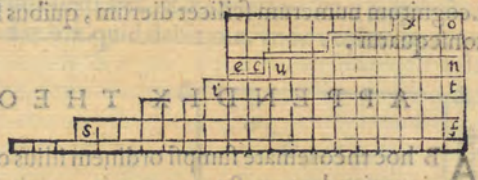
K                    Exempli

Exempli gratia, si primus octo diebus antequam secundus iter arripuisset, confecissetq́; singulis diebus.20.milliaria, tum secundus.25.quotidie perfecisset, multiplicandus esset numerus.8.cum. 20. ex quo darentur. 160.milliaria à primo solo ambulâte confecta, quibus diuisis per.5.differentiam motuum diurnorum, daretur. 32.numerus quæsitus dierum.

Cuius ratio apertissima est. Sint enim duo rectanguli.a.n. et. u.i. æquales inter se, quibus motus itinerarium significentur, quorum.a.n.sit primi,et.u.i.secundi, præterea.a.c.numerum milliarium diurnorum primi.et.u.e.secundi, ex quo. a. c. minor erit.u.e.per.o.e. atque ita. o. e. cognoscetur.Tum.c.o.numerum dierũ primi soli iter agentis denotet, cũq; constituamus.a.n.æqualem esse. u. i. o.i.æqualis erit.o.a.atque.o.a. cognitus ex suis producentibus. a.c.et.c.o. itaque.o.i.etiam cognitus, qui diuisus per latus cognitum.o.e. dabit. e.

i.cognitum numerum scilicet dierum, quibus secundo ambulandum est, vt primum consequatur.
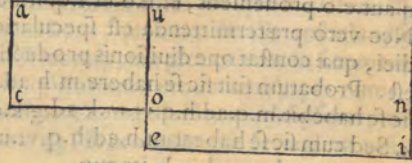
## APPENDIX THEOREM. CXIII.

A B hoc theoremate sumpsi ordinem illius operationis, numeris mediantibus, ad inueniendam exactam temporis quantitatem, seu interuallum, q̃ transit, vel intercedit inter vnam mediocrem coniunctionem & aliam proximam sequentẽ duorum planetarum, vt patet in epistola nostra ad Illustrem Bernardum Trottum contra Benedictum Altauillam repræhensorem Ephemeridum. Verùm tamen est q̃ cum praxis huiusmodi theorematis sit multiplex, visum fuit vnam proponere, quę non ita perspicua sit, sed subobscura, non quòd aliquid voluerim latere illum amicum mihi dilectissimum, cui priuatim omnes modos prius ostenderam, sed vt cerebrum illius mei aduersarij in laberintum conijcerem inextricabilem vt feci, quamuis modus ille egregius etiam sit, vt nunc ostendam .

In dicta epistola igitur mente cogitaui medium motum tardioris planetæ, puta saturni, illius temporis quo velocior planeta, scilicet Iupiter, percurrit suo medio motu totum zodiacum , incipiendo ambo eodem temporis puncto, nec non ab vna eorum media coniunctione, hoc est ab eodem zodiaci puncto, in quo coniunctę fuerunt eorum lineæ mediorum motuum , vbi inueni vi regulæ de tribus, quòd Saturnus spacio dierum vnius mediocris reuolutionis Iouis, qui sunt. 4328. progreditur medio motu gra.145.min.4.hoc est min.8704.posito quòd ipse Saturnus perficiat vnam mediam reuolutionem spacio dierum. 10746. vt dixi. Incipiendo igitur iterum Iupiter aliam reuolutionem percurrere, reperto Saturno per min.8704. ante ipsum spacio.4328. dierum, certus eram hos dies significatos esse à linea.a.u. vel.c. o.(æquales enim inuicem sunt ) in figura huiusmodi theorematis, & quòd rectangulum.a.o.præbebat summam graduum.145.min.4.hoc est min. 8704. et quòd. a. c. vel.o.u.significabat iter vnius diei ipsius Saturni , et.u.e.iter vnius diei Iouis. Cogitemus nunc.u.x.significari dies.30.& à puncto.x.productam esse.x.f.parallelam ipsi u.o.e.vnde certi erimus rectangulum.e.x.significare iter Iouis spacio temporis dierum.30.rectangulum verò.o.x.iter Saturni eodem temporis interuallo, vnde rectan-
gulum.

gulum.e.x.erit minutorum.149.& secundorum.43.et.o.x.minutorum.60.& secun.
20.vt in dicta epistola, vnde rectangulum.o.f.erit min.89.& secun.23.& quia re-
ctangulum.o.i. æquale est rectangulo.a.o.ergo.o.i.similiter continebit min.8704.
Nunc quia.a.c.vel.o.u. denotat iter vnius diei Saturni et.u.c.vnius diei Iouis vt di-
ximus ergo.u. o. erit minutorum. 2. secun. o. & tertiarum. 40. videlicet tertiarum.
7240.supposito periodo totali ipsius Saturni dierum.10740.et.u. e. erit minutorū.
4. secun.59.& ter.27.vel circa hoc est tertiarum.17967.vnde. o. e. erit tertiarum.
10727. Nunc si dixerimus cum.o.e.tertiarum.10727.dat.o.u. vel.a.c. (nam tam
vna quam altera est tertiarum.7240.) quid dabit.a.u.vel.o.c. (quia tam vna quam
altera est partium.4328.) clarum erit quod dabit.o.n.vel.u.t.uel. e.i. quia tam vna
quam altera erit partium.2921.quæ partes coniunctæ cum fuerint cum partibus ip-
sius.a.u.dabunt totam.a.t.partiū.7249.quæ erunt tot dies, hoc est periodus quæsita.

  Alia methodo similiter possumus idem cognoscere, scilicet dicendo si rectangu
lum.f.o. quod est minutorum.89. & secun.23.hoc est secundorum.5363.dat rectan
guluni.o.x. minutorum.60.& secun.20.hoc est secun.3620.quid dabit.a.u.partium
4328.vnde veniet.u.t.partium.2921.similiter, eo quod eadem proportio est rectan
guli.f.o.ad.o.x.quæ.e.o.ad.o.u.ex prima sexti, vel.18.19.septimi seu.15. quinti.

  Posset etiam aliquis dicere si. f.o.dat.o.x.quid dabit.o.a. vnde veniet.o.t. quo
diuiso per.o.u. daret. u.
t. quia ita se habet. a. o.
ad.o.t.vt.a.u.ad. u. t. ex
supra hic iam citatis.

  Sed ego, in dicta epi-
stola, aliam methodum
obseruaui, quæ est multi
plicando minuta. 8704.
per. 30.productumq; di
uisi per min.5363.quasi
dicens. Si.o. f. dat. o. i.
quid dabit.e.f.Vnde ex iam supradictis propositionibus veniet. e. i. & quia permu-
tando ita se habet.o.f.ad.e.f.vt.o.i.ad.e.i.ideo dixi,si min.89.cum secun.23.dat.30
quid dabit min.8704.

## THEOREMA CXIIII.

PRoponunt veteres & quærunt aliud, nempe si duo iter agentes, eodem in-
   stanti diuersis è locis proficiscantur, ita vt vnus locum vnde alter profectus
est petat, alterq; altero velocior sit, quo loco quáue die sibi inuicem occurrent.
  Exempli gratia, Patauio profectus quidam Taurinum petit, eodem instanti al-
ter Taurino Patauium, estq; iter.400.milliarium, ille tamen vndecim diebus, hic
9.motu regulari & vniformi appellit. Quærimus quot milliaria quisque confece-
rit, quotq; diebus iter egerit, priusquam sibi occurrant.
  Iubent nos veteres dies vtriusque inuicem inter se multiplicare, eritq; produ-
ctum.99.item etiam in summam colligere, eritq; summa.20.per quam productū.
99.diuiserimus dabuntur dies.4.cum.19. vigesimis vnius diei. At pro milliaribus
vtriusque, pro eo qui.11.diebus iter conficit, multiplicatis.400.per.4.et.19.vigesi
mis, tum diuiso per.11.dabitur numerus.180.à Patauio Taurinum & è contra, qui

                                            K  2    Tau-

Taurino Patauium. 2 2 0. quæ quiſque confecerit.

   Dum autem hæc ſpecularer attentius, occurrit alius ſoluendi modus, quàmuis pro-
lixior. Is aût eſt eiuſmodi. Accipiat medietas minoris numeri dierũ, népe. 4. cũ dimi-
dio, & per. 4 0 0. multiplicetur, productũ q̄; per maioré numerum diuidemus ſcilicet
1 1. ex quo dabuntur. 1 6 3. cum. 7. vndecimis, quo proueniente è dimidio milia-
riorũ itineris. 2 9 0. detracto, & preſiduũ népe. 3 6. cũ. 4. vndecimis multiplicato pro-
ducto q̄; diuiſo, p ſummã dimidij itineris. 2 0 0. cũ primo prouétu. 1 6 3. et. 7. vndecimis
népe p. 3 6 3. et. 7. vndecimas partes ,pueniet. 1 6. cũ. 4. vndecimis, quo cóiuncto pri-
mo ,puenicti, primus. 1 8 0. milliaria cófecerit, quæ è. 4 0 0. detracta ſupererunt. 2 2 0.
pro itinere ſecundi, qui. 9. diebus iter abſoluit. Ad hæc ſi tempus ſcire velimus
eius, qui. 1 1. diebus appellit, multiplicabimus. 1 1. cum. 1 8 0. productumq̄; per. 4 0 0.
partiemur, prouenientq̄; paulominus, quam quinque dies, nempe. 4. cum. 2 2. horis
et. 4 8. minutis, quod tempus vtrique viatori inſeruiet, quandoquidem idipſum pro-
uenit multiplicato. 2 2 0. per. 9. productoq̄; per. 4 0 0. diuiſo.

   Huius autem, qui à me preſcribitur modi, ſpeculatio talis eſt. Duo termini duabus
rectis lineis æqualibus, & parallelis inter ſe. b. p. et. d. q. ſignificentur, quæ alijs dua-
bus. b. d. et. q. p. coniungant, que parallelæ & æquales erunt ex. 3 3. primi, quibus ſigni-
ficentur duo itinera. Viator primus quidem lentior à. b. in. d. velocior à. q. in. p. Iam
ſumatur punctũ medium. q. p. ſit q̄;. k. & ab ipſo ad. b. d. ducatur. k. i. parallela. d. q. aut
b. p. quod idem eſt, ex quo. b. i. æqualis erit. p. ĸ. ex. 3 4. primi, hoc eſt. q. k. certiq̄; eri-
mus primum viatorem. q. p. in dimidio itineris. q. k. occurrere non potuiſſe viatori ip-
ſius. b. i. quandoquidem eo tempore, quo is, qui ipſius. q. p. mouetur per. q. k. (cum ſit
altero velocior) qui per. b. d. nondum peruenerit ad. i: Sit itaque punctum. c. in quo
lentior reperitur, dum velocior eſt in. ĸ. ex quo certi erimus eos inter. c. et. i. ſibi in-
uicem obuiaturos eſſe. Cogito deinde rectam lineam ductam. ĸ. c. & ut ſe habet. i.
c. ad. c. b. ita cogito ſe habere. u. ĸ. ad. ĸ. q. & à puncto. u. ad. i. duco. u. i. quæ, vt maniſe-
ſtum eſt, lineam. ĸ. c. in puncto. e. interſecabit, à quo cum fuerit ducta. e. o. n. parallela
ĸ. i. habebimus. o. n. ea ſcilicet puncta, quibus occurrunt ſibijpſis, nam cum ſic ſe ha-
beat. q. k. ad. ĸ. u. vt. b. c. ad. c. i. et. ĸ. u. ad. ĸ. n. vt. c. i. ad. c. o. ex ſimilitudine manifeſta
triangulorum, ex æqualitate proportionum ſic ſe habebit. q. ĸ. ad. ĸ. ñ. vt. b. c. ad. c. o.
& permutando ita. ĸ. q. ad. b. c. vt. ĸ. n. ad. c. o. & cum. q. ĸ. et. b. c. ſpatia ſint tempori-
bus æqualibus confecta, itaque ſpatia. ĸ. n. et. c. o. ex communi ſcientia temporibus
æqualibus conficientur.

   Quare recte dicimus, ſi tot diebus à. b. in. d. aliquis peruenit, quot milliaria in di-
midio temporis alterius viatoris idem conficiet? ex quo ex regula de tribus quam
primum iter. b. c. cognoſcitur, quo ex dimidio itineris detracto, remanet. c. i. cogni-
tus, ſed cum probauerimus. q. ĸ. ad. ĸ. n. hoc eſt. i. o. (cùm ſint æquales inter ſe, ex. 3 4.
primi) ita ſe habere. vt. b. c. ad. c. o. permutando ſic ſe habebit. q. ĸ. ad. b. c. vt. i. o. ad.
o. c. & cóponendo, q. ĸ. et. b. c. ad. b. c. vt. i. c. ad. c. o. quare recte dicimus ſi ſumma. q.
ĸ. cum. b. c. dat. b. c. quid dabit. i. c? nempe dabit. c. o. quo coniuncto cum. b. c. cogno-
ſcitur. b. o. quo. b. o. detracto ex. b. d. remanet cognitus. o. d. nempe. q. n. illi æqualis
ex. 3 4. prædicta. Gratia vero téporis patet nos recte dicere ſi. b. d. tot diebus abſolui-
tur, aut etiam. q. p. quo. b. o. aut. q. n. abſoluetur.

   Vt autem ad ſpeculationem regulæ antiquorum deueniamus, cogitemus pri-
mum viatorem ipſius. q. p. velociorem eo, qui per. b. d. iter agit, tanto tempore præ-
tergredi. p. quanto alter. b. d. abſoluit. Is autem ad. g. pertingat, ex quo eadem pro-
portio ſpacij. q. g. ad. q. p. hoc eſt. b. d. dabitur, quæ temporis quo. b. d. abſoluitur ab-

                       co

eo qui per. b. d. ad tempus quo. q. p. folum, qui per. q. p. mouetur (motus enim continui regulares & vniformes conftituuntur) eadem ratione itaque ea erit proportio. q. κ. ad. b. c. quæ. q. g. ad. q. p. & cum probatum fuerit ita fe habere. κ. n. ad. c. o. vt. q. κ. ad. b. c. itaque fic fe habebit. κ. n. ad. c. o. ut. q. g. ad. q. p. probatum etiam fuit ita fe habere. q. κ. ad. κ. n. vt. b. c. ad. c. o. ex quo componendo fic fe habebit. q. n. ad. n. κ. vt. b. o. ad. o. c. & permutando ita. q. n. ad. b. o. vt. κ. n. ad. c. o. hoc eft. q. g. ad. q. p. nempe vt tempus lenti ad tempus velocis itinerantis, & componendo ita. q. n. cum. o. b. hoc eft. b. d. ad. b. o. vt fumma dierū vnius & alterius viatoris ad minoré numerū dierū velocioris. Breuiter itaq; obtineremus in tentū qñ diceremus fi fumma dierum, quibus iter agitur à viatoribus talis eft (20) refpectu numeri dierum velocioris (9) qualis & cui refpōdebit totum fpacium. b. d? vnde dabitur fpacium. b. o. vnde reliqua omnia nobis cognita emergent.

Cum autem antiquorum regula iubeat numerum dierum vnius, cum numero dierum alterius multiplicari, ac poftmodum diuidi productum per fummam omnium dierum, recte id quidem fit. Nam cum fic fe habeat. b. d. ad. b. o. vt fumma omnium dierum ad minorem quantitatem dierum velocioris fcilicet. Ideo temporis proportio à mobili per. b. d. abfumpti ad tempus mobilis per. b. o. eadem erit, quæ fummæ omnium dierum ad numerum dierum velocioris. Quare recte dicemus, fi eiufmodi fumma talem refpectum habet ad minorem numerum dierum, quem numerum refpiciet dies ipfius. b. d? ex quo proferentur dies refpondentes ipfi. b. o. cætera iam dicta fuerunt.

Huiufmodi verò fpeculationis amplitudo ad pauciffima verba reduci poteft, in cuius gratiâ fit fubfcripta figura pars inquâ precedentis, in qua côftituam°. o. n. locū eū effe quo fibi viatores obuient, ex quo fpacium. q. n. à fuo viatore conficietur, eo ipfo tempore, quo à fuo fpacium. b. o. itaque eadem erit proportio. q. n. ad. b. o. quæ. q. g. ad. b. d. eadem erit inquâ proportio. d. o. ad. o. b. quæ numeri dierum eius, qui à. b. pergit in. d. ad numerum dierum alterius qui à. q. in p. proficifcitur, & componendo eadē erit proportio. d. b. ad. b. o. quæ fummæ dierum ad minorem numerum ipforum, & eadem quæ dierum. b. d. ad dies ipfius. b. o.
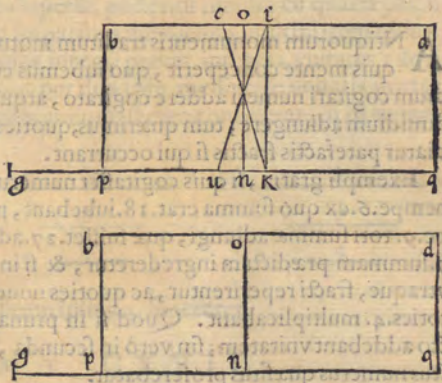
## THEOREMA CXV.

CIRCA hæc ipfa itinera aliud quæritur peruenufte, in quo quæfito illud conftituitur cognitum effe, nempe interuallum inter duo diuerfa loca, è quibus duo viatores eodem inftanti vt fibi occurrant proficifcuntur, certaq; milliaria fingulis diebus conficiant, ita tamen, ut unus ordinatè plura altero ambulet, quæritur deinde quoto die fibi occurrent. Hoc autem fit diuifo toto interuallo locorum per fummam milliariorum quam vterque quotidie abfoluit.

Exem-

78　　　I O. B A P T. B E N E D.

Exempli gratia, diſtant loca. 100. milliaribus à ſe inuicem ; vnus autem viator ſingulis diebus. 15 milliaria, alter. 10. conficit ſi itaque. 15. cum. 10. coniũgamus, ſumma erit. 25. per quã diuiſis milliaribus. 100. tótius interualli proferetur. 4. numerus quæſitus dierum quo viatoribus iter agendum erit priùs quam ſibi obuient.

In cuius ſpeculationis gratiam totum iter ſignificetur linea. a. u: primi autem viatoris iter diurnum ſit. a. e. & alterius. u. o: terminus verò. i. ſit occurſus ita vt eodem tempore, alter ſpacium. a. i. alter. u. i. contecerit, ſpacij autem. a. e. tempus per. b. ſignificetur & tempus ſpacij. u. o. per. c. quæ tempora erunt inter ſe æqualia, porrò ſpacij. a. i. tempus per. d. & ſpacij. u. i. per. f. denotetur, æqualibus inquam, ex quo eadem proportio erit. a. e. ad. a. i. quæ. b. ad. d. et. o. u. ad. u. i. quæ c. ad. f. vnde permutando eadem erit proportio itineris ipſius. b. ad iter ipſius. c. quæ itineris. d. ad iter ipſius. f. & componendo itinerum ipſius. b. c. ad iter. c. vt itinerum. d. f. ad iter. f. & permutando itinerum b. c. ad itinera. d. f. vt itineris. c. ad iter ipſius. f. meritò itaque quæritur ſi itinera. b. c. dat itinera. d. f. quid dabit tempus. c. nempe dabit tempus. f. ſed. c. ſignatum eſt pro vna die, quare in propoſito exemplo. f. ſignificabit 4. dies.

## T H E O R E M A　C X V I.

ANtiquorum monumentis traditum motum reperimus diuinandi numeri quem quis mente conceperit, quo iubemus eum qui numerum cogitauerit, dimidium cogitari numeri addere cogitato, atque huic ſummæ, rurſus eiuſdem ſummæ dimidium adiungere, tum quærimus, quoties noueratius tótam eam ſummam ingrediatur patefactis fractis ſi qui occurrant.

Exempli gratia, ſi quis cogitaſſet numerum. 12. iubebant huic dimidium addi, nempe. 6. ex quo ſumma erat. 18. iubebant, præterea dimidium huius ſummæ nempe. 9. toti ſummæ adiungi, quæ fuiſſet. 27. adhæc quærebant ſibi patefieri quoties. 9. ſummam prædictam ingrederetur, & ſi in prima aut ſecunda diuiſione aut etiã vtraque, fracti reperirentur, ac quoties nouem vltimam ſummam ingrediebatur, toties. 4. multiplicabant. Quod ſi in prima diuiſione fracti erant, vltimo producto addebant vnitatem; ſin verò in ſecunda, binarium adiungebant, ex quo exactus numerus quæſitus proferebatur.

Pro cuius rei ratione ſit. a. numerus cogitatione compræhenſus. et. e. ipſius. a. cum eiuſdem medietate ſumma. et. i. ipſius. e. cum eiuſdem medietate itidem ſumma, vnde. i. e. a. tres numeri continui proportionales, in ſeſquialtera proportione cuadent. Sumantur nunc tres numeri. 4. 6. 9. in eadem proportionalitate. Vnde ratione æqualitatis proportionum ita ſe habebit. i. ad. a. quéadmodum. 9. ad. 4. & permutando. i. ad. 9. quemadmodum. a. ad. 4. & ob id. 4. toties ingredietur. a. quoties. 9. ipſam. i. Sed quia ſæpe contingit, vt in ſecunda diuiſione, aut in ambabus etiam diuiſionibus reperiantur numeri fracti, animaduertendum eſt numerum animo compræhenſum. a. ſcilicet aut parem aut imparem ſemper futurum. Si par eſt, aut multiplex erit ad. 4. aut non. Si priori modo ſe habebit in duabus diuiſionibus, nullus numerus fractus admittetur; ſed ſi ad. 4. multiplex non erit, à multiplicibus per duo ſemper differet, & ſi per medium diuidatur, eiuſdem medietas impar ſemper erit, vnde prior

-xem-　　　　　　　　　　　　　　　　　　　　　　　　　　　quo-

quoque summa par nunquam existet, cuius medietatem aliquod medium semper ingredietur, & hanc ob causam posterior summa cum fracto semper erit, & numerum desumptum maiorem esse multiplici ad quatuor per duo significabit.

At verò si inter impares reponatur, aut eorum erit qui superant multiplicem ipsius quatuor per vnum, seu per tria, quod hinc innotescet, nempe, quia si eorum erit qui dictum multiplicem per vnum tantum vincunt, sua medietate ipsi numero addita, & præter hanc medietatem medio etiam integro adiuncto, tota hæc prior summa in numerum parem semper euadet, vnde in posteriori summa nullus numerus fractus conspicietur, & hanc ob causâ multiplici ipsius. 4. vnitas semper addetur.

Sed si numerus desumptus, in serie eorum, qui multiplicem ipsius. 4. per tria superant, collocabitur, hinc compræhendetur, quia primæ summæ numerus cum media vnitate semper impar erit, vnde secunda summa præter integras cum media vnitate nobis semper occurret.

Quod autem nobis prodere faciamus an in prima diuisione, & secunda numerus aliquis fractus consistat, eò tantum nobis inseruit, quò deueniamus in cognitionem an numerus animo conceptus multiplicem ipsius. 4. per vnum, per duo, aut tria supe ret. Quòd etiam medias eas vnitates ad integros reducere faciamus, eò tantum re fertur, vt minori labore eum, qui numerum imaginatione compræhendit, oneremus, quia reuera numerus impar nunquam mente concipi potest, quin aliquis fractus in prima diuisione, aut in secunda sequatur: vnde à numeris imparibus, qui multiplicem ipsius. 4. vnitatis tantum excessu superât, posterior summa cũ quarta parte vnitatis, præter integros numeros, & ab imparibus qui dictum multiplicem ipsius. 4. per tria vincunt, cum tribus quartis vnius integri præter integras vnitates; & à numeris paribus, qui multiplicem ipsius. 4. per duo cum medietate vnitatis, præter integros semper procedit. Ita cum is qui numerum secum considerat, si in numeris fractis versatus esset, qui eum interrogat prudenter se gereret, si sibi declarari curaret, quis nam ex fractis super integros secũdæ sũmæ remane ret, quia p quot quarta integros secũ dæ summæ superaret, per totidê inte gros numerus mente conceptus multiplicem ipsius. 4. superaret.

$$\frac{9}{6}\frac{e}{4}$$

## THEOREMA CXVII.

VNDE fiat, vt si aliquis quemuis numerum animo compræhendat, eique numero alium etiam quemlibet numerum propositum addat, & à tertia par te huius summæ tertiam partem numeri imaginati detrahet, residuum secundi numeri adiuncti, idest propositi, tertia pars erit.

Vt exempli gratia, si aliquis de numero denario cogitasset, huicq; 24. adderet, vnde triginta quatuor efficerent, detrahendo nunc tertiam partem numeri de narij cogitatione concepti, idest. 3. cum tertia parte vnius, à tertia parte huius sum mæ idest ab vndecim & vna tertia parte remanerent. 8. idest tertia pars numeri additi. Id quod mihi inter iocos in honestorum hominum cætu in mentem venit.

Pro cuius ratione, prior numerus ima ginatus mediante linea. a. b. et is, qui additus est intercedête linea. b. d. è directo con-

coniunctis denotetur, et.b.e.sit tertia pars ipsius.a.b.prioris numeri imaginati,et.b
c.tertia pars ipsius,b.d.secundi numeri propositi, vnde coniunctum vnius harum ter
tiarum partiũ cũ alia sit.e.c. quod quidem. e.c.esse tertiam partem summæ duorum
primorum idest.a.d.assero. Iam manifestum est ipsius. d.b. ad.b.c.esse quemadmo
dum ipsius.a.b.ad.b.e.vnde vicissim ipsius. d.b.ad.b.a. erit quemadmodum ipsius.b.
c.ad.b.e.& coniunctim ipsius.d.a.ad.a.b.quemadmodum ipsius.c.e.ad.e.b. & vicis-
sim ipsius.d.a.ad.c.e.quemadmodum ipsius.b.a.ad.b.e.sed proportio ipsius.b.a. ad.
b.e.est tripla, ergo ea quæ est ipsius.a.d.ad.e.c. erit quoque tripla; vnde sumendo. e.
c.pro tertia parte ipsius.a.d.& ab ipsa .e.c.subtrahendo tertiam partem ipsius . a . b.
tertia pars ipsius.b.d.remanebit.b. c'.
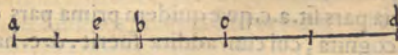
　　Aut alio hoc modo, supponendo.e.c.tertiam partem ipsius. a.d.et.e.b. ipsius. a.
b. exister. Dico.b.c.tertiam partem ipsius.b.d.futuram:quia si totius. a.d. ad totum
e. c. ita se habet,quemadmodum.a.b.à toto.a.d. dissecti atque diuulsi ad.e.b.à toto.
e.c.distractum, ergo ex. 19.lib.quinti Eu-
clid.residui.b.d.totius.a.d.ad residuum.b.c.
totius.e.c.erit,vt totius.a.d.ad totũ. e. c.at-
que hic quidem modus rem propositã spe-
culandi mihi aptior & commodior esse videtur.

## T H E O R E M A　C X V I I I.

**P**Ermulta ac varia problemata inuenerunt antiqui,longioribus verò vijs resolu-
　ta, propterea quòd nõ semper nobis succurrit breuissima in vnaquaque re ex-
plicatio. Vt exempli gratia, proponitur numerus. 50.diuidendus in tres tales par-
tes,quod secunda dupla sit primę,& adhuc eam superet tribus vnitatibus , tertia ve
rò æqualis sit aggregato primæ cum secunda,& amplius ipsum aggregatum superet
quinque vnitatibus.

　　Ad hoc autem quæsitum soluendum antiqui vtebantur regula falsi, quod reuera
breuiori modo potest solui, videlicet detra hendo illud secundum excessum, quin-
que scilicet ex.50.ita vt nobis.45. remaneret,cui medietati hoc est.22.cum dimidia
vnitate,si addiderimus illud quinque habebimus. 27.cum dimidia vnitate pro ter-
tia parte quæsita ipsius numeri. 50. deinde si ab eodem numero.22. cum dimidia
vnitate detractum fuerit illud. 3. primus excessus datus, remanebit. 19. cum dimi-
dia vnitate, cuius tertia pars,hoc est. 6. cum dimidia vnitate, prima pars, ex tri-
bus quæsita erit, quæ quidem si detraxerimus ex.19.cum dimidia vnitate, reli-
quum erit.13.cui cũ additus fuerit primus excessus idest.3.Iam propositum re-
sultabit nobis. 16. pro secunda parte quæsita.

　　Ratio verò huiusmodi operationis talis est, sit verbi gratia totalis numerus pro-
positus significatus per lineam.a.b.cuius secundæ partis numerus datus significetur
per lineam.g.& numerus tertiæ partis propositus per lineam.h.Nunc dempta. h. ex
a.b.nobis cognita,remanebit.f.a. qua quidé per æqualia imaginatione diuisa in pun
cto. e. & ipsi. e. f. addita. f. b. tota. e.b.nobis cognita erit, quæ quidem tertia pars
quæsita ipsius.a.b.erit,propterea quòd.a.e. (quæ æqualis est ipsi. e. f.) erit summa
primæ, & secundæ partis. Detrahatur postea.g.ex.e.a. & remanebit.d.a.cuius ter
tia pars sit.a.c.quæ quidem prima pars quæsita erit , & nunc cognita , & ita.c.d.
cognita, cui cum addita fuerit. d. e. habebimus secundam partem quæsitam, quæ
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　compo-

componitur ex.d.c.dupla. ad. a. c. primam partem, & ex.d. e.numero dato.
tertia verò pars.e.b.composita est ex.
e.f.æquali.a.e.hoc est æquali composito ex prima, & secunda parte, & ex.f.
b. numero dato vt proponebatur.
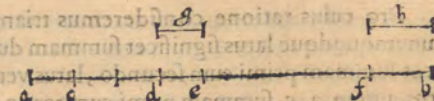
## THEOREMA CXIX.

INter alia problemata ab antiquis inuenta, hoc etiam ponitur. Aliquis interrogat quot sint horæ, alius verò respondit tot esse, quot duæ tertiæ præteriti temporis simul iuncta cum tribus quintis futuri temporis totius diei naturalis efficiunt. Nunc quæritur quot sint horę.

Antiqui, hoc etiam problema soluebant mediante regula falsi, sed mihi alio modo soluendum esse dictum problema videtur. Accipio enim ex quinque, tres vnitates, pro parte futuri temporis, quas quidem in tres vnitates præteriti temporis duco, vnde proueniunt mihi nouem vnitates, quod productum coniungo cũ quinque futuri temporis, vnde veniunt.14.vnitates, ex regula postea de tribus ita dico si ex.14. mihi prouenit. 9. quid resultabit ex. 24. & prouenient mihi horæ. 15.cum tribus septimis vnius horæ, hoc est minuta ferè. 26.

Pro cuius ratione, quinque vnitates, seu partes temporis futuri significentur à linea.e.u.quarum trium significentur à linea.e.i.sumpta deinde sit linea.e.o.æqualis lineæ.e.i. et.e.a.tripla sit ad.o.e.vel ad.e.i.quod idem est, vnde.a.e. composita erit ex.a.o.(hoc est ex duabus tertijs ipsius.a.e.) & ex.o.e.(hoc est ex tribus quintis ipsius.e.u.)vnde.a.u.ad.a.e. eandem rationem obtinebit, quæ.14.ad.9. propterea igitur possumus recte ratiotinari
si.14.nobis dat.9.quid dabit.24.
qui quidem.24. nobis dabit.15.
cum min.26. quod recte factum
erit ex.20.septimi Euclidis.
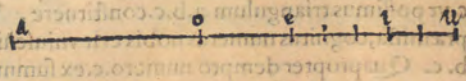
## THEOREMA CXX.

SVpponunt etiam antiqui tres socios nummos habere, quorum summa primi & secundi cognita sit, item summa primi & tertij cognita & summa secundi & tertij item cognita, atque ex huiusmodi tribus aggregatis veniunt in cognitionem particularem vniuscuiusque illorum.

Gemafrisius soluit hoc problema ex regula falsi. At ego tali ordine progredior. Sit verbi gratia, summa primi cum secundo.50.& secundi cum tertio. 70. & primi cum tertio. 60.harum trium summarum accipiantur duæ quæuis, vt puta. 50. &. 70 quæ coniunctæ simul dabunt. 120.à qua summa detrahatur reliqua, idest. 60. & restabit nobis. 60. cuius medietas erit. 30. hoc est numerus nummorum secundi socij quo numero detracto à. 70.hoc est à summa secundi cum tertio remanebit.40. hoc est numerus tertij socij, & hic numerus desumptus à. 60. residuus erit numerus primi socij.

L

Pro

Pro cuius ratione consideremus triangulum hic subnotatum . a . b . c . cuius
unumquodque latus significet summam duorum sociorum, vtputa latus.a. b. signifi-
cet summam primi cum secundo , latus verò.b.c.summam secundi cum tertio , la-
tus autem. a. c. summam primi cum tertio, et.a.e.seu.a.o.sit numerus primi socij, et.
e.b.vel.b.u.sit secundi socij, et.c.u.seu.c.o.sit tertij, cum autem.a. e. æqualis sit. a. o.
et.b.e:æqualis.b.u.et.c.u. æqualis. c. o.
ex supposito si dépta fuerit summa seu
latus.a.c.datum ex aggregato laterum.
a.b.cum.b.c.reliquarum summarum, re
linquet nobis cognitum aggregatum
ex.b.e.cum.b.u. Quare & eius medie-
tas.b.e.siue.b.u.nobis cognita erit,qua
detracta ex summa.b.a. relinquetur no
bis cognitus numerus.a.e. detracto ve-
rò numero.a.e.hoc est.a.o.ex.a.c. sum-
ma,seu latus, aut.b.u.ex.b.c.remanebit
o.c. seu.c.u.cognitus.
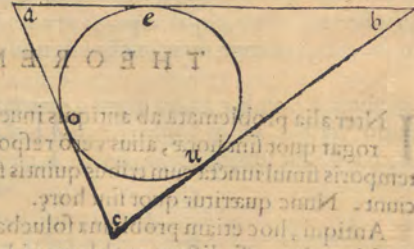
### THEOREMA CXXI.

HAC etiam methodo hoc facere possumus non solũ de tribus socijs, sed etiã
de omnibus quotquot volueris,vt exempli gratia,
sint sex socij.a.b.c.d.e.f. quorum summa per binos co-
gnita,vtputà summa numeri.a.cum.b.cognita nobis sit,
& summa numeri.b.cum.c.& summa.c. cum. d. & sum-
ma.d. cum.e.& summa.e.cum. f. necesse est etiam scire
summam duorum vno relicto, vtputa summam.a.cum
c.vt possimus triangulum.a.b.c.constituere, Vnde ex
præmissa,cognitus numerus nobis erit vniuscuiusque.a.
b. c. Quapropter dempto numero.c.ex summa.c.cum
d.& numero.d.ex summa.d.cum.e.& numero.e.ex sum
ma.e.cum.f.habebimus intentum.

### THEOREMA CXXII.

CVM aliquando, illud quod Archimedes inuenit, vt furtum Regi ab aurifa-
bro in regia corona factum, quemadmodum scribit Vitruuius,proderet, con-
templarer,mihi etiam visum est,vt aliquem modum scientificum inuestigarem,quo
proportio auri ad argentum, quod in aliquo proposito corpore ex ipsis misto cogni
ti ponderis cognosci posset. Et cum multos diuersis temporibus excogitarim offi-
cio meo deesse nolui in ijsdem literarum monumentis mandandis, quorum hic
vnus erit:proposita nobis sint tria corpora.A.M.V.æqualia inter se , sed diuer-
sarum specierum materiei, vtputa quod. A. sit argenteum, & omogeneum. V. ve-
rò aureum omogeneum, & M.mixtum ex auro, & argento, idest heterogeneum,
cupimusergo scire iustà quantitatem auri & argenti, quæ est in ipso corpore.M.
misto. Ita igitur faciamus. Videamus primum quantum sit pondus vniuscuiusque
ipsorum corporum, ponamus autem pondus corporis. V.auri esse vt. 234. pondus
                                                                          autem

autem corporis. M. mifti. vt. 216. argentei verô. A. vt. 156. detrahatur nunc pon-
dus. A. ex pondere. V. Reliquum erit. 78. quod vocetur prima differentia feruan-
da, dematur etiam pondus. M. ex pondere. V. reliquum erit. 18. pro fecunda diffe-
rentia, etiam feruanda, multiplicetur poftea pondus. A. per fecundam differen-
tiam, productum verô diuidatur per primam differentiam. Vnde in præfenti exem
plo proueniet nobis. 36. quiquidem prouentus erit quantitas argenti ipfius corpo-
ris mifti. M. quo etiam detracto ex pondere totali ipfius. M. reliquum erit quanti-
tas auri eius corporis, hoc eft. 180.

In cuius operationis fpeculatione, aliquid natura fua prius cognitum præcedere
oportet hoc eft, quod omnia corpora omogenea eandem proportionem obtinent
inter quantitates, quam inter pondera. Quo fuppofito denotetur corpus. A. li-
nea. o. a. corpus autem. V. linea. o. c. & corpus. M. linea. e. u: fed. e. o. fignificet par-
tem argenti, et. o. u. partem auri in corpore mifto. M. vnde ex communi conceptu
habebimus. o. e. æqualem. u. c. cùm ex hypothefi. e. u. æqualis fit. o. e. et. a. o. fimiliter.
Significetur poftea pondus. a. o. ab. f. & pondus. e. u. ab. b. x. & pondus. o. c. ab. f. g. pon
dus verô. o. e. ab. b. pondus autem. o. u. ab. x. pondus enim. u. c. ab. b. d. et. g. fit diffe-
rentia, qua. f. g. maior eft. f. et. d.
differentia qua. b. d. maior eft. b.
Vnde ex ratione omogeneitatis ea-
dem proportio erit. a. o. ad. e. o. vt.
f. ad. b. et. o. c. ad. u. c. quæ. x. b. d. feu
f. g. (quod idem eft). ad. b. d. Quare
ex. 11. quinti eadé. erit proportio.
f. ad. b. vt. f. g. ad. b. d. & permutan-
do ita erit. f. ad. f. g. vt. b. ad. b. d. &
feparando ita. f. ad. g. vt. b. ad. d. Sed. g. cognita nobis eft, vt differentia in
ter. f. g, et. f. cognita nobis eft etiam. fi. cognofcimus itidem. d. vt differentiam inter.
x. b. d. et. b. x. quapropter cognofcemus. b. ex. 20. feptimi Eucli. & fic. x. refiduum.
ex. b. x.
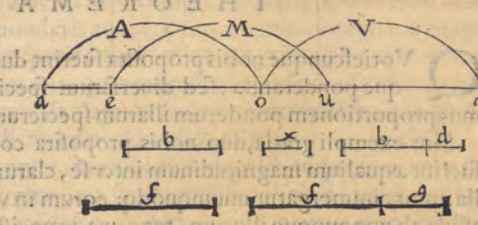
## THEOREMA CXXIII.

NVNC ex methodo præcedentis propofiti deuenire poffumus in cognitio-
nem veræ quàntitatis auri, & argenti confufi in corona Hieronis conftituen-
do primum duo corpora fimplicia æqualia inter fe, & coronæ hoc modo videlicet,
immergendo coronam, feu corpus miftum in aliquod vas aqua plenum, & diligen-
ter colligere aquam, quæ ex eo effundetur, poftea verô oportet aliud vas inuenire
præcifæ capax illius aquæ collectæ, in quod demum infundatur tantum auri, & po-
ftea tantùm argenti, quantum fieri poteft, vnde vnumquodque horum duorum cor
porum fimplicium æquale erit mixto, feu coronæ, & fic quod dictum eft in præce-
cedenti theoremate exequemur.

## THEOREMA CXXIIII.

SED vt breuiori methodo idem præftemus, quod in antecedenti propofito di-
ctum eft, quædam theoremata præmittenda funt, videlicet quòd quotiefcunque
fuerint tria corpora, quorum duo inuicem æqualia fint in quantitate, fed diuerfa-

rum fpecierum materiæ,tertium verò corpus maius,vel minus fit in quantitate vtro-
que illorum, fed eiufdem materiæ vnius quod vis illorum, ponderis verò alterius,
séper eadem proportio erit inter pondera æqualium corporum, quæ inter quátita-
tem corporis inæqualis,& eam quæ vnius cuiufuis æqualium.

    Exempli gratia, fit.b.corpus aliquod aureum æquale corpori.u. argenteo, fit
etiam corpus.a.argenteum maius corpore.b.vel.u.fed ponderis eiufdem, quod au-
ri.b. Tunc dico eandem effe proportionem ponde-
ris.b.ad pondus.u.quæ eft magnitudinis. a. ad ma-
gnitudinem.u.Quod ratiocinemur hoc modo, nam
cum proportio corporeitatis.a.ad corporeitatem.u.
eadem fit,quæ ponderis.a.ad pondus . u . ex ratione
omogeneitatis, ponderis verò.b.ad pondus.u.ex.7.
quinti,eadem quæ ponderis.a.ad pondus.u.ideo ex
1 1.eiufdem proportio ponderis.b.ad pondus.u.eadem erit, quæ corporeitatis. a.
ad corporeitatem.u.vel ad corporeitatem.b.quæ æqualis eft alteri.

## T H E O R E M A   C X X V.

Q Votiefcunque nobis propofita fuerint duo corpora cuiufuis magnitudinis æ-
    que ponderantia, fed diuerfarum fpecierum materiæ, cum fcire volueri-
mus proportionem ponderum illarum fpecierum inter ipfas hoc modo faciemus.

    Sint exempli gratia,duo nobis propofita corpora.a.et.b.(vt dictum eft) quæ fi
fuerint æqualium magnitudinum inter fe, clarum erit quod quæritur, fed inæqua-
lia erunt, immergatur unumquodq; eorum in vas aqua plenum,& collecta fit aqua
effufa ab vnoquoque illorum, tunc vnaquæq; iftarum aquarum æqualis magnitudi-
nis erit fui corporis impellentis, & proportio ponderofitatis illarum eadem erit,
quæ earum magnitudinum ex omogeneitate,quapropter fi vnamquamque illarum
ponderabimus, habebimus propofitum ex præcedenti theoremate.

## T H E O R E M A   C X X V I.

S E D cum fcire voluerimus pondus alicuius magnitudinis aquæ æqualis alicui
    corpori ponderofo,breuiffimus modus erit ponderando ipfum corpus tam in ae-
re,quàm in aqua , & quia femper leuius erit in aqua, tunc differentia ponderum ip-
fius corporis, erit pondus quæfitum, hoc eft vnius corporis aquei æqualis magnitu-
dinis magnitudini corporis propofiti ex. 7 . propofitione lib.Archimedis de infi-
dentibus aquæ.

    Quare ex præmiffis quotiefcunque immerfa fuerint in aquam dicti vafis duo cor
pora æquè ponderantia, fed diuerfarum fpecierum, vt dictum eft, proportio pon-
deris aquæ maioris ad pondus aquæ minoris magnitudinis eadem femper erit, quæ
ponderis minoris corporis ad pondus alicuius corporis eidem æqualis, fpeciei verò
maioris, vel eadem proportio ponderis alicuius corporis æqualis maiori,fpeciei ve
rò minoris ad pondus ipfius maioris.

    Vt puta fit corpus.a.argenteum æqualis ponderis corpori.b.aurei, & corpus.u.
argenteum æqualis magnitudinis corpori.b.aurei, corpus verò.n.aureum æqualis
magnitudinis corpori.a.argentei,corpus verò.f.aqueum æqualis magnitudinis cor-
                                                        pori

pori.a.argentei,corpus autem.e.aqueū æqualis ma-
gnitudinis corpori.b.aurei. Tunc dico proportio-
nem ponderis.f.ad pondus.e.eadem esse, quæ pon-
deris.b.ad pondus.u.vt in præcedenti theoremate
iam dictum est,vel quæ ponderis.n.ad pondus.a.ex
11.quinti Euclidis . Proptereà quòe ponderis.
n.ad pondus.a.est vt poderis.b.ad pondus.u.eo
quòd permutando ponderis.n.ad pondus.b.est vt
ponderis.a.ad pondus.u.ex corporum omogenei-
tate, & ex æqualitate magnitudinum corporum antecedentium & consequentium.

## THEOREMA CXXVII.

SCire etiam nos oportet,quòd quotiescùmque fuerint duo corpora aquea,quo-
rum vnum æqualis magnitudinis sit alicui misto, quod quidem mistum graue
sit tam in aere, quàm in aqua, alterum verò corpus aquem æqualis sit magnitudi-
nis alicui corpori simplici, quod quidem corpus simplex æqualis ponderis sit dicto
corpori misto. Tunc proportio ponderis aquei, cuius magnitudo æquatur magni-
tudini corporis misti, ad pondus corporis aquei, cuius magnitudo æqualis est ma-
gnitudini corporis simplicis, eadem erit, quæ proportio ponderis alicuius corpo-
ris simplicis, cuius magnitudo æqualis sit magnitudini corporis misti superius dicti,
sed speciei corporis simplicis iam dicti, ad pondus dicti misti.

Exempli gratia, sit corpus aqueum.e.magnitudinis æqualis corpori.m.mixto,
corpus verò aqueum.i.æqualis magnitudinis sit corpori simplici.a. quod quidem
corpus.a.æqualis ponderis sit cum corpore.m.& corpus.u.sit æqualis magnitudinis
cum corpore.m.sed speciei corporis.a. Tunc dico proportionem ponderis.e.ad
pondus.i.eādem esse,quæ ponderis.u.ad pondus.m.primum nulli dubium est, quin
eadem proportio sit magnitudinis.e.ad magnitudinem.i.quæ magnitudinis.m.ad
a.sed.m.ad.a.est vt.u.ad.a.ex.7.quinti quare
ex.11.eiusdem proportio.e.ad.i.erit vt.u.ad.a.
de ipsius magnitudinibus loquendo,sed propor-
tio ponderis.u.ad pondus.a.eadem est, quæ ma-
gnitudinis.u.ad magnitudinem.a.ex omogenei-
tate.Idem dico de pondere.e.ad pondus.i.Qua-
re proportio ponderis.e.ad pondus.i.eadem erit
quæ ponderis.u.ad pondus.a. Sed ponderis.u.
ad pondus.m.eadem est quæ ponderis.u.ad pondus.a.ex.7.quinti, ergò ex.11.
eiusdem proportio ponderis.e.ad pondus.i.eadem erit, quæ ponderis.u.ad pon-
dus.m.quod est propositum.

## THEOREMA CXXVIII.

NVNC ad cognoscendam proportionem duarum diuersarum specierum in
corpore misto proposito, tribus corporibus aqueis mediantibus; quæ quidē
corpora æqualium magnitudinum sint alijs tribus corporibus vnius & eiusdem pon
deris, quorum vnum sit mixtum, reliqua verò duo simplicia,sed specierum mixti,
hoc ordine procedemus.

Sint

Sint exempli gratia, tria corpora æquè ponderantia, & vnumquodque illorum
fitquinque librarum, quorum vnum fit aureum, aliud argenteum, reliquum verò
mixtum ex ijs duobus metallis, vnde corpus aureum fimplex minus erit, & argen
teum maius corpore mixto, quod nulli dubium est, fit nunc pondus corporis aquei
equalis corpori aureo, librarū.3.aquei verò equalis mifto,fit librarū 3.cū quarta par
te, aquei demum æqualis argenteo, librarum.4.cum dimidia, vnde ex ijs, quæ in præ
cedenti theoremate, & in.126.theoremate diximus,fi imaginatione concipiemus
alia duo corpora fimplicia, auri, & argenti, fed æqualium magnitudinum mixto,
habebimus proportionem ponderis aurei ad pondus corporis mixti vt triū librarum
cum quarta vnius ad.3. libras, & proportio ponderis mixti ad pondus argentei erit,
vt proportio librarum.4.cum dimidia ad tres libras cum quarta parte vnius libræ,
& proportio ponderis aurei ad pondus argentei vt librarum.4. cum dimidia ad li-
bras.3: hoc est aurei ad mixtum, vt.13.ad.12.& mixti ad argenteum,vt.18.ad.13.
& aurei ad argenteum, vt.3.ad.2.ideft, vt.18.ad.12.

Nunc inueniantur duo numeri ita inter fe proportionati, vt.3.ad.2.habentes ta-
men inter ipfos numerum ita proportionatum ad maximum, vt.12. fe habet ad.
13.& ita proportionatum ad minimum, vt fe habet.18.ad.13. quod hoc modo in-
ueniemus, multiplicabimus.18.per.12.& proueniet nobis.216. pro numero me-
dio, poftea multiplicabimus.18.per.13.& proueniet.234.pro maximo, demū multi
plicando.12.per.13.proueniet.156.pro minimo, ita quod.234. correfpondebit
ponderi corporis aurei:216.verò ponderi mixti, et.156. ponderi argentei æqua-
lium magnitudinum.

Cum autem proportiones horum trium corporum inuenerimus,fi ordinem theo
rematis.122.fequemur, habebimus quod quærebamus, & inueniemus in præfenti
exemplo proportionem ponderis auri ad pondus argenti in corpore mixto effe, vt.
180.ad.36.fed quia fuppofitum fuit corpus mixtum effe quinque librarum,propte-
reà dicemus. Si.216. hoc est toti corpori mixto correfpondent quinque libræ:tunc
parti.180.hoc est auro in ipfo corpore mixto, correfpondent libræ.4. cum duabus
vncijs, ex règula de tribus, refiduum verò quinque librarum, ideft vnciæ decem,
correfpondent parti.36. hoc est argento in dicto corpore mixto.

Sed fi tria corpora dicta fuiffent inuicem ita proportionata, vt.40.47.60.tunc
proportio auri ad argentum in corpore mixto effet vt.13.ad.7.quapropter cū pon
dus mixti fuiffet.120.librarum, tunc aurum ipfius effet librarum.78. argentum ve-
rò librarum.42.ex eadem regula.

Pro quarum rerum fpeculatione nil aliud oportet nunc dicere cum fatis dictum à no
bis fuperius fuerit, vno excepto, hoc est rationem reddere, qua motus fui ad inue
niendos illos.3.numeros ita inter fe difpofitos, vt dictum est,quæ quidem ratio fuit,
vt haberemus.3.numeros ita inter ipfos ordinatè difpofitos, vt funt pondera trium
illorum corporum æqualium magnitudinum. Propterea quòd quamuis inter pri-
mos.3.numeros ponderum corporum aqueorum eædem fuerint proportiones pon
derum corporum metallicorum, nihilominus medius numerus extra proprium lo-
cum, & inordinatè inueniebatur, respectu extremorum, vnde medius numerus in
fuo vero fitu inter.18.et.12.fuiffent.16. cū.8.tertijs decimis, fed vt fractorū incom
moditatem euitemus, præcepi, vt multiplicarentur extrema per.13. vnde produ-
cti fuerunt numeri.234.et.156.in eadē proportione, quæ est.18.ad.12.ex.18.fepti
mi, iuffi etiam multiplicari.18.per.12.vt nobis prodiret.216.ad quem numerum,
numerus.234. ita fe haberet,ut.13.ad.12.ex.19.feptimi,quod autem ita fit propor
Sint                                                                    tionatus

tionatus. 216. ad. 156. vt. 18. ad. 13. manifestum est ex ijsdem, nam tam. 18. quam. 13. multiplicatus fuit per. 12.

## THEOREMA CXXIX.

ALIVD proponitur problema hoc modo: supponitur obsidio alicuius loci, vbi alimento ad nutriendos. 10000. homines sufficiunt pro quinque mensibus tantum, sed quia eum locum obsidione non liberari putatur nisi. 18. mensibus exactis, quæritur, quot homines eo tempore illis alimentis nutriri possint, hoc est. 18. mensibus.

Præcipit regula, vt multiplicetur primus numerus, hoc est hominum. 10000. cum secundo, hoc est mensium quinque, productum verò diuidatur per. 18. hoc est mensium, tunc proueniet. 2777. cum. 7. nonis.

Cuius operationis ratio est hæc, sint exempli gratia duo hic subscripta producta superficialia. a. n. et. o. u. inuicem æqualia, sed tali figura delineata, vt proportio. u. x. ad. x. o. sit, vt. 10000. ad quinque, & proportio. a. x. ad. x. o. sit vt. 18. ad quinque, et. x. n. sit nobis ignota, quæ quidem est illa, quæ indagatur, ita ϕ vnumquodque istorum productorum significabit alimentum, et. u. x. significabit numerum hominum. 10000. qui quidem homines comederent totum alimentum. u. o. spacio temporis. x. o. quinque mensium, propterea quòd u. o. supponitur productum esse ab. u. x. in. x. o. Deinde supponēdo. a. x. tempus esse. 18. mensium, ergo. x. n. significabit numerum hominum, qui eo temporis spacio ali possunt, hoc est. x. a. alimento. n. a. eo quòd. a. n. producitur ex. n. x. in. a. x. vnde ex. 15. sexti, seu ex. 20. septimi proportio. x. u. ad. x. n. cadē erit, quę. a. x. ad. x. o. quapropter rectè factum erit accipere productū. u. o. quod idem est in quantitate, quod productum. a. n. & ipsum diuidere per. a. x. vnde nobis proueniat. n. x.
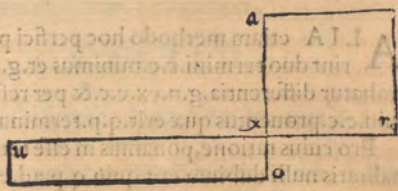
## THEOREMA CXXX.

QVotiescunque nobis propositum fuerit inuenire tertium terminum, trium terminorum continuè proportionalium armonicæ proportionalitatis, quorum duo nobis cogniti sint, ita agemus.

Sint, exempli gratia, tres termini. q. p: a. g. et. e. c. continuæ proportionalium armonicæ proportionalitatis, quorum. q. p. maior et. a. g. medius sint nobis cogniti, cum ergo voluerimus tertium. e.

c. cognitum nobis esse: a. g. detrahatur ex. q. p. differentia verò. d. p. addatur. q. p. quorum summa erit. q. o. cognita, qua mediante diuidatur productum, quod ex. a. g. in. d. p. exurgit, & proueniet nobis. n. g. hoc est minor differentia, eo quòd productum. q. o. in. n. g. æquale est producto
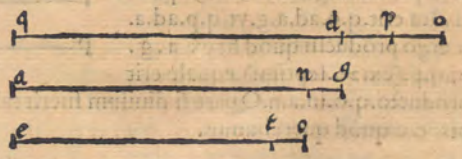
ducto.a.g.in.d.p.ex.20.feptimi, proptereà quòd proportio.q.o.ad.o.p.hoc eft ad. d.p.eft vt.a.g.ad.g.n.coniunctim cum difiunctim ita fit. q. p.ad.p.o.vt.a.ŋ. ad . n.g. permutãdo eo quòd.q.p.ad.a.n.(ideft ad.e.c.)ita fe hēt ut.p.o.(hoc eft.d.p.)ad.n.g. ex côditionibus armonicæ proportionalitatis . Deinde fi detraxerimus.n.g.ex.a. g. remanebit.e.c.minor terminus.

Sed fi.e.c.tertius terminus nobis propofitus effet fimul cum.a.g.medio , & volue rimus maiorem inuenire.q.p.fcilicet, oportebit.e.c.ex.a.g.detrahere, differentiam verò. n. g. fimiliter demeremus ex.e.c.unde remaneret nobis.e.t. cognitum, quo refiduo.e. t. me- diante diuidemus productum, q furgit ex.a.g.in.t.c. & prouentus. d.p.erit differentia maior , eo q productũ quod fit ex.e. t. in.d. p. æquale eft producto quòd fit ex.a.g.in.t.c.per 20.feptimi Eucli.eo quòd.a.g. (id- eft.q.d.)ad.d.p.eft ut.e.t.ad.t.c.difiunctim, cum coniunctim ita fit.q.p.ad.d.p.vt.e. c.ad.t.c.permutando, quia.q.p.ad.e.c.eft vt.d.p.ad.t.c. hoc eft ad. n. g. **ex legibus** dictis.
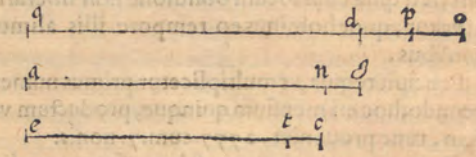
## THEOREMA CXXXI.

ALIA etiam methodo hoc perfici poffe comperi.Propofiti enim cum nobis fue rint duo termini.c.e.minimus et.g.a.medius,maximus verò quærendus fit, de trahatur differentia.g.n.ex.e.c.& per refiduum.e.t.diuidatur productum q fit ex.a. g.in.e.c.prouentus quæ erit.q.p.terminus quæfitus .

Pro cuius ratione,ponamus in effe terminum.q.p.tunc ex forma huius proportio nalitatis nulli dubium erit quin.q.p.ad.e.c.fit vt.d.p.ad.n.g.hoc eft ad. t. c. vnde ex 19.quinti vel 12.feptimi ita effet.q.d.ad.e.t.vt.q.p.ad.e.c.quare ex.20.feptimi pro ductum q nafcitur ex.p.d.(hoc eft.a.g. )in.e.c.æquale erit producto.e.t.in.q.p. qua propter fi diuiferimus id per.e.t.proueniet nobis.q.p.

Sed cũ nobis propofiti fuerint duo termini.q.p. maximus,et.a.g.medius,fi mini mũ.e.c.voluerim⁹ inuenire. Termino.q.p.maximo,iũgat̃.p.o.equalis,p.d.differētie propofitæ, diuidatur poftea productum q ex.q.p.in.a.g.generatur per.q.o. prouen tus autem fit.e.c. qui quidem erit terminus quæfitus.

Cuius operationis fpeculatio hæc erit, fupponatur terminum.e.c.inuentum effe vnde.n.g.differentia fit inter. e. c. et.a.g. ex forma igitur armonicæ proportionalitis ita erit.q.p. ad. a. n.vt.p.o.ad.n.g.vnde ex.13.quin- ti. Ita erit.q.o.ad.a.g.vt.q.p.ad.a. n.ergo productũ quod fit ex. a. g. in.q.p.(ex.20.feptimi) æquale erit producto.q.o.in.a.n.Quare fi diuifum fuerit tale productum per.q.o.proueniet no- bis. e.c.quòd querebamus.

THEO-

THEOR. ARITH.                    89
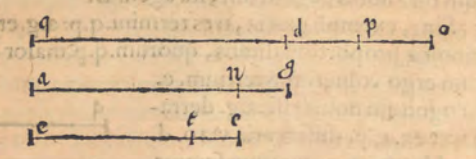
## THEOREMA CXXXII.

SED quia aliquis posset in dubium reuocare, an possibile sit inuenire tertium terminum rationalem, seu communicantem duobus datis terminis inter se communicantibus in tali proportionalitate, hoc est harmonica. Vt hoc ostendatur.

Sint duo termini dati.a.o.et.a.e. inter se communicantes, tertius verò inuentus sit.a.c.qui maximus, primò, sit in ea proportionalitate, quem dico communicantem esse cum primis datis.

Nam ex conditionibus huiusmodi proportionalitatis, habebimus primum eandem proportionem esse. a.c.ad.a.o. quæ est. e.c.ad.e.o.vnde permutando ita erit. a. c.ad.e.c. vt.a.o.ad.o.e. & quia ex.9.decimi Euclid.a.o.communicat cum.o.e.quare ex.10.eiusdem.a.c.communicabit cum.e.c. & per.9.cum.a.e. et per.8. cum.a.o. quod est propositum.

Sed si datus fuerit maximus.a.c.cum medio.a.e. inter se communicantes minimum verò. a.o.probabo cômunicantem cum illis esse. Cogitemus ergo.c.f.æqualem esse differentiæ.c.e.cognitæ, vnde habebimus proportionem,a.c.ad.c.f.vt.a.o. ad.o.e.& componendo.a.f.ad.f.c.vt.a.e.ad.e.o.& quia ( ex supposito ).a.c.communicat cum. e. c. hoc est cum . c.f. quare ex eadem.9.dicti decimi.a.f.et.f.c. erût inter se communicantes, & per.10.a.e. communicabit cum.o.e.& per.9.a.e.cô municabit cum. a.o.vnde per. 8.a.o.communicabit cum.a.c.similiter .

## THEOREMA CXXXIII.

SED si nobis duo extremi termini propositi fuerint, & medium inuenire desideremus in dicta proportionalitate, ita faciendum erit.

Sint, exempli gratia, duo termini dati.q.b. et.b.r.minor. b.r.ex maiori.b. q. detrahatur, residuum verò.q.x.multiplicetur per.b.r.productum posteà diuidatur per q.r.vnde proueniet nobis.x.l.pro differentia minori, quæ addita cum.b. x. minimo termino, dabit nobis. b.l.medium terminum harmonicum.

Pro cuius ratione cogitemus dictum medium terminum. b.l.iam inuentum esse, vnde ita erit proportio.q.l.ad.l.x.vt.q.b.ad.b.r.ex forma huius proportionalitatis, quare coniunctim ita erit.q.r.ad.r. b. vt q.x.ad.x.l.& propterea ex. 20 . septimi productum.quod fit ex.q.r.in.x.l.æquale erit producto.q.x.in.b.r. Rectè igitur fit cum diuiditur hoc productum per.q.r.vt proueniat nobis. x. l. differentia minor.

## THEOREMA CXXXIIII.

POssumus etiam harmonicè diuidere vnam datam proportionem absque aliqua diuisione productorum, ne nobis fractiones proueniant, hoc modo videlicet.

Nobis propositum sit diuidere harmonicè sesquialteram proportione inueniantur primo minimi termini huius proportionis ut putà. 3. et. 2.quarum summa, hoc est quinque,multiplicetur per minorem idest. 2. vnde proueniet nobis.10.qui quidem erit minor terminus trium quæsitorum,quorum maximus erit productum summ

M          mæ

mæ iam dictæ in maiorem eorum, hoc est quod fit ex quinque in.3. quod erit.15. Vt
autem medium terminum harmonicum inter istos habeamus, accipiatur duplū pro-
ducti, quod fit ex primis minimis terminis, quod erit.12.

Cuius rei speculatio est ista: significentur duo termini datæ proportionis ab. q. b.
et. b. r. quorum summa erit. q. r. cuius quadratum sit. q. o. sit etiam imaginata. b. e.
parallela ad. o. r. Sitq̀; . b. x. æqualis. b. r. et. q. u. similiter, & ducatur. x. y. parallela ad
r. o. et. u. l. ad. q. x. Tunc habebimus. b. o. æquale ei producto, quod fit ex. q. r. in. b. r.
et. b. y. eidem etiam æquale, et. q. e. pro producto, quod fit ex. q. r. in. q. b. et. q. l. pro
eo, quod fit ex. q. x. in. b. r. Vnde. q. l. cum. b. y. æquale fiet duplo ei, quod fit ex. q. b.
in. b. r. Dico nunc. b. o. esse minimum terminum eorum, quos quærimus, et. y. b. cum.
x. u. medium. q. e. verò maximum huiusmodi proportionalitatis.

Primum ergo certi scimus ex prima sexti vel. 18. septimi eandem existere pro-
portionem. q. e. ad. b. o. seu ad. b. y. quæ. q. b. ad. b. r: sed. u. y. ad. u. x. est vt. y. l. ad. l. x.
hoc est vt. q. b. ad. b. r. idest vt. q. e. ad. b. o. & summa. u. y. cum. u. x. idest. q. y. minor est
quam. q. e. maximus terminus per. b. y. minimum ter-
minum. & cóniunctim. q. y. ad. q. l. vt. y. x. ad. x. l. hoc est
vt. q. r. ad. r. b. Vnde ex speculatione præcedétis theo-
rematis, sequitur. u. y. esse differentiam inter maximū
& medium terminum, et. u. x. esse differentiam inter
medium & minimum dictæ proportionalitatis. Nam
eadem proportio est. q. e. maximi termini ad. b. o. mi-
nimi. quæ. u. y. ( differentia inter. q. e. & gnomonem.
u. b. y.) ad. u. x. ( differentia inter dictum. u. b. y. et. b. y.
minimum terminum, quia sunt ambæ ut. q. b. ad. b. r.
vt diximus. Quare. b. y. coniunctū cum . x. u. medius
terminus erit, qui quidem (vt dictum est ) duplus est ei
quod fit ex. q. b. in. b. r.
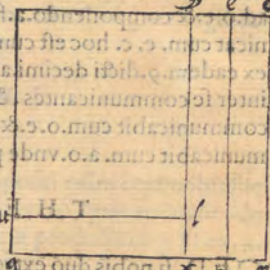


## THEOREMA CXXXV.

ALIVM etiam modum ab antiquis traditum ad hoc problema perficiendum
inueni, qui talis est. Inueniatur primo inter datos terminos extremos, me-
dius terminus in arithmetica proportione, per qué
multiplicetur vnusquisque dictorum extremorum,
deinde multiplicentur ipsi extremi inter se, vnde
habebimus tria producta eadem proportione inui-
cem existentia, vt quærebatur.

Exempli gratia, ponamus duos propositos ter-
minos esse. 3. et. 2. quorum medius arithmeticè
esset. 2. cum dimidia vnitate, per quem cum vnum
quemque priorum multiplicauerimus, emergét no-
bis duo producta, quorum primum idest maius esset
7. cum dimidia vnitate, reliquum verò esset
quinque, productum postea quod ex ipsis extremis
prouenit, erit. 6. quod quidem est harmonicè collo-
catum inter. 7. cum dimidia vnitate, & quinque.
Cuius rei speculatio omnis à præcedenti theore-
mate dependet. Sint exempli gratia, duo termini
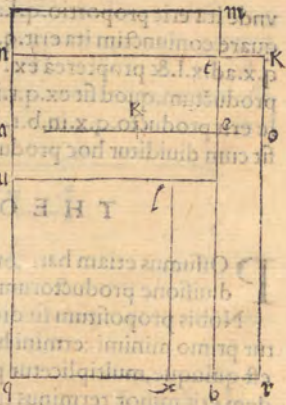


pro

propositi.a.e.maior,et.e.o.minor, Sitq;.o.k.medius arithmeticus inter dictos, vn-
de clarè patebit.o.k.esse dimidium summæ dictorum terminorum ex.75.theorema
te huius libri . Sit ergo productum a.t.id quod fit ex.a.e.in.o.k.et.o.t. sit productū
quod fit ex.e.o.in.o.k.et.n.m. sit productum quod fit ex.a.e.in.e.o. quorum vnum-
quodque erit dimidium vniuscuiusque producti præcedentis theorematis ,
ex. 18. et.19.septimi Eucli. vnumquodque sui relatiui . Quare argumentando per
mutando à conclusionibus præcedentis theorematis ad has præsentis , habebimus
productum .

## THEOREMA CXXXVI.

MEDIVM autem contra harmonicū inuenire cum quis voluerit inter duos
propositos terminos , ita faciendum erit, hoc est per summam datorum ex
tremorum diuidatur productum quod fit ex minimo termino in differētiam dato-
rum,prouentus posteà erit differentia inter maximum & medium quæsitum.

Vt exempli gratia, si nobis propositi fuerint hi duo termini. 3. et.2. summa eo-
rum erit quinque, per quam cum diuiserimus productum , quod nascitur ex mini-
mo.2.in differentiam eorum, quæ est vnum, quod quidem erit. 2.tunc duæ quintæ
partes prouenient, quæ si demptæ fuerint ex maximo termino, reliquum erit.2. cū
3.quintis, hoc est medius terminus contra harmonicus.

Pro cuius ratione cogitemus.u.d.et.x.c.esse duos terminos nobis propositos , in-
ter quos desideremus inuenire.o.s.medium ita illis relatū, vt proportio excessus ip-
sius supra.x.c. (qui sit.e. n.) ad excef-
sum. u.d. supra. o. s. (qui sit.n.d.) ea-
dem sit quæ.u.d.ad.x.c.

Cogitemus igitur.x.c. coniunctum
e sse cum.u.d. & hæc summa vocetur.
b.d. vnde habebimus proportionem.
u.d.ad.u.b. vt.e.n.ad n.d. Quare cō-
ponendo ita erit.d.b.ad.u.b.ut.e.d.ad.n.d.sed quia.d.b:u.b.et.e.d. quantitates no-
bis cognitæ sunt , ideò.d.n.ex.20.septimi cognita nobis erit.

## THEOREMA CXXXVII.

SVpponunt antiqui aliquot mercatores dantes pecunias lucro in diuersis vnius
anni temporibus, tunc in fine anni summa totius lucri datur cognita , sed quæ-
ritur quantum vnicuique illorum ex ipsa summa debeatur.

Exempli gratia , primus in principio anni posuit.100.aureos, secundus verò.100
diebus post primum posuit.50.aureos tertius autem.200.diebus post primum po-
suit.25.aureos summa lucri posteà in fine anni fuit aureorum.60

Nunc vt sciamus quantum huius summæ vnicuique illorum proueniat , præcipit
regula, vt faciamus tria producta, quorum primum sit ex numero dierum totius an-
ni in numerum aureorum primi, vnde tale productum in præsenti casu erit.36506.
secundum verò fit ex numero dierum à primo die in quo ipse secundus posuit usque
ad finem anni , in numerum ipsorum nummorum, quod erit.13250. tertium autem
productum ex diebus tertij in numerum suorum aureorum, quod quidē erit. 4125.
quæ producta simul collecta faciunt. 53875. deinde multiplicetur vnumquodque
M 2 ipso-

ipforum productorum per fummam lucri hoc eft per. 60. vnde multiplicatio primi
producti erit. 2190000. multiplicatio verò fecundi producti erit. 795000. tertij po
ftea erit. 247500. quarum multiplicationum vnaquæque diuidatur per fummam
53875. productorū, & proueniet ex prima diuifione. 40. cū fractis. 35000. vnius in-
tegri diuifi in partes. 53875. quòd erit lucrum primi, prouentus autem fecundæ di-
uifionis erit. 14. cum fractis. 41050. vnius integri diuifi in partes. 53875. lucrum fecū
di. prouentus verò quartæ diuifionis erit. 4. cum fractis, 32000. vnius integri, vt fu
pra diuifi in partes. 53875 . hoc eft lucrum tertij.

Cuius rei fpeculatio ex fe in fub fcripta figura patet, vbi. a.q. fignificat numerum
dierum totius anni prò primo mercatore. q. n. autem fignificat numerum dierum fe
cundi mercator is. e. q. poftea fignificat numerum dierum tertij fit etiam. s. a. pro nu-
mero denariorum primi, et. o. n. pro numero fecundi, et. e. t. pro numero
tertij, productum autem. q. s. fignificet valorem primi lucri, et. q. o. fecundi,
et. q. t. tertij. x. y. autem fignificet fummam lucri omnium, et. x. i. fignificet
partem primi, et. i. p. fecundi, et. p. y. tertij. vnde clarè patebit ex commvni
fcientia quòd eadem proportio erit. x. y. ad. x. i. quæ aggregati omnium producto-
rum. q. s: q. o. et. q. t. ad. q. s. & ita. x. y. ad. i. p. vt aggregati dicti ad. q. o. et. x. y. ad. p. y.
vt dicti aggregati ad. q. t. Rectè igitur ex regula de tribus multiplicatio. q. s. in. x. y.
diuiditur per aggregatum omnium
productorum, ita vt fi aliquis dice-
ret, fi ex dicto aggregato, prouenit
x. y. quid proueniet vnicuique illo-
rū productorū. Nā fi numerus dena-
riorum fecūdi æqualis effet numero
a. s. primi vt putà. n. b. tunc eius lucrū
fignificaretur à rectangulo. q. b. & ita
de tertio dico q̄ fignificaretur à re-
ctagulo. q. c. vel fi ftantibus ijfdē denariorū quantitatibus. n. o. et. e. t. omnes fuas pe-
cunias eodem tempore pofuiffent, tunc rectangula fignificantia eorum lucra effent
q. s: q. d. et. q. f. fed cum nec eodem tempore, nec eandem quantitatem pofuerunt re
ctè eorum lucra fignificantur à rectangulis. q. s. q. o. et. q. t. q̄ ex prima. 6. vel. 18. aut.
19. feptimi ratiocinando clarè patebit.
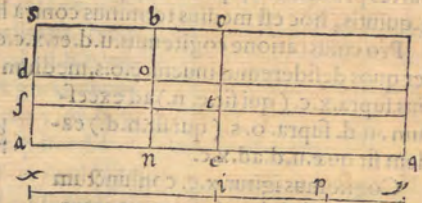
# THEOREMA CXXXVIII.

N Icolaus Tartalea in primo libro vltimæ partis numerorum ad. 35. quæfitum
docet inuenire quantitatem laterum vnius propofiti trianguli, cuius la-
terum proportio nobis data fit fimul cum area fuperficiali ipfius trianguli, fed quia
ipfe Tartalea vtitur regula algebræ, mihi vifum eft breuiori methodo hoc idem fa
cere, & etiam vniuerfaliori via.
Supponamus igitur duo triangula, quorum vnum. u. n. i. fit nobis propofitū, &
cognitæ fuperficiei, proportiones fimiliter laterum. i. n. ad. n. u: et. u. n. ad. u. i. fint no
bis datæ, alterū verò triangulū fit. a. o. u. à nobis tamen ita confectū, vt latera fint in
ter fe proportionata eodem modo, quo latera prioris trianguli, fed hæc nobis etiā
cognita fint, q̄ facillimum eft. Nunc verò fi demptū fuerit quadratū. a. o. minimi
lateris, ex quadrato. o. u. maximi, relinquet nobis duplum producti. o. u. in. u. e. per
penultimā. 2. Eucli. fupponēdo. a. e. perpendicularem ad. o. u. vnde tale productum
quòd fit ex. o. u. in. u. e. confequenter nobis cognitum erit, & quia. o. u. nobis cogni-
tum eft,

tum est, ideo cognoscemus.e.u.sed cũ.e.u.minor sir.a.u.ex.18.& penultima primi,
si demptũ fuerit quadratum.e.u.ex quadrato.a.u.remanebit nobis cognitũ quadra-
tũ.a.e.& sic nota erit nobis perpendicularis.a.e.ex penultima primi, quæ quidem.
a.e.si multiplicata fuerit in dimidium.o.u.dabit nobis superficiẽ trianguli.a.o.u.ex
41.dicti libri. Et quia proportio trianguli.a.o.u.ad triangulum.u.i.n.(propter simi
litudinem) est vt quadrati. o. u.ad quadratum.n.i.ex communi scientia cum vna-
quæque istarum proportionum dupla sit proportioni.o.u.ad.n.i.ex. 17.et.18. sexti,
deinde cum nobis cognitæ sint tres istarum quatuor quantitatum hoc est superficies
trianguli.a.o.u.superficies trianguli.u.n.i.& quadrati. o.u.quare ex regula de tribus
cognoscemus etiam quadratum.n.i.& sic.n.i.latus primi trianguli, vnde reliqua la
tera illicò nobis innotescent ex ipsa regula de tribus, cum dixerimus, si.o.u.dat nobis
u.a.tunc.i.n.dabit.u.n.quòd etiam infero de.u.i.

Possemus etiam ita hoc perficere,
scilicet inuenire.x. quantitatem me-
diam proportionalem inter duas su-
perficies triangulorum, vnde super-
ficies trianguli. i. a. u. o. ad.x. se ha-
beret ut.o.u. ad. i. n. & ita ex regula
de tribus cognoscemus.i.n. Multo tẽ
pore postquàm hoc theorema construxi, ipsum conscriptum inueni in decimo
secundi libri Ioannis de monte Regio, satis tamen obscurè expressum.

## THEOREMA CXXXIX.

IN eodem primo libro vltimæ partis numerorum, Tartalea probat, via algebrę
quòd quælibet duo latera trianguli orthogonij, angulum rectum continentia,
sint tertio longiora per diame-
trum circuli inscriptibilis in ip-
so triangulo. sed hoc breuius
geometricè potest demõstrari,
quemadmodum in subscripta
hic figura videre est, propterea
quòd cum anguli. A. o. u. et.n.
omnes sint recti et. A.u.æqualis
o.n.et. A.n.æqualis.u.o.ipsæ.A.
u.et. A.n.æquales erunt diame-
tro ipsius circuli. Sed eædem.
A.u.et. A.n.sunt superfluum, quo. A.B.et.A.C.sunt maiores.B.C.cum.B.u.et.C. n.
sint æquales.B.C.ex penultima tertij Eucli.
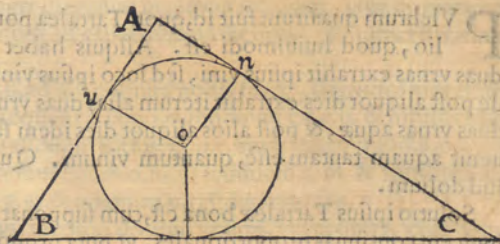
## THEO. SEQVENS THEO. CXXXIX.

SImiliter in nono capite secundi libri nouæ scientiæ poterat ipse Tartalea breuio
ri methodo absque vlla operatione ipsius Algebræ inuenire.A.H.respectu.A.
E.esse vt.4.cũ vno septimo ad vnũ. Nã ipse supponit.A.E.decimã parte esse ipsius
A.I.

A. I. vnde quadratum lineæ. A. I. erit. 100. idem dico de quadrato lineæ. I. L. quare
ex penultima primi. A. L. erit radix quadrata quadrati. 200. ideſt. 14. cum vno ſepti-
mo ferè. quare. A. L. iuncta. A. O. erit. 28. cum duobus ſeptimis. ſed. L. O. ex ſuppoſi-
to erit. 20. eo quòd. L. I. æquatur ipſi. A. I. ſimiliter et. I. O. vt ipſe etiam probauit. qua
dempta ex. L. A. O. relinquetur. H. A. M. (nam. L. H. cum. O. M. æquatur ipſi. L. O. ex.
35. t ertij ipſius Eucli. partium. 8. cũ duabus ſeptimis. cuius dimidiũ hoc eſt. A. H. erit
4. cum una ſeptima, quod eſt propoſitum . Reſpice figuram ipſius Tartaleæ.

## T H E O R E M A C X L.

Q Vadrageſimum nonum quæſitum ſimiliter poſſumus alio modo ſoluere , vt
putà cum vnumquodque latus rhombi ſimul cum area cognitum, ſeu datum
nobis ſit cognitũ ſimiliter nobis erit quadratum lateris. a. d. hoc eſt ſumma duorum
quadratorum. a. o. et. o. d. ex penultima primi Euclid. cúmque nobis cognita etiam
ſit totalis ſuperficies rhombi, cognita etiam nobis erit eius medietas, hoc eſt produ-
ctum. o. d. in. o. a. vnde ex methodo. 37. Theorematis cognoſcemus. a. o. et. o. d. & ſic
etiam eorum dupla , quod quærebatur.
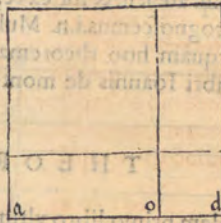


## T H E O R E M A C X L I.

P Vlchrum quæſitum fuit id, quod Tartalea ponit pro. 18. noni libri in quarto fo-
lio , quod huiuſmodi eſt . Aliquis habet dolium mero plenum , ex quo
duas vrnas extrahit ipſius vini, ſed loco ipſius vini infundit duas vrnas aquæ. Dein
de poſt aliquot dies extrahit iterum alias duas vrnas illius miſti , & iterum infundit
duas vrnas aquæ, & poſt alios aliquot dies idem facit , & hac vltima tertia vice in-
uenit aquam tantam eſſe, quantum vinum . Quæritur nunc quot vrnas capiat il-
lud dolium .

Solutio ipſius Tartaleæ bona eſt, cum ſupponat illas quatuor quãtitates vini eſſe
inuicem continuas proportionales, vt putà primò totum vinum merum , poſtcà re-
ſiduum pro ſecunda quantitate , deinde pro tertia in ſecunda , & pro quarta in ter-
tia extractione, hoc eſt quòd proportio totius vini meri ad vinum in prima ſit, vt hu
ius ad vinum in ſecunda , & vt huius ad vinum in tertia miſtione . Sed quia ipſe
non probat hanc continuam proportionalitatem ex methodo ſcientifica, mihi viſũ
eſt hoc loco illam deſcribere.

Cogitemus igitur. a. u. pro capacitate dolij , et. a. i. pro quantitate duarum vrna-
rum . Nunc uerò ſupponamus quamlibet partem huius miſti omogeneam eſſe ſuo
toto, quapropter ſequetur eandem proportionem eſſe vini ad aquam in qualibet
parte, quæ erit in toto, & ideò imaginemur. e. o. æqualem. a. i. Sed in puncto . i. tali
modo diuiſam, vt proportio. i. e. ad. i. o. eadem ſit quæ. i. a. ad. i. u. Supponamus etiã

e. o.

e.o.esse duas primas vrnas vini misti hoc est primæ mistionis, vnde cum eadem pro
portio sit.a.i.ad.i.u.vt.e.i.ad.i.o.ita erit (ex.19.quinti).a.e.ad.o.u.ut. a.i. ad. i. u. &
componendo ita erit.a.e.cum.o.u.hoc est.i.o.u. (propterea quòd.i.o.æqualis est. a. e.
vt residua totorum æqualium) ad.o.u.quemadmodum.a.i.u.ad.i.u. Quare. i. u. erit
media proportionalis inter. a.u.et.o. u. vnde proportio.a.u.ad.o.u.dupla erit pro
portioni.i.u.ad.o.u. Nunc autem cum extracta fuerit quantitas.e. o. ex primo mi-
sto, & postea infusa aqua vsque ad plenitudinem dolij, proportio ingredientium
huius secundi misti erit ea, quæ est inter.o.u.et.o.a.eo quòd in prima mistione pro-
proportio ingredientium erat ea, quæ est inter. o. u. et. a. e. vel inter. a. e. et.o.u.
vt demonstrauimus. Accipiamus ergo.t. m.huiusmodi secundi misti, magnitudi-
nis.a. i. vel . e .o .significantis duas vrnas, & permutemus eum in tantam aquam,
sitq; punctum.o. quod nobis diuidat.t.m.in.o.m.et.o.t.partes simplices, tali propor
tione inuicem relatas, vt sunt.o.u.et.o.a.vnde habebimus ex supradictis rationibus
eandem proportionem ipsius.a.t.ad.m.u.vt.a.o.ad.o.u.& componendo.a.t.cum.m.
u.hoc est.i.m.u.(eo quòd cum.t.m.æqualis sit.a.i.per consequens. i. m.æqualis erit.
a.t.)ad.m.u.vt.a.o.u.ad.o.u.sed proportio.a.o.u.ad.o.u. dupla erat proportioni.i.o.
u.ad.o.u.quemadmodum supra diximus. Ergo proportio.i.m.u.ad.m.u.erit dupla
similiter proportioni.i.o.u. ad. o.
u.quapropter.o.u.erit media pro
portionalis inter.i.u. et.m. u. Ec-
ce igitur quomodo eadem est pro
portio.a.u.ad.i.u.quæ.i.u.ad.o.u.& quæ.o.u.ad.m.u.qui quidem modus necessarius
est vt intellectus acquiescat, id quod experientia non facit.
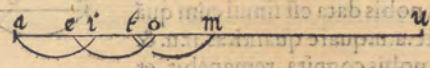
## THEOREMA CXLII.

PRæcedens Tartaleæ quæsitum elegans quidem est, sed pulchrum etiam vide-
tur quærere proportionem ingredientium in ultima mistione, cum cognita fue
rit nobis proportio continentiæ dolij ad capacitatis vrnæ simul cũ numero vitium
extractionum & impletionum.

Exempli gratia, si proportio.a.u.ad.a.i. cognita nobis fuerit, cognoscemus etiam
e.i.ex regula de tribus & per consequens etiam.i.o.residuum ex.e.o. & similiter ag-
gregatum.a.i.cum.i.o. & sic.o.u.residuum totius, et.o.t.similiter, eo quòd.a.u. ad.a.
o.est ut.t.m.ad.o.t.vnde cognoscemus etiam.o.m.vt residuum.t. m. & similiter ag-
gregatum.a.o.cum.o.m.hoc est. a.m.& etiam.m.u.residuum totius.

Cognoscere autem proportionem totius dolij ad vrnam, vel econtra, cum cogni
ta nobis fuerit proportio ingredientium in vltima mistione simul cum numero vi-
tium extractionum, & repletionum, quod scribit Tartalea, hoc etiam modo
possumus.

Exempli gratia, si proportio.m.u. ad . m . a. cognita nobis fuerit, illicò scie-
mus proportionem.a.u.ad.m.u.& cum sciuerimus numerum vitium extractionum,
& impletionum illicò cognosci-
mus multiplicitatem proportio-
nis.a.u.ad.m.u.ad proportionem.
o.u.ad.m. u. quapropter propor-
tio.o.u.ad.m.u.nobis cognita erit
hoc est.a.u.ad.i.u.& similiter ea, quæ est. a.u.ad.a.i. & è conuerso similiter.
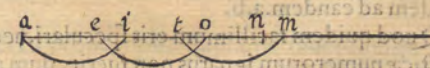
Vnde

Vnde cum aliquis diceret priori modo, dolium habeo vrnarum. 400. vini, & per vices. 25. extraxi & impleui ipfum, vt dictum eft. Nunc verò velim fcire proportionem vini ad aquam hac vltima vice. Nunc igitur fi procedemus iuxta doctrinam primi exempli huius theorematis, obtinebimus quod quærebamus.

Sed fi diceret iuxta Tartaleæ quæfitum, hoc eft dolium habeo, quod ignoro quot nã urnas contineat, volo tamen per. 25. vices extrahere, & implere vt fupradictũ eft, ita vt vltima vice proportio vini ad aquam fit fefquialtera. Tunc fi iuxta modum fecundi exempli huius theorematis procedemus habebimus quod cupimus.

Alio etiam modo aliquis quærere poffet, hoc eft, habeo doliũ quod capit. 400. vrnas. Habeo etiam vas trium vrnarum, quo mediante me oportet extrahere, & implere. Velim tamen fcire quoties me hoc facere oporteat, ita vt poftrema vice vinum fe habeat ad aquam in proportione fefquialtera, vnde multoties accidet vltimam extractionem, & impletionem mutilatam, feu imperfectam, euadere.

Exempli gratia, fi proportio vini ad aquam in vltima miftione deberet effe vt. n. u. ad. n. a. ita vt extrema vice fuiffet. t. m. quæ quidem. t. m. excederet terminum per. n. m. quæ. n. m. reuera effet nobis cognita, eò quòd ex priori modo hic fupra dicto proportio. a. m. ad. m. u. nobis innotefceret, & proportio. n. a. ad. n. u. nobis data eft fimul cum quã titate. a. u. quare quantitas. n. u. & m. u. nobis cognita, remanebit, et n. m. eorum differentia fimiliter, etiam, et. t. n. refiduum vafis, quo metimur, vnde neceffe erit, quod vltima vice vas contineret folum. t. n. reliqua uerò perfe patent.
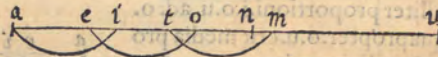
## THEOREMA CXLIII.

Hieronymus Cardanus in lib. fuæ arithmeticæ cap. 66. quæftione. 56. quam Cardanicam vocat, ita inquit.

Quidam perambulauit prima die certam quantitatem fpatij, & fecunda die, tã tò plus proportionaliter, quantò diameter eft maior cofta, & tertia die tantò plus fecunda, quantò proportionaliter portio lineæ diuifæ fecundum proportionem habentem medium, & duo extrema excedit minorem portionem, & quarta die in proportione ad tertiam vt fecunda ad primam, & quinta die proportionaliter tantò plus quarta, quantò in tertia plus fecunda, & ita alternatis vicibus in diebus nouem peregit nouem milliaria. Quæritur igitur quantum ambulauit die prima.

Hoc autem nihil aliud eft, quàm fi aliquis diceret, propono tibi, exempli gratia, lineam. a. l. nouem partibus inuicem non æqualibus ita diuifam. a. c. c. d. d. e. & cæteris, quarum partium proportiones tibi etiam do, vt putà. a. c. ad. c. d. et. c. d. ad. d. e. et. d. e. ad. e. f. & fic de cæteris vfque ad poftremam. k. l. quæ quidem proportiones fint etiam inuicem diffimiles, feu inæquales, do tibi etiam proportionê totius lineæ. a. l. ad. a. b. fuam partem, quæ vt in propofito exemplo nonupla eft.

Quæro nunc quam proportionem habebit. a. c. ad. a. b. & fic de cæteris partibus eiufdem ad eandem. a. b.

Quod quidem facillimum erit fpeculari, nec non operari vnicuique, qui omnino practicæ numerorum ignarus non fuerit, dum ab ordine fcientifico non difcedat.

Cum enim cognofcimus proportionem. a. c. ad. c. d. confequenter cognofcemus etiam proportionem aggregati. a. c. d. ad. c. d. cum autem cognouerimus proportionem.

nem.c.d.ad.d.e.si.c.d.accipiemus, vt medium inter.a.d.et.d.e.cognoscemus etiam proportionem.a.d. ad. d. e. quare etiam eam quæ. a.e.ad.d. e. collocando posteà. d.e.inter.e.f.et.a.e.innotescet ea,quæ est.a.e.ad.e.f. & ita gradatim accedemus ad perfectam cognitionem proportionis totius.a.l.ad.k.l. Nunc autem mediante.k.l. cognoscemus proportionem totius.a.l.ad.i.k. & hac mediante, eam cognoscemus, quæ totius.a.l.ad.g.h. & hac mediante eam quæ totius.a.l.ad.f.g. & sic gradatim, cognita nobis erit proportio totius lineæ.a.l.ad suam partem.a.c. beneficio posteà totius lineæ.a.l. cognoscemus proportiom.a.c. ad a.b. & sic aliarum respectu lineæ.a.b.vt quærebatur, quæ quidem propositio, etsi cardanica uocetur leuissima tamen est.
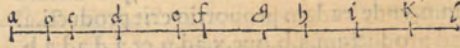
## THEOREMA CXLIIII.

QVamuis multi de modo in summa colligendi,subtrahendi, multiplicãdi, & diuidendi proportiones scripserint,nullus tamen ( quod sciam ) perfectè, ac scientificè speculatus est has operationes, quapropter hanc rem cum silentio transire nolui , quin aliquid de ipsa conscribam à summa dictarum proportionum incohando.

Quotiescunque igitur volunt duas proportiones inuicem aggregare , simul earum antecedentia multiplicant, & similiter earum consequentia . Tunc proportio terminata ab illis productis euadit in summam illarum duarum propositarum proportionum.

Vt exempli gratia,si voluerimus colligere proportionem sesquialteram cum sesquitertia, multiplicando.3.cum.4.antecedentia scilicet,productum erit. 12.posteà multiplicando.2.cum.3.consequentia, tunc productum erit. 6. Proportio igitur, quæ inter.12.et.6.reperitur.(quæ dupla est) est summa propositarum proportionũ.

Cuius rei speculatio erit huiusmodi sint.x.et.u. duo antecedentia quarumuis proportionum.t. verò et. n sint eorum consequentia, productum autem antecedentium sit.a.g.illud verò quod cõsequentium sit.d.a.vnde proportio.a.g. ad. a . d. composita erit ex proportione.x. ad. t. & ex ea, quæ est.u.ad.n.per.24.sexti vel quintam octaui. Patet igitur ratio rectè faciendi,vt suprà dictum est.
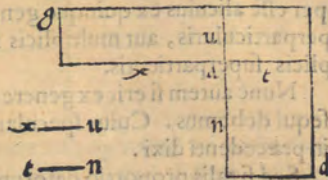
## THEOREMA CXLV.

QVotiescunque deinde detrahere volunt vnam proportionem ex altera multiplicant antecedens vnius cum consequenti alterius. Tunc proportio, quę inter talia duo producta inclusa reperitur, est residuum,seu differentia illarum duarum proportionum datarum .

Vt exempli gratia , si aliquis vellet ex proportione dupla detrahere sesquialteram , multiplicaret.2.antecedens duplæ cum. 2.consequenti sesquialteræ , quorum productum esset.4.pro antecedenti residuę proportionis . Deinde multiplicaret.3 antecedens sesquialteræ cum. 1. consequenti duplæ, & productum esset.3.pro cõ-

sequenti residuę proportionis; quæ quidem residua proportio esset vt.4. ad. 3. hoc est sesquitertia, & sic de cæteris.

Pro cuius ratione, sit proportio. x.ad.n.ea quæ ( exempli gratia ) maior sit, à qua volumus demere proportionem. t . ad. u. minorem scilicet. Nunc autem productum . x. in. u. sit. a. g. illud verò . t. in. n.sit.a.d. Tunc dico proportionem.a.g. ad. a. d. esse residuam quæsitam. Sit.b.a.productum u.in.n.vnde eadem proportio erit producti.a. g.ad productum,a.b.quę.x.ad.n.et.a.d.ad.a.b. quæ.t.ad.u.ex prima sexti, seu. 18. vel. 19. se-ptimi, sed proportio.a.g.ad.a.b.hoc est. x. ad. n.componitur ex ea, quæ est.a.g.ad.a.d.& ea, quæ est.a.d.ad.a.b.hoc est.t.ad.u.ergò ea, quę est.a.g.ad.a.d. erit quàm quærebamus.
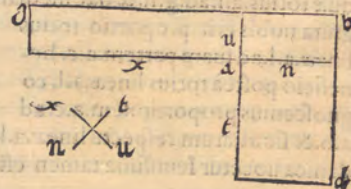
## THEOREMA CXLVI.

R ATIO verò, quòd rectè fiat, quotiescunque aliquam proportionem dupli-care volentes, quadramus terminos ipsius proportionis, vel si eam triplicare voluerimus, cubamus ipsos terminos, vel si eam quadruplicare voluerimus inuenimus censicos censicos terminorum ipsius proportionis,& sic de singulis,in.17 Theo. huiusmodi tractatus manifesta est.

## THEOREMA CXLVII.

Q Votiescunque nobis propositi fuerint duo numeri ad libitum,desideraremus que duas proportiones tali relatione inuicem refertas, quali sunt hi duo pro-positi numeri inter se, ita faciendum erit.

Sciendum primo est proportionem maioris numeri propositi ad minorem sem-per esse alicuius ex quinque generum, hoc est aut erit generis multiplicis, aut su-perparticularis, aut multiplicis superparticularis,aut super partientis, aut multi-plicis superpartientis.

Nunc autem si erit ex genere multiplici,iam ab antiquis traditus est modus, quę sequi debemus. Cuius speculatio à me inuenta patet.in. 17. Theo. huius libri, vt in præcedenti dixi.

Sed si talis proportio datorum numerorum erit alicuius aliorum generum, ita agemus, si fuerit superparticularis.

Sit exempli gratia, sesquialtera, tunc sumantur duo numeri inuicem inæquales, quos à casu volueris.o.et.c.qui quidem cubentur, & eorum cubi sint.a.et.e.Inuenia tur postea.u.ita proportionatus ad.o.vt.o.est ad.c.ex regula de tribus, hoc est diui-dendo quadratum ipsius.o.per.c.vnde nobis proueniat.u. & quia proportio.a.ad.e. tripla est proportioni.o.ad.c.& proportio.u.ad.c.dupla est eidé, quæ.o.ad.c. ideo proportio.a.ad.e.sesquialtera erit proportioni.u.ad.e.

Sed si proportio numerorum propositorum fuerit sesquitertia, faciemus.a. et.e. esse censica censica ipsius.o.et.c.tunc sumemus.u.consequentem ad.o. vt dictum est, deinde inueniremus.i.consequens ad.u.ita ut.u.consequens ipsius. o. tunc habebi-mus proportionem.i.ad.c.triplam, & eam quæ est.a.ad.e. quadruplam proportio-

ni.o.

ni.o.ad.c. Idem dico de reliquis proportionibus superparticularibus.

Sed si data proportio numerorum fuerit ex superpartientibus, vt exempli gratia de quinque ad tria, efficiemus, vt.a.et.e.sint prima relata ipsius.o. et. c. vnde proportio.a.ad.e . ita se habebit ad proportionem. o. ad. c. vt quinque ad vnũ & proportio.i.ad.c.ut tria ad vnũ.Quare proportio. a. ad . e. ad proportionem.i.ad.c. se habebit, vt quinque ad tria,& sic de reliquis.
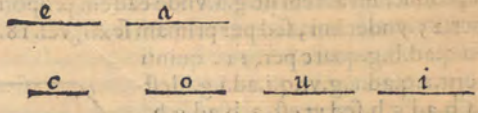
Pro alijs, eundem ordinem seruando, obtinebimus quod volumus.

## THEOREMA CXLVIII.

QVamuis in. 16.sexti et.20.septimi manifestè pateat ratio, quare rectè fiat accipiendam radicem quadratam illius producti, quod sit ex duobus datis terminis, vt medium proportionale geometricè inter ipsos habeamus : nihilominus, quia per aliam methodum hoc idem scire possumus, inconueniens non erit aliquid circa hoc dicere.

Cogitemus igitur exempli gratia, tres numeros continuè proportionales geometricè.a.b:c.d.et.e.f.quorum.a.b. et.e.f.tantummodo nobis cogniti sint, imaginemur etiam.g.a.esse productum quod sit ex.a.b.in.e.f.et.d.k.quadratum.c.d.et. a.h. id quod sit ex.a.b.vnde eandem proportionem habebimus.a.h.ad.a.g.quæ est. h. b. ad.b.g.ex prima.6. aut.18.vel.19.septimi, sed per.11.octaui ita est quadrati. a. h.ad quadratum . k. d. vt. a.b.ad.e.f.hoc est vt.h.b.ad.b.g.ergo per.11. quinti ita erit.a.h.ad.a.g.vt ad.k.d.vnde.a.g. æquale erit.k.d.per.9.quinti. Rectè ergo erit accipere radicem quadratam.a. g. pro.c. d.quod etiam est diuidere vnam datam proportioné per æqualia, hoc est in duas æquales partes, non dubito quin posset aliquis dicere non oportere vti posterioribus Theorematibus ad demonstrandum priora illis, sed hoc. 148. dictum sit ludendi loco.
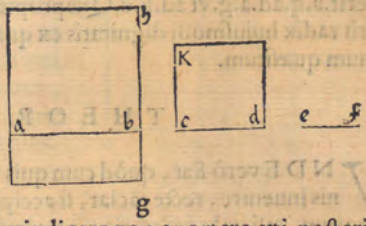
## THEOREMA CXLIX.

VNDE fiat ꝗ si quis inuenire voluerit secundum terminum ex quatuor nume ris continuè, & geometricè proportionalibus,quorum duo extremi tantummodo nobis cogniti sint, rectè factum sit quadrare primum eorum , & hoc quadratum posteà per alium terminum cognitum multiplicare,cuius producti demum accipere radicem cubam pro secundo termino quæsito, hoc loco videbimus.

Imaginemur quatuor terminos continuè proportionales, vt dictum est, esse.

N 2 a.b:

a.b:c.d:e.f.et.g.h.quorum.a.b.et.g.h.nobis tantummodo cogniti sint, sitq́; imagina
tione descriptus cubus.a.q.primi termini, cubusq́;.d.k.secundi termini, considere-
mus etiam basim.a.i.quadratam ipsius cubi.a.q. hoc est præcedentem dignitatem ip
sius cubi eiusdem radicis, quæ quidem basis.a.i.multiplicetur per quartum terminū
g.h.productum autem sit.g.a.vnde eadem proportio erit.a.q.ad.a.g.quæ.b.q. ad.b.
g.per.25.vndecimi, sed per primam sexti, vel.18.aut.19.septimi ita est,q.i.ad.i.g.
vt.b.q.ad.b.g.quare per.11. quinti

ita erit.a.q.ad.a.g.vt.q.i.ad.i.g.idest
vt.a.b.ad.g.h.sed vt est. a. b.ad.g.h.
sic est.a.q.ad.k.d.per.36. vndecimi,
seu per.11.octaui,vnde per.11.quin
ti sic erit.a.q.ad.a.g.vt ad.k. d.Qua-
re per.9.eiusdem.a.g.æqualis erit.k.
d.Vnde rectè erit accipere radicem
cubam.a.g.pro secūdo termino. c. d.

id, quod nobis inseruit ad inueniendam tertiam partem vnius propositæ propor-
tionis.
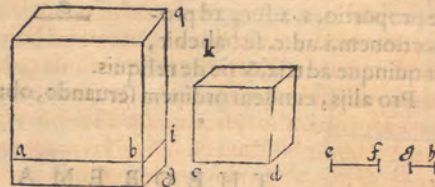
## THEOREMA CL.

SED vt speculatio ista ita vniuersalis fiat vt ad oēs dignitates applicari possit;
Supponamus.a.q.et.k.d.esse duas dignitates quas volueris vnius, sed eiusdem
speciei, et.a.i.dignitas præcedens dignitatem.a.q. à cuius multiplicatione in. a. b.
eius radix producitur dignitas.a.q.& ab ipsius. a.i.multiplicatione in.g.h.resultet.a.
g.vnde ex.18,vel.19. septimi eadem proportio erit. a.q.ad.a.g.quæ.a.b.ad.g.h. sed
eadem etiam est.a.q.ad.k.d.ex ijs, quæ in.17.theoremate dixi, vnde ex.11.quinti,
ita erit.a.q.ad.a.g.vt ad.k.d.Quapropter.a.g.æqualis erit.k.d. & ideo cum inuenta
fuerit radix huiusmodi dignitatis ex quantitate.a.g.habebimus.c. d. secundum ter-
minum quæsitum.

## THEOREMA CLI.

VNDE verò fiat, quòd cum quis voluerit dimidium alicuius datæ proportio-
nis inuenire, rectè faciat, si accipiat radices quadratas illorum datorum ter-
minorum, etsi voluerit tertiam partem, accipiat radices cubas : si autem quartam,
accipere radices censicas censicas ipsorum,& sic de singulis in.17. Theoremate om-
nia patent.

## THEOREMA CLII.

VNDE autem fiat, vt cum quis voluerit multiplicare aliquam proportionem
per fractos, rectè faciat prius multiplicando eam per numeratorem , dein-
de productum diuiserit per denominationem ipsorum fractorum.
Vt exempli gratia, cum aliquis voluerit multiplicare proportionem sesquiquar-
tam per duo tertia, multiplicabit prius ipsam proportionem per numeratorem. 2.
& productum, erit proportio.25.ad.16.qua postea diuisa per. 3. denominatórem,
prouentus erit proportio radicis cubæ.25.ad radicem cubam.16. vel vt proportio.

                                                                         25.ad

25.ad radicem cubam. 10000. quæ quidem proportiones æquales inuicem sunt, cũ tam vna, quàm alia, sit tertia pars totius.

Pro cuius ratione cogitemus.a.b. esse aliquod totum, quod multiplicare cupimus per duas tertias, quod quidé nihil aliud est, quàm accipere duas tertias partes vnius totius superficialis, imaginemur igitur hoc totum. a. b. lineare diuisum esse in tertias partes mediantibus.e.et.d.& tunc multiplicando ipsum per 2.tertias lineares produ-ctum erit.a.c.sex vnitatum superficialium, quod quidem productum postea diuisum per.3.dabit. d. c. hoc est duas tertias superficiales (quæ est tertia pars ipsius. a. c.) & æquales numero.c.b.duabus vnitatibus linearibus,idest duabus tertijs ipsius.a.b. No tandum etiam est,quòd cum fere omnia reducantur ad regulam de tribus,propterea etiam multiplicatio alicuius quantitatis per aliam quantitatem, nihil aliud est quàm quædam operatio ipsius regulæ de tribus, vt exempli gratia volo multiplicare. 25. per 20.hoc nihil aliud est nisi quærere alium numerum ita proportionatum ad. 25. vt 20.se habet ad vnum,vnde multiplicando.25.cum. 20. & productum diuidendo per vnum ex regula de tribus,prouentus est idem numerus ipsius producti,& propte rea cum volumus multiplicare aliquem numerum per fractos hoc nihil aliud est quàm quærere aliquem numerum ita proportionatum ad ipsum numerum datum, vt se habet numerator ad denominatorem,exempli gratia si.24.aliquis voluerit mul tiplicare per duo tertia hoc idem est vt si quæreret numerum ad quem. 24.ita se habeat,vt.3.ad.2.& idem dico de proportionibus,hoc est quod aliud non est multi-plicare aliquam proportionem per fractos, quàm aliam proportionem quærere ad quâ data se habeat,vt denominator se hét ad numeratoré;& hoc ex regula de tribus perficitur,cõstituẽdo denominatoré in primo loco,qui locus est diuisoris,numerato ré verò in secũdo loco,multiplicãdo postea pro portionem per numeratoré, & productũ diuidẽ do per denominatorem , prouentus demum erit proportio, ad quam data se habebit, vt denomi-nator se hét ad numeratorem ex ratione ipsius re gulę de tribus.Ratio verò methodi diuidẽdi vnã datam proportioné per fractos,ex se satis patet , cum idem sit modus diuidendi quemlibet nume rum integrum per fractos. Quare, quæ vnius, & alterius est ratio.
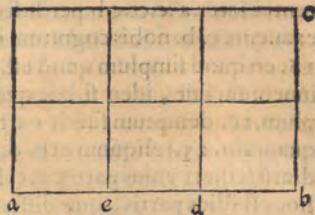
## THEOREMA CLIII.

Nicolaus Tartalea in . 3. lib. quintæ partis numerorum soluit . 24.quæsitum si-bi propositum à Hieronymo Cardano,via particulari & non generali.Quæ-situm autem tale est quamlibet propositam rectam lineam in duas partes ita diuide re via Euclidis,ut cubus totius lineæ ad cubos partium se habeat in proportione tripla.

Tartalea igitur inquit quòd vt satisfiat speculatiuis ingenijs soluendum sit huius-modi quæsitum,secando lineam propositam.a.b.in tres æquales partes,quarum vna sit.c.b.vnde problema solutum erit.

Verum dicit , sed hæc non est methodus generalis , propterea , quod cum tale problema alterius fuisset proportionis quàm triplæ, talis methodus nihil valeret.

lem

Quapropter non tacebo quod mihi in mentem venit circa hoc problema.

Sit ergo linea. a.b. diuisibilis in puncto.c.ita vt cubum totius dictæ. a. b. lineæ ad summam cuborum suarū partium. a.c.et.c.b.oporteat eam proportionem habére, exempli gratia, vt. 125. ad.65. vt vitemus fracta pro nunc, notantes talem proportionem quadrupla nunquam maiorem esse posse, vt quilibet ex se contemplari potest, constituendo punctum.c. in medio loco inter. a. et .b. vnde proportio totalis cubi ad summam partialium esset omnium maxima quæ possint esse, collocando. c. vbi volueris in dicta linea.a.b. & hæc esset quadrupla.

Sed vt ad propositum reuertamur, considerabimus cubum totalem ipsius . a . b . esse vt. 125. & summam partialium vt.65. quam detrahemus ex cubo totali & nobis remanebit.60. pro summa trium solidorum inuicem æqualium, quorum longitudo vniuscuiusque erit tota linea.a.b. nobis cognita vt radix dati cubi totalis, quæ erit in hoc exemplo quinque partium, latitudo verò vniuscuiusque dictorum solidorū erit.a.c. pars maior ipsius.a.b. quæ quidem.a.c. adhuc nobis ignota est, profunditas seu altitudo vniuscuiusque illorum solidorum, erit.c.b. pars reliqua ipsius.a.b.& etiā nobis incognita, sed quia summa horum trium solidorum nobis manifesta superius fuit, quæ erat. 60. propterà nobis cognita erit quantitas vniuscuiusque illorum solidorum, vt tertia pars totius summæ ipsorum quæ erit. 20. in proposito exéplo, dein de cum vnumquodque illorum solidorum producatur à superficie contenta seu producta ab.c.a.in.c.b. in tota linea.a.b. sequitur quòd si diuiserimus hoc solidum . 20 per lineam.a.b. quinque partium prouenict nobis cognita superficies producta ab. a.c.in.c.b. quatuor partium, sed cum quadratum totius.a.b. nobis cognitum sit, eo quod.a.b. vt eius latus etiam cognitum est. Tunc dictum quadratum erit. 25. quod quidem æquale est quadruplo illius quod fit ex.a.c.in.c.b. simul cum quadrato differentiæ inter.a.c.et.c.b. per.8. secundi Eucli. Vnde quia quadruplum illius quod fit ex.a.c.in. c. b. nobis cognitum est, vt

16. eo quod simplum quod est. 4. iā     b|————|c——————————|a

inuentum fuit , ideo si hoc quadruplum.16. demptum fuerit ex totali quadrato. 25. reliquum erit. 9. qua dratū scilicet vnius partis. a. c. ipsius hoc est illius partis, quæ differentia est inter.a.c.et.c.b. quæ quidem erit. 3. partium quæ differentia cum subtracta fuerit ex.a.b. reliquum erit du plum ipsius.c.b. duo scilicet. Quare.

c.b. erit vt. 1.et.a.c. vt. 4. & productum. a. c.in.c. b. erit. 4. vnitatum superficialium. &c.
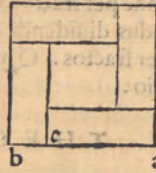
# APPENDIX
# DE SPECVLATIONE
## REGVLAE FALSI.

VNC idem ferè mihi accidit, quod & Michaeli Stifelio, à quo cum Petreius Tipographus nuper totam suam Arithmeticam recepisset, mox postea per literas petijt explicatione regulæ falsi.

Similiter post incisas omnes superiorum Theorematum figuras, opereq; Typographo commisso, amicus quidam omnium scientiarum ornatissimus maxima necessitudine mecum coniunctus monuit me, vt aliquid de regula falsi scribere vellem, cuius suasu hæc, quæ sequuntur appendicis vice ponere libuit, ne lector, quidpiam quod ad hanc rem pertinet iure merito à nobis desiderare posset ; vt autem ad ipsam regulã accedamus Ego sicut, & in alijs multis, ita & in huiuscæ regulę inuentione cum ipso Stifelio maximè conuenio, putans regulam falsi, seu falsarum positionum inuentam fuisse per paruos numeros in quæstionibus facillimis & cognitis, eodem ferè modo, quo ipse monstrat illis duobus exemplis, quæ quamuis ipse appellet theoremata, nihilominus theoremata ego illa non vocarem, nisi adiuncta fuerit speculatio ab ipso præterita, & non experientia tantummodo, vt ipse fecit. Primum eius exemplum est, quôd

Quorumcumque duorũ numerorum differentia, si fuerit multiplicata in aggregatum eorum, producit ipsam differentiam, quæ est inter quadrata eorum.

Secundum verò exemplum est, quod

Datis tribus numeris secundum progressionem arithmeticam dispositis, facit multiplicatio medij in se, quátum multiplicatio extremorum inter se cum multiplicatione differentiarum inter se.

Talia enim exempla ipse aliter non probat nisi experientia in aliquibus numeris, arbitratus ex eo inuentam esse regulam falsi, experientia tantummodo confirmatam, quod quidem etiam & ego credo. At experientia in philosophia mathematica, aut nullã prorsus facit scientiã, aut omnino superfluus fuit Euclides in multis suis propositionibus, & præcipuè in eius secundo libro, si sufficeret experientia. Idcirco quo magis ad euidentiam ipsius veritatis, quam profiteor, deuenire possim,

accipiã primò primum exemplum ipsius Stifelij hic superius citatum, & pro numero maiori, in prima hic subscripta figura. AE. accipio. a. i. cuius quadratum sit. a. c: pro minori verò numero capio. a. e. parté ipsius a. i. cuius quadratum sit. a. t. differentia autem horum numerorum erit. e. i. reliqua pars ipsius. a. i: & differentia ipsorum quadratorum erit gnomon. e. c. o: Nunc autem protraho. i. c. latus quadrati maioris quousque c. n. æqualis sit. a. e. numero minori, perficioq; rectangulum. c. n. quod produ-

producitur ex.i.e.differentia in.i.n.aggregatum amborum numerorum, sed hoc pro
ductum excedit productum.e.c: partem gnomonis dicti per.u.n. quod quidem . u.
n. æquatur ipsi.u.o. reliquæ scilicet parti ipsius gnomonis, nã.e.u. æqualis est. i.c. qua
re et. a.i.sed.e.t. equatur.e.a. vnde.t.u. æqualis erit.e.i. quare et.u.c: at cum.c.n.æqua
lis sit ipsi.a.e. erit etiam æqualis ipsi.
o.t. quare. u. n. æqualis erit ipsi. u . o.
& tunc intellectus quiescit , & absq;
aliqua alia experientia verè scientifi
ceq; dicere potest, quòd
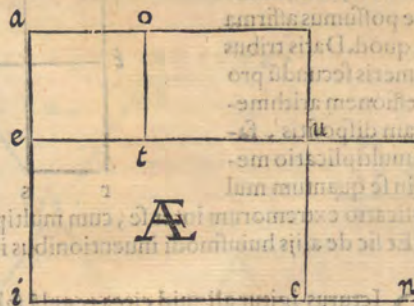
Quorumcumque duorum nume-
rorum differentia , si fuerit multipli-
cata in aggregatum eorum, producit
ipsam differentiã , quæ est inter qua-
drata eorum.

Hæc autem propositio à me ipso
etiam in. 60. Theoremate huius libri
aliter demonstrata fuit.
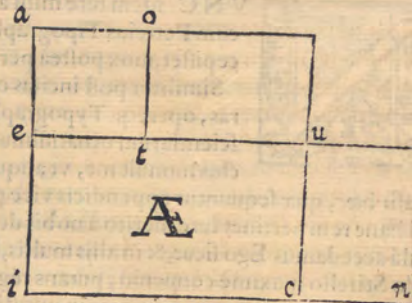
D E speculatione autem, et scientia secundi exempli, in secunda hic subscripta
figura. ⌣. cogitemus lineam.u.a. tribus in partibus arithmeticè diuisam, qua
rum maxima sit.u.o. media.sit.o.e. minima verò sit.e.a. multiplicatio autem mediæ.
o.e. in se sit quadratum.o.t. abscindatur deinde ex.o.e: e. i. æqualis.e.a. tunc. o. i. erit:
differentia inter.o.e. et.e.a. & æqualis differentiæ inter.o.e. et.o.u. ex hypotesi , qua:
quidem.o.i. in se ducta procreabit quadratum.o.c. quod erit productum ex differen
tijs ipsarum partium , & erit pars quadrati . o.t. superius dicti , vt ex se patet. Nunc
autem dico gnomonem.i.t.n. æqualem esse ei quod sit ex.a.e.in.o.u. Producatur igi
tur.e.t. quousque.t.r. æqualis sit ipsi.o.i. tunc.e.r. erit æqualis.o.u. quod etiam clarum
est . Claudatur ergo rectangulum. i.r. quod erit æquale producto ipsius. e.a. in.o. u.

Nam.e.i. sumpta fuit
æqualis.e.a. sed ex ra
tionibus in priori exé
plo allatis, ˒ductum.
i . r. æquale erit gno-
moni.i.t.n. Nunc au
tem verè, scientifice-
que possumus affirma
re, quòd. Datis tribus
numeris secundũ pro
gressionem arithme-
ticam dispositis , fa-
cit multiplicatio me-
dij in se quantum mul
tiplicatio extremorum inter se , cum multiplicatione differentiarum inter se.
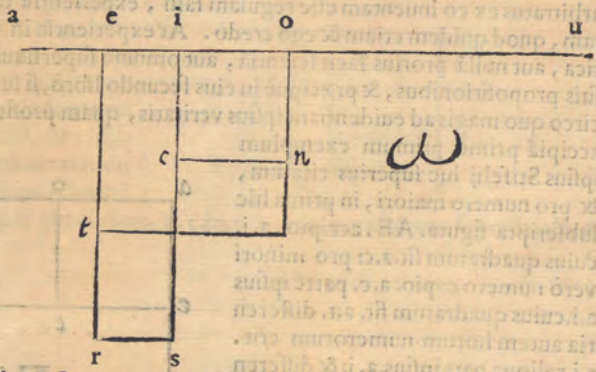
Et sic de alijs huiusmodi inuentionibus infero.

D Icturus igitur aliquid circa regulã falsi, videtur mihi nullam oportere facere
mentionem de origine huiuscæ regulæ , cum in hoc Stifelius satisfecerit, sed

potius

potius veras rationes propriaq; fundamenta huiufmodi operationis oftendere, fu-
mendo eadem exempla propofita ab ipfis practicis, & maximè à Nicolao Tartalea
viro accuratiffimo, qui vbicunque potuit fpeculatus eft caufas ipfarũ operationum,
etfi de huiufmodi falfi regula circa finem cap. 8. lib. 17. promittat poftea loqui, nul-
libi tamen loquutus eft. Monendum etiam cenfeo, me nihil de rationibus regulæ
falfi fimplicis dicturum, cum ex feipfis fatis appareant, quod non ita eft de pofitio-
nibus duplis. Incipiam ergo à primo problemate lib. 17. ipfius Tartaleæ, quo etiã
ipfe vtitur pro exemplo docendi gratia, ipfam regulam duplæ pofitionis, quod qui
dem problema aliter à me folutũ fuit in. 118. Theoremate huius mei lib. quod fimi
liter ob hanc demum occafionem mihi oblatam, alia etiam via, fpeculatus fum idem
poffe fieri, quæ quidem via feu methodus generalis erit, & ita fe habet.

Accipio enim propofitum numerum diuifibilem, à quo detraho fummam
datorum numerorum, primo duplicato, eo quòd tam in fecunda quam in
tertia parte reperitur, vt in propofito exemplo, datus numerus eft 50. à
quo detraho fummam dictorum numerorum, quæ eft. 11. nam tres, & tres, &
quinque funt vndecim, eo quòd primus ingreditur in fecunda, & in tertia parte,
dempto igitur hoc numero. 11. ex. 50. remanet. 39. qui quidem numerus intelligen-
dus eft pro fumma trium partium fimplicium adhuc incognitarum, à quo extrahen
da eft prima, eo modo quo nunc proponam ex regula de tribus, hoc eft aggregan
do dictas partes fimplices fine aliqua additione vtcunque volueris (fed commodius
erit in minimis numeris) iuxta propofitum, quod quidem propofitum eft, vt fecun
da pars dupla fit primæ, tertia verò æqualis fit primæ & fecundæ, quæ partes in di-
ctis minimis numeris, ita difpofitæ erunt. 1. 2. 3. quarum fumma erit. 6. Nunc fi ex
regula de tribus dixerimus, cum hæc fumma proueniat nobis ab vno, à quo proue-
niet. 39. et veniet nobis. 6. cum dimidio pro prima parte quæfita in propofito nume-
ro. 39. cum ergo habuerimus primam parté, reliquas poftea illicò cognofcemus.

Huiufmodi verò operationis ratio ex fe manifefta patet, eo quòd proportio fum
mæ partium in minimis numeris ad primam eorum partem eadem effe debet, quæ
ipfius. 39. ad primam partem quæfitam huiufmodi aggregati partium fimpliciũ, fed
quia nemo adhuc, quod fciam, fatis animaduertit rationem modorum, qui ab anti-
quis obferuati funt, qui quidem modi duo funt circa hoc Helcataym duplæ falfæ
pofitionis, igitur non prætermittam aliquid de hac re fpeculari, & primo de pri-
mo modo.

In primis igitur fciendũ eft, ợ
veritas ita inueniri poterit eo-
rum modo, mediantibus fimpli-
cibus partibus, vt etiã median-
tibᵒ cõpofitis, ut in prefenti exẽ
plo pro primis pofitionibus ac-
ceperunt. 10. et. 8. pro fecundis
verò compofitis cũ numero. 3.
inuenerũt. 23. et. 19. pro tertijs
aũt cõpofitis cũ quinq;, notaue
runt. 38. et. 32. vnde prima fum
ma refultauit. 71. fecunda verò.
59. ita ợ prim°error remanebai
21. fecũd° aũt. 9. vt in figura. A.

Compofitorum.

$$\begin{array}{ccc} 90 & 168 & 78 \\ 10 & 8 & 6\frac{1}{2} \\ 23 & 19 & 16\frac{1}{2} \\ 38 & 32 & 27\frac{1}{2} \\ 7 & 12 & 59 & 50 \end{array}$$

A

21   12   9
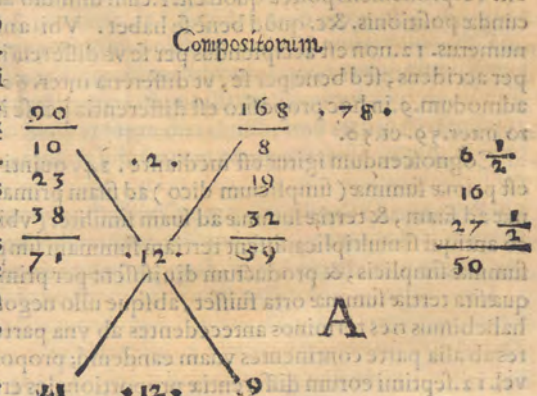
O   Sed

SED ijdem errores proueniunt ex summis partium simplicium.
Vt exempli gratia, in figura. B. summa proposita partium simplicium est. 39.
vt diximus, eo quòd ab ipso.50.detraxerimus.11.summa scilicet numerorum adij
ciendorum ad efficiendas partes compositas, summa posteà simplicium partium
primæ positionis, erit.60.eo quòd prima pars erat.10.secunda autem simplex.20.
tertia verò simplex.30.iuxta ordinem propositi. Summa deinde simplicium partiũ
secundæ positionis esset.48.quia prima eius pars erat.8.secunda verò simplex.16.
tertia autem simplex.24.vnde prima summa excederet datam.39.per.21.differen-
tiæ, secunda verò per.9. vt supra vidimus de summis compositis à dato.50.compo-
sito, & hoc quidem mirandum non est, quod scilicet tres summæ simplicium par-
tium sint inuicem inæqua-
les, ijsdem differentijs me-
diantibus, quibus differũt
dictæ tres summæ composi
tæ, cum ab vnaquaque có
positarũ ablatus sit nume-
rus.11.æqualiter, vnde ex
necessitate, permutando,
earũ differentiæ relinquẽ
dæ erant æquales inuicem
ex.78. theoremate  hu-
ius nostri lib.summæ enim
compositæ erant.71.59.et
50.simplices verò.60.48.
et.39.differentes à primis
per.11. vt dictum est, qua

**Simplicium.**



re veritas ita manabit à compositis, quemadmodum à simplicibus, sed à simplici-
bus per se, & à compositis per accidens vt iam iam videbimus.

ANtiquorum igitur primus m odus vtitur regula de tribus, hoc ordine, multi-
plicando scilicet secundum errore m, qui est.9. cum differentia primarum par
tium positarum, quæ est.2.& productum diuidendo per differentiam errorum, quæ
est.12.proueniens posteà quod est.1.cum dimidio additur hoc loco primæ parti se-
cundæ positionis. &c. quòd benè se habet. Vbi animaduertendum est, quod ille
numerus.12.non est accipiendus per se vt diffe rentia errorum hoc est.21.et.9. nisi
per accidens, sed benè per se, vt differêtia inter.60.et.48.simplices summas, quẽm
admodum.9.in hoc proposito est differentia per se inter.48.et.39.per accidens ve-
ro inter.59.et.50.

Cognoscendum igitur est mediante.24. quinti Eucli. quod eadem proportio
est primæ summæ (simplicium dico) ad suam primam partem, quæ secundæ sum-
mæ ad suam, & tertiæ summæ ad suam similiter (vbi rectè etiam fecissent hoc in lo-
co antiqui si multiplicauissent tertiam summam simplicem cum prima parte prioris
summæ simplicis, & productum diuisissent per primam summam, vnde prima pars
quæsita tertiæ summæ orta fuisset, absque ullo negotio ipsius plus vel minus) Quare
habebimus tres terminos antecedentes ab vna parte, & tres terminos consequen-
tes ab alia parte continentes vnam eandemq; proportionem, vnde ex.19. quinti,
vel.12.septimi eorum differentiæ proportionales erunt, hoc est, q̃ eadem propor

tio
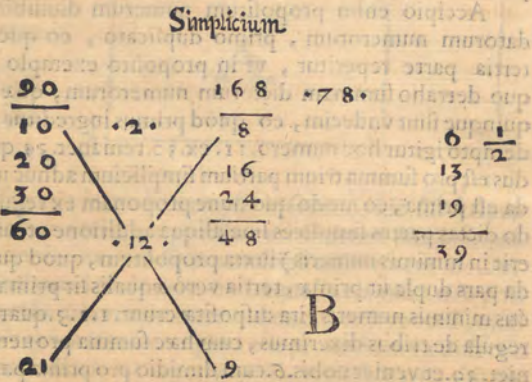
tio erit eius differentiæ, quæ est inter primam & secundam summam, ad differentiam quæ est inter primas earum partes, quæ illius differentiæ, quæ est inter secundam & tertiam summam, ad differentiam, quæ est inter primas illarum partes, sed harum.4.differentiarum, tres nobis cognitæ sunt, idest.1 2.et.9.ergo ex regula de tribus ab Eucli.in.20.septimi speculata inueniebatur quarta differentia, quæ est.1. cum dimidio.

A compositis summis idem etiam proueniet, sed non vt ex proprijs causis, & per se, sed per accidens. Nam quamuis eadem differentia sit inter 7 1 . et. 5 9. quæ inter.60.et.48.& eadé inter.5 9.et.5 0.quæ inter.48.et.3 9.Nihilominus non est eadé proportio (propriè)ipsius.7 1.ad.5 9.quæ ipsius.60.ad.48.nec ea quæ ipsius.5 9.ad. 5 0.est quæ ipsius.48.ad.3 9: Vnde non erit eadem proportio ipsius.7 1. ad. 5 9. quæ ipsius.1 0.ad.8.nec ea quæ est ipsius.5 9.ad.5 0.quæ ipsius.8.ad.6 cum dimidio. Sed minores illis . Nam ex æqualibus additamentis diminuuntur proportiones maióris inæqualitatis.

A simplicibus igitur summis pendet ratio huiusmodi effectus . Si vero prima pars secundæ positionis esset. 4. tunc secunda eius pars esset.8. & tertia. 1 2. quarum summa esset.24. (harum simplicium partium scilicet)& minor vera (39.)per.1 5.& differens à summa primarum. (60.) per. 36. & differentia primarum partium esset.6.differentia vero primæ partis secundæ positionis,a prima parte qúe sita esset. 2.cum dimidio . Vnde in huiusmodi exemplo videre est quare colligantur errores inuicem,quando alter eorum eccedit,réliquus vero deficit à numero proposito . Quod quidem ob aliam causam non sit,nisi vt cognoscatur differentia. 36. differentia scilicet simplicium summarum ipsarum positionum.

Secundus autem modus ab antiquis magis exercitatus est, quod multiplicabant diametraliter errores cum primis partibus, hoc est primum errorem cum prima parte, hoc est cum numero secundæ positionis, secundum vero errorem cum prima parte, hoc est cum numero primæ positionis, differentiam postea vel aggregatum horum duorum productorum diuidebant per differentiam vel aggregatum dictórum errorum, proueniens postea erat prima pars quæsita numeri propositi . Vnde oriebantur tria producta,quorum tertiú, hoc est differentia, seu aggregatum illorum constituebatur ex differentia seu aggregato errorum, & ex numero quæsito.

Vt in præsenti exemplo, primus error est. 2 1. qui multiplicatus cum prima parte secundæ positionis, quæ est.8.producit.1 68.secúdus verò error est.9.,qui multiplicatus cum prima parte primę positionis producit.90.differentia autem horum productorum est.78.quæ diuisa per differentiam errorum,quæ est 1 2.dabit.6.cú dimidio,pro prima parte quæsita dati numeri diuifibilis, qui erat.5 6.

Hæc omnia rectè se habent . Sed, vt supra dixi diuisor non est per se differentia errorum,neque etiam differentia per se summarum compositarum , sed bene simplicium.

Pro cuius rei speculatione,accipiendæ sunt summæ simplices , quarum differentiæ per se vtiles sunt in huiusmodi operatione ; & quia etiam ratiónes veritatis ex istis, & non ex illis fluunt; quamuis tam vnæ, quam aliæ sint eædem in quantitate, idest æquales.

cum                                                    O 2        Dispo-

Disponantur igitur huiusmo-
di numeri tali ordine, vt sim-
plex summa, quæ ab vna reli-
quarum superatur, & aliam supe-
rat, medium locum teneat; vt
in proposito exemplo summa
mediocris est. 48. quę à summa.
60. superatur, & superat sum-
mam. 39. locata igitur sit hęc.48.
inter illas, suæ verò primæ partes
similiter constitutæ sint supra di-
ctas summas, cum suis differētijs,
& tria producta iam dicta, vt in fi
guris. C. et. D. arithmeticis
clarè patet: figura enim. C. est
pro exemplo ipsius plus simpli-
citer: figura verò. D. pro exem-
plo ipsius plus, & minus. Et sic
in figura. C. habebimus tres
numeros consequentes. 60. 48.
39. & tres antecedentes. 10. 8.
6. cum dimidio, vnam, & ean-
dem proportionem terminantes,
ex. 24. quinti, vt diximus; qua-
re eorum differentiæ similiter
proportionales erunt, quod etiam
vidimus. Supponamus nunc nos
ignorare æqualitatem maximi
producti cum reliquis duobus
accipiendo solum pro hypotesi,
quòd dicta producta oriantur
ex lateribus iam dictis.

$$\frac{90}{10} \qquad \frac{168}{8} \qquad \frac{78}{6\frac{1}{2}}$$

$$2 \qquad 1\frac{1}{2}$$

$$3\frac{1}{2}$$

C

$$60 \qquad 48 \qquad 39$$

$$12 \qquad 9$$

$$21$$

$$\frac{63}{7} \qquad \frac{78}{6\frac{1}{2}} \qquad \frac{15}{5}$$

$$\frac{1}{2} \qquad 1\frac{1}{2}$$

$$2$$

D

$$42 \qquad 39 \qquad 30$$

$$3 \qquad 9$$

$$12$$
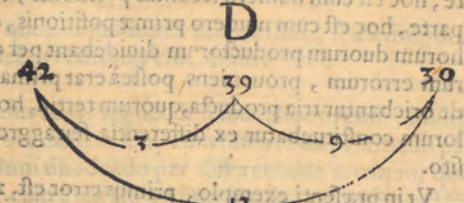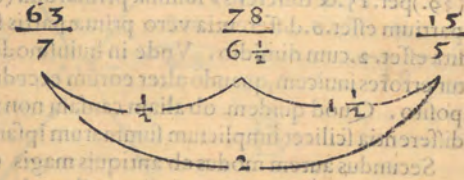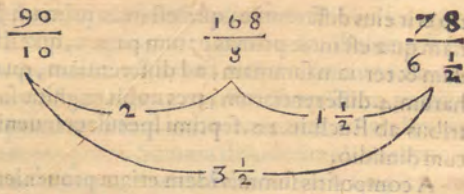
Demonstrandum nobis nunc relinquetur, maximum productum æquale esse re-
liquis duobus, hoc est productum. 168. æquale esse productis. 90. et. 78. quorum
duorum productorum alterum. 90. scilicet, generatur à differentia. 9. quæ est se-
cundę, & rertię summæ, in primum numerum antecedentem, qui est. 10. alterum ve-
ro productum. 78. scilicet, generatur à differentia. 12. quę est primę, & secundę sum
mę in tertium numerum antecedentem, qui est. 6. cum dimidio, maximum vero
productum. 168. scilicet generatur à differentia maxima. 21. quę est primę, & tertię
summę (& semper ęqualis prioribus duabus differentijs. 12. et. 9.) in secundum nu-
merum antecedentem, qui est. 8.
　　Constituantur igitur duo producta simul iuncta ęqualia duobus. 90. et. 78.
lateralibus supra vnam aliquam rectam lineam. q. p. sitq; productum. f. g. ęquale.
90. productum verò. g. n. ęquale. 78. sit etiam basis. g. p. vt. 9. et. g. q. vt. 12. vnde. g. i.
vel. q. n. erit vt. 6. cum dimidio. et. g. d. vel. p. f. vt. 10. & ideo. i. d. differentia erit. 3.
　　Dispo-　　　O 2　　　　　　　　　　　　　　　　　　　　　　　cum

cum dimidio, ut in figura.C.geometrica hic subscripta videre licet,et.q.p.erit . 2 1.
Cogitemus nunc differentiam.d.i.diuisam esse in puncto.e. ita vt eadem proportio
sit ipsius.d.e.ad.e. i. quæ ipsius.q.g.ad.g.p.hoc est vt.1 2.ad.9.quapropter.d. e. erit.
2.et.e.i.erit. 1.cum dimidio , vt in dicta figura.C.arithmetica reperiuntur esse dif-
ferentiæ ipsorum antecedentium numerorum,deinde à puncto.e.ducatur imagina-
tione.u.e.o.æquidistans ipsi.q.p. & producatur. q.n.vsque ad.u.vnde ita se habebit
u.e.ad.e.o.ut.q.g.ad.g.p.quare vt.d.e.ad.e.i.ideo ex.1 5.sexti vel. 20.septimi. n. e.
rectangulum æquale erit ipsi.e.f.quapropter rectangulum. q. o. æquale erit duobus
rectangulis.f.g.et.g.n:sed cum.g.i.sit vt.6. cum dimidio, et.i.e.vt. 1.cum dimidio,er
go.g.e.erit ut.8.qui quidem numerus multiplicatus cum.q. p.2 1.producit.168. ve
rum est igitur quod dictum fuit, hoc est ꝙ maximum productum æquale sit reliquis
duobus.
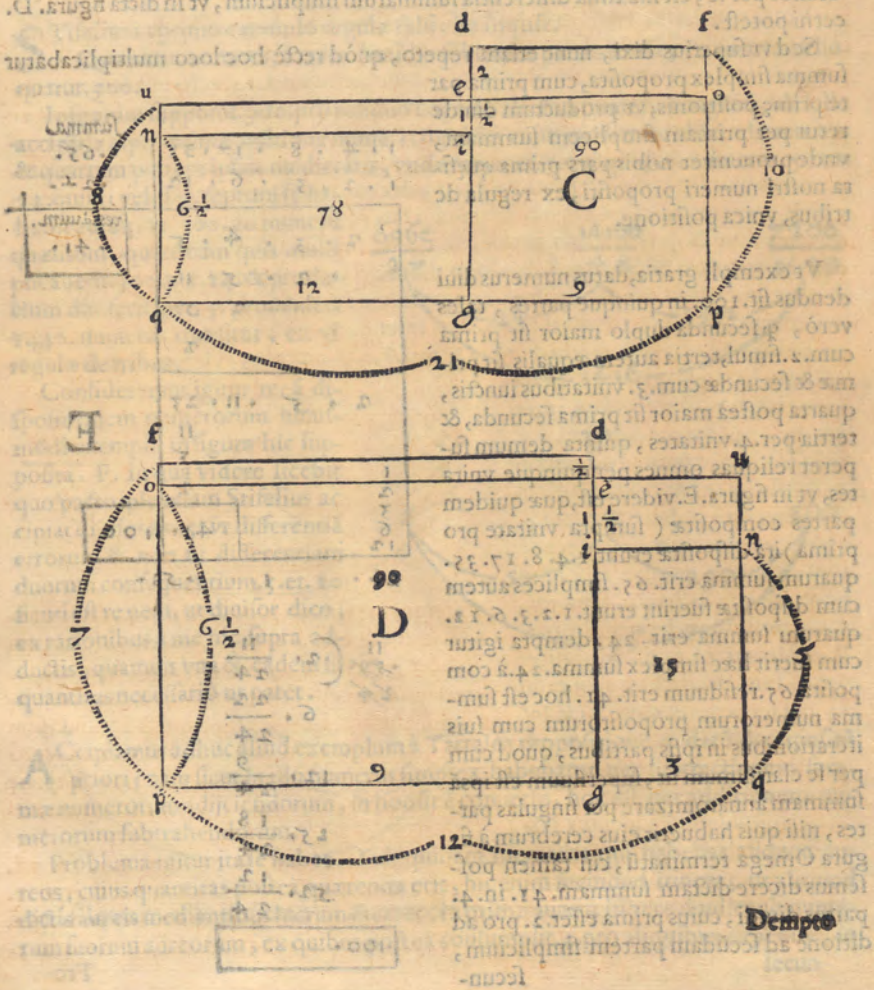
Empto posteà quo volueris horum altero productorum ex maximo, diuisóq; reliquo per differentiam consequentium, ipsi diametraliter oppositam, pro ueniet tibi numerus antecedens correspondensq; illi.

Animaduertendum tamen est, quòd si in figura à me ita ordinata, summa simplex proposita medium locum occuparet, vt in figura. D. arithmetica videri potest; tunc vt habeatur eius productum, addenda simul erunt circunstantia producta. eo q eius secundum latus esset antecedens medio loco constitutum, & prima pars quęsita numeri propositi: in qua figura. D. manifestè patet ratio, quare colligendi sint tam errores, quam producta, dum eorum alterum est plus, reliquum verò minus.

Speculatio figurę. D. arithmeticę videbitur in figura. D. geometrica, eodem fe rę modo quo fecimus in figuris. C. mutatis mutandis, respectu ipsius plus, & minus.

Collectio namque errorũ similiter accidentalis est, eo quòd essentialis numerus diuisor per se, est maxima differentia summarum simplicium, vt in dicta figura. D. cerni potest.

Sed vt superius dixi, nunc etiam repeto, quòd rectè hoc loco multiplicabatur summa simplex proposita, cum prima parte primę positionis, vt productum diuideretur per primam simplicem summam, vnde proueniret nobis pars prima quęsita nostri numeri propositi, ex regula de tribus, vnica positione.

Vt exempli gratia, datus numerus diuidendus sit. 100. in quinque partes, tales verò, q secunda duplo maior sit prima cum. 2. simul, tertia autem æqualis sit primæ & secundæ cum. 3. vnitatibus iunctis, quarta posteà maior sit prima secunda, & tertia per. 4. vnitates, quinta demum superet reliquas omnes per quinque vnitates, vt in figura. E. videre est, quæ quidem partes compositæ (sumpta vnitate pro prima) ita dispositæ erunt, 1. 4. 8. 17. 35. quarum summa erit. 65. simplices autem cum dispositæ fuerint erunt. 1. 2. 3. 6. 12. quarum summa erit. 24. dempta igitur cum fuerit hæc simplex summa. 24. à composita. 65. residuum erit. 41. hoc est summa numerorum propositorum cum suis iterationibus in ipsis partibus, quod cum per se clarissimum sit, superfluum est ipsã summam annatomizare per singulas partes, nisi quis habuerit eius cerebrum à figura Omega terminatũ, cui tamen possemus dicere dictam summam. 41. in. 4. partes diuidi, cuius prima esset. 2. pro additione ad secũdam partem simplicium,

secun-

secunda verò esset.5.pro additione ad tertiam partem simplicium, tertia autem es-
set.11.pro additione ad quartam partem simplicium, quarta demum esset.23.pro
additione quintæ partis simplicium, quarum partium.2.5.11.23. summa est.41. vt
diximus. Hæc igitur summa.41.subducenda est à numero.100. proposito, vnde re-
linquetur.59.pro summa partium simplicium numeri propositi, quarum prima erit
.2.cum vndecim vigesimisquartis ex diuisione huiusmodi.59.per.24. summam par-
tium simplicium ex vi regulæ de tribus, dicendo si.24.prouenit nobis ab.1. prima
partium simplicium, à quo proueniet nobis.59?vnde proueniet à.2. cum vndecim
vigesimisquartis pro prima parte quæsita, secunda verò iuxta propositum, erit.6.
cum.22.vigesimisquartis, tertia autem.12.cum nouem vigesimisquartis, quarta po
steà.25.cum.18.vigesimisquartis, quinta demum erit.52.cum.12.vigesimisquartis,
quarum omnium summa erit.100.

Tifelius in primo exemplo regulæ falsi, ita inquit.

Quæratur numerus, à cuius dimidio subtractæ partes tertia, & quarta relin-
quatur.300.

Ipse enim supponit.300.pro residuo cognito alterius numeri incogniti, deinde
accipit.24.pro prima positione numeri cogniti, à cuius medietate abscindit tertiam
& quartam partem ipsius medietatis, vnde remanet.5.qui quidem numerus.5. ex.
22.quinti vel.15.septimi se ha-
bebit ad.24. vt.300. ad numerū
quæsitum, quare cum quis multi
plicauerit.300.per.24. & produ-
ctum diuiserit per.5. prouæniet.
1440. numerus quæsitus, ex vi
regulæ de tribus.

Consideremus igitur meā di-
spositionem numerorum huius-
modi exempli, in figura hic sup-
posita. F. in qua videre licebit
quo pacto ipse etiam Stifelius ac
cipiat diuisorem.5. vt differentiā
errorum & non ut differentiam
duorum consequentium.5.et.10
sicuti est re vera, ut diuisor dico,
ex rationibus à me hic supra ad-
ductis, quamuis vna & eadem sit
quantitas necessariò ut patet.



Accipiamus adhuc aliud exemplum à Tartalea propositione.9. datū,& oppositū
priori; nam sicut in illo numerus simplex habebatur per subtractionem sum-
mæ numerorum adijciendorum, in hoc fit è conuerso, hoc est per additionem nu-
merorum subtrahendorum.

Problema igitur ita se habet. Fuit quidam mercator qui habebat aliquot au-
reos, cuius quantitas posteà quærenda erit, hic enim fecit duo itinera, ut aliquod
dictis aureis mediantibus lucrum faceret, in primo autem itinere duplicauit nume-
rum suorum aureorum, ex quibus posteà consumpsit.4.pro aliquibus expensis, in

secun-

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secundo itinere iterum duplicauit fuos aureos, ex quibus etiam poftea confumpfit. 8.numeratis poftea pecunijs reperit tantummodo.24.aureos in eius marfupio, quę ritur nunc quot habebat aureos in principio primi itineris.

    In tali cafu, cum ipfe quolibet itinere duplicabat eius pecuniam, nulli dubium eft quòd in fine fecundi itineris ipfe habuiffet pecuniam fuam quadruplicatam, fi ex ipfa nihil detractum fuiffet, fed quia in fine primi itineris confumpfit.4. aureos, quibus alios.4. lucratus effet in fecundo itinere, poftea confumpfit iterum. 8.aureos, ita ꝙ ex quadruplo fuæ primæ pecuniæ, rectè dici poteft, quod confum- pferit.16.aureos; qui quidem numerus ex communi conceptu erit differentia in- ter.24.& quadruplum prioris pecuniæ, cum qua profectus fuit in principio eius iti- neris; quapropter fi addiderimus.16.ipfi.24. habebimus.40. pro quadruplo eius prioris pecuniæ. Rectè igitur dici poteft, fi.4.prouenit ab vno, à quo numero pro ueniet.40.

    Videamus igitur nunc quo pacto hoc refpondeat cum methodo antiquorum. Ego enim inueni duas pofitiones fcriptas à Tartalea pro prima pecunia hoc eft.12. et.14.fed à.12.pro primo errore reperi.8.more antiquo à.14.verò pro fecundo er- rore proueniebat.16.producta autem horum numerorum diametraliter, funt.112. et.192.quorum differentia eft.80.pro tertio producto, quo diuifo per differen- tiam erroru.8.fcilicet, præbet nobis.10.pro pecunia quæfita, vt etiam ego inueni.

    Sed hoc mihi vifum eft fubtilius examinare mea methodo mediante, vt in figu- rà. G. videre eft, prius enim fuo loco pofui tria producta dicta, deinde duas pofitio nes.12.et.14.& quia fciebam productum.112. oriri à multiplicatione.14. cum.8. ideo pofui talem numerum.8. fuo loco diametraliter oppofito ei producto.112. & quia fciebam etiam productum.192.nafci ex.12.et.16.ideo fuo loco pofui hunc numerum.16.qui eft maxima differentia inter duos confequentes( ita à me fupra nominatos ) à qua differentia dempta priori.8.iam inuenta, reliqua.8. mihi daba-

tur, quá fuo loco notaui, fuo etiã loco fcripfi.2. differentiam inter 12.et.14. antecedentium. fed quia fciebam eandem proportio nem effe inter hanc differentiam & differentiã.8. huic fuppofitã, quæ reperitur inter.12.antecedé tem, & fuũ confequentem; ideo pofui.48.pro dicto confequenti, diuifi poftea productum.80.per. 8. differentiam ei diametraliter oppofitam, vnde prouenit mihi 10.cui ita proportionatus eft fuᵍ numerus confequens.40.vt.48. ad.12.et.56.ad.14.ex ijfdem ra- tionibus à me fupra dictis. In tali igitur figura videntur nu- meri naturaliter correfpondétes ipfis pofitionibus,& hac metho- do poffumus inuenire tales numeros confequentes in omnibus alijs exemplis à no- ftris maioribus fcriptis.

$$\frac{112}{14} \qquad \frac{192}{12} \qquad \frac{80}{10}$$

2      2

4

56      48      40

8      8

16

PRoponitur etiam quoddam vas, cuius pes sit quarta pars totius vasis cum oper culo, pars autem media sine operculo, sit quinta pars ipsius pedis, operculum verò. 18. libras pendeat. quæritur nunc quantitas dicti pedis.

Ex methodo enim antiquorum inuentus est pes. 4. cum. 14. decimisnonis talium partium, seu librarum, qualium operculus est. 18. Videamus igitur & nos ex nostra figura, quo pacto hoc respondeat veritati.

Inuenta enim sunt tria producta, iã orta ex dicta methodo. 10. 100. 90. quæ suis locis notaui, vt in figura. H. subscripsi etiam duas illorum positiones. 5. et. 10. cum sua differentia. 5. & cum productum. 10. oriretur ab vno latere. 10. reliquum erat. 1. quod suo loco notaui, similiter quia. 100. productum, pro vno eius laterum erat. 5. reliquum autem. 20. suo loco posui, & quia differentia inter. 20. et. 1. duo latera, quę est. 19. æqualis est ei, quæ inter duo consequentia duarum positionum, etiam suo loco ipsam constitui, sed quia hæc differentia est vnum laterum producti. 90. ergo reliquum latus quæsitũ erit. 4. cum. 14. decimisnonis, rectè igitur operatur. sed cum eadem proportio sit inter differentiam. 5. superiorem, et. 19. inferiorem, quæ est vnius antecedétis ad suum consequens, quare. 10. antecedés habebit pro suo consequenti. 38. et. 5. habebit. 19. et. 4. cum. 14. decimisnonis habebit. 18. rectè igit dictum fuisset si. 19. prouenit. à. 5. à quo proueniet. 18 ?

Huiusmodi autem rei ratio ita se hét, esto linea. a. e. u. cuius pars a. sit quarta reliquarum. e. u. iuncta rum, sed. e. sit quinta ipsius. a. Tunc clarum erit quod. e. erit vigesima dictarum. e. u. quare erit decimanona ipsius. u. sed cũ u. sũpta sit vt. 18. rectè igitur dici potest, si. u: ut. 19. prouenit ab. a. ut quinque, à quot ipsius. a. proueniet. u. ut. 18.
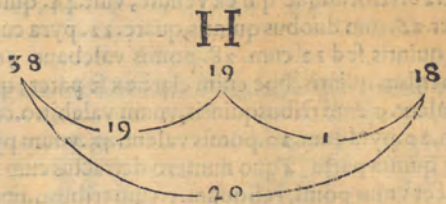
Quis enim non uidet quod diui sa cum fuerit. u. in partes. 19. quod quinque illarum æquabuntur ipsi. a. cum quælibet fuerit æqualis. e. quintæ parti ipsius. a.
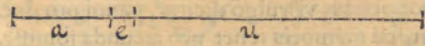
HAc igitur mea numerorum dispositione mediante reperiuntur ipsi numeri in feriores naturaliter consequentes, correspondentesque ipsis superioribus an tecedentibus; quamuis multoties cõtingere possit, ut generationes seu com positiones ipsorum ignorentur: & quia tam à differentijs errorum, quam ab illis, quę sunt inter ueros consequentes numeros (propter eorum æqualitatem) elicitur ipsa ueritas, proptereà rectè antiqui illis vsi sunt, quamuis sint potius sensum sequuti, uel experientiam, quam rationem: quæ quidem ratio pendet ab ipsis naturalibus numeris consequentibus (ut supra uidimus) etsi incognitis ut plurimum, quod si ipsos inuenire primò nobis datum fuisset, unica tantũmodo positio sufficeret

P ret

ret, mediante ipſa regula de tribus, vt iã ſepius dictũ eſt, quod etiã clarè patet ex di-
uerſis problematibus. 17.lib.ipſius Tartaleæ, vt ex primo, quod aſſumpſimus pro
noſtro etiam primo exemplo, ex. 9.15. 16. 17.18.19.20.27.28.29. 30. 33. & ex
alijs multis, vbi facillimè inue nitur conſequens ipſius poſitionis, qui quidem nume-
tus eſt diuiſor producti ipſius numeri propoſiti in numerum poſitionis, vnde poſteà
prouenit ſecundũ latus huiuſmodi producti, hoc eſt numerus quæſitus, per regulã
de tribus, vt dixi.

Alia verò multa problemata inueniuntur, pro quorum reſolutione poſſumus ali
qua methodo vti, in qua manifeſtè pateant eorũ rationes abſque regula falſi, cuius
regulæ rationes non ita promptè ipſi intellectui ſe offerunt, vt ſupra vidimus.

Accipiamus pro exemplo.21.problema ipſius Tartalæ in dicto. 17. libr. vbi ſup-
ponit vnum hædum diuiſum in. 4.partes, quarum quælibet vendebatur eodem pre
cio, interiora vero.6. denarijs minus quam quælibet dictarum partium, ſumma
autem omnium iſtorum denariorum fuit. 127. quæritur nunc precium cuiuſque
partis.

Tale enim problema hoc etiam alio breuiori modo poteſt ſolui, vt rationes ma-
gis pateant, quam ex regula falſi.

Nam ſi illi numero.127.denariorum, additus fuerit numerus.6.ſumma erit.133.
qua diuiſa per quinque, illico proueniet. 26.cum tribusquintis pro precio vniuſcu-
iuſque quatuor partium, à quo.26.cum tribusquintis dempto.6.remanebit.20.cum
tribusquintis pro precio interiorum.

Simili modo in.24. problemate inquit.

Duodecim pyra cum.28.pomis venduntur. 36.denarijs, et.20.pyra. cum.200.po
mis vẽduntur.44.denarijs, quærĩt nunc, quod nã fuerit preciũ vniuſcuiuſq; illorum.

Hoc etiam problema, hac alia methodo ſolui poteſt, dicendo ex regula de tribus,
ſi ex.20.vtrorunque qui ea vendit, vult.44. quid volet ex.12? manifeſtũ erit quod
volet.26.cum duobus quintis, quare.12. pyra cum.12.pomis valebũt.26. cum duo
bus quintis, ſed 12.cum. 28. pomis valebant. 36.ergo. 16. poma ſola valebunt. 9.
cum tribus quintis, hoc enim clarè ex ſe patet; quare cum dixerinus, ſi.16. poma ſo
la valent.9.cum tribusquintis, vnum valebit.0.cum tribusquintis, ſed quemadmo-
dum.20.pyra cum.20.pomis valent.44.vnum pyrum, cum vno pomo valebunt. 2.
cum quinta parte, à quo numero detractus cum fuerit. 0.cum tribus quintis, precio
ſcilicet vnius pomi, reliquum.1.cum tribusquintis, erit precium vnius pyri.

Idem etiam dico de. 28. problemate, vbi ſupponit quod quidam comparaſſet
quatuor petias, vt vulgo dicitur, panni pro ducatis.96.quarum primæ precium ob-
litus ſit, ſed memoria tenet pro ſecunda ſoluiſſe.6. plusquam pro prima, & pro ter-
tia ſoluiſſe.8.plus quam pro ſecunda, & pro quarta ſoluiſſe. 10. plus quam pro ter-
tia, quæritur nunc quantum fuerit precium vniuſcuiuſque illarum.

Quod quidẽ problema
breuius eſſet ita ſolui, vt in
ſubſcripta figura. I. videri
poteſt, addẽdo ſimul omnes
exceſſus. Nam exceſſus ſecũ
dæ ſupra primam eſt. 6. ſed
cum exceſſus tertiæ ſupra ſe
cundam ſit.8. ergo exceſſus
tertiæ ſupra primam erit. 14
ſed
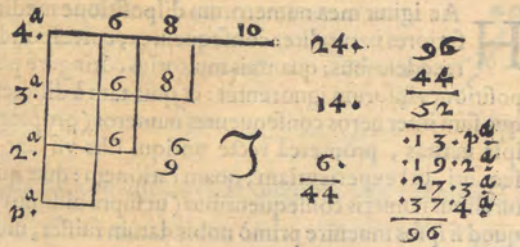
fed exceſſus quartæ ſupra tertiam eſt. 10. vnde ſupra ſecundam erit. 18. & ſupra pri-
mam erit. 24. quæ omnia ſimul addita erūt. 44. & in qualibet harum triū m remane-
bit una pars æqualis primæ quantitati, quare ſi ex. 96. detractus fueri nꝰ merus. 44.
reliquus. 52. erit quadruplus primæ, quare prima pars valebit. 13. ſecunda. 19. ter-
tia. 27. & quarta. 37. quarum omnium ſumma eſt. 96.

E X poſitionibus autem Tartaleæ in noſtra figura. K. digeſtis, videre poſſumus
quo pacto colligātur huiuſ
modi conſequētes numeri ſimpli-
ces. 36. et. 52. more figuræ. E. quia
colliguntur primò partes compoſi
tæ. 9. 15. 23. 33. ex quarum ſumma
80. ſubtrahitur. 36. ſumma ſim-
plex ex ſimplicibus partibus. 9. 9.
9. 9. & reſiduū quod eſt. 44. ſubdu
citur ex. 96. ſumma compoſita &
propoſita, vnde remanet. 52. pro
ſumma ſimplici, ex numero dato,
cuius proportio ad. 13. eadem eſt
quæ. 36. ad. 9. & proptereà ſuper-
flua eſt ſecunda poſitio, quādo ſci
mus inuenire tales duos numeros
conſequentes, vt in hoc exemplo
ſunt. 36. et. 52. quia ex regula de
tribus poſtea elicitur veritas quæ-
ſita. Idē dico de 33. problemate.



$$\frac{36}{9} \qquad \frac{260}{13} \qquad \frac{224}{14}$$

$$4 \qquad\qquad 1$$

$$5$$

K

$$36 \qquad 52 \qquad 56$$

$$16 \qquad\qquad 4$$

$$20$$

$$9 \; 15 \; 23 \; 33 \qquad 80 \; 96 \; 52$$
$$9 \; 9 \; 9 \; 9 \qquad 36 \; 44$$

P R O quo. 33. problemate acci-
piantur poſitiones primi exē-
pli Tonſtalli hoc eſt. 33. et. 31. vt in figuris hic ſubiectis. P. Q. facile quis poteſt vi-
dere, vbi in figura P. vidēbit nume-
ros compoſitos, in figura verò Q. cer
net numeros ſimplices, à quibus pro
ueniunt rationes per ſe huiuſmodi
operationis, in figura autem R. vide
bitur meus ordo, & iſtæ tres figuræ ſi
miles erūt tribus illis primis. A. B. C.
ita quòd cum quis illas intellexerit, il
licò etiam iſtas cognoſcet, vbi etiā
videbit quam confuſè ratiocinētur ij
qui ignorant hunc meum ordinem
ſimplicium numerorū, à quibus fluit
tota ratio ( vt ſupra dixi) huiuſcemo
di operationis.

$$\frac{99}{33} \qquad \frac{279}{31} \qquad \frac{186}{30}$$

$$36 \qquad 34 \qquad 33$$
$$40 \qquad 38 \qquad 37$$
$$\overline{109} \qquad \overline{103} \qquad \overline{100}$$

$$9 \qquad 6 \qquad 3$$

P

P 2            Idem

$$\frac{99}{33} \qquad \frac{279}{31} \qquad \frac{180}{30}$$

$$\frac{99}{33}$$

$$33$$

$$33$$

$$\frac{33}{99} \qquad \begin{array}{c}31\\31\\\hline 93\end{array} \qquad \begin{array}{c}30\\30\\\hline 90\end{array}$$

Q    R

9    3          99    93    99

6    3

9
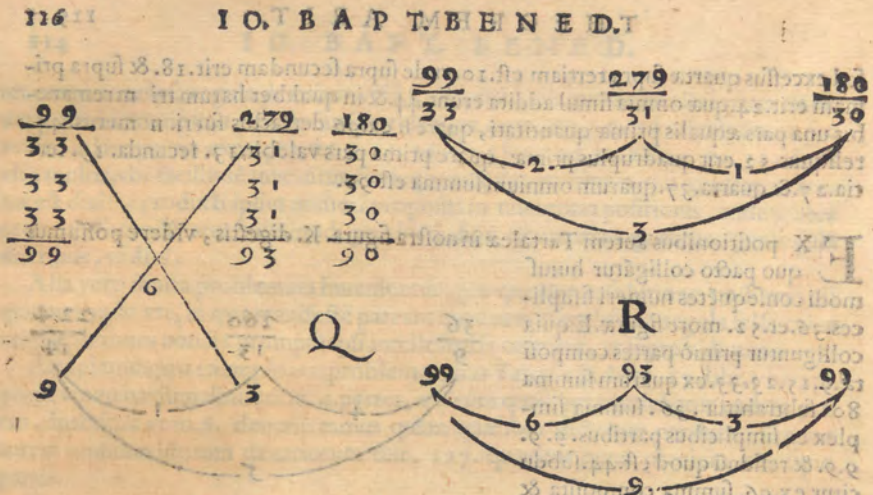
Idem etiam poteſt dici de. 15. problemate ( ſicut de alijs multis ) vbi ponit tres homines habentes. 40. aureos quorum primus habet duas quintas partes ſecundi, ſecūdus verò quinq; octauas tertij, quærit nūc quot ducatos habeat vnuſquiſque.

Quis non videt quæſo, ꝗ omnes partes erunt. 15. quare cum dixerimus ſi. 15. dat nobis. 2. ( pro prima portione primi hominis) quid dabit. 40? vnde nobis proueniet 5. cum tertia parte.

Et de. 29. ſimiliter aſſero, vbi ponit aliū emiſſe tria fruſta panni pro ducatis. 48. quarum ſecundam habuit pro dimidio precio primæ, tertiam autem pro quarta parte ipſius ſecundæ, quare omnes partes erunt. 13. quapropter precium tertiæ petiæ erit tertiadecima pars ipſius. 48. hoc eſt. 3. cum. 9. tertijs decimis.

Adhuc duo exempla videtur mihi proponere, quorum primum eſt. 38. eiuſdem lib. vbi ſupponitur operarium quendam velle perficere opus quoddam ſpacio dierum. 36. tali pacto, quod qualibet die, in qua ipſe operaturus ſit lucretur ſolidos. 16. qualibet verò die, in qua nihil agat perdat ſolidos. 24. Tunc accidit, vt exacto termino perfectoq; opere, tantum lucratus ſit, quantum perdiderit. Quæritur nūc quot fuerint dies lucri, quotúe perditionis.

Huiuſmodi problematis operatio breuiſſima abſque vlla falſa poſitione ita erit, hoc eſt diuidendo productum. 36. in. 24. per. 40. ideſt per aggregatum ipſius. 24. cū 16. & prouentus erit. 21. cum tribus quintis pro diebus lucri, vnde reliquum ex. 36. erit. 14. cum duabus quintis pro diebus perditionis.

Cuius operationis ratio ex ſe ſatis patet, cum duo producta, vnum lucri, alterum vero perditionis æqualia eſſe debeant, vnde ex duodecima, & vigeſimaſepti mi ex regula de tribus reperiun tur partes ipſius 36. eodem mo do

$$21 \cdot \frac{3}{5}$$

M

16          24

$$14\ \frac{2}{5}$$
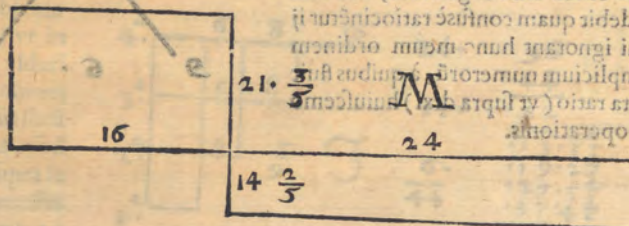
do se inuicem habentes, vt. 24. et. 16. quæ sunt. 21. cum tribus quintis, et. 14. cũ dua bus quintis, ex quo sequitur, vt quod fit ex. 21. cum tribus quintis, in. 16. ęquale sit ei quod fit ex. 14. cum duabus quintis, in. 24. & ita reperiuntur duo producta æqualia, vnum lucri, reliquum verò perditionis, vt in figura. M. clarè videtur.

Aliud verò exemplum est. 39. quod quidem à superiori non differt, nisi quod in fine operationis, operarius dictus lucratus est solidos. 60: quęritur nũc vt supra, quot fuerunt dies lucri, & quot perditionis. Hoc etiam absque vlla falsa positione dicto citius potest solui, hoc modo, diuidẽ do scilicet illos. 60 solidos per. 40. idest per aggregatum. 24. cum. 16. proueniens autem, quod erit. 1. cum dimidio, adde ad latus superius inuentum, hoc est. 21. cum tribus quintis, & summa erit. 23. cum decima parte pro numero dierum lucri, deinde idem prouentum deme ex alio latere superius reperto, 14. cum duabus quintis, & residuum erit. 12. cum nouem decimis, vnde habebis numerum dierum perditionis.

Pro cuius rei speculatione cogitemus in figura. N. duo dicta producta inuicem æqualia, o. b. et. n. c. existente latere. u. c. vt. 24. u. o. ut. 16. b. u. vt. 21. cum tribus quintis, et. u. n. vt. 14. cum duabus quintis. Nunc verò si mente concepta fuerit recta. e. a. t. æquidistans. o. c. ita vt rectangulum. o. e. sit. 60. tunc rectangulum, seu productum b. t. superabit rectangulum seu productum. n. e. per idem. 60. ex communi conceptu, eo quòd ex producto. n. c. sublatum est productum. a. c. 24. & producto, o. b. additum est productum. e. a. 16. rectè igitur feci cum diuiserim. 60. per. 40. vnde prouenit mihi. u. a. idest. 1. cum dimidio, quod additum ipsi. b. u. composuit. b. a. & dempto ex. u. n. relinquit. a. n. pro lateribus duorum productorum. b. t. et. n. e.

Sed si idem operator perdidisset. 60. tunc cogitaremus parallelam dictam. e. a. t superius ductam esse ita vt secaret. b. u. & non. u. n. vnde adderet. 24. ipsi producto. n. c. & demeret. 16. à producto. b. o.
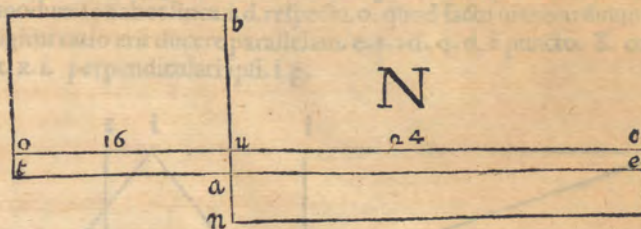


CIRCA verò talia quæsita videtur mihi non inutile fore si aliquid notatu dignum aduerterim, hoc est quod sæpe accidere poterit ut casus impossibiles proponantur. Quemadmodum si aliquis diceret, cupio mihi uestimentum conficere ex duobus pannis colore & pretio differentibus, quorum unus exempli gratia.

tia fit albus, rubeus ueró alter, deinde albus fit pretij. 40. folidorum uniufcuiuf-
que cubiti, rubeus ueró precij. 50. uellemque omnes cubitos effe. 8. nec plus
nec minus. Vellem etiam foluere folidos. 450. neque minus.

Hic igitur cafus impoffibilis eft, eo quòd. 8. cubiti totius rubei effent precij fo-
lidorum. 400. tantummodo, unde ex alio panno albo minoris precij fumere ali-
quid non poffumus.

Idem etiam eueniret fi uoluiffet foluere folidos. 320. neque plus, eo quòd. 8. cu-
biti illius minoris precij, hoc eft. 40. folidorum, efsent ualoris. 320. folidorum tan
tummodo, quare pro alio panno nullus efset locus. Animaduertendum igitur erit
quod numerus poffibilis ad foluendum tale quæfitum erit inter. 400. et. 320. & non
extra iftos terminos, vt vnicuique patere poteft.

Similiter idem in hoc alio cafu accidere poterit, ut fi quis diceret.

Emi quinque petias panni pro aureis. 55. pretium tamen primæ oblitus fum, fed
memoria teneo, quòd fecunda altioris pretij erat quam ipfa prima per. 4. & ter-
tia preciofior fecunda per. 7. et quarta carior tertia per. 9. quinta veró fuperabat
quartam per. 2.

Hic etiam reperitur impoffibilitas quædam, eo quòd aggregatum omnium ha-
rum rerum, dato etiam quòd pro prima nihil folutum effet, fuperat aureos. 55. quòd
quidem nullo pacto fieri poteft, vt veri fint fupra dicti excefsus, fi verus eft numerus
totalis aureorum. 55. Nam. 4. cum. 7. faciunt. 11. qui quidem. 11. cum. 9. efficiunt.
20. & hic cum. 2. facit. 22. fed. 22. cum. 20. et. 11. et. 4. dant. 57. qui numerus maior
eft quam. 55.

**FINIS THEOREM. ARIT.**

# DE RATIONIBVS
## OPERATIONVM
### PERSPECTIVAE.

#### CAP. I.

VM nullus adhuc ( quod fciam ) veras internasq́; caufas operationis perfpectiuæ perfectè docuerit, operæprecium exiftimaui aliquã de ijs difputationem fufcipere. Multi enim eorũ, qui huiufmodi operationis regulas præfcribunt, cum eius effectuum veras caufas ignorent, varios diuerfofq́; errores committunt, vt exempli gratia in fubfcripta figura fuperficiali. A. volentes degradare (vt dicunt) rectangulum. q. a. in triangulo. i. d. q. ducunt parallelã ipfi. q. d. à puncto. B. interfecationis lineæ. o. l. cum latere. i. d. trianguli, & (idem) indifferenter, eandem quoque à puncto. Z. interfecationis ipfius. o. l. cum perpendiculari. x. i. ducunt. nefcientes hunc folum effe verum modum, non item alium, quia fi alius, talis effet, hic, verus non exifteret, nam fi vellent fefe excufare, quòd ducendo dictam parallelam à puncto. B. hoc fiat præfupponendo planum ipfius. i. d. q. verfus rectangulum. q. a. orizontale inclinatum, fecundum angulum. i. d. q. hæc excufatio accipienda non effet, quia horum confenfu, præfupponendo planum. i. d. q. inclinatum, anguli inferiores rectanguli degradati, non tam acuti, quam funt duo. i. d. q. et. i. q. d. effe deberent, quod facilè eorum ratione innotefcet, quæ de figura corporea. A. hic fubfcripta mox proponam, præter id, quòd volentes deinde afpicere quadratum degradatum, oporteret huiufmodi planum refpectu oculi ita collocare, quemadmodum fe habet linea. i. d. refpectu. o. quod factu nimis arduum effet.

Vera igitur ratio erit ducere parallelam. e. r. ad. q. d. à puncto. Z. communi ipfis. o. l. et. x. i. perpendiculari ipfi. l. p.

Pro cuius rei fpeculatione imaginemur in figura corporea. A: q. a. effe figuram re-
ctangulam orizontalemq́; ad degradandam fuper aliquod planum perpendiculare
orizonti, & cum eo primum coniunctam in linea.q.d.cuius plani triangulum. i. q. d.
pars erit, fit autem oculus refpicientis. o. cuius altitudo.o.p. ab orizonte, qui quidē
confpicit rectangulum dictum orizontale. q. a. in pyramide . o. q: o. u: o. a. et. o. d.
terminata quatuor triangulis . o. q. u: o. u. a: o. a. d. et. o. d. q. fit verò primum ita
collocatus pes. p. eius qui refpicit, vt linea. p. l. perpendicularis ipfi. u. a. lateri re-
ctanguli, medio loco pofita fit, inter. a. n. et. u. s. Idq́; primum nobis erit exem-
plum.

Imaginemur nunc lineas. u.q. et.a.d. indefinitè productas effe, quæ in fuperficie-
bus duorum triangulorum.o. u. q. et. o. a. d.& rectanguli orizontalis. q. a. ex prima
vndecimi Euclid. pofitæ erunt. Imaginemur etiam lineam. p. s. n. perpendicula-
rem ipfi.p.l. quæ etiam cum duabus. u. q. s. et. a. d. n. ex. 34. primi Euclid. angulos
rectos conftituet, cum ex. 28.duæ.u.q. s. et. a.d.n.fint parallelæ ipfi.p.l. et.s.n. ipfi.u.
a. & quia fupponitur.o.p.perpendicularis plano orizontali, Angulus ergò.o. l. re-
ctus erit ex fecunda definitione. 11. Euclid. Imaginemur quoque ductas effe
duas.o.s.et.o.n. vnde.l.p. ei fuperficiei, in qua funt duæ lineæ . o. p. et. s. n. ex. 4.
11.perpendicularis erit,& fuperficies orizontalis.a.s.perpendicularis erit cum dicta
o.s.n.ex.18.eiufdem lib.vnde ex dicta definitione.o.s.u.et.o.n. a. erunt anguli recti
et.o.s.et.o.n.ex communi fcientia,in fuperficiebus duorum triangulorum.o. u. q. et.
o.a.d.erunt,fi noluerimus cogere aduerfarium ad confitendum duas lineas rectas in-
cludere fuperficiem, quemadmodum cogere-
tur facere, fi opinaretur duas alias rectas per
eadem puncta . o. s. n. tranfire, quæ funt in di-
ctis fuperficiebus.Vnde.o.s.et.o.n. communes
erunt fectiones duarum dictarum fuperficierū
cum fuperficie. o. s. n. Imaginemur nunc has
duas fuperficies. o. u. et. o. a. quarum commu-
nis fectio fit,o.t.(quæ erit linea recta ex. 3. lib.
11.)quæ erunt perpendiculares fuperficiei.o.s.
n. ex. 4. et. 14. iam dictis. & ex. 19. eiufdem
o. t. perpendicularis eidem fuperficiei. o. s. n.
erit,& ex.6.eiufdem hæc linea. o. t. duabus. u.
q.s.et.a.d.n. parallela exiftet,& ex. 9. eiufdem
hæc linea.o.t.duabus.u. q.s.et.a. d.n. parallela
exiftet,& ex eadem. 9. erit parallela ipfi. p. l.
Imaginemur nunc planum, fuper quod defide
remus videre quadrangulum orizontale, quod
planum,exempli gratia,fit primo,vt iam dixi-
mus, locatum in linea. q. d. ad angulos rectos
cum plano orizontali, cuius communes fectio
nes cum fuperficiebus. s. t. et. n. t. vifionis la-
terum. u. q. et. a. d. fint. i. q. et. i. d. & com-
munis fectio trianguli. o. u. a. ideft vifionis
lateris.a.u.cum dicto plano, fit. r. e. Vnde ex
communi fcientia rectangulum orizontale,
oculo.o.feipfum patefaciet in plano.i.q.d.fe-
cundum

cundum figuram quadrilateram.q. d. r. e. Communis autem fectio fuperficiei. p. t.
cum dicto plano,fit.i.x. quæ.i. x. perpendicularis erit. s. a. fuperficiei orizontali ex
19.lib.11.quia.p.t.eft etiam orizonti perpendicularis ex.18.eiufdem,cum. o. p. ei-
dem perpendicularis exiftat. Vnde.i.x.erit altitudo trianguli.i.q.d. & æqualis ipfi.
o. p. ex. 34. primi. Sit deinde.o.l.cõmunis fectio fuperficiei triangularis.o. a. u. cũ
fuperficie.p.t.quæ.o.l.fecando lineam.e.r. in puncto.Z.nobis oftendet quantum di-
ftare feu eminens effe debeat latus.e.r. in plano ab.q. d. medio ipfius.z. x. Et quia
præfuppofuimus.p.l. in eodem medio,inter.u.s. et. a. n. ideo. x. q. equalis erit. x. d.
& ex.4. lib.primi.i.q.ipfi.i.d.et.e.r.parallela ipfi.q.d.ex.6.lib. 11.cum ipfa quoque
fit perpendicularis fuperficiei.p.t. ex. 19. eiufdem. Hucufque igitur in figura cor-
porea. A. prodeunt in lucem omnes caufæ effectuum figuræ fuperficialis. A. ideft vn
de fiat,vt in ipfa figura fuperficiali, triangulum.o.p.l.tale confurgat,& quid figni-
cet.o.et.o.p.et. p.l. et. o.l. & quam ob caufam tale quoque formetur triangulum. i.
q.d.atque in tantam altitudinem, quantam obtinet.o.p. & quid fint latera.i. q. et.i.
d.& quare erigatur.x.i.parallela ipfi.p.o.ab eadem. p. o. tanto fpatio diftans, & qua
ratione producatur à puncto.Z.ipfa.Z.r.e.parallela ipfi. q.d.

Nunc obferuandum eft, quòd fi planum ipfius. i. q.d. in figura corporea aliquan-
tulum inclinatum effet orizontem verfus,anguli.i.q.d.et.i. d. q. maiores exifterent,
quàm cum idem eft ipfi orizonti perpendiculare,quemadmodum clarè demonftra-
tum fuit in. 39.primi Vitelionis.

Non igitur rectè fit fi in figura fuperficiali ducatur à puncto.B. parallela ipfi. q. d.
abfque maiori apertura angulorum.i.q.d.et.i.d.q.

SVPERFICIALIS.



CAP. II.

CVM verò duæ præcedentes figuræ intellectæ erunt, facilè quoque erit intel-
ligere duas fubfequentes.B.B. in corporea quarum.p.l. extra lineas.u.s.et.a.n.
reperitur,vbi enim aduertendum erit oportere fumere femper. p: x. figuræ fuperfi-
cialis æqualem ei,quæ eft corporeæ,& eidem fuperficiali, adiungere. x. d. æqualem
ei,quæ eft corporeæ,& compofito. p.d. ex dictis duabus lineis, in figura fuperficiali,
addere.d.q.æqualem ei,quæ eft figuræ corporeæ, deinde accipere punctum. l. in fu-

Q perficiali

CORPOREA.

perficiali,ita diſtans à. p.vt in corporea reperitur.Po-
ſteà. x. i. erigetur æqualis lineæ. p. o. & ſuis terminis
concludetur triangulum.i. q. d. & id quod remanet.
Vnde ſi longius diſerendo progrediare, patebit ex.
4. primi. i. d. ſuperficialem, futuram æqualem. i. d.
corporeæ.Idem dico de.i. q. & de reliquis.

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B

B
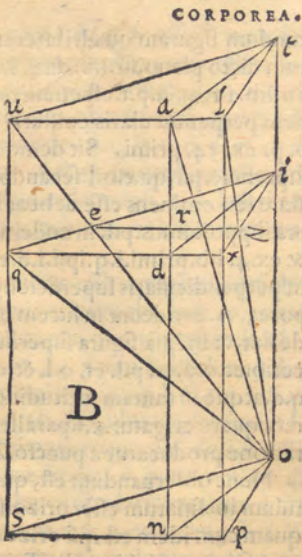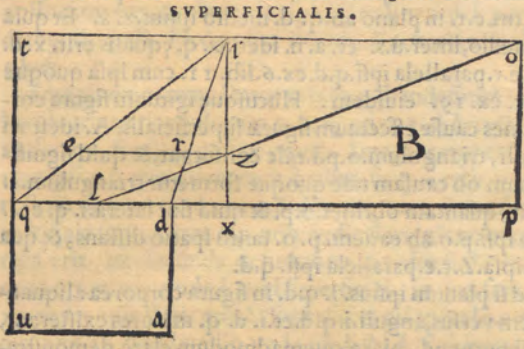
## CAP. III.

MOᴅᴠs hic, proprius eſt, & vniuerſalis, licet in figura ſuperficiali. A. ſupe-
rius poſita, ſecundum communem antiquorum conſuetudinem exemplum
dederim, effectus enim idem eſt. Sed ſi quis vellet conſiderare dictam figuram ſu-
perficialem. A.ſecundum eum modum, quem de figura. B. ſuperficiali preſcripſi, id
poterit in ſubſcripta figura. C. ſpeculari in hunc modum. Accipiet enim. p.x.ſuper
ficialem, æqualem corporeæ, & quia in ipſa corporea. A. ſuppoſita fuit linea. p.l.
ideſt punctum.x. inter duas.u.s.et.a.n.ſecabimus.p.x. ſuperficialem in puncto. d. ita,
vt.d.x.ſuperficialis,æqualis ſit corporeæ,& ipſi ſuperficiali.p.x.addetur.x. q. æqualis

corporeæ. vnde.q.d. ſuperfi-
cialis æqualis erit corporeę,
et. p. x. ſuperficiali addetur.
x. l. æqualis corporeæ. De
ijs poſtea quæ dicenda ſuper-
ſunt, iam ſatis ſuperque di-
ximus.
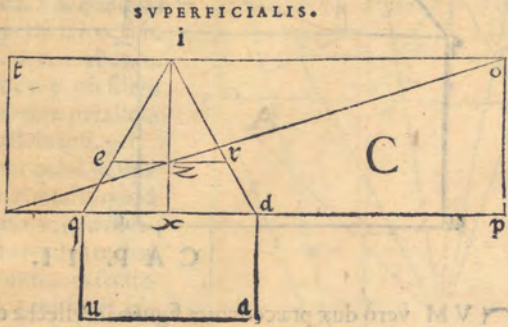
Quamobrem, punctum.
x. aut intra,aut extra lineam. l
q.d. veniat, hunc modum ſe-
quentes, in errorem non in-
cidemus, imò efficietur qua-
drilaterum. q. r. ſuperficiale,
ſimile,& æquale corporeo.

SVPERFICIALIS.

i

C

## CAP. IIII.

PVɴᴄᴛᴠᴍ verò.i. (quod verum eſt punctum perſpectiuæ, vt practici dicere ſo
lent) quid ſit, hac via & ratione ſub noſtram cognitionem cadit: quòd nihil
aliud

aliud eft, quàm punctum, varijs fectionibus commune, & huiufmodi punctum, ocu
lus non eft, quemadmodum multi Pictores, Sculptores, Architecti, & Perfpectiui
ignari, ipfum punctum, oculum appellando, falsò crediderunt, quafi punctū.i. per-
fpectiuæ oculus effet.

In fupradictis igitur figuris manifefte elucefcit caufa diminutionis obiectorum,
& altitudinis triangul. æqualis ei,quæ eft oculi à plano orizontali,vt etiam diftantiẹ.
p.l.p.x.& cuiusuis tandem rei. Sed vt huius effectus fcientia magis in vniuerfum pa-
retur. Volo duas hic fubfcriptas figuras.D. corpoream, & .D. fuperficialem à vo-
bis confiderari, in quarum corporea,linea. p.l.fit extra duas.u.s. et. a. n.vt in figu-
ra. B. locata, ita tamen vt planum trianguli.i.q.d. difiunctum fit à rectangulo fuper-
ficiali,ideft,vt feparatum exiftat à linea.q.d.latere ipfius rectanguli, & fit etiam obli
quum.refpectu ipfius rectanguli,ideft vt communis fectio dicti plani cum fuperficie
a.s.orizontalis ipfi.u.a.parallela non fit, fed fit obliqua, fi tamen idem planum per-
pendiculare dictæ fuperficiei orizontali.a.s. erit: & dicta communis fectio exprima
tur characteribus.q. u. a. d. x. nunc in figura corporea habebimus figuram. e. r. c. m.
in plano, quod vifualem pyramidem fecat, medio cuius figuræ.e.r.c. m. oculus po-
fitus in. o.rectangulum orizontale confpicit.Volentes vero nunc in figura.D.fuper-
ficiali eam defcribere, faciem us. p. x. fuperficialem, æqualem corporeæ, eíque
addemus. x. l. æqualem corporeæ, aut fumemus. p. l. eidem corporeæ ẹqua-
lem, quam fecabimus in puncto. x. eodem planè modo, quo corporea reperi-
tur diuifa;erigemus deinde.p.o.et. x.i.æquales corporeis. Secabimus deinde. x. q.
æqualem corporeæ,& ducemus.q.i.et.l.o.vnde habebimus triangulos. o.p.l. et.i.x.
q.fimiles & æquales corporeis ex. 4. primi Eucli. Secabimus deinde. q.x. in pun-
cto.d.eadem ratione,qua fecta fuit corporea,& ducemus lineam. d. i. vnde habebi-
mus triangulos.i. d. q. et. i. d. x. fimiles corporeis. & mediante triangulo.i.q.d.hu-

CORPOREA.

cufque habebimus fitus duorum laterum figurẹ
rectanguli degradati,ideft fitus ipfius.e.m. et.r.
c.etiam fi adhuc nefciatur in qua parte ipfius. i.
q. & ipfius. i.d. effe debeãt. Quod fi fcire volue
rimus fecabit. p.l.in pūcto. g. fimilis corporeæ,
fi in ipfa tamen corporea prius protraxerimus
lineam.q.d.latus rectanguli vfque ad.p.l.in pun
cto.g. Ducetur deinde linea.o.g.fuperficialis,
quæ fecabit lineam.i.x.in puncto. f. linea vero.
o.l.in puncto. z.punctis fitis in.i.x. fuperficiali,
prẹcisè vt in corporea, quemadmodū quilibet
ex fe facilè cognofcere poteft. Deinde in cor
porea,in fuperficie orizontali ducatur. p. q. et.

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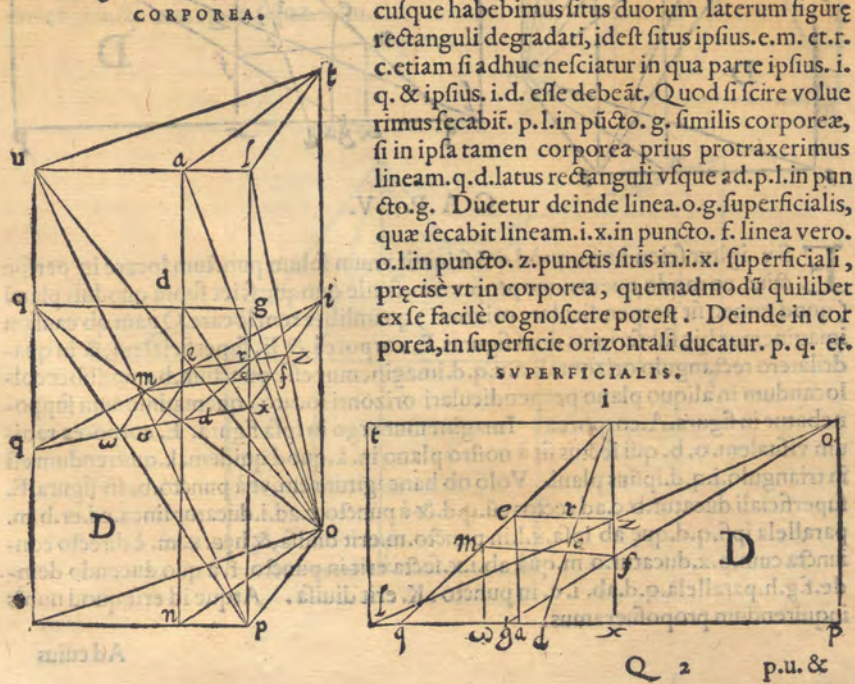
p.u.& imaginemur.o.q.in fuperficie.t.s.vnde trianguli.o.p.q.et.o.p.u.erunt perpen
diculares orizonti ex.18.lib.11.et.ω.m.et.ℓ.e. commutes fectiones dictorum duorũ
triangulorum cum plano trianguli.i.q.x.ipfi quoque plano ex.19. eiufdem lib.erunt
perpendiculares. Nunc autem fecetur.q.x. fuperficialis in punctis.ω.et.ℓ. eadem ra-
tione; qua corporea fecta fuit à duabus.p.q.et.p.u.à quibus punctis.ω.et. ℓ.fuperficia
libus ductæ fint duæ.ω.m.et.ℓ.e.perpendiculares vfque ad latus.i.q. in punctis. m.et.
e. quę fitum habebunt in. i. q. fuperficiali preciſe, vt in corporea,ex. 26. primi,du-
cendo deinde in fuperficiali duas.m.f.et.e. Z. eæ æquales erunt corporeis ex. 4. pri-
mi,& fic anguli.i.e.z.et.i.m.f.& eę duę lineę.e.z.et.m.f.ſectę erunt à linea.i.d.in duo

CORPOREA.

bus punctis.r. et. c. vnde. e.r. et. m. c. æquales
erunt corporeis ex. 26. primi, fed ita quo-
que fe habent duę.e.m.et.r.c.fi verum eſt ꝗ dif
ferentię rerum æqualium fint adinuicem etiam
æquales. Hac ratione igitur habebimus figu-
ram quadrilateram.m.e. r.c. fuperficialem om
ninò fimilem,& ęqualem corporeæ. Is tamen
modus prolixus eſt , & arduus, quam ob cauſ-
fam neque ego vnquam eũ vſui accommo-
darem, neque alijs, vt eodem vterentur ſua-
derem.

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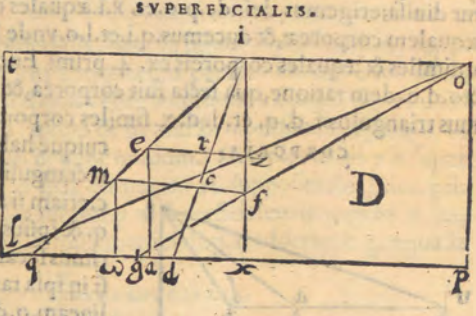


C A P. V.

ES T igitur ſciendum, quòd qui ſciuerit vnum ſolum punctum locare in perſpe
ctiua,eo modo quem nunc proponam,facilè quoque ſciet ſupra quoduis planũ
(quod tamen ſit perpendiculare orizonti) quamlibet rem locare.Quam ob cauſam
imaginemur hic ſubſcriptas duas figuras.E. corporeã,& E. ſuperficialem, & in qua-
drilatero rectangulo orizontali.a.u.q.d.imaginemur eſſe punctũ.b.quodlibet col-
locandum in aliquo plano perpendiculari orizonti locato, quemadmodum ſuppo-
nebatur in figura.A.corporea. Imaginemur ergo in ipſa figura. E. corporea radi-
um viſualem. o. b. qui ſectus ſit à noſtro plano in. k.quod quidem. k.quærendum eſt
in triangulo.i.q. d.ipſius plani. Volo ob hanc igitur rem,vt à puncto.b. in figura.E.
ſuperficiali ducatur.b.c.ad rectos cũ.q.d.& à puncto.c.ad.i.ducatur linea.c.i.et.b.m.
parallela ipſi.q.d.quę ab ipſa.x.l.in puncto.m.erit diuiſa,& hęc. x. m. è directo con-
iuncta cum.p.x.ducatur.o m.quæ ab.i.x.ſecta erit in puncto. f. à quo ducendo dein-
de.f.g.h.parallela.q.d.ab. i. c. in puncto. K. erit diuiſa. Atque id erit quod nobis
inquirendum propoſueramus.

Ad cuius

Ad cuius rei ſpeculationẽ, imaginatione con
cipiamus lineam. b. c. corpoream, protractam eſ
ſe vſque ad. y. lineæ. s. n. & imaginatione ſit com
præhēſa linea. y. o. et. b. ℞. parallela eidem, ideo
ob rationes iam dictas de figura. A. hæ tres li-
neæ. o. y: i. c: et. ℞. b. ſimul cum linea. o. b. erunt
in vna eademque ſuperficie plana, quam cha-
racteribus. y. ℞. notemus. et. i. c. eius erit ſe-
ctio communis cum plano, in quo quæritur pũ-
ctum, et. f. k. ipſius plani cum triangulo. o. b. m.
erit ſectio communis, & parallela ipſi. q. d. ex.
6. lib. 11. quia. k. f. perpendicularis eſt ſuperfi-
ciei. p. t. ex. 19. eiuſdem cum triangulus. o.
b. m. eidem ſuperficiei. p. t. ex. 18. eiuſdem
perpendicularis exiſtat. Vnde perſpicuè pa-
tet ratio quare protracta ſit parallela. b. c: et
quare ducta ſit. i. c. et coniuncta. x. m. cum. x.
p. directè, & quare ducta ſit. o. m. et. f. k. Lau-
do igitur vt ſemper præſupponatur. p. x. perpen-
dicularis baſi ipſius plani & præſupponatur, (vt
rem totam vno verbo complectar) ſuperficies.
p. t. perpendicularis plano, & orizonti. Quod
reliquum eſt, neceſſarium non eſt, niſi ad ſpe-
culandum. Neceſſariæ ergo non ſunt aliæ li-
neæ, quàm. p. x: p. o. x. i: b. c: et. x. m. è dire-
cto coniuncta cum. p. x. (quæ. x. m. coniuncta
aequalis



CORPOREA.

ſVPERFICIALIS.

æqualis fit ipfi. b. c.)
o. m. etiam. i. c: et. f.
k. vt in figura. F. cla
riffimè patet . Alias
autê multas lineas in
alijs figuris non aliã
ob cãm duxi, quã ad
facilius eruêdas è te-
nebris ignorantiæ, &
in cognitionis lucem
proferendas horum
effeċtuum caufas, vt
dixi.

## CAP. VI.

SED vt locum altitudinis, in noftro plano perpendiculari orizonti,& ita locatũ, vt poftremo diximus, inueniamus; duas hîc fubfcriptas figuras confiderabimus. G. corpoream,& G. fuperficialem, fimiles duabus. E. E. proximè præcedentibus, in quarum corporea fit linea, b. M. altitudinis perpendicularis orizonti . Quare fi defiderabis inuenire in noftro plano fitum puncti. M. ideft punċtum radij. o. M. vi-fualis in quo ipfe radius à plano eft diuifus, quod fit . R. quamuis extra triangulũ i. q. d. tibi imaginatione confige duċtam effe lineam. p. b. quæ erit feċtio commu-nis orizontis cum fuperficie. o. p. b. M. quæ fuperficies erit perpendicularis ipfi ori-zonti ex. 18. lib. 11. Quòd autem non minus. o. p. quàm. M. b. fit in vna eademꝗue fuperficie dubitandum non eft, quia fi imaginabimur duċtam effe lineam. p. M. ha-bebimus triangulum. o. p. b. cum triangulo. M. b. p. communibus partibus in vna ea-demꝗue fuperficie conftantem, vt triangulum quoque. o. p. M. cum triangulo. M. b. o. & triangulum. o. p. b. cum triangulo. o. p. M. & triangulum. M. b. p. cum triangulo. M. b. o. Vnde cum quilibet triangulus in vnica tantum fuperficie fit ex. 2. lib. 11. fe-quetur fuperficiem. o. p. b. M. planam effe,& vnicam, cuius communis feċtio cum no-ftro plano fit. θ. ĸ. R. quæ perpendicularis orizonti exiftet ex. 19. lib. 11. eritꝗue pa-rallela ipfi. i. x. ex. 6. eiufdem . Imaginare nunc ereċtam effe. m. T. æqualem ipfi. b. M. orizonti perpendicularem, quæ extenfa erit in fuperficie. p. t. quod ex fe ad confiderandum admodum facilè, clarumꝗue exiftit, reducendo ad impoffibilia quemlibet hæc negare volentem . Imaginemur quoque duċtam effe lineam . M. T. quæ. b. m. ex. 33. primi erit parallela, quia. m. T. ꝗqualis. b. M. parallela eft ipfi . b. M. ex. 6. lib. 11. præter hæc. b. m. parallela eft ipfi. q. d. quia fic fuit duċta fuperius, vnde. M. T. parallela erit ipfi. q. d. ex. 9. vndecimi, & ob id perpendi-cularis erit fuperficiei . b. t. ex. 8. eiufdem . Nunc fit. R. V. communis feċtio trian-guli . o. M. T. cum noftro plano, vnde. R. V. perpendicularis erit fuperficiei. p. t. ex. 19. lib. 11. quam ob caufam parallela erit ipfi. q. d. ex. 6. aut ex. 9. eiufdem quia ex. 6. diċta, parallela eft ipfi. M. T. At fi. R. V. parallela eft ipfi. q. d. etiam. f. ĸ. probatum iam fuit parallelam effe eidem, ergo. R. V. parallela erit ipfi. ĸ. f. ex. 30. primi, Vnde ex. 34. æqualis erit ipfi. ĸ. f. Accedamus nunc ad figurã. G. extruċtã fupra figuram. E. fuperficialem,& erigamus. m. T. perpendi-cularem ipfi. m. p. fed æqualem perfeċtæ altitudini, & ducamus. T. o. vt fecet li-neam. i. x. in punċto. V. ab ipfo ducentes. V. R. parallelam ipfi. q. d. ducendo de-

inde

índe. k. R. parallelam ipſi.i. a. habebimus altitudinem . k. R. quam quærebamus in
noſtro plano . Quod cum ſui natura clarum euadat , laborem ratiocinandi de eo ,
CORPOREA . cuilibet vel mediocriter in præclariſſima hac ſcien-
tia erudito relinquo . ideſt, vt probetur. k. R. ſu-
perficialé, æqualé eſſe corporeæ. Sed tollédo ſuper-
fluitaté linearum, & hoc accómodantes vt in figura.
F. diligenter conſideretur figura. H.

SVPERFICIALIS. CAP. VII.

### C A P. V I I.

ALiam tamen inueni viàm breuiorem vt in figura. H. H. in qua sit punctus
b. perfecti, & k. degradati plani. Nunc ducatur b. c. s. ad rectos cum
p. m. indefinitè, quæ quidem abscindatur in puncto.s. ita quòd. c. s. æqualis sit alti
tudini perfectæ, deinde coniungatur recta.s.cum.i. Tunc si ab.k. vsque ad protractâ
i.s. ducta fuerit. k. R. parallela li-
neç. c. s. hæc. R. k. erit altitudo
quæsita seu degradata.

Quod ita probo. Iam nulli du
bium est quin.f.V.sit æqualis alti-
tudini quçsitæ seu degradatç, quo
tiescunq; ergo pbauerimus. k. R.
æqualem esse lineæ. f V. habebi-
mus propositum. Quare certum
nobis erit eandem proportionem
esse lineæ.c.s.ad.k.R.quam.c.i.ad
k.i.et.c.i.ad.k.i.vt.x.i.ad. f.i. et.x.
i.ad.f.i.vt.m.o.ad.f.o. et. m.o. ad.
f.o.vt.m.T.ad.f.V. ex similitudine
triangulorum.Ergo. m.T. ad.f.V.
erit vt. c. s.ad.k. R. ex.11.quinti,
sed.c.s.sumpta fuit æqualis. m. T.
quare.c.s.ad.f.V. erit , vt. m.T.ad
eâdé.f.V.ex.7.qnti,& ex.11.eiuf-
dem.c.s.ad.f.V.erit vt.c.s.ad.k.R.
quapropter ex.9.eiufdem.k.R.æqualis erit.f.V.
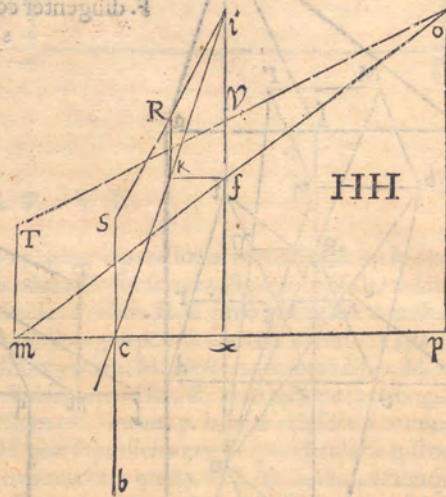


### C A P. V I I I.

MOdus ab antiquis philosophis obseruatus, est etiam vtilis, compendiosaq; via
progreditur, cuius speculationem, in subscripta figura, quadam ex parte secû
dum morem antiquum, quadam etiam ex parte secundum ingenij mei vires côstru-
cta,cognoscemus.In qua ego diuisi.x.i.in puncto. s. ab. x. ita eleuato , quanta est
vera

## DE PERSPECT.     129
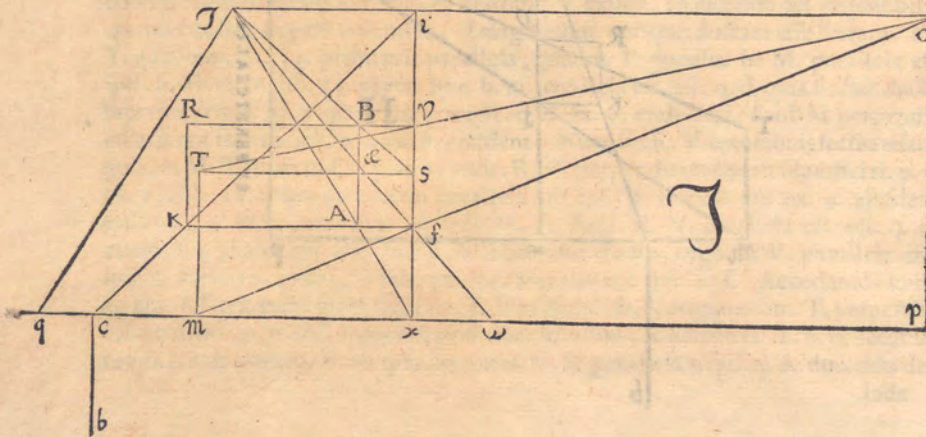
vera altitudo ipsius. M. T. et I.s.duxi supponendo esse.I.punctū pspectiuæ secundū antiquos,idest angulum supremum trianguli antiquorum à punctoque.k. meo duxi k.f.parallelam ipsi.c.m.p.vsque ad.i.x.in puncto.f.& à puncto à communi ipsis. k. f. et.i.x.vsque ad.I.s.duxi quoque.A.B.parallelam ipsi.i.x. atque hæc omnia ex more antiquo præstiti.

Nunc verò eum considerans modum, quem ego de figuris.G.H. antecedentibus præscripsi,videndum est, an punctum.B.tribus lineis.A.B.I.s.et.R.V.quarum hęc vl tima à me iam ducta fuit,commune existat,idest vtrum.A.B. æqualis existat ipsi. K. R.quam secundum modum à me adinuentum, reuera scimus esse desideratam altitu dinem in perspectiua. Quod tunc à nobis probatum erit, quando rationibus clarè patebit ipsam.A.B.æqualem esse ipsi.f.V. Quamobrem ducamus. I. f. vsque ad. ω. lineæ.c.p.vnde ratione similitudinis triangulorum manifestè intelligemus, eandem proportionem esse ipsius.m.T. ad.f.V.quæ est.m.o.ad.f.o.& eius,quæ est.m.o. ad.f. o.quæ est.ω.I.ad.f.l. & eius,quæ est.ω.I.ad.f.I.quæ est.x.I.ad.A.I.& eius, quæ est. x. I.ad.A.I.quæ est.x.s.ad. A.B.idest vt eius,quæ est.m.T.ad.A.B.sed idem quoq; erat de. m.T.ad.f.V. Vnde sequitur.A.B.æqualem esse.f.V.ex.9.quinti Eucli.atq; etiam ipsi. k. R. quod à nobis propositum est inquirendum.

### CAP. IX.

INSTITVENS etiam sermonem de figuris superficialibus orizontalibus , seu de plantis, pulcherrimum quendam modum, quem ego ad locandum quodlibet punctum in perspectiua, (degradatum cum fuerit parallelogrāmum quod dam rectangulum , in nostro plano perpendicula ri orizonti,quemadmodum in superioribus figuris. A. demonstrauimus ) consideraui , silentio haud prætereundum esse.

Sit igitur in subscripta hîc figura. K. in paralle logrāmo perfecto pūctum.b. quod locari debeat in degradato.e.q.d. r. Nunc à duobus quorumlibet quatuor angulorum.q.u.a.d.ducuntur duæ lineæ occultæ.q.g.et.u.f. per punctum. b. vsque ad latera.q.d.et.u. a. ita tamen vt eorum extremitates . g.et.f.intus cadant inter.q.d.et. u.a. ipsorum laterum,idest vt non secent duo latera.q.u.aut.d. a.Deinde punctum.f.inter.q.et.d.cōiungatur occultè cum angulo degradato.e. qui correspōdet. u.perfecti,mediante linea.e.f.quæ erit.u.f. degra data in nostro plano . Deinde sumatur punctum. n.in linea.q.d.tam distans à.q.quàm. g. distat ab. u.ducaturque linea.i.n.quæ lineam.e.r.in puncto c.diuidet,quod ex ijs, quæ superius iam diximus ad ipsum.g.referetur.Ducendo postea lineam oc cultam.q.c.patebit eam correspondere lineæ. q.g.quæ secans lineam.e.f.in puncto. t. hoc, communi scientiæ ratione,respondebit ipsi.b.vt omnes cognoscent.



         R    Sed

Sed fi fortè punctum . b. effet in aliquo
laterum, puta. q. u. volo vt in rectangulo per-
fecto. q. d. a. u. ducta fit vna diagonalis quam
volueris puta. q. a. deinde à puncto. b. ad reli-
quum angulum oppofiti lateris ducta fit recta.
b. d. ita quod à diagonali fecetur in puncto. ⍵.
per quod punctum demum à reliquo angulo la
teris . q. u. ducta fit. u. ⍵. vfque ad latus. q. d. in pũ
cto. f. quo facto , ita faciendum erit in rectangu
lo degradato, hoc eft ducenda erit diagonalis.
q. r. quę correfpondet diagonali. q. a. perfecti
deinde. f. e. quæ correfpondet rectæ. f. u. perfe-
cti , quæ etiam interfecabitur à diagonali. q. r.
in puncto. o. correfpondens. ⍵. perfecti, per quẽ
o. à puncto. d. cum ducta fuerit. d. o. vfque ad. t.
in latere. q. e. hoc punctũ. t. correfpondebit pun
cto . b. perfecti.

Idem eueniet fi loco diametri. q. a. fumpta
fuerit diameter. u. d. & loco. b. d. protracta fue
rit. b. a. deinde loco. u. ⍵. f. ducta fuerit. q. ⍵. f. vn-
de punctum correfpondens ipfi. f. in figura de-
gradata erit in latere fupremo. e. r. correfpon-
dens lateri. u. a. & ita ducenda erit diameter. d.
e. correfpondens diametro. d. u. et. q. f. furfum
verfus correfpondens. q. f. imum verfus deinde.
r. o. refpondens. a. ⍵. quæ terminabitur ab eodé-
met puncto. t. vt prius.



## C A P.  X.

EX mea figura. F. fuperficiali perfpectiuæ facillimum modum locandi quoduis
punctum in perfpectiua elicui. Iuffi enim vt aptaretur tabula quædam rectan-
gula exactè plana , triplo aut quadruplo, aut quanto volueris maioris longitu-
dinis, quàm latitudinis protenfa , quæ quidem latitudo erat ad duos circiter pedes
defignata ab. A. B. C. D. cuius duobus lateribus. A. B. et. B. C. iuffi, vt duæ regulæ affi
gerentur, quæ fuperficiem eiufdem tabulæ excederent , vt vnũ ex lateribus alicuius
anguli recti materialis, qui appellaf norma (vt inferius dicam) ei adherere poffit, cu-
raui poftea, ut iuxta angulum. D. in puncto. o. fixo mobilis regula. o. Q. affigeretur
tantæ longitudinis, aut paulò minoris, quantam occupabat latus. D. A. quæ circum .
o. volueretur, in rectitudine poftea. o. i. parallela ipfi. D. A. in puncto. i. duobus pedi-
bus longè à latere. A. B. aliam quoque mobilem appendere feci. i. M. in tantam ferè
longitudinem extenfam, quanta conftat. A. B. côftitui etiam, vt quoddã angulum re
ctum materiale tantæ magnitudinis, quanta nobis vfui effe poterat fuper eadem ta-
bula; necnon regula quædam materialis neceffariæ longitudinis ftatuerẽt, atq; hæc
omnia tenuiffima , vt fierent curaui.  Quandam deinde lineam ad. o. i. parallelam,
ideft. p. E. fuper eadem tabula adeò diftantem ab. o. i. vt inter. E. p. et. B. c. perfe-
ctæ res, quæ degradari debebant, locari poffent, fignaui.  Hæc autem diftantia, quæ
inter

inter.o.i.et.p.E.intercedebat,altitudinem oculi ab orizonte fignificabat. Signa-
ui etiam lineam.i.G. perpendicularem lineæ. E.p. cui affigi poffet non nihil chartæ
quotiefcunque volebam in perfpectiua aliquid delineare. Quod cum facere defi-
derabam,ponebam perfectum optimè affixum in quadrangulo. E.G:& in quadran
gulo.E.i.aliquod folium papyri affigebam. Ponamus nunc, me voluiffe conftituere
punctū.b. fumebā angulū rectū materialē,feu normā,& eius vnum latus, iuxta latus.
B.c.ponebam,atque aliud per punctum.b. tranfire faciebam, & vbi hoc latus lineā
E.p.diuidebat, punctum.c fignabam per quod efficiebatur,vt regula.i.M.tranfiret,
quiefceretque aliquantulum aliquo modo in huiufmodi fitu, opera deinde circini
interuallum.b.c.fumebam,& in.p.E.à puncto.x.verfus.E. punctum.m.fignabā : per
quod faciebam , vt tranfiret regula.o. Q. quæ lineam.x.i. in puncto. f. diuidebat.
Angulum deinde rectum materialem accipiebam,cuius vnum latus.A.B.ponebam,
aliud verò per punctum.f.tranfibat,quod quidem latus regulam. i. M. in puncto. k.
( quod ftatim fuper folio papyri fignabatur ) interfecabat , atque hoc erat punctū ,
quod quærebam,puncto.b. correfpondens.Huiufmodi effectus rationes ab ijs, quæ
fuperius dixi eliciuntur. Atque hæc ad bafes rerum,vt in fubfcripta figura elucefcit,
fpectabant.



## CAP. XI.

AD degradandas deinde altitudines , vfus fum mea figura tam.H. quā ēt. H. H.
vt mihi fefe offerebat occafio. In primis ratione modi figuræ.H.curabā, vt

vnum ex lateribus anguli recti, seu normæ regulæ. B. C. anniteretur, aliud verò
per.m.in rectitudine cuius signabam.m. T. interuallum æquale altitudini perfecti,
idest punctum. T.æqualiter distans ab.m.transire faciebam, deinde regulam. o. Q.
per punctum. T. transire quoque faciebam: & notabam intersectionem ipsius cum
linea.i.x.in puncto. V. efficiebam deinde vt vnum ex lateribus anguli recti, lateri ta-
bulæ. B. C. anniteretur, aliudque per punctum. k. transire faciebam, & in huiusmodi
rectitudine à puncto.k.signabam quandam mensuram æqualem lineæ.f.V. quę erat
k.R.pro altitudine degradata .

## ALITER IDEM.

MEDIANTE deinde figura. H. H. vnum ex lateribus anguli recti, lateri tabu-
læ.B.C.vt anniteretur faciebam; aliud verò per punctum.b.perfecti, idest ba
sis eiusdem perfecti transire faciebam . Et in huiusmodi rectitudine signabam. c. s.
æquale interuallum altitudini perfecti, idest punctum.s.ita distans à.c. efficiendo de
inde, vt latus anguli recti, lateri. B. C. tabulæ anniteretur, aliudque per punctum.k.
transire faciens signabam . k.R.indeterminatam. Faciens deinde transire regulam
i.M. per punctum.s.notabam punctum.R. intersectionis eiusdem cum linea. k. R. iã
ducta . Itaque altitudinem. k. R. degradatam habebam . Hæc autem via aliquan-
tulum breuior, expeditiorque altera.



IACOBO

## JACOBO SOLDATO MEDIOLANENSI
### Sereniſsimi Ducis Sabaudiæ Architecto peritiſsimo.

### CAP. VII.

SVPERIORIBVS diebus non diu poſtquam de perſpectiuis inter nos ſermonem habuimus, dum animus totus adhuc in his eſſet. Illud in mentem venit quòd eximius ille vir, & profundiſſimæ doctrinæ, nec vnquam ſatis laudatus Daniel Barbarus ſe accepiſſe profitetur à Ioanne Zamberto patritio Veneto, qui ad verbum omnia deſumpſerat à Ioanne Cuſino Pariſienſe. Nec parum mirabar peritiſſimum illum Cuſinum, quod in capite quarto ſecundæ partis perſpectiuæ, vt quodpiam planum quadrilaterum in quadratam figuram redigeret, ſuper vnam datam lineâ quadratam compoſuiſſe. Non animaduertens diſtantiam aut interuallum.b.c. degradatum in linea.b.f.(quod eſt.b. E.) ita eſſe poſſe latus parallelogrammi rectanguli magis longi quam lati, aut magis lati quam longi, vt etiam latus quadrati, quod beneficio ſubſcriptæ hic figuræ facilè depræhendi poteſt. Vbi. b. c. latitudo eſſe poteſt, tam perfecti degradati in triangulo.b.n.m. aut in triangulo.b.q.t. quam in triangulo.b.a.c. Sed perfectum degradati in triangulo.b.n.m. magis longum quam latū & perfectum degradatum in triangulo.b.q.t. magis latum quam longum, & perfectum degradati in triangulo.b.a.c. quadratum erit quemadmodum à meis etiam figuris.A. ſcientificè intelligi poteſt. Hinc, ad inueniendum perfectum alicuius plani degradati, non ſufficere degradationem ſolum interualli inter duos terminos ſolos, ideſt.b. E. aſſignare, apertè patet, quia non omnia parallelogramma perfecta ab vno tm̄ interuallo producuntur, eo ꝙ non ſunt omnia quadrata. Ad inquirendū igitur perfectum alicuius plani parallelogrāmi, alicuius propoſiti degradati, oportet vniuerſam degradationem tam latitudinis, q̄ longitudinis, & nō ſolius longitudinis aſſignare; Vt exēpli gratia, in ſubſcripta hic figura, volēdo inuenire perfectum parallelogrammum degradati.b.h.l.m. dando diſtantiam orizontalem.b.d. à pede.d. hominis
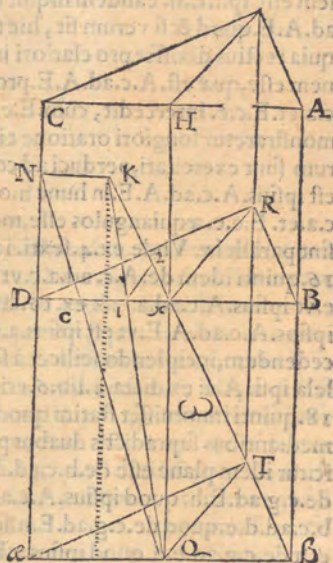
minis vfque ad plani fitum in quo degradatio facta fit : ftatim altitudo. A. oculi à pe
de , quæ tanta femper effe debet quanta eft altitudo trianguli.b.n.m. qui clauditur,
protrahendo. m. l. et. b. h. vfque ad concurfum in. n. in lucem prodibit . Oportet
deinde erigere lineam.b.o. perpendicularem lineæ.d.b.m. & vfque ad eandem pro
ducere lineam.l.h.in puncto.E. et à puncto.A.per.E. vfque ad.c.ipfius.d.b.m.produ
ctæ ducere. A.E.c.atque deinde protrahere lineam.o.b.vfque ad.T.ita vt.b T.equa
lis fit ipfi.b.c.& ad ipfam à puncto.m.ducere parallelam.m.R.& à puncto.T.ducere.
T.R.parallelam ipfi.b.m. Vnde ex.34.primi Eucli.m.R. æqualis erit ipfi.b T.et.R.
T. ipfi.m.b.& anguli in rectos euadent,atque hoc parallelogrammum rectangulum
erit verum perfectum degradati. b.m.l.h.ob rationes à me circa figurā. A.adductas .



Sed eft hic quod magis nos commoueat,quia cum ex linea.b.c.quadratum. b.g.pro
duxerit,vult eum poftea degradare. Quod vt faciat ( hanc figuram videbis in cap.
4. fecundæ partis Danielis Barbari ) oculum . A. in eadem fuperficie extenfa,
quadrati. b. g. collocat . Quod recte fieri non poteft , quia oculum hoc mo
do locantes, vifualesque radios beneficio vnius plani fituati in. b. f. fecantes in ipfo
plano,nihil aliud quam dictã lineã.b. f. & nullã degradatione inueniét.Id quod,& fi
natura fua fit omnibus notum, ponit tñ id ipfum Vitelio pro quinta propofitione
quarti libri de perfpectiua . Præter hæc,credit latera.b. d. et. c. e. quadrati degra-
dati femper videri mediantibus angulis. b. A.c.et.f. A.g.quod fieri nõ poteft,quem-
admodum ex mea figura corporea.A.facilè cognofcere poffumus, propterea quòd
latera.d.r.et.q. e.meæ figuræ,mediantibus angulis.d.o.r.et.q.o.e. qui extra fuperfi-
ciem. s. a. exiftunt videntur , vnde fi quis imaginaretur in puncto.p.oculum effe , &
ab ipfo ad.u.et.q.duas lineas duceret, angulus.q.o.u.nunc maior , nunc minor effet
angulo.q.p.u.aliquando etiam æqualis,quamuis rariffime ; Sub diuerfis igitur an-
gulis,pro maiori parte,deteguntur latera,à partibus quadrati tam degradati, quàm
perfecti,quæ non funt anguli.b. A.c.et.f. A. G. Quod vero idem poftea dicat eam
proportionem effe ab.b.E.ad.f.h. & fimul ad.c.g.quæ eft ab.a.g.ad.h.g. id tuo relin
quam

quam iudicio . Tibi quoque confiderandum relinquo; cum rationabilis degrada-
tio effe debeat, qua ratione neceffarium fit , vt diftantiæ refque , in vna & eadé pro-
portione cum altitudine oculi ad rem degradatam exiftant? Cum poftea degrada-
uerit quadratū , is fcriptor, in figura. d. b. c. e. eum bene & ex perfpectiuæ optimis
legibus degradatum fuiffe probare nititur; folum probans. d. e. æqualem effe ipfi.
E. h. q̃. E. h. fecundū ipfum eft degradatio lateris. c. g. & cū fuperius dixerit , fe tria
quadrati plana degradauiffe , quia. b. E. degradat. b. c. et: E. h. degradat. c. g. et. f.
h. degradat. f. g. nec quidem de lateribus. b. d. et. c. e. loquitur , quia fi. c. g.
perfecti, degradatum eft in . E. h : et. d. e. rectè protracta exiftit , cum fit æqua-
lis ipfi. E. h. cum etiam. b. d. et. c. e. rectè protractæ effe debeant : qua de cau-
fa ipfis. b. E. et. f. h. quæ, ex ipfo, funt degradationes. b. c. et. f. g. æquales effe non de-
bent ? Poffet is mihi quidem refpondere, q̃ hoc pacto nulla fuperficies clauderetur.
Ergo tria latera. b. c: c. g. et. g. f. nõ benè funt degradata, eiufq̃; pportionalitates ma
lè intellectæ nil probant. quia fi dictæ proportionalitates , nobis tutò promitterent
degradationes , ab eo primum effectas, in linea. b. f. effe bonas, ergo duæ. b. d. et. e. c.
falfæ exifterent, quarum quælibet maior eft. b. E. et. f. h. ex. 18. primi Eucli. Omitta-
mus etiam quod vbi is fcribit eam effe rationem, aut comparationem ab. A. d. ad. b.
E. quæ eft ab. d. c. ad. b. c. eandemque effe ab. E. h. ad. c. g. quæ eft ab. A. E. ad. A. c. nil
probet; nec fimilitudinem triangulorum, nec aliquam propofitionem Eucli. citans.
In quo excufari non poteft, quòd non foleat Euclidem, aut alium quemuis autorem
citare, cum vel in ipfo operis principio capite. 3. primæ partis, Apollonium Pergeū
Euclidemq̃;, & fi etiam præter rem, citet. Deinde quū idem probare vult. d. e. æqua
lem effe ipfi. E. h. eandem inquit effe proportionem. a. b. ad. a. d. quæ eft ipfius. A. c.
ad. A. E. quod & fi verum fit , hic tamen modus ratiocinandi nullo ordine nititur,
quia rectius dixiffet pro clariori intelligentia ipfius. a. c. ad. a. e. eandem proportio-
nem effe, quæ eft. A. c. ad. A. E. propter fimilitudinem, quæ inter duos triangulos. A.
c. a. et. E. c. e. intercedit, cum. E. e. fupponatur parallela ipfi. A. a. quod etiam vt de-
monftraretur longiori oratione ei opus fuiffet fi voluiffet intellectum eorum, qui pa
rum funt exercitati, perduci ad cognofcendū idem planè futurum de. a. c. ad. a. e. vt
eft ipfius. A. c. ad. A. E. in hunc modum, ideft probando primùm duos triangulos. A.
c. a. et. E. c. e. æquiangulos effe, mediante. 29. primi Eucli. cum. A. a. et. E. e. inuicem
fint parallelæ. Vnde ex. 4. fexti. idem extitiffet de. A. c. ad. E. c. vt. a. c. ad. e. c. et. ex
16. quinti idem de. A. c. ad. a. c. vt ipfius. E. c. ad. e. c. & ex. 19. eiufdem de. A. E. ad. a.
e. vt ipfius. A. c. ad. a. c. & ex. 16. iam dicta de. A. E. ad. A. c. vt ipfius. a. e. ad. a. c. ideft
ipfius. A. c. ad. A. E. vt eft ipfius. a. c. ad. a. e : Aut hoc alio modo , qui breuior eft pro-
cedendum, incipiendo fcilicet à fecunda fexti Eucli. dicendo q̃ exiftente. E. e. paral
lela ipfi. A. a: ex dicta. 2. lib. 6. erit idem de. c. E. ad. E. A. vt de. c. e. ad. e. a. vnde ex.
18. quinti innotuiffet ftatim quod de. c. A. ad. E. A. vt de. c. a. ad. e. a. extitiffet. Nunc
mediantibus fupradictis duabus propofitionibus ideft. 29. primi, &. 4. fexti, cogno-
fcitur idem planè effe de. b. c. ad. d. e. quod ipfius. a. c. ad. a. e. & ex eifdem idem effe
de. c. g. ad. E. h. quod ipfius. A. c. ad. A. E. vnde ex. 11. quinti bis repetita idem erit de
b. c. ad. d. e. quod de. c. g. ad. E. h. fed cum ex fuppofito. c. g. fit æqualis ipfi. c. b. idem
erit de. c. g. ad. e. d. quod ipfius. c. b. ad eandem ex. 7. quinti, vnde ex. 11. idem erit de
c. g. ad. E. h. quod eiufdem. c. g. ad. e. d. ex. 9. igitur eiufdem. d. e. æqualis erit ipfi. E.
h. atque hic verus eft modus ducendi intellectum parum exercitatum in cognitio-
nis campum. quem quidem mihi obferuandum proponerem fi onus fcribendi fu-
fciperem ijs , qui in fcientijs parum verfati funt, quos tanquam puerulos manu du-
cere

cere oportet. Ratio verò ab ipfo adducta propter quam.E.reprefentatur oculo al-
tius quàm.b.nempe eo quod.A.fuperftet ipfi. E. nihil valet, quia fi inferius eſſet,
idem contingeret, ſed hoc euenit eo quod.E.altius eft ipfo.b. Idem dico de.h.
vbi ſimiliter decipitur. Idem etiam in.7.cap.fallitur in ſecundo modo,quem often
dit pro ſecundo quadrato aliquo degradato à parallelogrammo degradato magis
longo quàm lato,cum ducat parallelam.l.m.ad.b.c.à puncto.l. interſectionis ipſius.
o.c.id,quod non rectè efficitur quemadmodum ex rationibus à me allegatis circa
meas figuras. A.A. facilè innoteſcit.

　　Nono deinde cap. contrario planè ordine, quam oporteret proceſſit, quia cũ
angulus.2. trianguli perfecti magis diftet à plano ſuper quod degradari debet
triangulum,quàm latus.1.3. oppoſitum dicto angulo.2. & per conſequens longè re
motior ſit ab oculo, ipſe in degradato, eũ magis propinquum eſſe facit, è con-
tra cap.10.rectè fecit contra id, quod capite.9. tradiderat.

　　Quod autem deinceps in prima parte.11.& vltimi capitis aſſerit eſt,admittendũ.
Quod verò in ſecunda parte ab eo traditur, ideſt alius quidam modus, quem de trãſ
ferendis punctis à perfecto in degradato proponit, non eſt modus vniuerſalis ; quia
ſi altitudo.T.Q. oculi à plano orizontali, non eſſet æqualis medietati lateris. B. D.
perfecti, interualla.a.b.c.d.e.lateris. B.D. admittenda non eſſent.

　　Pro cuius rei intelligentia ſit in ſubſcripta hic figura corporea. ω. parallelogram-
mum rectangulum A.B.C.D. in plano orizontali, & linea. Q.H.illud per medium
diuidat,quæ ſit parallela duobus lateribus. A.B.et. C. D. in cuius quolibet puncto.
Q. ſit infimus terminus altitudinis oculi, & in.
T. ad perpendiculum ipſius. Q. ſit verus ſitus
eiuſdem, tantum eleuatus à. Q. quanta eſt
medietas ipſius. D. B. ſitque figura corpo-
rea finita ſimilis meæ. A. vnde. Q. T. æqualis
erit ipſi.Q.æ.& planum perpendiculare orizó-
ti, ſuper quod punctum. k. perfecti duci debet
ſit. R. D. B. ſintque ductæ per imaginationem
lineæ.T.K.Q.K. et ſit.K.N. perpendicularis la-
teri.C.D. à quo puncto.N. imaginatione ſit có
præhenſa linea.N.Q. atque hæ tres lineæ ſectæ
ſint à plano in punctis.c.i. et.2. quorum punctũ.
2.erit quæſitum plani. Imaginemur nunc duos
triangulos. K.T. Q. et. N. Q. æ. qui ſecti erũt
à plano.R. B. D.quorum communes ſectiones
erunt.1.2.et D.c. & quia. N.K. D. i. et. æ. Q.
inuicem ſunt parallelæ, ſequitur eandem pro-
portionem futuram ipſius. Q.K.ad.K.i. quæ eſt
ipſius. æ. N. ad.N.D. imaginatione concipien
do à puncto.K. vſque ad.æ. Q.quandam paral-
lelam ipſi.N.æ.quemadmodum ex te ipſo intel
ligere potes. Sed ratione ſimilitudinis trian-
gulorum ita ſe res habet de.æ.Q.ad.D.c.vt de.
æ. N. ad.N.D.vt quoque de. T.Q. ad.2.1. quemadmodum ipſius. Q.K.ad.K.i.vn-
de ex.11.quinti,idem erit de.Q.T.ad.1.2.quod de.Q.æ.ad.c.D.& ex. 16. eiuſdem
de. Q.T.ad.Q.æ.quod de.1.2.ad.c. D.& exiftente.æ. Q. ex ſuppoſito æquali ipſi.

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　I.2.

## DE PERSPECT. 137

.1.2. Vnde huiufmodi regula tunc bona redditur, quando T. Q. æqualis eft ipfi. œ. Q. ideft medietati ipfius. D. B. at verò fi æqualis non effet hoc minimè fequeretur, vt facilè patet. Quòd verò. 2. R̃. z. &. fint benè difpofita, dubitandum non eft, quia punctum. i. meæ hic fubfcriptæ figuræ, quod corefpondet K. eius figuræ adeò diftat à medio. R. X. trianguli. R. B. D. vt. 2. cum. 1. 2. dicto medio. R. X. ex. 6. Vndecimi fit parallela. Idem de reliquis dico. quod manifeftè cognofci poteft, ab eo, quod in fuperius pofitis figuris corporeis dixi. Huiufmodi modus ducendi res in perfpectiua, non folum à Gallis, fed à Germanis etiam in vfum reducitur. Sed quia ad hæc vfq; tempora eiufdem perfectionis ratio, quam ego fuperius propofui, nódum in lucem emerfit, factum fuit, vt errorũ laqueis irretirentur, fumentes. T. Q. modo maiorem, modo minorem mediate lateris. D. B. Cum hunc igitur modum hic Autor vniuerfalem effe putet, labitur in errorem, cum debuiffet longitudinem ipfius. T. Q. debere effe æqualem medietati ipfius. D. B. proferre. Afferit deinde diftantiam ipfius. T. Q. à latere. B. D. æqualem effe debere lateri. C. D. quod neceffarium non eft, quia in quibuslibet diftantijs, iufta operatio fieri poteft, quemadmodum in fubfcripta hîc figura facile patet, ideft, quòd quibufcunque modis. c. D. æqualis remaneat ipfi. 1. 2. & fic interualla, quæ p̃ tranfuerfum aguntur vfq; ad mediũ trianguli. D. R. B. Neque etiam probandus eft auctor ille, cum pro oculo, fuum. T. loco. Q. à me pofiti, ponit, cum is locus fit verus fitus pedis eius qui refpicit, & non oculi. Quòd autẽ Auctor ifte, modo vniuerfali intelligat, vt iam diximus, cõfideretur figura tertij modi primi cap. tertiæ partis, in qua fuum oculum (vt ita dicam) ponit in. o. altius feu diftans à rectitudine lateris. c. d. plus quam fit totum latus. d. b.

## AD EVNDEM IACOBVM.
### CAP. XIII.

TVAs accepi literas omnis humanitatis & officij plenas, in quibus requiris caufam, quæ me in alijs meis literis impulit ad dicendũ, angulũ. q. o. u. modo maiorem, modo verò minorem futurum angulo. q. p. u. meæ figuræ corporeæ. A. hanc igitur ob caufam imagineris in fubfcripta hîc figura duo triangula. q. o. u. et. q. p. u. quorum. q. p. u. perpendiculariter fit fuper fuperficie trianguli. q. o. p. collocatum, præcifè vt in mea figura corporea. A. fuperficies verò trianguli. q. o. p. fit exempli-

gratia. V. M. & trianguli. u. o. p. fit. V. D. quarum cómunis fectio fit. V. p. o. x. non eft enim dubitãdum quin triangulum. q. p. u. fit perpendicula-re triangulo. q. o. p. cũ hoc ex. 18. lib. 11. Eucli. perpendicula-re fit fuperficiei. a. s. in qua reperitur triã-gulum. q. p. u. & hoc



ex linea. o. p. perpendiculari dictæ fuperficiei. a. s. Nunc dico angulum. q. o. u. modo maiorem, modo minorem effe angulo. q. p. u. Notiffimum igitur primum nobis eft

S

est angulum.p.q.u.obtuſum eſſe; Imaginemur ergo circa triangulum.p.q.u.circun-
ſcriptum eſſe circulum, cuius portio.p.q.u. minor erit medietate eiuſdem medij cir-
culi,vt iam ex 30.Eucli.lib.tertij nouiſti.nunc imaginemur dictum circulum circum
lineam.q.u.loco axis verſus.x.moueri,vnde girus eiuſdem,per quem tranſibat linea
V.x.remouebitur ab eadem linea non nihil cum motus erit à primo ſitu vſquequò
ad ſecandam dictam lineam.V.x.in alio quodam puncto inter.p.et.x.redibit; quod

quidem punctum ſi
erit inter.o.et.x.angu
lus. q. o. u. maior erit
angulo. q. p. u. Sed ſi
idem punctū erit in-
ter.p.et. o. dictus an-
gulus. q. o. u. minor
erit. q. p. u. de qua q-
dē re tu ipſe median-
te.20.lib.3.et. 16.lib.
primi certior fieri po-
tes . Valde miror q
hæc Ioannis Cuſini di



cta ad hæc vſque tempora tanto in prætio ſint habita , vt ab excellentibus ſcriptori-
bus quaſi ſi proprij eorum ingenij partus eſſent,de verbo ad verbum vt theſauros,in
ſuis ipſorūmet libris reſcripta fuerint,quemadmodum iam omnes admonui in mea
gnomonica Orontium, Munſterum, aliosq̃; permultos feciſſe.

## CAP. XIIII.

EX ijs, quæ de nonnullis effectibus ducendo in perſpectiua tertium corpus regu
lare , q octo triangulis æquilateribus eſt termīnatum,ſcire deſideras,hoc vnū
eſt caput: vnde fiat, aut quomodo probetur quaſlibet duas facies oppoſitas eiuſ-
dem corporis octoaedri inuicē æquidiſtantes eſſe . Quamobrem ſit hic ſubſcriptū
octoaedrū, cuius diameter vna ſit.b.q.et.b.p.
l. vna ex faciebus, cui opponatur facies.q. k.
d.quas adinuicē æquidiſtantes eſſe contendo
ſint aliæ duæ facies , quæ inter has ponuntur.
b.d.k.et.q.p.l.& à punctis extremis.b.q. dia-
metri.ductæ ſint quatuor lineæ.b.a:b.u:q.a:q.
u.ad puncta.a.et.u.diuidentia.k.d.et. l. p. per
medium , vnde ex 4.primi Eucli. quatuor hæ
lineæ adinuicem equales erunt ſumēdo eas vt
baſes triangulorū.a.d.b:u.l.b:a.d. q. et.u.l. q.
adinuicē quoq; æqdiſtabūt.a.b. ab.u. q. et.b.
u.ab.q.a.ex.27.primi;qa ſi imaginabimur dia
metrum.b.q.tunc ex.4.aut ex.8.eiuſdem lib.
habebimus angulos.a.b.q.et. u. q. b. æquales
inuicem; ſed ob eaſdem rationes.p.l.paralle-
la eſt ipſi.d.k.vnde ex 15.lib.11.facies. b.p. l.
parallela ſit,aut æquidiſtans ipſi. q.d. k. ideſt
primam propoſitum.



Ad

Ad habendam deinde quantitatem distantiæ, aut interualli simul cum situ, in facie.q.d.k.quem latus.p.l.perpendiculariter respicit. Imaginemur à puncto. u. super q.a. cadere lineam perpendicularem. u. o. quæ illico reperitur cum triangulum. a. u.q.ex lateribus datis & cognitis constet, quodquidé triangulum, medietas est quadrilateri, seu.rumbi.q.a.b. u. cui vnaquæque dictarum quatuor facierum perpendicularis existit ex.4.et.18.lib.11.& ob id linea.u.o.extensa in superficie dicti quadrilateri, & perpendicularis lineæ. q. a. perpendicularis erit faciei. q. d. k. & ex.29. primi, angulus.b.u.o.rectus erit, ut etiã angulus.o.u.l.ex.2.definitione lib.11.vnde ex.4.eiusdem lib.o.u.perpendicularis erit faciei.b.p.l. Habebimus ergo situm in facie.q.d.k.qui respicietur ad angulos rectos à linea.p.l.quiquidem erit in perpendiculari à puncto.o.ad.q.a.ducta.

Quòd autem.a.o.sit latus exagoni æquilateris circumscriptibilis ab eodem circulo, qui vnam ex faciebus triangularibus æquilateribus propositi corporis circunscribere potest,ita ostenditur. sit côprehensum imaginatione,triangulum.a.q. u.separatim,cuius latus.a.u.æquale est vni ex lateribus triangulorũ eiusdem corporis ex.33. primi,quodlibet verò aliorum duorum æquale perpendicularibus dictorum triangulorum, in quo triangulo.a.u.q.ducta sit perpendicularis. u. o. ab vna extremitatũ lateris maioris,ad vnum ex minoribus lateribus,quę perpendicularis intra triangulum cadet,quia dictum triangulum oxigonium est. quod autem attinet ad duos angulos. a.et.u.cum æquales sint ex quinta lib.primi;17. nos certiores facit;quod verò angulus.q.sit etiã acutus:30.lib.tertii nos certos reddit,q.a.a.u.minor est diametro sphę ræ datum corpus circumscribentis, cum. q. dictæ sphęrę superficiem tangat.

Ad probandum.a.o.equalem esse lateri exagoni dicti, satis erit probare.a.q.sesqui alteram esse ad. a. o. quia si in subscripto hîc circulo ducemus duas semidiametros. n.p.et.n.l.ad. angulos triãguli equilateri.p. et.l.& cum quodlibet laterum ipsius exagoni, equale sit semidiametro circuli ex. 15. lib. 4. habebimus ex. 8.primi,angulum.n. p.l.æqualem angulo.q.p. l. Vnde ex.4.eiusdem.o.n. equaliserit ipsi.o.q.idest.q.a. sesquialtera erit ad.a. o.

Ad probandum nunc in triangulo. a.q. u: a.q.sesquialteram esse ad. a. o. est quoq; sciendum primò omne latus trianguli equilateri in potentia sesquitertium esse ad perpendicularem eiusdem trianguli, quod vndecima lib.14. Eucli. breuiter demonstratum est.
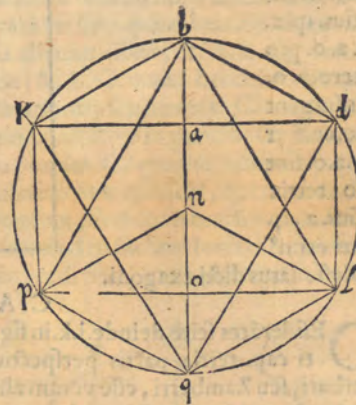
Ponamus nunc quadratum lateris.a.u.effe.12.clarum erit quodlibet quadratum
aliorum duorum laterum.a.q.et.u.q.futurum nouem,ex ijs quæ pofteriore loco dixi
mus, & quia quadratum ipfius. q. a. eft tantò minus aliorum duorum quadratorum
fumma,quantum eft duplum producti ipfius.q.a.in.a.o.ex.13. fecundi, fed alia duo
quadrata fimul collecta faciunt.21.à quo numero fubtrahendo quadratum ipfius.a.
q.ideft nouem,remanebit numerus.12.pro duplo producti ipfius.q.a.in. a. o. cuius

dupli me-
dia pars,id-
eft fimplex
productum
ipfius. q. a.
ī a.o.e rit 6.
Sed qa qua
dratum ip-
fius.q. a. eft
nouem ,
eius radix.
q.a. erit. 3.
per quā di-
uidendo. 6.
productum
ipfius. q . a.
in. a. o. pro
latere. a. o.
confurgent
duo,cum er
go.a.o. fint
duo tertia
ipfius. a. q.
certi erim'
a.o.effe latus dicti exagoni.
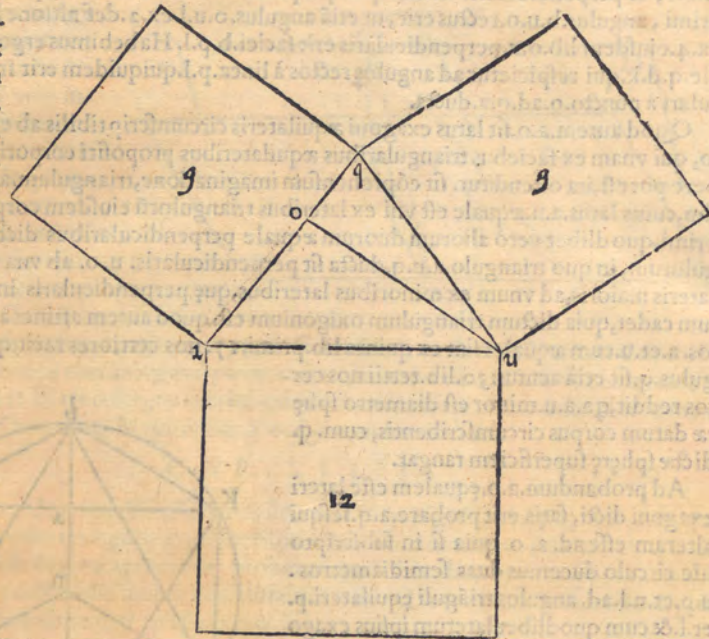


## CAP. XV.

DEfiderātes fcire deinde.l.k.in figura. M. quar
ti cap. tertiæ partis perfpectiuę Danielis
Barbari, feu Zamberti , effe veram altitudinē cor-
poris octoaedri,primū fcire debemus ợ exiftēte.b.
h.vt etiā.b.l.tripla ad.b.k.vt ex ijs, quę fuperius iā
diximus,facile percipi poteft;ex penultima primi.
b.l.in potentia,fefquioctaua erit ad. k. l. ipfa ét. k.
l.dupla in potētia ad.h.k.& ob id ducta cū effet. h.
l.exifteret in potentia tripla ad.h.k. & fefquialtera
ad.l.k.& fefquitertia ad.l.b.& fic ad.h.b. vnde.l.h.
æqualis effet vni ex lateribus triāguli ęquilateri di-
cti còrporis. Ex rationibus igitur fuperius hîc pofi-
tis.l.k. erit altitudo dicta,id eft diftantia inter duas
facies inuicem oppofitas, octoaedri.
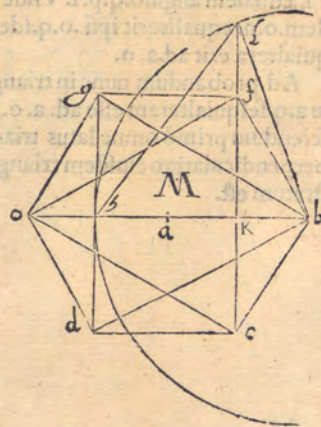


Neq; volo te ignorare aliū nó paruū fuiffe errorē
illius Zamberti : cum eodē capite affirmet angulos
octoaedri rectos effe cū fint acuti,nā vnufquifq; minor eft angulo cubi folido.   DE

# DE MECHANICIS.

*CRIPSERVNT multi multa, & quidem scitissimè, de mecha-
nicis, at cum natura ususq; aliquid semper vel nouum, vel
latens in apertum emittere soleant, nec ingenui aut grati sit
animi, posteris inuidere, si quid ei contigerit comperuisse prius
tenebris inuolutum : cum tam multa ipse ex aliorum diligentia
sit consequutus. Paucula quædā futura, vt reor, non ingrata his
qui in hisce mechanicis versantur, nusquam ante hac tentata,*
*aut satis exactè explicata in medium proferre volui : quo vel iuuandi desiderium, vel
saltem non ociosi ingenioli argumentum aliquod exhiberem: atque vel hoc vno modo me
inter humanos vixisse testatum relinquerem.*

## De differentia situs brachiorum libræ.

### CAP. I.

OMNE pondus positum in extremitate alicuius brachij libræ maiorem, aut mi-
norem grauitatem habet, pro diuersa ratione situs ipsius brachij. sit exempli
gratia.B. centrum, aut, quod diuidit brachia alicuius libræ, &. A. B. Q. vertica-
lis linea, aut, vt rectius dicam, axis orizontis, &. B. C. vnum brachium dictæ li-
bræ, & in. C. sit pondus, &. C. O. linea inclinationis, seu itineris. C. versus cen-
trum mundi, cum qua. B. C. angulum rectum constituat in puncto. C. Existente
igitur in huiusmodi situ brachio. B. C. dico pondus. C. grauius futurum, quam
in alio quolibet situ. quia supra centrum.B. omninò non quiescet, quemadmodum
in quouis alio situ faceret. Ad quod intelligendum, sit dictum brachium, in situ.B.
F. cum eodem pondere in puncto. F. & linea itineris seu inclinationis dicti ponderis
sit.F.u.M. per quam lineam dictum pondus progredi non potest, nisi brachium.B.F.
breuius redderetur. Vnde clarum erit
quòd pondus. F. aliquantulum supra cen
trum. B. mediante brachio. B. F. nititur.
Est quidem verum, quòd pondus. C. nec
ipsum etiam per lineam.C.O.proficisce-
tur, quia iter extremitatis brachij est cir-
cularis,&.C. O. in vno quodā puncto est
contingens. Sit hoc iter. A.C.Q. Opor-
tet nunc præsupponere pondus extremi-
tatis brachij debere tanto magis cētro. B.
inniti, quanto magis linea suæ inclinatio-
nis ( ponamus.F.u.M.) propinqua erit di
cto centro. B. quod sequenti cap. proba-
bo, vt exempli gratia, sit.F.super.u.pun-
ctum medij ex æquo inter. C. et. B. qua-
propter. u. B, æqualis erit.u. C. vnde se-

quetur

quetur dictum pondus grauius futurum pro parte. F.C.quam pro ea,quæ eſt. A.F. & minus ſupra centrum. B. pro dicta parte. F. C. quam pro parte. A. F.quieturum ; & dictum brachium quanto magis orizontale erit à ſitu. B.F.tantò minus ſupra dictum centrum. B.quieſcet,& hac ratione grauius quoque erit, & quanto magis vicinum erit ipſi. A.à dicto.F.tantò magis ſuper centrum. B.quoque quieſcet,vnde tātò quoque leuius exiſtet. Idem dico de omni ſitu brachij per girum inferiorem. C. Q. vbi pondus pendebit à centro. B.dictum centrum attrahendo, quemadmodum ſuperius illud impellebat. Hæc verò omnia cap. ſequenti melius percipientur.

## De proportione ponderis extremitatis brachij libræ in diuerſo ſitu ab orizontali.

### CAP. II.

PROPORTIO ponderis in. C. ad idem pondus in F. erit quemadmodum totius brachij.B.C.ad partem.B.u.poſitam inter centrum & lineam.F.u.M.inclinationis , quam pondus ab extremitate. F.liberum verſus mundi centrū conficeret.Quod vt facilius intelligamus imaginemur alterū brachium libræ.B. D. & in extremo. D. locatum aliquod pondus minus pondere. C. vt. B. u. pars.B.C.minor eſt. B.D.clarè cognoſcetur ex.6.lib.primi de ponderibus Archimedis, quòd ſi in puncto.u.collocatum erit pondus ipſius.C.libra nihil penitus à ſitu orizontali dimouebitur. Sed perinde eſt quòd pondus.F.æquale.C.ſit in extremo. F.in ſitu brachij. B.F.quā vt ſit in puncto.u.in ſitu ipſius.B.u.orizontali. Ad cuius rei euidentiam imaginemur filū. F.u.perpendiculare, & in cuius extremo.u.pendere pondus,quod erat in.F.vnde clarum erit quòd eundem effectum gignet, ac ſi fuiſſet in.F.quod, vt iam diximus remanens affixum puncto.u.brachij.B.u.tantò minus graue eſt ſitu ipſius.C. quantò.u. B.minus eſt ipſo.B.C.Idem aſſero ſi brachium eſſet in ſitu. e. B. quod facilè cognoſcere poterimus, ſi imaginemur filum appenſum ipſi. u.brachij. B. C. & vſque ad. e. perpendicularè,in quo extremo appenſū eſſet pondus æquale ponderi. C. & liberū ab.e.brachij.B.e.vnde libra orizontalis manebit. Sed ſi brachium.B. e. conſolidatum fuiſſet in tali ſitu cum orizontali.B.D.

& appēſo pōdere.C.in. e. libero à filo,nec aſcēderet, neq; deſcenderet. quia tantum eſt quod ipſum ſit appenſum filo,ꝙ pendet ab.u.quantum quòd ab ipſo liberum appē nſum fuiſſet.e.brachij.B. e.& hoc procede ret ab eo quòd partim penderet à centro. B. & ſi brachiū eſſet in ſitu. B.Q.totum pō dus centro.B.remaneret appenſum,quemadmodū in ſitu.B. A. totū dicto centro anniteretur. vnde fit vt hoc modo pondus magis aut minus ſit graue , quò magis aut minus à centro pendet,aut eidem nititur: atq; hæc eſt cauſa proxima , & per ſe, qua fit vt vnum idemq; pondus in vno eodemq; medio magis aut minus graue exi-

ſtat.

stat . Et quamuis appellem latus.B.C.orizontale , supponens illud angulum rectum
cum. C.O. facere,vnde angulus.C.B.Q.sit vt minor sit recto , ob quantitatem vnius
anguli equalis ei,quem duæ.C.O.et.B.Q.in centro regionis elemétaris constituût,
hoc tamen nihil refert,cum dictus angulus insensibilis sit magnitudinis. Ab istis au-
tem rationibus elicere possumus , quòd si punctus.u.erit ex æquo medius inter cen-
trum.B.& extremum.C.pondus.F.aut.M.pendebit,aut nitetur pro medietate dicto
centro.B.& si dictum.u.erit propius.B.quam puncto.C.pendebit ab ipso,aut nitetur
ipsi amplius quã ex medietate, & si magis versus . C. minus quã ex medietate niteî.

*Quòd quantitas cuiuslibet ponderis,aut uirtus mouens re-*
*spectu alterius quantitatis cognoscatur beneficio*
*perpendicularium ductarum à centro*
*libræ ad lineam inclinationis .*

### CAP. III.

EX ijs,quæ à nobis hucusque sunt dicta, facilè intelligi potest, ꝙ quantitas. B. u.
quæ ferè perpendicularis est à centro. B.ad lineam. F.u. inclinationis, ea est,
quæ nos ducit in cognitionem quantitatis virtutis ipsius. F.in huiusmodi situ, consti
tuens videlicet linea. F.u.cum brachio. F.B.angulum acutum.B.F.u. Vt hoc tamen
melius intelligamus, imaginemur libram.b.o.a.fixam in centro.o. ad. cuius etrema
sint appensa duo pondera,aut duæ virtutes mouentes.e.et.c.ita tamen ꝙ linea incli-
nationis.e.idest.b.e.faciat angulum rectum cum.o.b.in puncto.b. linea verò inclina
tionis.c.idest.a.c.faciat angulum acutum, aut obtusum cum.o.a.in puncto.a.Imagi-
nemur ergo lineam.o.t.perpendicularem lineæ.c.a. inclinationis, vnde. o. t. minor
erit.o.a.ex.18. primi Euclidis. secetur deinde imaginatione o.a. in puncto. i. ita ut
o.i.æqualis.sit.o.t.& puncto.i.appensum sit pondus æquale ipsi.c.cuius inclinationis
linea parallela sit lineæ inclinationis ponderis.e.supponendo tamen  pondus aut vir
tutem.c.ea ratione maiorem esse ea,quæ est.e.qua.b.o.maior est.o. t. absque dubio
ex.6.lib.primi Archi.de ponderibus.b.o.i.non mouebitur situ,sed si loco.o.i. imagi
nabimur.o.t.consolidatam cum.o.b.& per lineam.t.c.attractam virtute. c. similiter
quoque continget ut b.ó.t; communi quadam scientia,non moueatur situ. Est ergo
quod proposuimus verum quantitatem alicuius ponderis respectu ad eam , quæ est
alterius debere depræhendi à perpendicularibus, quæ à centro libræ ad lineas incli
nationis exiliunt. Hinc autem innotescit facillimè, quantum vigoris,& vis  pondus,
aut virtus.c.ad angulum rectum cum.o.a.minîmè trahens, amittat. Hinc quoque co
rollarium quoddam sequetur, quò d quantò propinquius erit centrum.o.libræ cen-
tro regionis elementaris,tantò quo que minus erit graue.
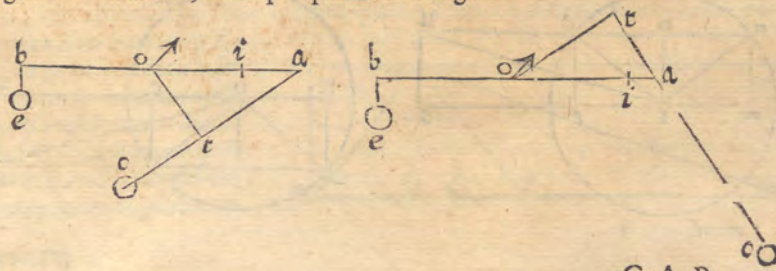


CAP.

*Quemadmodum ex supradictis causis omnes staterarum &*
*uectium causæ dependeant.*

## CAP. IIII.

VIs brachij longioris alicuius stateræ, aut vectis, maior breuioris, ab ijs, quæ in su
periobus capitibus diximus, idest ꝗ nitatur pendeatué magis aut minus à
centro pondus in extremitate brachij maioris positum, oboritur. Quamobrem illud
à nobis primò est cognoscendum , stateras, aut vectes, puras mathematicas li-
neas non esse, sed naturales, hincque existere corpora cum materia coniuncta. Nunc
igitur imaginemur. n. s. eam superficiem esse, quæ secundum longitudinem axem sta
teræ scindit. & supponamus ipsius centrum esse primum in. i. & maius brachium esse
.i. u: minus autem. i. n. & lineam verticalem. i. o. quæ tanta sit , quanta est spissitu-
do, aut crassities ipsius stateræ à superiori latere ad inferius, ad faciliorem intelligen-
tiam, supponendo. n. s. parallelogrāmam. Positis igitur duobus ponderibus æquali-
bus in extremitatibus brachiorum, experientia innotescit, ꝗ pondus ad. u. s. appen-
sum, violentiam faciet ponderi appenso ad. n. x. sed nos volumus inuestigare causā
huius effectus, quæ à nemine vnquam literarum monumentis , ꝗ sciam , consignata
fuit. Iam diximus stateram, aut vectem materialem esse &. n. s. eius superficiem me-
diam, supponendo. i. esse centrum quo nititur dicta statera aut vectis; Cum hoc er-
go ita se habeat, sint. u. s. et. n. x. lineæ inclinationum ponderum , & imaginemur, ꝗ
dicta pondera pendeant à punctis. u. et. n. vt reuera pendent, etiam si appensa essent
sub. s. et. x. quia punctum. u. & punctum. n. ita coniuncta sunt cum. s. et. x. ut qui vnū
trahit alterum quoque trahat. Imaginemur quoque duas lineas. i. u : i. n. et. i. e. quę
i. e. faciat angulum. o. i. e. æqualem angulo. o. i. n. Hinc clarè nobis patebit, si quis ipsi
e. pondus ipsius. u. (ꝗ æquale est ponderi. n.) appenderet, id eandem planè vim habe
ret, quam pondus ipsius. n. habet, & stateram neque sursum, neque deorsum moue-
ret, quia ambo pondera ad centrum. i. mediantibus lineis. e. i. et. n. i. ex æquo annite-
rentur , sed dicto pondere posito in. u: linea. u. i. per quam pondus centro annititur,
magis orizontalis quam. e. i. fit, & linea. u. s. inclinationis longius distans à centro. i.
quam linea. e. t. vnde huiusmodi pondus magis quoque liberum à centro. i. resultat.
magisque ponderosum, quam cum erat in. e. ratione eorum, quæ primo & secundo
capitibus diximus, & ob hanc causam superat pondus positum in. n . Sed si centrum
fuerit. in. o. imaginabimur duas lineas. o. s. et. o. x. & supponemus quòd pondera po-
sita sint in. s . et . x. vnde existente magis orizontali linea. o. s. quam erit. o. x. & linea
u. s. inclinationis longius distante à centro. o. quàm linea. e. t. eius pondus erit quoq;



grauius
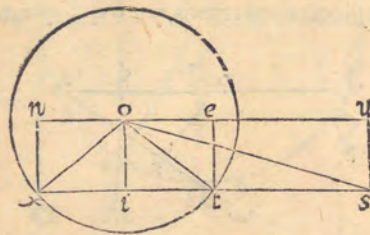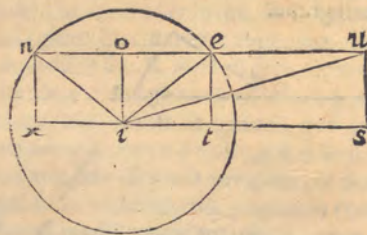
grauius, quia tantò minus pendebit à centro. o. & ratiocinando, vt superius diximus, inueniemus eundem effectum verum esse. In stateris, rectè & propriè appellari potest. x. i. s. aut. n. o. u. orizontalis, sed in omni vectium specie, hoc tátum per quandam similitudinem dicetur. Idem contemplari licet supponendo centrum in medio inter. o. et. i. qucd vnusquisque ex se absque alterius auxilio facile præstare poterit.

## De quibusdam rebus animaduersione dignis.

### CAP. V.

NOn omittenda mihi vidétur quædam, quæ ad tractatione vectium admodum sunt necessaria. Quod autem quærimus, in eo consistit, quòd aliqui vectes adhibeantur ad opus, quorum centrum, quod Græci hypomochliô appellant vnum est ex extremis ipsius vectis, & pondus, quod sursum eleuari debet, inter ipsamet extrema iacet, propinquum tamen hypomochlio, vt exempli gratia, si vectis esset infrascripta figura. o. s. u. x. cuius hypomochlion esset in puncto. o. & pondus in puncto. n. clarum erit, ꝗ cum eleuari debeat. n. oportebit quoque opera manus eleuari. u. Nunc considerandum est quomodo pondus. n. annitatur ad. u. Hanc ob causam imaginabimur rectas lineas. n. o: n. i: n. e: n. t. et. n. u. quarum. n. i. versus mundi centrum sit posita, et. n. t. faciat angulum. i. n. t. æqualem angulo. i. n. o. Nunc ponendo aliquam virtutem in. i. æquali inclinatione ad superius constante, vt. n. ad inferius (remota tamen grauitate materiæ vectis) huiusmodi virtus, totum pondus ipsius. n. communi quadam scientiæ notione sustinebit. & si pódus ipsius. n. esset in. x. è directo super. o. totum pondus super hypomochlio se haberet, & tanta virtus ipsius hypomochlij sufficeret ad resistendum pro sustinendo, quanta est grauitas ipsius ponderis, sed ipsum iterum ponamus in. n. ibi clarum erit, quòd si alia virtus à parte inferiori ad superiorem vectis non opponitur, excepto tamen hypomochlio, oportebit virtute cuiusdam partis ponderis. n. (absque consideratione tamen, vt iam dixi, ponderis materiæ vectis) vt vectis à parte. s. u. deprimatur, & dixi vnius cuiusdam partis ponderis. n. quia alia eiusdé ponderis pars annititur ipsi hypomochlio. o. mediáte linea o. n. quæ angulos rectos cum. o. x. non facit. Si autem à puncto. t. opponet sese huiusmodi resistentia, vt vectis non deprimatur, clarum erit communi scientia, ꝗ virtus ponderis. n. diuisa erit per medium æqualiter, cuius vna medietas super. o. quiescet, & alia super. t. mediantibus duabus lineis. n. o. et. n. t. Imaginemur nunc resistentiam t. ablatam esse, positamꝗ in. e. clarum quoque erit, ꝗ maior pars ponderis. n. ipsi. e. annitetur beneficio lineæ. n. e. quàm ipsi. o. cum linea. n. i. inclinationis ipsi. e. sit propinquior quam. o. quia omnis resistentia aut in. i. aut in. e. aut in. t. aut in. u. est loco centri, quemadmodum est. o. & alter alterius opera iuuatur. Si verò eadem resisten tia posita erit in. u. clarum quoque erit, ꝗ minor pars ponderis. n. annitetur ipsi. u. quá ipsi. o. cum dicta. n. i. à centro. u. longius quam à centro. o. distet, & proportio partis ponderis. n. in. o. ad proportionem partis ponderis. n. in u. non erit secúdum proportionem angulorum. u. n. i. et o. n. i. sed secundum proportionem. u. i. ad. i. o. quod claré compræhendi potest ab
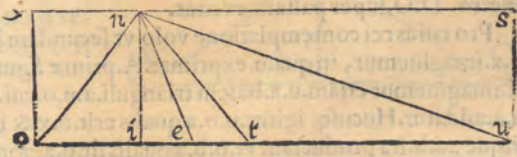


T huius

huius effectus conuerſo, ideſt, vt quemadmodum nunc ſupponuntur.o.et.u.eſſe duo
centra quibus ſuſtineť pondus.e. ipſius.n. imaginemur. n. eſſe quoddam centrum à
quo pendeant duo pondera. o. et. u.ſic inuicem proportionata, ut ſunt.u.i.et.i. o.
certe horum ponderum cauſa ſtatera. o.s.quam vectem appellabamus à nulla parte
inclinabitur. Redeuntes nunc ad propoſitum, dicemus ꝗ annitente pondere ipſius.
n.minus ad.u.quam ad.o.ideſt ad.t.minori vi opus erit in. u. quàm in.t. ad attollen-
dum pondus ipſius.n. & ſic per conſequens quantò longius erit punctum.u. ab.t.tan
tò minori quoque vi egebit , & conſequenter quando vis, aut reſiſtentia in.u.ita pro
portionata erit illi, quæ eſt ipſius.o.vt eſt.o.i.ad.i.u.vectis non mouebitur . Sed quan
do erit proportio maior, reſiſtentiæ ipſius.u.ad eam , quæ eſt ipſius . o.ea, quæ eſt.o.

i. ad. i. u. tunc vectis à par-
te ipſius.u.s . eleuabitur , ſi
vero proportio minor eſſet
quàm.o.i. ad. i. u. tunc ve-
ctis ab eadem parte depri-
metur.
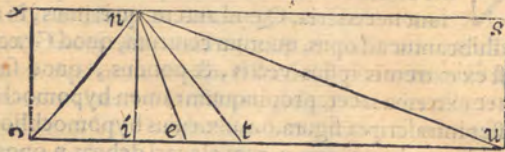


## De ratione cuiuſdam uis adaucta.

### CAP. VI.

QVibuſdam in locis vtuntur quidã quodã inſtruméto piſtorio ad ſubigédã pa-
ſtam, vnius tantum hominis ui adhibita, quæ quidem machina cum mihi di-
gna contemplatione eſſe videatur , eius aliquam rationem proponere volui, pro cu-
ius deſcriptione imaginemur planum, in quo ſedet ille, qui voluit paſtam , & in quo
ipſa paſta eſt repoſita.T. S. D.& triangulum.T. A. S. immobile perpendiculare-
que ſuperficiei dicti plani, angulo autem. A.coniunctum lignum. A.E.vt ſemidiame
trum mobilem, & æqualem perpendiculari ipſius trianguli, unde. A. loco centri erit
et. D. O.ſit ſemidiameter, qui paſtam contundit, & ab eius extremo.O. ( quod. O.
quando. D. O.orizontalis eſt, in baſi dicti trianguli reperitur ) veniat lignum. O. V.
quod cum. A.V.ſit æquale perpendiculari imaginatæ ab angulo. A.baſi.T.S. deno-
datũ tñ utvulgo diciť ſeu flexile in.O.& in.V.vt elleuare atꝗ; deprimere ſemidiame
trum. D.O.poſſit, et.V.O.ſit æqualis. A.V.et.V. medium ſit inter. A.et.E.vnde. A, V.
cum.O.V.æquales erunt. A.E.ſunt deinde duo ligna perpédicularia ab. A.ad baſim
fixa, & immobilia inter ſe adeò diſtantia, vt inter ipſa pertráſeát.O.V.et. D.O.ſupra
& infra , ne deuiet ſemidiametrum. D.O.In extremitate deinde ipſius.E.ſit lignum
quoddam tenue, vt digitus polex, ad angulos rectos cum. A.E.quod ab aliquo, qui
antedictam machinam ſtet , manibus teneatur, qui quidem homo idipſum lignum ,
ideſt ſemidiametrum. A.E.à ſuperficie trianguli dicti, ad ſe trahendo, & deinde ver
ſus eundem triangulum impellendo, vim quandam maximam mediante ſemidia
metro. D.O.ſuper paſtam excitat.

Pro cuius rei contemplatione volo vt ſecundam hanc ſubſcriptam figuram. b. a.
u.x.imaginemur , in qua.u. exprimat. A.primæ figuræ, & .a.denotet.O.& .o. V. & . x.
E.imaginemur etiam.u.a.baſem trianguli.a.u.o.cui.o.t.perpendicularis dictæ baſi.
u.a.addatur.Hucuſꝗ; igitur.u.o.æqualis erit.o.x.& ipſi.o.a.imaginemur etiam. a. o.
vſque ad.b.ita productam vt.o.b.æqualis ſit.o.a.ponamus etiam pondus in.a.impel-
lere

lere verſus.u.vnde linea eius inclinationis ſit ſemper.a.o.b.
eſſe librã, aut ſtateram, aut vectem, &. o. eius centrum, vnde vis, aut virtus ipſius.a.
proportionalis erit ipſi. o. t. reſpectu virtutis, aut vis imaginatæ in. b. inclinationis
perpendicularis ipſi.b.a.quæ quidem virtus, aut vis in.b. proportionalis erit ipſi. b.
o. ex tertio capite huius tractatus; Si ergo fuiſſet poſita in. b. virtus quædam ad an-
gulum rectum, trahens lineam.b.o.tam proportionatam virtuti perpendiculari ip-
ſius.a.quam eſt.o.t.proportionata ipſi.o.b.ſtatera.b.o.a.non moueretur, ſed quæuis
portio maior in.b.ſuperaret.a.cum autem fuerit.o.x.æqualis ipſi.o.b.idē planè eue-



niet, communi quadam ſcientia, ponen-
do virtutem.b.in.x.Quantitas ergo virtu
tis in.x. quæ ſuperare debet reſiſtentiam
in.a.quæ ipſi.u.contraponitur, debet ha-
bere aliquantulum maioris proportionis
ad reſiſtentiam, quæ in. a. angulum re-
ctum efficeret cum. a.o. ea, quæ eſt. o. t.
ad.o.x.

*De quibuſdam erroribus Nicolai Tartaleæ circa pondera*
*corporum & eorum motus , quorum aliqui deſumpti*
*fuerunt à Jordano ſcriptore quodam antiquo.*
C A P.    V I I.

Vm magis amici veritatis eſſe debeamus quàm cuiuſquam hominis,quemad-
modum Ariſto.ſcribit,detegam hoc loco quoſdam errores Nicolai Tartaleę
de ponderibus corporum,& velocitatibus motuum localium. Et primum decipitur
is in. 8. lib.ſuarum diuerſarum inuentionum in ſecunda propoſitione,cum non ani-
maduerterit quanti momenti ſint extrinſecæ reſiſtentiæ.

Subiectum quoque tertiæ propoſitionis eſt malè demonſtratum , quia idem pla-
nè ex eius demonſtratione iam dicta corporibus hætereogeneis , aut figura diuerſis
contingeret,quod ad velocitates attinet.

In quarta propoſitione,quod ad diſputādū proponit nō concludit melius.autē id
ab eo ſequiꝶ, quod Archimedes in.6.propoſitione lib. primi de pōderibus,ꝓbauit.

Sed in ſecunda parte quintę propoſitionis non uidet ꝙ uigore ſitus eo modo,quo
ipſe diſputat, nulla elicitur ponderis differentia. quia ſi corpus. B.deſcendere debet
per arcum.i.l.corpus.A.aſcendere debet per arcum.u.s.æqualem,& ſimilem.eadem
quoque ratione ſituatum,vt eſt arcus.i.l. vnde vt eſt facilè corpori. B. deſcendere
per arcum.i.l.difficile ita erit corpori.A.aſcendere per arcum.u.s. Hęc autem qnin
ta propoſitio Tartaleæ eſt ſecuuda quæſtio à Iordano propoſita.

Quòd autem ad primum corollarium dictæ propoſitionis attinet,verum ille qui
dem ſcribit,eius tamen effectus cauſa & à Iordano prius,& ab ipſo poſtea citata,na-
tura ſua vera non eſt.quia vera cauſa per ſe ab eo oritur,ꝙ à centro libræ dependeat
vt primo cap.huius tractatus oſtendi. Secundum verò corollarium falſum eſſe, ijs ra
tionibus quas nunc ſubiungam,patebit . Imaginemur. u. pro centro regionis ele-
mentaris,& libram.b.o.a.obliquam reſpectu ad.u. & brachiis æqualibus conſtātem,
& pondera in.a.et in.b.etiam æqualia.lineæ autem inclinationum ſint. a. u. et. b.u.
imaginemur etiam lineam.o.u. & à centro.o.libræ duas.o.t.et.o. e. perpendiculares
inclinationum lineis; vnde pondus ipſius.a.in huiuſmodi ſitu tam erit proportiona
tum ponderi.b.quam proportionata erit linea.o.t.lineæ.o.e.ex eo ꝙ tertio cap. hu-
ius tractatus probaui, ſed linea.o.t.maior eſt linea.o.e.quod ſic probo.Imaginemur
triangulum.u.a.b. circunſcriptum eſſe à circulo.u.a.n.b.cuius.c. ſit centrum , ꝙ erit
extra lineam.u.o.cum ſupponatur.a.o.b.obliquam eſſe reſpectu ad. u. o . Imagine-
mur deinde à centro.c.lineam.c.o.s.vſque ad circunferentiam , quæ perpendicula-
ris erit ipſi. a. b. ex tertia lib.3. Eucli.ſi poſteà imaginemur duas lineas.c.a.et.c.b. ha
bebimus ex.8.lib.primi,angulum.a.c.o.æqualem angulo.b.c.o.Vnde ex. 25.lib. 3.
arcus.a.s.æqualis erit arcui.b.s.ſed ſi imaginabimur. u. o. ad circunferentiam vſque
productam, clarum erit ꝙ arcum.s.b.ſecaret in puncto. n. vnde arcus. n.b.minor erit
arcu.n.a.& ſic etiam angulus.n.u.b.minor erit angulo.n.u.a.ex ultima lib. 6. Imagi-
nemur nunc alium quendam circulum,cuius.o.u.ſit diameter , cuius circunferentia
per duo puncta.e.ct.t. prætergradiaꝶ,cum in ipſis ſint anguli recti, quòd quilibet ex
ſe ratiocinando colligere poteſt,ſi.30.lib.3.in mentem reuocauerit.Sed cum angu-
lus.o.u.t.ſit maior angulo.o.u.e.arcus.o.t.maior erit arcu.o.e. ex vltima.6.vnde cor
da.o.t.maior erit corda ipſius.o.e.ex conuerſo.27.lib.3.quod eſt propoſitum. Pon-
dus igitur ipſius.a.in huiuſmodi ſitu,pondere ipſius.b.grauius erit.Quod è directo ijs
repugnat quæ Tartalea in 2.parte quinræ propoſitionis ediſerit , & per conſequens
2.corollarij falſitatem oſtendit,vt eam quoqne,quæ in 6. propoſitione latet.quia cū

                                                                        pro-

proportio póderis. a. ad pon
dus ipſius. b. eadem ſit cum
ea quę eſt. o. t. ad. o. e. ſub co
gnitioné noſtram cadere po
teſt, primum cognoſcendo
angulos obliquitatis librę,
ideſt angulos. b. θ. u. et. a. o.
u. quia oportet ſemper ſup-
ponere ſitum aliquem no-
tum. Si nobis deinde co-
gnita erit proportio ipſius.
o. u. ad. o. b. et. ad. o. a. aſſe-
quemur cognitionem angu
li. b. et. o. a. u. & per conſe-
quens ipſius. o. a. t. eius reſi-
dui, vnde poſtea beneficio
angulorum. e. et. t. rectorum
& laterum. o. b. et. o. a. cogni
torum in cognitionem. o. t.
et. o. e. facile deueniemus.
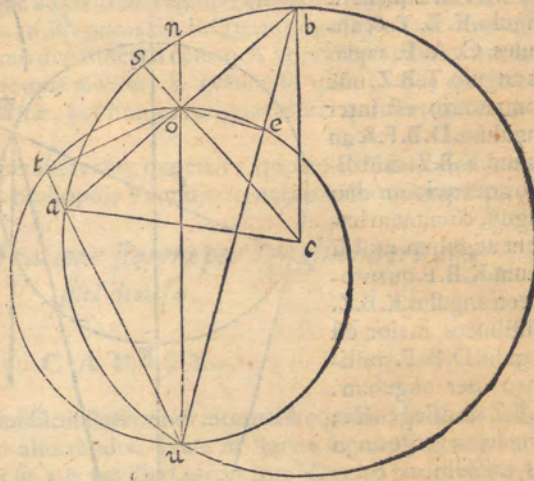
## CAP. VIII.

Q Vod autem idem Tartalea in. 6. propoſitione, & Iordanus in ſecunda parte
ſecundæ propoſitionis ſcribunt, maximum quoque errorem in ſe continet.

Dicunt enim angulũ
h. a. f. differentem ab
angulo. d. b. f. alia ra-
tione non eſſe quàm
per angulum conta-
ctus duorũ circulorũ,
vt in ſua figura ſcribit
Tartalea; id quod fal-
ſiſſimum eſt. Quã ob
cauſam in ſubſcripta
figura ſit libra. B. A.
& eius centrum. C. et.
u. centrũ regionis ele
mentaris, et. A. u. et. B.
u. lineæ inclinationũ.
Imaginemur deinde
lineam. B. K. parallelã
ipſi. A. u. quæ gyrum.
B. F. A. in puncto. K.
communi ſcientiæ prę
cepto ſcindet, & habe
bimus angulum. K. B.
Z. æqualem angulo.
H. A. F. ideſt. u. A. F.
(quia. H. u. et. D. unũ
ſunt) cum ex. 29. libr.
primi Euclidis angu-
lus.
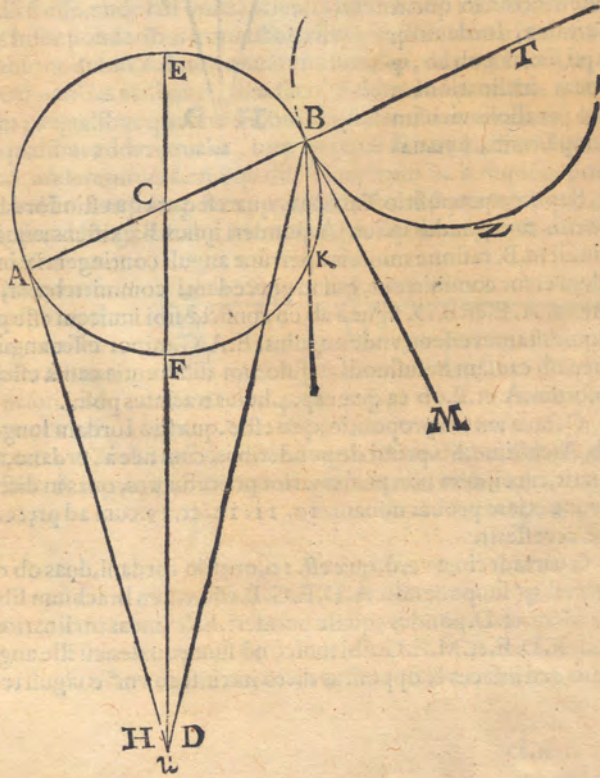
lus u. A. C. æqualis fit
angulo. K. B. T. & an-
gulus. C. A. F. æqua-
lis angulo. T. B. Z. nūc
comparatio eft inter
angulum. D. B. F. & an
gulum. K. B. Z. miftili-
neos, qui quidem duo
anguli, cómunem ha-
bent angulum miftili
neum. K. B. F. quapro-
pter fi angulus. K. B. Z.
miftilineus maior eft
angulo. D. B. F. mifti-
lineo per angulum.
K. B. Z. contingentiæ,
circulorum ergo angu
lus miftilineus com-
munis. K. B. F. æqualis
erit miftilineo, angu-
lo. D. B. F. pars vide-
licet fui toto. Omnis
autem error in quem
Tartalea, Iordanusq;
lapfi fuerunt ab eo, φ
lineas inclinationum
pro parallelis viciffim
fumpferunt, emana-
uit.

Septima propofitio Tartaleæ, quæ eft qnta quæftio Iordani mihi videt excipien-
da rifu, cum pondus ipfius. A. ponderi ipfius. B. exiftens æquale, grauius fit pondere
eiufdem. B. ratione minoris aperturæ anguli contingentiæ in. A. quam in. B. in quo
idem error committitur, qui in præcedenti committebatur, cum fcilicet ipfe putet
lineas. A. E. et. B. D. figuræ ab eo confictæ fibi inuicem effe parallelas, quæ etiam fi
æquidiftantes effent (vnde angulus. E. A. G. minor effet angulo. D. B. F.) non eam ta
men ob caufam huiufmodi angulorum differentia caufa effet differentiæ grauitatū
ipforum. A. et. B. ob ea quæ cap. 4. huius tractatus pofui.

Octaua autem propofitio, quæ eft. 6. quæftio Iordani longè melius demonftratur
ab Archi. in. 6. lib. primi de ponderibus, cum nec à Iordano, nec à Tartalæa probata
fuerit, cum ijdem non probauerint præcedentes, quas in dicta. 8. Tartalęa citat, qui
neque etiam probat nonam. 10. 11. 12. et. 13. cum ad pręcedentes probandas mini
mè accefferit.

Quartadecima verò, quæ eft. 10. quęftio Iordani, duas ob caufas eft falfa, quarum
vna eft, φ (fupponendo. A. D. E. G. B. effe vnum brachium librę, et. A. punctum cētri
eiufdem, et. D. pondus equale ponderi. E. & lineas inclinationum. D. K. et. E. M.) an
guli. K. D. E. et. M. E. G. fibi inuicè nó funt equales; cū ille angulus fit intrinfecus, hic
verò extrinfecus & oppofitus dicto intrinfeco vni⁹ triãguli terminati à. D. E. à. D. K.
et. E.
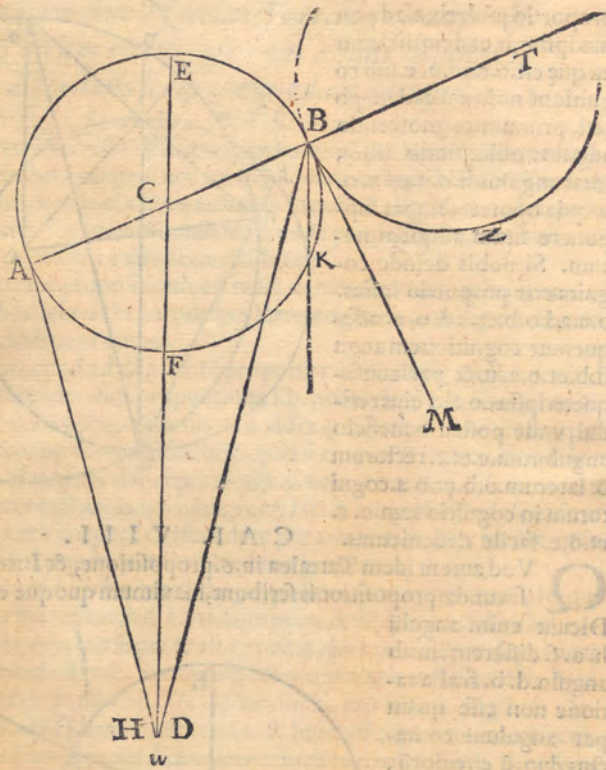
et. E. M. lineis productis vſque ad centrum regionis elementaris, vnde dictus angu-
lus. M. E. G. maior eſt alio, ex. 16. lib. primi Eucli. Qua ratione fit, vt hanc ob cauſam
E. grauius ſit ipſo. D. cum minus dependeat à centro. A. vt primo cap. huius tractatus
iam dixi. Alia quoque eſt ratio, qua dictum . E. grauius ſit ipſo. D. quę quidem eſt
maior diſtantia à centro. A. libræ , per ſimiles rationes capit. 4. huius tractatus ci-
tatas.

Decimaquinta quoq; nil penitus valet, quę eſt. 11. quęſtio Iordani, cuius Autho-
ris opuſculum opera Traiani Bibliopolę Venetijs è tenebris in lucem emerſit.

*Quòd ſumma ratione ſtaterę per æqualia interualla*
*ſint diuiſa .*

## CAP. IX.

Magna cum ratione diuidūtur ſtaterę per interualla ęqualia , in libras , aut in
vncias , aut quoquo alio modo . Nam ſit ſtatera exempli gratia . a. b.
& punctum, ꝙ eam ſuſtinet ſit. c. & vas illud, ꝙ continet id, quod ponderari debet
f. Imaginemur nunc quod pondus brachij. c. b. ab una parte, & pondus brachij. c. a. cū
eo, ꝙ eſt dicti vaſis. f. ab altera parte, ſint cauſę, quibus ſtatera. a. b. c. ſtet orizonta-
lis. cui ſic orizontali manenti imaginemur ad punctum. a. adiunctum eſſe pondus,
veluti vnius librę. & ad punctum. d. tam diſtanti à. c. ut eſt. a. ab ipſo. c. aliud quoque
pondus vnius libræ additū eſſe, vnde cōi quadā ſcientia ſtatera, non mouebitur ſitu.
qa exiſtentibus duobus hiſce ponderibus æqualibus, altero in. d. & altero in. a. remo-
ta cum eſſent. d. b. et. f. abſque dubio. a. d. non mutaret ſitum, ſed. d. b. et, f. in ſitu, in
quo reperiuntur, à centro paribus viribus pręditta ſunt. Addendo igitur. d. b. ipſi. d.
et. f. ipſi. a: ſumma earum, æqualibus quoque viribus conſtabunt . ex communi ſen-
tentia, quæ habet ſi ęqualibus addas ęqualia, tota quoque fient ęqualia. Si verò
ponderi ipſius. a. aliud adderetur eidem ęquale, haberemus in. a. duplum pon-
dus ei ꝙ eſt ipſius. d. ſed volentes vt ſolum cum pondere ipſius. d. ſtatera ſtet orizon-
talis, ſi dictum pondus ipſius. d. longè diſtabit à centro. c. per duplum ipſius. c. a. ideſt

ipſius. c. d. id ꝙ volumus aſſeque-
mur, beneficio ſupradictarum ra
tionum, adiuti opera ſextę lib. pri
mi de pōderibus Archimedis. Et
ſi quis aliud quoq; pondus adiun
geret ipſi. a. æquale illi priori, ad
efficiēdum, vt ſtatera ſemper ori



zontalis maneret, oporteret , vt pōdus ipſius. d. ab. c. longè diſtaret, ita vt huiuſmodi
diſtantia tripla eſſet primæ, & ſic per quoſdam quaſi gradus interualla redderentur
æqualia .                                                                   Quòd
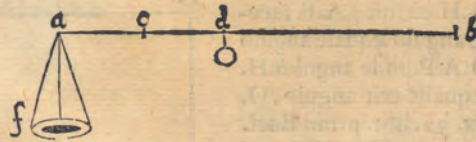
## *Quòd linea circularis non habeat concauum cum con-*
## *uexo coniunctum, & quod Aristo. circa proportio*
## *nes motuum aberrauerit.*

### CAP. X.

ARiftoteles in principio quæftionum Mechanicarum ait lineam, quæ terminat circulum videtur conuexum habere coniunctum cum concauo, quod falfum eft: quia huiufmodi linea partes nullas fecundum latitudinem habet, ( vt ipfe etiam confirmat ) fed eft idem conuexum circuli: linea verò quæ terminus eft fuperficiei ambientis , & ampletentis circulum eft eadem concauitas dictæ fuperficiei eundem circulum ambientis, quæ nullam conuexitatem habet. & hæ duæ funt lineæ , quarum vna diuerfa eft ab alia, neque altera alterius, quod ad conuexum, & ad concauum attinet.

Sed illud, quod Ariftoteles fcribit de duplici refpectu motus vnius puncti fecundum vnam datam proportionem, non fufficit, ille enim fic ait.

Sit proportio fecundum quam latum fertur, quam habet. A.B. ad. A. C. et. A. qui dem feratur verfus.B: A.B. verò fubterferatur verfus. M.C. latum autem fit. A. quidê ad. D. vbi autem eft. A. B. verfus. E. Quoniam igitur lationis erat proportio, quam. A.B. habet ad. A. C. neceffe eft &. A.D. ad.A.E. hanc habere rationem. Simile igitur eft proportione paruum quadr ilaterum maiori. Quamobrem etc.

Cui refpondeo, punctum. A. quod mouetur in linea. A.M. ab. A. verfus. M. vfque ad.F. non moueri ab aliqua proportione determinata magis quàm ab alia: vnde nó folum poffumus imaginari dictum punctum. A. moueri ab. A. vfque ad. F. eiufdem velocitatis fub alia quadam proportione, fed etiam fub alia, quæ iam datæ contraria fit, vt eft proportio ipfius. A.C. ad. A.B. imaginâtes moueri. A. verfus. C. et. A. C. ver fus.B.M. delatam. Dico etiam idem. A. moueri vfque ad. F. fecundum proportionem ipfius. A.O. ad. A.N.Quamobrem imaginemur à puncto.F. lineam. F. H. cum

linea.F.A. efficere angu- lum æqualem angulo. O. P. A. & à puncto. A. lineâ A.H. cû linea. A. F. face- re angulû æqualê angulo O.A.P. unde angulus.H. æqualis erit angulo. O. ex. 32. libr. primi Eucl. & triangulû. A.H.F. ęqui angulum erit triangulo. A.O.P. Quam ob causâ eadê proportio erit ipfiʾ A.H.ad.F.H. quę ê ipfius A. O. ad. O.P. punctum



igitur. A. vfque ad. F. mouetur fecundum proportionem etiam ipfius. A.O. ad. O. P. Huiufmodi igitur confideratio, ab Ariftotele facta, nullius eft momenti.
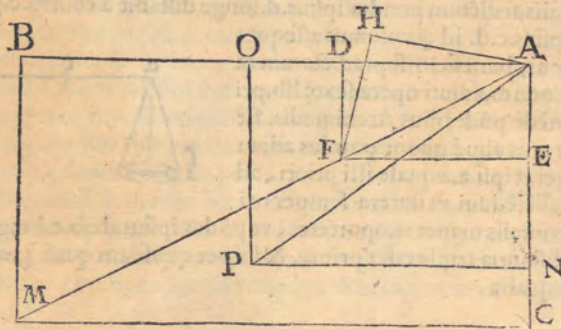
Quòd

## Quod Aristo. in prima mechanicarum quæstionum eius quod inquirit, ueram causam non attulerit.

### CAP. XI.

QVærens Aristoteles vnde fiat, vt eæ libræ, quæ brachia habent alijs longiora, sint exactiores cæteris, ait hoc euenire ratione maioris velocitatis extremorum earundem. Quod verum non est; quia hîc effectus nil aliud est, quam clarius proponere ob omnium oculos obliquitatem brachiorum à linea orizontali, & ostendere etiam facilius à dicto orizontali situ exire brachia iam dicta. Quæ quidem per se neque à velocitate, neque à tarditate motus, sed à ratione vectis, & à maiori interuallo inter secundum situm extremorum à primo proficiscuntur. Vt exempli gratia, imaginemur magnam libram. A. B. orizontalem, cuius centrum sit. E. et pondus. B. maius sit pondere ipsius. A. vnde conceditur, quòd ob hanc rationem dicta libra situm mutabit, qui secundus situs sit in. H. F. Imaginemur etiam paruã quãdam libram. a. e. b. orizontalem, quæ pondera habeat. a. et. b. æqualia duobus ponderibus alterius libræ & secundus situs sit in. h. f. ita tamen vt anguli circa. e. æquales sint ijs, qui sunt circa. E. idest. b. e. f. sit equalis. B. E. F. Nunc dico situm. H. F. exactiorè futurum & clariorem situ. h. e. f. ratione interualli. B. F. maioris, interuallo. b. f. quod. B. F. in eadem proportione maior est ipso. b. f. in qua. B. E. maius est. b. e. quod autem interuallum. B. F. breuiori, aut longiori temporis spacio quam. b. f. sit factum, nil planè refert. Ratione vectis deinde, dico φ si supponemus duas libras pares æqualesq́; in omni alio respectu, præter quàm in brachiorum longitudine, pondus. B. maiorem vim habebit ad deprimendum brachium. E. B. quàm pondus. b. quia libræ materiales, cum sustineantur ab. E. e. & non à puncto mathematico, sed à linea, aut superficie naturali in materia existente, vnde aliqua resistentia ipsi motui brachiorum oritur, & hanc ob causam, supponendo hanc resistentiam æqualem tam in. E. quàm in. e. clarum erit ob ea, quæ in cap. 4. huius tractatus ostendi. B. cum minus dependeat ab. E. aut minus quoque eidem. E. annitatur, ponderosum magis futurum, quam. b. & hac de causa mouebit ad partem inferiorem, maiori cum agilitate, brachium. E. B. multo magis etiam illud ipsum deprimet, idest maiorem etiam angulum. B. E. F. quàm erit angulus. b. e. f. faciet.



V        DE

## De uera causa secunda, & tertia quæstionis mechanicæ ab Aristotele non perspecta.

### CAP. XII.

ARistoteles in secunda quæstionum mechanicarum quærens illius rationem sic scribit.

Cur siquidem sursum fuerit spartum quando deorsum lato pondere quispiam id admouet rursus ascendit libra : si autem deorsum constitutum fuerit non ascendit, sed manet? an quia sursum quidem sparto existente plus libræ extra perpendiculum sit (spartum enim est perpendiculum) quare necesse est deorsum ferri id, quod plus est, quare & cætera.

Sed vera causa, vnde fiat, vt si spartum fuerit sursum, & brachium vnum ipsius libræ deprimendo, & idem liberum deinde permittendo, ad situm orizontalem redeat, non solum est maior quantitas ponderis brachiorum quæ iam præ tergressa est vltra verticalem lineam, sed etiam est longitudo brachij eleuati, quæ vltra verticalem lineam reperitur, vnde eius extremi pondus redditur grauius in proportione, quam in hoc exemplo proponam, sit. A. B. libra in situ orizontali, cuius spartum sit. E. super ipsam. & deprimentes brachium ipsius. A. vsque ad. F. eius situs sit in. F.H. vnde medium pŭctum. G. prætergressum erit lineam verticalem. V.Z. versus. B. quæ. V. Z. secabit brachium. F.G. in puncto. D. vnde. D. H. longius erit ipso.

F. D. Nunc nobis supponendum est id, quod verissimum existit, dictam scilicet libram in situ. F.H. etiã si sustineatur à puncto. E. idem tamen futurum ac si sustentaretur in puncto. D. vnde sequitur, quod pondus appensum ex ipsa. H. ita grauius reddatur, ipso . F. in eadem proportione, quæ maior est. D. H. ipso. D . F. ob rationes quas in primis huius tractatus capitibus posui, vt etiam si. D.H. quod materiale esse supponitur, nullam planè grauitatem haberet, solus tamé excessus vis ponderis in. H. positi, longè maior pondere in F. collocato pro maiori longitudine ipsius D. H. sufficiat. ad præstandum vt libra ad situm orizontalem redeat.

In secunda deinde huius quęstionis parte, in qua scribit libram in situ, in quo posita est, firmam manere, toto cęlo aberrat, quia necessariũ est, vt omninò cadat, eòusq; quò spartum sursum remaneat : ablato tamen omni impedimento, quod nulla eget probatione, cum natura sua clarissimè pateat.

Causa, deinde, vera tertiæ quæstionis non est ea, quam Aristoteles ponit, sed huiusmodi effectus ab eo, quod capitibus. 4. et. 5. huius tractatus proposui originem habet.

Quod

*Quòd Aristotelis ratio in 6. quæstione posita non sit admittenda.*

### CAP. XIII.

VOlens Aristoteles rationem proponere, vnde fiat, vt nauis velocius moueatur cum antennam altiorem quàm cum depræssiorem habet, id ad vectis rationem refert, quod verum nó est. Huiusmodi enim ratione nauis tardius potius, quàm velocius ferri deberet, quia quantò altius est velum, vi venti impulsum, tátò magis proram ipsius nauis in aquam demergit. Sed huiusmodi effectus à maiori potius quantitate venti quam recipit, quàm ab alia aliqua causa oritur, quia ventus liberius vehementiusq; in altiore parte, quàm in depræssione vagatur & perflat.

*Quòd rationes ab Aristotele de octaua quæstione conficta*
*sufficientes non sint.*

### CAP. XIIII.

RAtiones etiam ab Aristotele propositæ pro indaganda octauæ quæstionis veritate, in qua quærit vnde fiat, vt corpora rotundæ figuræ, ad voluendû sint faciliora reliquis, quarum reuolutionum corporum tres species assignat, quarú vna est, vt rotarum curruú; altera vt rotarum puteorum, aut trochlearum, quibus hauritur aqua; & tertia, vt paruorum vasorum a figulis fabricatorum, sufficiétes nó sunt.

Incipiens autem à prima dico dubium non esse, quin tangente corpore aliquo rotundo aliquod planum mediante solo quodam puncto contingat, quemadmodum probat Theodosius in. 3. lib. primi & Vitellio in. 7 1. lib. primi, & ducédo per centrû sphæræ lineam vsque ad punctum contactus, ipsa erit perpendicularis plano contingenti sphæram dictam, vt probat idé Theodosius in. 4. lib. primi Alhazé in. 2 5. quarti, & Vitellio in. 7. primi. Verum etiam est omnem inclinationem ponderosam huiuf modi corporis homogenei totam hanc lineam æqualiter omni ex parte circundare; cuius quidem rei exemplum in carta describere possumus mediante figura circulari hîc subscripta. a. n. e. u. contigua lineæ rectæ. b. d. in puncto. a. vnde. e. o. a. perpendicularis erit ipsi. b. d. ex. 1 7. lib. 3. Eucli. & tantú ponderis habebimus à parte. a. u. e. quantum ab ipsa. a. n. e. Nunc igitur si imaginabimur ductum esse centrum versus. u. per lineam. o. u. parallelam ipsi. a. d. clarum nobis erit, q absq; vlla difficultate aut resistentia idé ducemus, quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distátiæ seu interualli, quę inter ipsum lineamque. a. d. intercedit, q quidem centrum in se colligit totum pondus figurę. a. n. e. u. & beneficio lineæ. e. o. a. illud ipsum puncto. a. in linea. b. a. d. committit, productum. a. nil refert, vt magis, aut minus versus ipsum. d. aut versus b. dirigat; ita vt cú non oporteat vt huius figuræ pódus, vna vice, magis eleuetur, quàm alia, sed semper equaliter super lineam. b. a. d. quiescat.
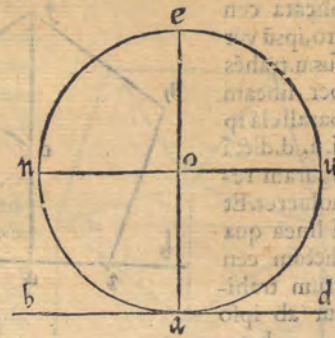


V 2      Sitq;

Sitq; femper diuifum à linea.a.o.e.per medium , fequitur communi quodam con-
ceptu , nullam nobis difficultatem oborituram, dictum centrum ad quam volueri-
mus partem ducendo , quemadmodum à qualibet alia figura, quæ perfectè rotunda
non effet,emergeret; Vt exêpli gratia, fi i maginabimur pentagonum.K.i. h.f.l.quie
fcere fup eande linea.a.b. K.ita ut primū totū latus. i.K.in linea.b.K.extêdaſ, ducê-
do poſteà centrum.o. (ponamus.) verfus.l.dubium non eſt, quin oporteat, vt dictum
centrum.o.à linea.b.d. eleuetur,ab eademq; magis diſtet, voluens fe per arcū vnum
circuli, q ꝓ fuo femidiametro habeat.o.K.quę maior eſt ipfa.o.a.ex.18. li. primi Eu
cli. vnde ſi a puncto.K.imaginabimur lineam. K. c. refpicientem centrum regionis
elementaris,dubium non eſt,quin ſi velimus transferre cêtrum hoc à priori fitu vfq;
ad dictam lineam,oporteat addere pondus parti ipſius.l.quæ à linea.K. c.fuit fecta ,
aut aliquid de ipfo pondere partis centri detrahere. quod quibufuis modis fiat, ar-
duum certè eſt ad efficiendum;neque hoc etiam accidit figuræ perfectè rotundæ ,
cum cêtrum ꝗ perfectè in medio ipſius ponderis eſt,reperiatur femper in línea per-
pendiculari ipſi plano, in quo animaduertendum eſt, ꝗ etiam ſi ipfum planum ap-
pellem;pro plano tamen perfecto intelligi nolo, fed pro fuperficie perfectè fphęri-
ca circa centrum à corporibus grauibus expetitum; nam ratione magnæ amplitudi-
nis huiufmodi fuperficiei,nullam differentiam notatu dignam à perfecto aliquo pla
no exigui interualli ad curuitatem eiufdem fuperficiei imaginari poterimus. Sed ut
redeamus ad fermonem de reuolutione figuræ rotundæ fufceptum , clarū igitur erit
quamlibet minimam vim(vt ita dicam)quę trahat,aut impellat centrum.o.verfus.u.
huiufmodi figuram reuoluturam,cuius media pars ad trahendum, aut impellendum
punctum.e.fufficiere; Imaginemur autem ꝗ li
nea.n.o. u. effet libra quędā in figura perfectè
rotunda.a.n.e.u.poſita,& vis, quę trahere cen
trum deberet, diuiſa effet per medium, cuius
medietas appenfa effet extremitati. u. diame-
tri.n.o.u.clarū erit , ꝗ abſque vlla difficultate
reuolueret figuram fuper lineam.b.a.d.verfus.
d. quia huius vis,aut pondus nullū contra pon
dus haberet vltra centrum.o.uerfus. n. ꝗ cen-
trum. o.perpetuo quiefcit fup. a.in linea. e. o.
a. per medium diuidente femper totum pon-
dus figurę fuppoſitę. Tantò facilius ergo tota
dicta vis ap
plicata cen
tro,ipfū ver
fus.u.trahês
per lineam
parallelā ip
fi. a. d.dictā
figuram re-
uolueret.Et
fi linea qua
dictum cen
trum trahi-
tur ab ipfo
b.a.
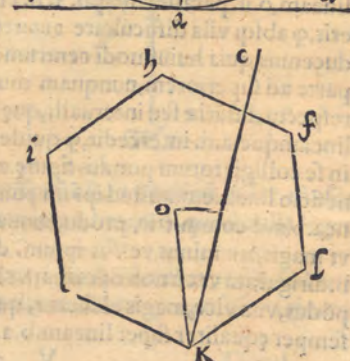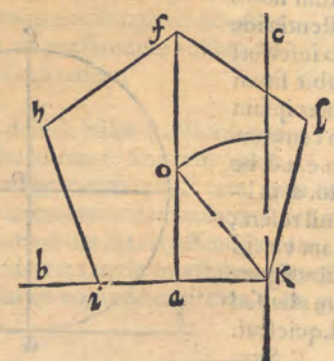
b.a.d. non æquediſtaret, ſed ſurſum traheret ſuper. u. aut ſubter, aliquid de ſua vi vir
tuteq́; amitteret, & tantò plus, quantò inclinata magis eſſet verſus. a.o.e. & tandem
cum eſſet vnita cum. a.o.e. aut ad ſuperius, aut ad inferius quantalibet ui, etiam ſi in-
finita, figuram extra ſitum primæ lineæ. a.o.e. non moueret, ſed ſi ſurſum traheret ſe
iungeret eam à linea. b.a.d. non ob id tamen efficeret, ut centrum. o. exiret extra pri
mam lineam. a.o.e.

Secunda verò ſpecies, tribus reuolutionum modis, abſque axis mutatione conſta
re poteſt, ideſt modo, quo reuoluuntur trochleæ mediante fune, & quo reuoluuntur
aliquæ rotæ, in quibus aliquod animal incedit; & quo reuoluuntur illæ, quæ in homi
nis manu circunuoluuntur medio alicuius manubrij inflexi. Hi omnes modi cum
circulari figura magis, quâ cum alia quauis, faciliores euadunt. Et primò ſi priorem
modum conſiderabimus, vt mediante fune quælibet figura, quæ circularis non ſit,
voluatur, ſupponamus exemplo debere reuolui pentagonum æquiangulum. a.e.i.o.
u. circa centrum. c. mediante fune. q.u.a.e.i.p. neceſſariò occurrent (in hac figura an-
gulorum, laterumq́; diſparium) plures inæqualitates, quæ reuolutionem eiuſdem fi-
guræ irregularem efficient; quarum vna erit, quod duæ partes funis, ideſt. u.q. et.i.p.
non erunt in vna eademq́; inter ſe diſtantia ſemper, quod facile intellectu erit, ſi ima
ginabimur ductas eſſe lineas. a.i: u. i: et. i. c. t. ſi funis duo pondera habebit alterum
altero maius, ſuis extremis appenſa, vnde debeat figura virtute ponderis maioris cir
cunuolui: dictæ duæ partes. u.q. et.i.p. eiuſdem funis, mûdi centrum, dum firmæ ma
nebunt, reſpicient; ſed permittentes pondera libera; maius, efficiens vt circunuolua-
tur figura; efficiet, vt aliquando vnum ex lateribus, eiuſdem figuræ mundi quoq; cen

trum reſpiciet, vt in
figura. A. ſicq́; etiam
linea. i. c.t. (pro exé-
plo) erit menſura di-
ſtantiæ funium inter
ipſas, & deinde circû
uoluendo etiam di-
ſtabunt inter ſe per li
neâ. i.a. aut. i. u. vt in
figura. B. ĩnotuit exé
plo, & ſic etiam ali-
quando erunt magis
diſtâtes, quàm linea
t. i. & minus quàm. i.



a: nunquam tamen minus quam. t.i. neque magis quâ
i.a. aut. i.u. quæ ſunt æquales; Quæ quidem varietas,
in hanc, & in illam partem impellet partes penden-
tes funis, vnde æqualiter non trahent. Idem dico, ſi
extrema. q. et. p. eſſent quoque ſemper in vna eadéq́;
diſtantia; neque à corpore póderoſo eſſent attracta,
quia aliæ partes ipſius. u.q. et.i.p. ex ſupradictis ratio-
nibus vnam eademq́; diſtantiam nó ſemper ſeruarêt.
vnde fieret vt cum diuerſis angulis tam. i. p. quâ. u.q.
traherét ſemidiametros. c. i: c. e: c. a: c. u. et. c. o. quia nó
ſemper traherent ope ſeu virtute anguli æqualis ipſi.
c. i. p. Hæc autem inæqualitas communis eſt omnibus
figu-

figuris rectilineis tam paris, quàm disparis numeri. Sed aliam quandam maiorem inæqualitatem habent hæ figuræ numeri disparis, quæ est, quòd quãdo linea. t.i. tam .u .q .quàm ipsi. i.p.
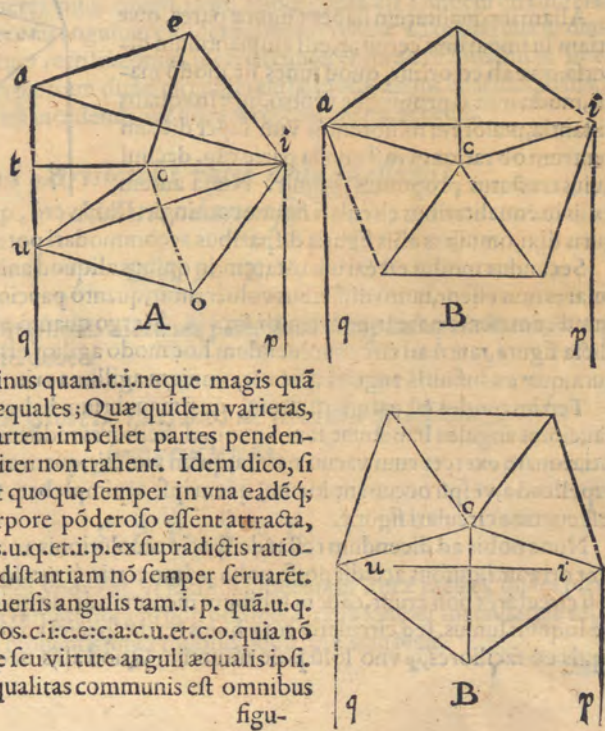
ppēdicularis fuerit, idest quãdo.t.i. cum dictis partibus funis angulos rectos constituerit, tūc ratione lõgitudinis ipsius . c. i. maioris quam . t. c.(quia cum sit.c.i.ę- qualis ipsi.c.a.et.c.a. maior ipsa. c. t : c. i. etiam maior sit ipsa. c.t.) pondus aut vis ipsius.p.superabit eã quæ est ipsius. q. sed quando.t. erit in opposita parte, et.i. in ea, quæ est ipsius.t:q. eãdem ob causam superabit.p.& sic motum faciet irregularem,& nõ vniformem; & ob id etiam perarduum, præterictus, quos infligunt anguli in partem pendentem ascendētem funis, quãdo vnum ex lateribus vnitur cum fune.

Aliam inæqualitatem habent figuræ pares, quæ etiam in imparibus cernitur, etsi aliquantulum diuersa; quæ ab eo oritur, quod funes sit modò magis, modo minus propinquę centro; quæ inæqualis distantia, maiorem minoremq; vim super dictum centrum ob rationes in secunda parte cap. decimi huius tractatus propositas, gignit . Nulla autem ex ijs inæqualitatibus circulari figuræ contingit. Illud verò , quod de pentagonis figuris dixi, omnibus aliis figuris disparibus accommodari potest.

Secundus modus est earum rotarum, in quibus aliquod animal incedit, quæ si circulares non essent, tantò difficilius voluerentur, quantò pauciores angulos haberent. quod cum per se pateat, non demonstrabo. Si ergo quantò plures angulos habebit dicta figura, tantò ad circunuoluendum hoc modo agilior erit. Circularis igitur figura, quæ ex infinitis angulis efficitur, omnium agillima erit.

Tertius modus est earum rotarum, quæ manubrium habent, quæ etiam quantò pauciores angulos habebunt, tantò quoq; difficiliores reddentur, tam ratione inimi citiæ: quam exercet cum vacuo natura, quàm violétię, quam anguli aëri faciunt, eum expellendo, vt ipsi occupent locum, quem ipse aër implebat . Quod nullo modo potest euenire circulari figuræ.

Nunc nobis ad dicendum restat de specie reuolutionis rotarum , quæ parallelæ sunt orizonti, quibus accidit posse volui primo tertioq; modo secundę speciei, & ob id si circulares non erunt, eadem subibunt incommoda, de quibus in secunda illa specie loquuti sumus. sed circulares rotæ huius tertiæ speciei ad reuoluendum erunt reliquis eò faciliores, q̃ vno solũ polo nituntur ; Quod alijs nequaquam conceditur.

Super

## DE MECHAN. 159.
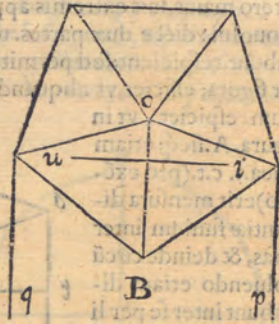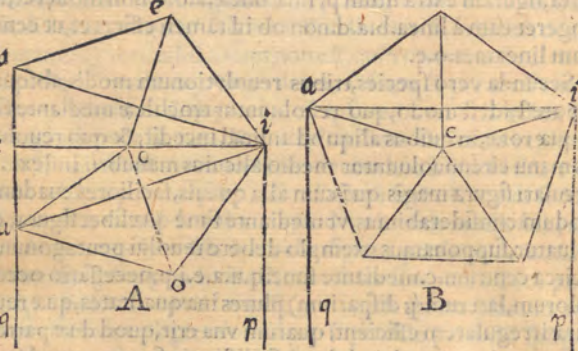
Super hac tertia specie formari potest problema , vnde fiat , vt quiescens huiusmodi rota parallela orizonti super vnum punctum, & quantò fieri potest existens ęqualis, si eam circunuoluamus maiore qua poterimus ui, & eãdem postea dimittentes non perpetuò circunuoluatur .

Hoc quidem, quatuor fit ob causas. quarum prima est, quia huiusmodi motus, eius rotæ non sit naturalis. secunda est, quia etiamsi rota super punctum mathematicum quiesceret, oporteret tamen vt superius alterũ haberet polum, qui ipsam orizontalẽ teneret , qui quidem munimento aliquo corporeo indigeret; vnde fricatio quędam consequeretur, ex qua resistentia prodiret.

Tertia est, quia aër contiguus eam perpetuò astringit, hocq; modo eius motui resistit .

Quarta est, quia quęlibet pars corporea, quę à se mouetur, impetu eidem à qualibet extrinseca virtute mouente impresso, habet naturalem inclinationem ad rectum iter, non autem curuum, vnde si à dicta rota particula aliqua suę circunferentiæ disiũ geretur, absque dubio per aliquod temporis spatium pars separata recto itinere ferretur per aërem , vt exemplo à fundis, quibus iaciuntur lapides, sumpto, cognosce re possumus , in quibus, impetus motus impressus naturali quædam propensione rectum iter peragit, cum euibratus lapis, per lineam rectam contiguam giro , quem primo faciebat, in puncto, in quo dimissus fuit, rectum iter instituat , vt rationi consentaneum est.

Eadem, quoque ratione fit, vt quantò maior est aliqua rota, tantò maiorem quo que impetum, & impressionem motus eius circunferentiæ partes recipiant , vnde sę pe euenit, vt dum eam sistere volumus, id cũ labore & cum difficultate agamus ; quia quantò maior est diameter vnius circuli, tantò minus curua est eiusdem circunferen tia, & tantò propius accedit angulum eiusdem circunferentiæ ad quantitatem duorum angulorum rectorum rectilineorum, idest circunferentia ad rectitudinem linea rem.  Vnde earundem partium dictæ circunferentiæ motus ad inclinationem sibi à natura tributam, quæ est incedendi per lineam rectam, magis accedit.

### Quod Aristotelis ratio nonæ quæstionis admittenda non sit.

### CAP. XV.

VEra ratio nonæ quęstionis à secunda parte decimi cap. huius tractatus , & non aliunde, accersiri debet.

### Quod Aristotelis rationes de decima quæstione sint reijciendæ.

### CAP. XVI.

ARistotelis rationes, vnde fiat, vt facilius moueantur libræ vacuæ, quàm plenè ad propositam disputationem non pertinent; quia semper ineunda est ratio proportionis virtutis mouentis super mobile; quod ipse non fecit.

Sit

Sit exempli gratia libra. a. i. e. quæ in vtraque extremitate vnciam vnam folum
ponderis obtineat, & fit libra.n.i.u.æqualis priori,quæ pro fingula extremitate vnã
ponderis libram habeat. Ariftoteles admiratur, quòd addendo ipfi.e.mediam pon
deris vnciam,brachium.i.e.velocius cadat, quàm adijciédo ipsã mediã vnciã ipfi. u.
brachij.i.u. Quod à duabus caufis proficifcitur, quarum prior eft,magna differentia
proportionis vnius libræ ad medietatem vnius vnciæ,ad proportionem vnius vnciæ
ad ipfam medietatem, quia fi pondus adiectum extremo.u.dimidiæ effet libræ , &
cum eadem tarditate brachium moueret,optimo iure in admirationem poffet Ari -
ftoteles duci.  Sed hoc fieri non poffet,quia ipfum deprimeret cum eadem quafi ve
locitate,qua media vncia brachium.i.e. Dixi autem quafi , quia nonnihil difcrimi-
nis intercederet,quod proficifcitur à fecunda ratione . Et hæc, refiftentia eft , quæ
oritur à fparto,  quia quantò maius pondus continet libra,tantò magis præmit fpar
tum in loco,in quo fuftinetur;vnde maior refiftentia in circunuolutione eiufdé fpar
ri,in loco,in quo quiefcit,exoritur,quia ipfum eft corpus materiale. Si quis autem
vellet,vt brachium.i.u.eadem agilitate,qua.i.e. defcenderet,oporteret, vt propor-
tio dimidiæ libre adiectæ ponderi ipfius.u.
quod eft vnius libræ, vim fuam haberet ,
quæ excederet refiftentiam fui fparti ( me-
dio brachiorum maiorum ijs qui funt. a. i.
e. ) ita proportionatam , vt proportionata
eft vis dimidiæ vnciæ ipfi e.iunctæ,refiften
tiæ fui fparti. Huiufmodi rationes cum ro-
tis grauioribus leuioribusqʒ,& ijs,que à cor
poribus quibuflibet grauibus impelluntur,accommodatæ fuerint,titubantem intel
lectum confirmabunt.
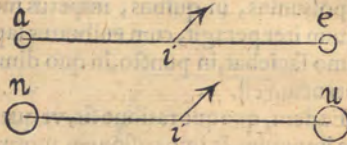
## De uera cauſa.12.quæſtionis mechanicæ.

### CAP. XVII.

VEra ratio , cur multò longius corpus aliquod graue impellatur funda , quam
manu,inde oritur, quòd circunuoluendo fundam , maior impræffio impetus
motus fit in corpore graui, quàm fieret manu, quod corpus liberatum deinde cum
fuerit à funda, natura duce,iter fuũ à puncto,à quo profilijt, per lineam contiguam
giro,quem poftremò faciebat,fufcipit. Dubitandumqʒ non eft , quin dicta funda
maior impetus motus dicto corpori imprimi poffit,cũ ex multis circumactibus,ma-
ior femper impetus dicto corpori accedat. Manus autem eiufdem corporis motus,
dum illud ipfum circunuoluitur ( pace Ariftotelis dixerim) centrum non eft, neque
funis eft femidiameter.  Immo manus quam maximè fieri poteft in orbem cietur;
qui quidem motus in orbem,vt circumagatur etiam ipfum corpus, cogit, quod qui-
dem corpus, naturali quadam inclinatione, exiguo quodam impetu iam incepto,
vellet recta iter peragere,vt in fubfcripta figura patet, in qua.e.fignificat manum.a.
corpus.a.b.lineam rectam tangentem girum.a.a.a.a.quando corpus liberum rema-
net. Verum quidem eft,impreſtum illum impetum,continuò paulatim decrefcere
vnde ftatim inclinatio grauitatis eiufdem corporis fubingreditur, quæ fefe mifcens
cum impreffione facta per vim,non permittit,vt linea.a.b.longo tempore recta per
                                                                    maneat,

maneat, fed citò fiat curua, cum dictum corpus.a. duabus virtutibus moueatur, qua-
rum vna eft, violentia impraffa, & alia natura, contra opinionem Tartaleæ, qui ne-
gat corpus aliquod motibus violen
to & naturali fimul & femel moueri
poffe. Neq; eft filétio prætereūdus
hac in re q dã notatu dign° effectus,
qui eiufmodi eft, ꝗ quantò magis
crefcit impetus in corpore.a.caufa
tus ab augmento velocitatis giri
ipfius.e.tatò magis oportet, vt fen-
tiat fe trahi manus à dicto corpore
a.mediante fune, quia quantò ma-
ior impetus motus ipfi.a.eft impref
fus, tantò magis dictum corpus. a.
ad rectum iter peragendum incli-
natur, vnde vt recta incedat, tantò
maiore quoque vi trahit.
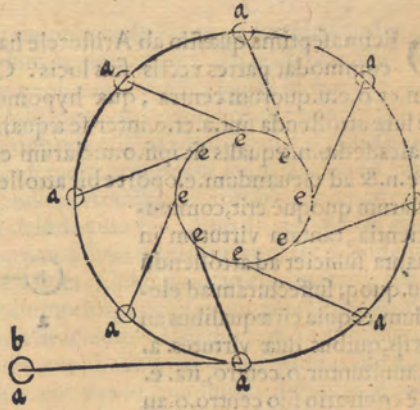
## De decimatertia quæstione.

## CAP. XVIII.

Ecimatertia quæftio ad vectem omnino eft referenda. Imaginari debemus
axem cylindrici iugi, hypomochlion effe. Quod reftat, illud ipfum totum de
pendet à.4.quintoꝗ; cap.huius tractatus. Vna tamen differentia inter hanc machi-
nam, vectemꝗ; reperitur, quæ eft, ꝗ iugum aliquam refiftentiam pro coniunctione
calcata in loco, in quo voluitur, magis quàm hypomochlion vecti efficiat.

## De decimaquarta quæstione.

## CAP. XIX.

Ationes etiam decimæquartæ quæftionis dependent ab ijs, quæ funt vectis, vt
exempli gratia fit lignum.a.b.c.d. frangendum in medio, annitendo genibus
in punctum.σ. clariffimè tunc videbimus, ꝗ tenentes manus longè à medio, in locis
a. et. c. facilius minoriꝗ; cum labore illum frangemus, quàm fi eafdem vicinas me-
dio eiufdem ligni in locis.e.et.i. poneremus. Cuius rei rationes cædé funt cū ijs, quæ
primis huius tractatus capitibus propofitæ fuerunt. Imaginemur lineas rectas ductas
à puncto.o.ad loca.a.e.i.et.c.hinc manifefte perfpiciemus eorum, quæ iam diximus
ratione, ꝗ loca.e.et.i.mediantibus duabus lineis.e.o.et.i.o.magis annitentur.o. cen-
tro, quàm loco.a.et.c.duarū linearū.a.o.et.c.o.beneficio; vnde vim quoque maiorem
habebūt
in.a.et.c.
quàm in
e.et.i.

X            De

162    IO. BAPT. BENED.
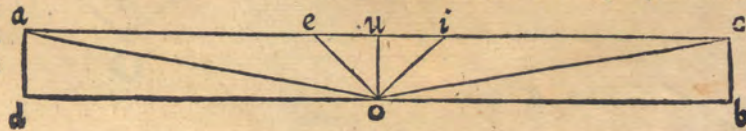
## De uera ratione. 17. quæstionis.

### CAP. XX.

Decimaseptima quæstio ab Aristotele haud benè percepta fuit, quia is non accommodat partes vectis suis locis. Quamobrem imaginemur duos vectes. a.o.n.et.o.e.u.quorum centra, quæ hypomochlia appellantur sint. o. & pondera, quæ sunt attollenda sint.a.et.e.inter se æqualia,& distantię sint.a.o.et.e.o.sibi inuicē æquales,sed.o.n.æqualis sit ipsi.o.u:clarum erit,ꝗ ad eleuandum.a.oportebit deprimere.n.& ad eleuandum.e.oportebit attollere.u.Et quia omnia supponuntur æqualia,clarum quoque erit,communi scientia, tantam virtutem in n.quanta sufficiet ad attollendū a.in.u.quoꝗ suffecturam ad eleuandum.e. quia cū æqualibus angulis ijs,quibus duæ virtutes. a. et.n.annituntur.o.centro, ita. e. et.u.è contrario suo centro.o.an nituntur. & omnes rationes pro vecte.a.o.n.quarto quintoꝗ huius tractatus capitibus citatæ,vecti.o.e. u. vt satis su perꝗ dixi in dicto capit.5.conuenire possunt.

Nunc sit aliqua pars ligni cindenda secundum venulas suas.d.e.f.g. & sit cuneus a.b.c.qui vi mallei. P. vsquę ad.t.x.penetrarit. Hinc clarum erit, quòd apertura i.m.r.ligni, postquam infigitur cuncus secundum venas,longior erit parte.x.b.t.cunei, quæ ingressa est. Oportet nunc imaginari duos vectes similes supradictæ. u. e. o. in hunc modum, vt puncta.i.r. ligni sint loco.u.extremi ipsi⁹ vectis, et.t.x.loco virtutis applicatæ ipsi. u. & resistentia circa punctum.m.loco ponderis.e.vectis.o.e. u. dicti, & pars. K. quasi immediata post.m. versus extremitatem.f.e.ligni,sit loco hypomochlij.o. Hinc fiet vt quanto longiores erunt lineæ.i.m.K.et.r. m.K.tantò quoque facilius virtutes.t.x.impellent.i.r.



De
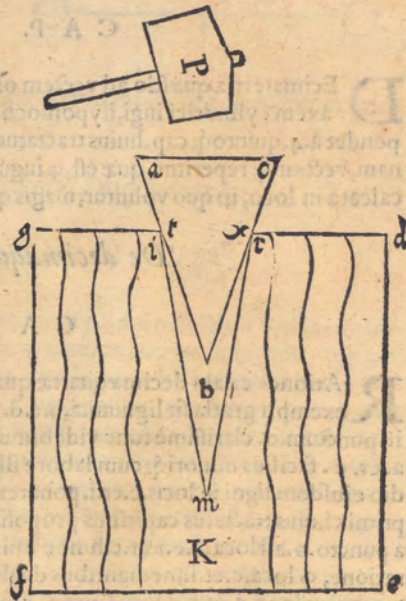
## DE MECHAN.

### De uera & intrinseca causa trochlearum.

#### CAP. XXI.

PRo intelligenda vera, & intrinseca ratione, vnde fiat ut multitudo rotularum in trochleis causa sit, ut exigua vis sursum moueat, aut attollat pódera magna. Imaginemur duas hîc subscriptas trochlæas explicatas transuersaliter in hunc modum, idest sit paruū tignū. a. b. fixum & parallelū orizonti. cui sint rotulæ appensæ ab inferiori parte ad superiorem huicq; è regione opposit°sit aliud tignū. c. d. quod moueri possit ab imo ad sumum, super quod totidem sint rotulæ aut radij, cū annexa postea fuerit funis puncto. b. fixo, eam faciendo pertransire per rotulas tam à parte superiore, quam ab inferiore; & appensum deinde cum erit paruo illi tigno. c. d. mobili pondus. E. ducendo postmodum extremum. f. funis transeuntis per rotulas, idem plainè fiet quod à trochlęis simul unitis fieri solet. Cuius quidem effectus ratio sub nostram cognitionem cadet facilius in huiusmodi figura. Imaginemur separatim stateram: g. h. cuius cétrum sit. K. ita situm, ut brachium. g. k. sit duplum ad brachium. K. h. supponendo igitur in puncto. g. pondus, aut virtutem mouentem unius libræ, & in h. duarum librarum, absq; dubio hæ duæ uirtutes in huiusmodi distantijs à centro equales inuicé erūt, ob rationes prioribus capitibus iam allatas, & statera orizontalis manebit. Vnde clarum erit, q quæuis etiam exigua virtus adiuncta ipsi. g. mouebit stateram extra orizontalem situm. Nunc si puncto. i. ex æquo medio inter. g. et. K. applicata erit virtus ipsius. h. non amplius considerato brachio. K. h. inclinante uirtute ipsius. i. eandem partem versus, in quam inclinabat, quando erat in. h. sed uirtus ipsius. g. inclinet contrario modo, diuersóq; ab eo, quo inclinabat prius; clarum quoq; erit, communi conceptu, & ob ea, quæ cap. 5. huius tractatus sunt dicta. g. h. semper in eodem situ absque motu mansuram, hancq; stateram appellabimus mobilem, & primam. Imaginemur nunc à puncto. e. fixo descendere funem. e. K. quæ fulciat punctum. K. extremum diametri. g. K. quam intelligo pro diametro vnius ex rotulis inferioribus trochleæ; & sit. n. l. m. diameter vnius ex rotulis superioribus alterius parui tigni defixi à parte inclinationis ipsius. g. & parallela diametro. g. K. cuius diametri centrum fixum sit. l. & sit coniunctum. g. punctum, à fune cum puncto. m. quæ tā perpendicularis sit primo diametro. g. i. K. quàm secundo. n. m. idest ita vt anguli. n. m. g.



X 2

164 IO. BAPT. BENED.
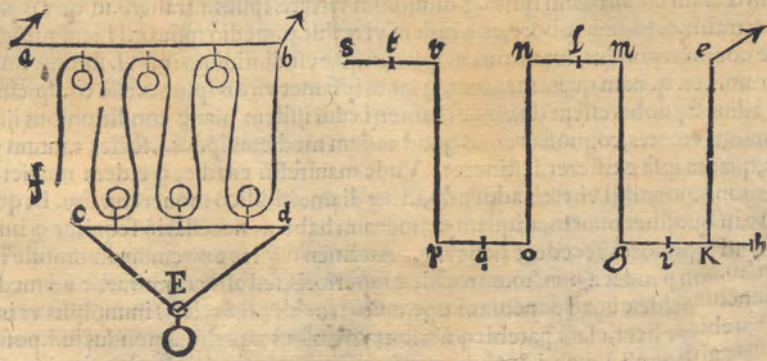
et.m.g.k.fint recti. Imaginemur quoq; virtutem ipfius.g.applicatam effe extremo. n.cum inclinatione tamen contraria,ideft ad inferiorem partem,quæ quidem virtus communi quodam conceptu eandem poffidebit vim fuftentandi immobilem diame trum.g.i.k. quam habebat, qñ erat in.g.cum inclinatione ad fuperiorem partem, & fic etiam diameter.n.l.m.non magis ab una,quàm ab alia parte declinabit, quia cum quædam virtus in.n.reperiatur æqualis medietati uirtutis ipfius.i. quæ uirtus ip fius.i.uim habet deprimendi ipfum.g.ideft.m.pro dimidia fui ipfius parte, fequitur. n.m.debere immobilem permanere. Nunc fi alia diameter rotulæ mobilis erit de- fumpta,quæ fit.p.q.o.cuius centrum fit.q.in fitu parallelo ipf.n.l.m.& fic collocata, vt coniungendo.o.cum.n.anguli.m.n.o.et.n.o.p.fint recti: fi imaginati fuerimus trãf latum effe ponduśipfiº.n.in.o.cũ eadẽ inclinatione ad depræffiorem partem,illud ip fum,ac fi effet in.n. communi conceptu,fine alicuius diametri mutatione præftabit. Et fi centrum.q.fixum effet,& extremo.p.appofitum fuiffet pondus ipfius.o.cum in clinatione ad fuperiorem partem,idem etiam planè prȩftaret,etiam fi nullum ullius diametri fitum,communi fcientia,mutaret, cum extremum. m. deorfum fit ductum à.g.uirtute dimidiæ partis ipfius.i.& ab alia huic fimili. m. quoque deorfum fit tra- ctum ab . o : quod quidem.o.deorfum eft alteratum,ob inclinationem ad fuperius à uirtute pofita in.p.fupponendo centrum.q.fixum. Sed fi loco centri fixi,imagina bimur in.q.pondus aliquod æquale ipfi. i. quod duplum erit in uirtute ad eam, quæ eft ipfius.p.& ipfius quoque. g: fequetur etiã eadem immobilitas horum trium dia- metrorum . Quia cum fit huiufmodi pondus feu virtus in.q. cum inclinatione con- traria virtuti in.p.quæ æquipollet dimidiæ parti ipfius.q.& fic ei quæ eft ipfius.o. fi- militer quia.o.tractum eft fupra ab.n.virtute ipfius.g.quod.m.deorfum trudit; idcir co quanta erit vis quam habebit virtus in.q.ferendi deorfum diametrum.p. o. tanta quoque virtutes ipforum.p.et.o.æquales,& æqualiter diftantes à.q.ipfum ad fupe- riorem partem inclinabunt. Quamobrem nec afcendet, nec defcendet,nec locum mutabit. Supponamus nunc quartum diametrum rotulæ.s.t.r. quæ fit fecunda rotu larum fixarum,parallela ipfi.p.o.& in eo fitu,quo coniungendo extrema.r.p.anguli o.p.r.et.p.r.s.fint recti,& imaginemur virtutem ipfius.p.reperiri in.s.cum inclinatio ne tamen contraria,ideft deorfum verfus,ex his idẽ quoque planè fequetur, ideft ꝗ nulla harũ quatuor diametrorum mouebitur. quia eundem effectũ cũ inclinatione deorfum verfus efficeret dicta virtus in.s.quem in.p.cum inclinatione furfum verfus. et iam dictum eft virtutem ipfius.g.dimidium virtutis ipfius.i.trahere. m. quæ mediã te.n.attrahit.o.eodem robore,et.s.eadem vi trahit.p.medio ipfius.r.Hucufque fciẽ- tificè nouimus pondus,aut virtutem ipfius.s.quæ eft dimidium ipfiº. i. fuftinere uim ipforum.i.et. q. nam quater tantum, quanta ipfamet virtus ipfius.s.effe confpicitur. Et fi adiunctȩ nobis effent duæ aliæ diametri cum ijfdem planè conditionibus ijfdẽ rationibus vtentes,cognofceremus quod eadem medietas ipfius.i.fexies tantum pó deris,quanta ipfa exifteret,feftineret. Vnde manifeftũ euadit, ꝗ eidem medietati ipfius.i.in.s.nonnihil virtutis addendo,dictæ diametri,illicò mouerétur fitu. Et quia rotulæ in quolibet puncto,aliquam diametrum habent, neceffariò fequitur ꝗ infe- riores ad fuperiores accedere debeant . Attamen fi forte extremum immobile ip- fius funis non pendet à puncto.e.trochleæ fuperioris,fed alligatum fuerit ad mediũ inferioris trochleæ ut ad punctum.i.ope unius trochlȩ fuperioris immobilis vt in fi gura. A.videre licet,clarè patebit ꝗ à tribus virtutibus æqualibus pondus in.i.pofitũ fuftinebitur:hoc eft à.g.ab.i.& ab.k. quarũ vnaquȩque tertia pars erit ipfius.i.in con contrariam partȩ,hoc eft tertia pars refiftentiæ.propterea ꝗ ex æquo inter fe diftát.

g.i.

DE MECHAN. 165

g. i. et. K: Quà propter augebitur virtus per numeros impares, hoc modo; Nam. g.
effet tertia pars refiftentię, quemadmodum prius media erat. Idem infero de. m. n.
o. p. r. et. s. Sed cum oporteat pondus. q. tantum effe vt fufficiãt refiftentiæ in. o. et. p.
ipfum fuftinere, idcirco ipfum pondus. q. fubfefquialter erit põderi in. i. pofiti. Qua-
propter. s. quinta pars erit ponderum. i. et. q. Deinde fi adhuc. duo diametri vnus in-
ferior, alter verò fuperior additi fuerint cum pondere æquali. q. ad medium diame-
tri inferioris, tunc pondus. s. erit feptima pars trium ponderum. i. q. & tertij additi, ex
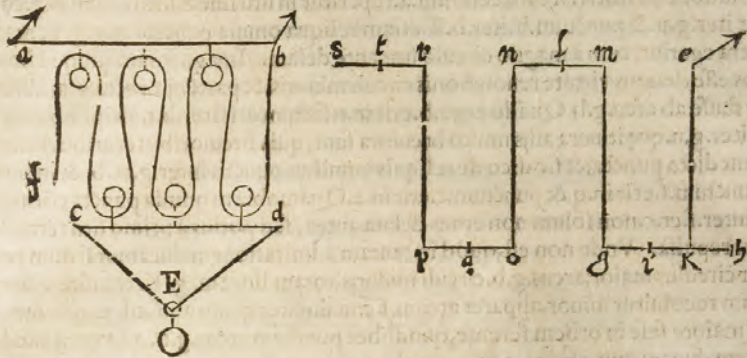


fupradictis rationibus. Et quia virtus fufti
nens totale pondus trochleæ inferiori ap-
penfum in tot diuiditur partes æquales,
quot funt diametri orbiculorum trochleæ
inferiori, quando extremum immobile fu
nis alligatum fuerit trochleę fuperiori, vt
puta in puncto. e. cum verò alligatum fue-
rit trochleæ inferiori, virtus primi diame-
tri. g. i. K. trochleæ inferioris femper fefqui
altera erit vnicuique aliorum diametrorũ;
ideò virtus refiftentię alterius extremi mo
bilis funis, puta. s. fubmultiplex erit totalis
ponderis, eo modo quo diximus, cuius vir
tus, feu grauitas diuiditur feu diftrubuitur
diametris inferioris trochleæ vt dictum eft.
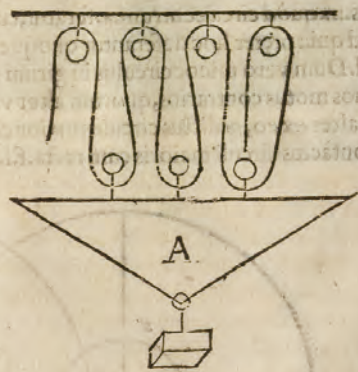
*De propria caufa.* 24. *quæftionis.*

CAP. XXII.

V Era caufa effectus, qui vigefimaquarta quæftione exprimitur, adhuc à nemine
(quod fciam) animaduerfa fuit, licet non fit admodum ardua vel obfcura. Ima
ginemur ergo duos circulos. c. f. et. b. g. concentricos, itaq; fimul coniunctos, vt fi ip
forum vnus feratur in orbem, alius quoque circumagatur, eo modo, quo curruum ro
tæ voluuntur. Et imaginemur primò fuper lineam. f. i. reuolui maiorem, & quando
idem circulus erit in. l. dictam lineam. f. i. tangere circunferentiam eiufdem in pun-
cto. c.

&to.c.vnde linea.g.m.mediante.K.continget circunferentiam circuli minoris in pun
&to.b:et.K.g.ex.34.primi Eucli.æqualis erit ipfi.f.l.quia ex.17.tertii, anguli.f. et. g.
funt æquales,vnde ex.28.primi.f.l.et.g.K.funt parallelæ.& fic erunt.k.l.cum. f.g. ex
eadem fupradicta.Ratio autem, qua arcus.g.b.tranfierit lineam.g. K. maiorem ipfa,
eft, quia dum mouetur,quodlibet punctum ipfius.g.b.virtute reuolutionis ipfius.f.c.
omne punctum eiufdem arcus.g.b.vlterius verfus.K.quam fi moueretur virtute re-
uolutionis ipfius.g.b.fuper lineam.g.m.defertur. vt exempli gratia , quando virtute
reuolutionis maioris circuli, centrum.a.reperitur in fitu lineæ.l.K.punctum.g.confe
cerit iter.g.u.& punctum.b.iter.b.K.etiam reliqua omnia puncta inter.g. b. magna
itinera egerint, cum à magno circulo fint ante delata. Imaginemur quoque hos cir
culos effe delatos virtute reuolutionis circuli minoris,& parté.g.t.rectè.g.m.dimen-
fam fuiffe ab arcu.g.b.Quãdo ergo.b.erit in.t.factum erit iter.b.t.ab ipfo.b.et.g.fa-
ciet iter.g.n.que itinera alijs multò breuiora funt, quia breuioribus cruribus reuolu-
ta funt dicta puncta;& fic dico de reliquis omnibus punctis inter.g.et.b. & in hoc ca
fu punctum.f.erit in.q.& punctum.c.erit in.e.Quamobrem omnia puncta cõtingen-
tiæ inter.f.et.c.non folum non erunt delata anteà, fed potius à primo fitu retrorfum
erunt repulfa. Vnde non eft,quòd in tantam admirationem ducamur fi dum reuol
uitur circulus maior,arcus.g.b.circuli minoris,totam lineam. g. K. tranfire videtur ,
& dum reuoluitur minor,apparet arcum.f.c:maius iter quam ab.f.ad. e. non facere ,
cum maiore fefe in orbem ferente,quodlibet punctum arcus.g.b. ad vnam eandéq;
partem duos motus obtineat.vt exempli gratia punctum.b.non folum mouetur ver
fus.m.quòd circa,centrum.a.feratur,cum ipfum etiam centrum moueatur verfus.m.
fed quia preter hoc deferantur quoque à circulo maiori verfus.m. vfque ad lineam.
k.l.Dum verò minor circulus in girum ducitur,habet quodlibet punctum arcus. f.c.
duos motus contrarios,quorum alter verfus.i. virtute reuolutionis circuli minoris ,
& alter ex eo, φ dictus circulus maior circa centrum.a. voluatur, vnde omne punctũ
contactus circuli maioris cum recta.f.i.te trorfum pellitur verfus.x.



De

*De uera causa. 3 0. quæstionis.*

## CAP. XXIIII.

VEra ratio, cur homo dum sedet ( non tamen Turcarum more velit sese in pedes erigere, calcaneos retrahit, vt efficiat angulum acutum, cum fęmoribus coxis à parte inferiori, & ventrem inclinat, ad constituendum etiam angulum acutum in superiori parte, ea est; vt totius corporis pondus, ex æquo, idest ab oppositis partibus circundet lineam rectam, quæ transit per locum, in quo conquiescunt pedes versus mundi centrum. idest, ut edatur æquilibrium ponderis ipsius corporis circum lineam illam, quę sub pedibus inseruit pro sparto . Vnde aperiendo, deinde dictos duos angulos circa dictam lineã, absque vlla difficultate erigitur corpus, & absque periculo in alterutram partem cadendi.

*De ratione. 3 5. & ultima quæstionis.*

## CAP. XXV.

VEra ratio, quare, quę reperiuntur in vorticibus aquarum , semper versus medium ipsarum vertiginum vniuntur , inde promanat , quod media vertiginum semper depressiora sunt . vnde quòd dicta corpora ad medium accedant, nihil aliud est, quàm ipsa corpora suo pondere grauitatéque descendere, figura enim vorticibus est quasi conica , & concaua cum angulo deorsum, & gyro basis sursum. Atque hæc vera est huius effectus causa, & non ea quam Aristoteles ponit, à quo aliarum omnium quæstionum, quas ego omisi rationes sunt benè propositæ.

DISPV-

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# DISPVTATIONES
## DE QVIBVSDAM PLACITIS
### ARISTOTELIS.

ANTA *est certè Aristotelis amplitudo atque authoritas, vt difficillimum ac periculosum sit quidpiam scribere contra quam ipse docuerit, & mihi præsertim, cui semper visa est viri illius sapientia admirabilis. Veruntamen studio veritatis impulsus, cuius ipse amore in seipsum si viueret excitaretur, in medium quædã proferre non dubitaui, in quibus me inconcußa mathematicæ philosophiæ basis, cui semper insisto, ab eo dißentire coegit.*

*Qualiter & ubi Aristoteles de uelocitate motuum naturalium localium aliter tractauerit quam nos*
*sentiamus.*

### CAP. I.

Olens Aristoteles probare vacuum non esse in rerum natura. 8. cap. lib. 4. phy sicorum ait, idem corpus per varia diuersaq́; media, vt per aerẽ, & per aquã si moueretur, proportionem velocitatis eiusdem corporis per aerem, ei, quæ per aquam fit, vnam eandemq́; futuram cum ea, quæ est subtilitatis aereę ad subtilitatem aquæ. In postrema autem parte eiusdem capitis sic scribit: Nam cum ea quę maiorem vel ponderis vel leuitatis prestantiam habent, si simili figura sint, spaciũ par, & æquale, maiore celeritate conficere cernamus, ea quam magnitudines inter se habent, proportione: profectò idem etiam per inane fieret. Aliam quoque rationem proponit phylosophus. 2. cap. sexti physicorum scribens eademmet proportione, qua tempus diuiditur, magnitudinem etiam diuidi. Sexto autem cap. primi de cœlo scribit, tempora eandem proportionem habere, quam habent è conuerso pondera; vt si media pars vnius ponderis, vnius horæ spatio moueretur, vniuersum pondus in media hora moueretur. Secundo cap. lib. 3. de cœlo duobus in locis apertè com monstrat velocitatem corporis minoris, maiori corpori comparatam, in eadem existere proportione, in qua dicta corpora adinuicem relata existunt. Quinto cap. eiusdem lib. idem affirmat, exemplo ab igne desumpto. Ex alijs etiam plurimis locis cognosci potest, sensisse Aristotelem duo corpora eadem specie, & figura prædita eandem planè proportionem in suorum motuum velocitatibus, quam in suis magnitudinibus habent, retinere. Alij quoque permulti eandem opinionem retinue runt, & omniũ postremus Nicolaus Tartalea, secunda propositione vigesiminoni quæsiti octaui libri, vbi profitetur se demonstratiuè probare hanc propositionem veram existere; neq; videt quàm magna resistentiarum sit differentia, quæ tam ex diuersitate figurarum, quàm ex magnitudinum varietate exoriri potest; quas qui dem diuersitates ne considerat quidem.

Quędam

*Quædam supponenda ut constet cur circa uelocitatem motuum
naturalium localium ab Aristotelis placitis
recedamus.*

## CAP. II.

CVM susceperimus prouinciam probandi quod Aristoteles circa motus
locales naturales deceptus fuerit, sunt quædam primo verissima & obie-
cta intellectus per se cognita presupponenda, ac primum quælibet duo corpora,
grauia, aut leuia, area æquali, similiq; figura, sed ex materia diuersa constantia, eodé
que modo situm habentia, eandem proportionem velocitatis inter suos motus loca
les naturales, ut inter suamet pondera aut leuitates in vno eodemq; medio, seruatu-
ra. Quod quidem natura sua notissimum est si considerabimus non aliunde maio-
rem tarditatem, aut velocitatem gigni, quàm à.4.causis (dummodo medium vnifor
mé sit & quietum) idest à maiori aut minori pondere aut leuitate; à diuersa figura;
à situ eiusdem figuræ diuerso, respectu lineę directionis, quæ recta inter mundi cen-
trum, & circunferentiam extenditur;& ab inæquali magnitudine. Vnde patebit,
quòd figuram non variando, nec in qualitate nec in quantitate, neque eiusdem figu-
ræ situm, motum fore proportionatum virtuti mouenti, quæ erit pondus aut leuitas.
Quod autem de qualitaté, de quantitate & situ eiusdem figuræ dico, respectu resi-
stentiæ ipsius medii dico: Quia dissimilitudo aut inequalitas figurarum, aut situs di-
uersus non parü alterat dictorum corporum motus, cum figura parua facilius diui-
dat continuitatem medij, quam magna; vt etiam cęlerius idem facit acuta, quàm ob
tusa;& illa quæ cum angulo, qui antecedat mouebitur velocius quàm illa, quæ secus.
Quotiescunque igitur duo corpora vnam eandemq; resistentiam ipsorum superfi-
ciebus, aut habebunt aut recipient, eorum motus inter seipsos eodem planè modo
proportionati consurgent, quo erunt ipsorum virtutes mouentes: & è conuerso, quo
tiescunque duo corpora vnam eandemq; grauitatem, aut leuitatem, & diuersas resi
stentias habebunt, eorum motus inter seipsos eandem proportioné sortientur, quá
habebunt eorum resistentiæ conuerso modo; quæ quidem resistentiæ inter seipsas,
eandem proportionem quàm ipsarum superficies habebunt, aut in qualitate sola fi
guræ, aut in quantitate sola, aut in situ, aut in aliquibus ex dictis rebus, eo tamen mo
do, qui superius positus fuit, vt scilicet corpus illud quod alteri comparatum, æqua-
lis erit ponderis, aut leuitatis, sed minoris resistentiæ, existet velocius altero, in eadé
proportione, cuius superficies resistentiam suscipit minorem ea quæ alterius est cor-
poris, ratione facilioris diuisionis continuitatis aeris, aut aquæ; Vt exempli gratia,
si proportio superficiei corporis maioris superficiei minoris sesquitertia esset, pro-
portio velocitatis dicti corporis maioris, velocitati corporis minoris, esset subsesqui
tertia; vnde velocitas minoris corporis, maior esset velocitate corporis maioris, qué
admodum quaternarius numerus ternario maior existit.

Aliud quoque supponendum est, velocitatem scilicet motus naturalis alicuius
corporis grauis, in diuersis medijs, propor-
tionatam esse ponderi eiusdem corporis in
ijsdem medijs; Vt exempli gratia, si pondus
totale alicuius corporis grauis significatum
erit ab.a.i.quo corpore posito in aliquo me-
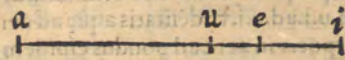
Y     dio

dio minus denso, quàm ipsum sit,(quia in medio se densiore si poneretur, non graue
esset, sed leue, quemadmodum Archimedes ostendit ) illud medium subtrahat par-
tem.e.i.vnde pars.a.e. eiusdem ponderis libera maneat;& posito deinde eodem cor
pore in aliquo alio medio densiore, minus tamen denso quam ipsum sit corpus,
hoc medium subtrahat partem . u . i . dicti ponderis , vnde pars. a . u. ei usdem
ponderis remanebit. Dico proportionem velocitatis eiusdem corporis per mediū
minus densum, ad velocitatem eiusdem per medium magis densum futuram vt.a.e.
ad.a.u.vt est etiam rationi consonum, magis quàm si dicamus huiusmodi velocitates
esse, vt.u.i.ad.e.i.cum velocitates à virtutibus mouentibus solum ( cum figura vna ,
eademq; in qualitate, quantitate situq; erit) proportionentur. Quæ nunc diximus,
plane similia sunt ijs, quæ supra scripsimus, quia idem est dicere, proportionem velo
citatum, duorum corporum hetereogeneorum, sed similium figura, & magnitudine
æqualium, in vno solo medio , æqualem esse
proportioni ponderum ipsorum, vt si dicam?
proportionem velocitatum vnius solum cor-
poris per diuersa media eandem esse cum ea,
quæ est ponderū dicti corporis in isidem medijs.
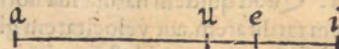
### *Posse uelocitatem alicuius corporis proportionem contrariam*
### *in diuersis medijs habere cum densitate eorum.*

### CAP. III.

Possibile est in rerum natura corpus aliquod huiusmodi densitate præditum re-
periri, vt velocitas eius motus naturalis per aerem, velocitati per aquam ita pro
portionata existat, vt est desitas aquæ densitati aeris. Densitas aquæ notetur(exem-
pli gratia) per.u.i.& ea, quæ aeris est per.e. i.& pondus alicuius corporis in aere per
c.a.& pondus eiusdem corporis in aqua per.u.a.ita tamen, quod eadem proportio
sit.e.a.ad.u.a.vt.u.i.ad.e.i.vnde per vltimam suppositionem præcedētis capitis, pro
portio velocitatis prædicti corporis per aerem,
proportioni eiusdem corporis per aquam erit, vt
c.a.ad.u.a.ergo per.11.quinti,vt.u.i.ad.e.i.
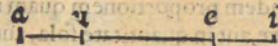
### *Oscitanter ab Aristotele nonnihil prolatum cap. 8.*
### *lib. 4. Physicorum.*

### CAP. IIII.

EX supradictis patet in vniuersum non esse verum quod Aristo.8.cap.4.lib.phy
sicorum scribit, velocitates scilicet motuum alicuius corporis per diuersa me-
dia, proportionatas esse densitatibus eorundem mediorum . Quocirca , sit propor-
tio.u.i.ad.e.i.vt desitatis aquę ad aereā desitatem.et.e.a.ad.u.a.vt ponderis alicuius
corporis in aere ad pondus eiusdem in aqua, ita tamen vt maior aut minor propor:
tio sit.e.a.ad.u.a.quam.u.i.ad.e.i.vnde existente proportione velocitatis per aerē

ad

ad velocitatem per aquam vt.e.a.ad.a.u.non erit ergo vt.u.i.ad.e. i. Ob hanc igitur
causam nimis diſſentaneum eſt rationi, opi-
nari proportionem velocitatis omnium cor
porum grauium per aerem vnam eandemq́;
eſſe cum velocitate eorundem per aquam,
quemadmodum Ariſtoteles senſit.
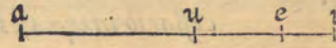
### Exempla dictorum.

### CAP. V.

Ponamus, exempli gratia, aquam eſſe in denſitate dupla ad aerem, & aliquod
graue corpus in aqua duplum ad denſitatem ipſius aquæ, vnde dictum corpus in
denſitate ad aerem quadruplum erit; quam ob cauſam, mediam ſui ponderis tota-
lis partem in aqua, & in aere quartam partem, ex.7.lib.de inſidentibus aquæ ab Ar-
chimede conſcripto, amitteret. Moueretur igitur in aqua virtute illius mediæ partis
póderis ſui, in aere aūt uirtute triū quartarū; vnde proportio facultatis mouétis dicti
corporis in aere ad facultatem mouentem eiuſde m in aqua ſeſquialtera erit. hocq́;
corpus appelletur. A. Sit aliud quoque corpus, quod.B.nominetur, ſimíle figura, &
magnitudine corporea corpori. A.ſed déſitate, in proportione ſeſquialtera ad aquā,
& denſius erit aere in proportione tripla. quamobrem corpus. A.grauius erit cor-
pore.B.in aere in proportione ſeſquialtera, vnde etiam velocius erit ipſo. B.in aere
in eadem proportione, ſed corpus.B.in aere, duplo maius pondus habebit, quá in
aqua, cum in aere remaneant ei duæ ponderis tertiæ partes, & in aqua vna tantum,
ita vt Ariſtoteli concedam corpus.B.in aere, quam in aqua velocius futurum in ea-
dem proportione, in qua, aqua eſt déſior aere, ex Euclidis vndecima propoſitione
lib.quinti. Sed præter hæc omnia, ſi corpus. A. eſſet etiam velocius in aere, quá in
aqua, in eadem proportione, ſequeretur ex.16.dicti lib.quinti proportionem velo-
citatis.A.in aqua ad velocitaté ipſius.B. in aqua etiam ſeſquialteram eſſe. Sed cum
corpus.A.in denſitate ad aquam duplū ſit, & corpus.B.ſeſquialterū ad ipſam aquā,
ſequetur proportioné ponderis ipſius. A. ad pódus ipſius. B.in aqua eſſe in propor-
tióne dupla; Vnde ex primo ſuppoſito capitis ſecundi proportio velocitatis. A. ad
velocitatem.B.in aqua dupla erit, non ſeſquialtera. Si ergo proportio velocitatis.
A.ad eam quæ eſt.B.in aqua dupla eſt, & ea, quæ eſt.B.in aere, ad eam, quæ eſt ipſius
per aquam eſt etiam dupla( vnde ea quę eſt.A.per aquam ęqualis erit ei, quæ eſt. B.
per aerem, ex.9.lib.quinti ) & cum ea, quæ eſt.A.ſit ei, quæ eſt.B.per aerem ſeſqui-
altera, erit ergo ea, quæ eſt. A.per aerem, ei, quæ eſt ſuimet ipſius per aquam ſeſqii
altera, non autem dupla, ex. 7. eiuſdem libr.quinti. Hiſce rationibus accedimus ad
confirmandam veritatem vltimi ſuppoſiti cap. 2.proportionem videlicet velocita
tis mot⁹ naturalis in diuerſis medijs alicui⁹ corporis póderoſi in ipſis medijs eſſe ean
dem cum ea, quæ eſt inter pondera
dicti corporis in dictis medijs. de ijs
tamen medijs intelligendo, quæ un-
ctuoſa, aut pinguia non ſunt, ut ſunt
oleum, lac, aut huiuſmodi alia, quæ à
qualibet minima qualitate frigoris
aut caloris alterantur, & impermea-
biles fiunt.

p aerem.A.6.

p aerem.B.4.    p aquam.A.4.

p quam B.2.    p aquam.B.2.

Y 2 Quod

*Quod proportiones ponderum eiusdem corporis in diuersis medijs pro*
*portiones eorum mediorum densitatum non seruant. Vnde ne-*
*cessariò inæquales proportiones uelocitatum*
*producuntur.*

## CAP. VI.

OMne corpus graue variat proportionem ponderis per diuersa media , vnde
proportiones velocitatum inæquales existunt . Vt exempli gratia , si fue-
rit corpus. A. cuius pondus totale sit. o. a. quod in aqua diminutum sit ratione partis.
e. o. ita vt ei solum relinquatur pondus. a. e. & in aere adempta sit ei pars. i. o. vnde so
lum remaneat pondus. a. i. Supponamus aliud quoq; medium in eadem proportio-
ne minus densum, quàm aer, quemadmodum aer minus densus est, aqua, in quo, cor
pus. A. ammittat partem. t. o. ponderis sui, vnde ex. 7. lib. de insidentibus aquæ Ar-
chimedis, eadem proportio erit. e. o. ad. i. o. quæ est. i. o. ad. t. o. Supponamus quoq;
eandem proportionem esse. a. i. ad. a. e. est. e. o. ad. i. o. tunc dico non futuram ean-
dem proportionem. t . a. ad. a. i. quæ est. i. o. ad. t. o. Cum sit ergo proportio. a. i.
ad. a. e. ut. e. o. ad. i. o. erit disiunctim. e. i. ad. e. a. vt. e. i. ad. i. o. Quare ex. 9. libr. quin
ti erit. a. e. æqualis. i. o. sed cum ita se habeat. e. o. ad. i. o. vt. i. o. ad. t. o. ita quoque
se habebit, ex vndecima quinti. a. i. ad. e. a. ut. i. o. ad. t. o. Cum autem (vt vidimus). a. e.
equalis sit ipsi. i. o. non poterit esse proportio. t. a. ad. i. a. vt est. o. i. ad. t. o. quia si
hoc esset, esset etiam disiunctim proportio. i. t. ad. i. a. vt est. i. t. ad. t. o. & ex supradicta
9. lib. quinti. a. i. æqualis esset. t. o. Maximum autem inconueniens esset. t. o. minorem
o. i. idest minorem. a. e. æqualem esse. a. i. quæ maior est. a. e. Ostensiuè tamen idem
hoc modo probari potest, vt existente. i. o. equali ipsi. a. e. per consequens quoq; erit
minor ipsa. a. i. cum. a. e. pars sit ipsius a. i. Per eandem tamen rationem. o. t. minor est.
o. i. Tanto magis igitur minor erit. t. o. ipsa. i. a. Vnde ex. 8. libri quinti maiorem pro-
portionem habebit. i. t.
ad. t. o. quam ad. i. a. &
ex. 28. eiusdé lib. i. o. ad
t. o. maioré proportio-
né habebit, quàm. t. a.
ad. i. a. ex. 1 2. igitur di-
cti quinti maiorem pro



portionem habebit. i. a. ad. e. a. quàm. t. a. ad. i. a. ita ergo se habebunt ipsorum velo-
citates.
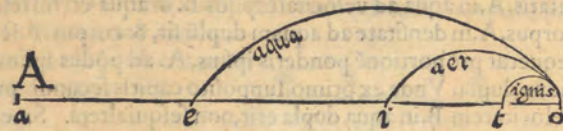
*Corpora grauia aut leuia eiusdem figuræ et materiæ sed inæqualis*
*magnitudinis, in suis motibus naturalibus uelocitatis, in eo*
*dem medio, proportionem longè diuersam seruatura*
*esse quam Aristoteli uisum fuerit.*

## CAP. VII.

ESt mihi nunc probandum φ in uno eodemq; medio duo corpora inæqualia, sed
simili figura & materia, mouebuntur naturali motu, diuersa tamen ratione ab

ea,

Quod

ea, quàm Ariftoteles præfcripfit.

Sint igitur corpora.a.et.o.inæqualia ,eadé tamen figura & materia prædita,quorum.a.maius fit,& per confequens in eadem quoque proportione grauius ipfo.o. in qua eft maius,communi omnium fententia.

Scribit ergo Ariftoteles proportionem velocitatis corporis. a. ad eam , quæ eft corporis.o.(naturaliter fe vnoquoque mouente) eandem futuram, quæ eft magnitudinis,aut grauitatis corporis.a.ad magnitudinem, aut grauitatem corporis.o. Imaginemur igitur corpus u. eadem magnitudine & figura,qua corpus.a.præditum eft , fed eandem grauitatem obtinere,quæ communicata eft corpori. o. quod ex quauis materia conftet . Hinc ex primo fuppofito fecundi capitis certi erimus proportionem velocitatis corporis.a.fi comparetur cum velocitate corporis.u.futuram, vt eâ, quæ eft ponderis corporis.a.ad pondus ipfius corporis.u. Ex.9.igitur lib.quinti Eucli.cogitur fateri Ariftoteles velocitatem corporis. o. effe vnam eandemᶐ; in fpecie,quæ eft corporis.u.Quod primo fuppofito cap.fecundi huius lib.planè repugnaret. Igitur hæc Ariftotelis opinio falfa eft. Idem quoque probaretur mediante corpore.i.æquali magnitudine, fimiliᶐ; figura cum corpore. o. prædito , fed, quòd ad quantitatem attinet,æquali corpori.a.vnde ex primo fuppofito cap.fecundi huius libri in eadem pro-

portione veloci⁹ effet corpore . o. in qua grauius eft. ex. 9. igitur quinti cogitur Ariftoteles affirmare tã velox effe corpus a. quâ eft corpus i. vnde idem pla-



nè inconueniens emergit ex fecundo fuppofito cap.fecundi huius lib.
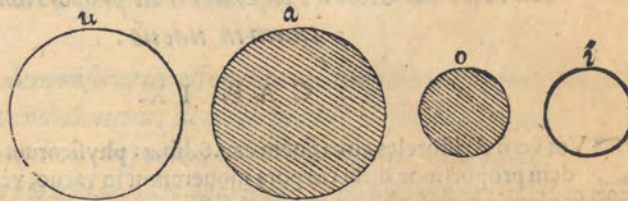
*Quod duo corpora inæqualia eiufdem materiæ in diuerfis medijs eandem uelocitatis proportionem retinebunt .*

## CAP. VIII.

Vælibet duo corpora inæqualia fimili tamen figura & eadem materia conftantia, naturaliter fe per diuerfa media mouentia , vnam eandemᶐ; femper proportionem velocitatum feruant.

Sint duo corpora.A.et.B.fibi inuicem inæqualia quorum.A. fit maius, fed fimile figura & idem materia, cuius pondus totale fit. x.o.& pondus totale ipfius.B.fit. u.s. Imaginemur quoque corpus.A. pofitum in aqua amittere parté.o. e. ponderis. O.X.
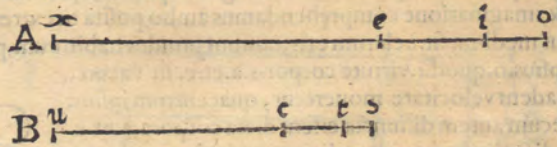
o.x.et.B.quoque in eodem loco amittere.c.s.et.A.in aëre partem.i.o.et. B.partem.
.t. s. Nunc quia corpus aqueum, cui correſpondet.e.o.æquale eſt ipſi. A. & corpus
aqueum,cui correſpondet.c.s.æquale eſt i pſi. B.vt eſt ab Archimede probatũ: com
muni quadam ſcientiæ ratione,ſequitur eandem proportionem futuram.o.x.ad.e.o.
quæ eſt.u.s.ad.c.s.ob eaſdemq́; rationes idem erit de. x. o.ad.i.σ.ut.u.s.ad. t.s.& idẽ
etiam erit de.o.x.ad.s.u.vt de.e.o.ad. c.s.vt etiam de. o.i.ad.s.t. Vnde ex. 19 .lib.
quinti erit de.x.i.ad.u.t.quemadmodum de.x.o.ad.u.s.idem dico de.x.e.ad.u.c.Ex
11.igitur dicti lib.erit.de.x.i.ad.u.t.quemadmodum de.x.e.ad.u.c.ex quibus quidẽ
proportionibus, ſi ſubtra
hantur proportiones ʼreſi
ſtétiarum extrinſecus ad-
uenétium , proportiones
quæ remanebunt, ex ter-
tio communi axiomate
ab Eucli.in principio pri
mi lib.poſito,ad inuicem



erunt æquales,ſecundum quas eorundem corporum ſunt velocitates.
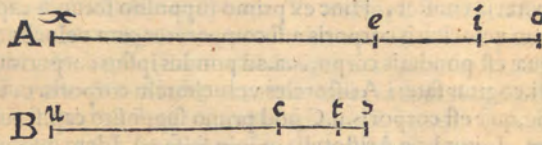
### An rectè Ariſtoteles diſeruerit de proportionibus mo-
### tuum in uacuo.

### CAP. IX.

CVm verò Ariſtoteles circa finem cap.8.lib.4. phyſicorum ſubiungit quod ea-
dem proportione dicta corpora mouerentur in vacuo, vt in pleno,id pace ei⁹
dictũ ſit planè erroneũ eſt.quia in pleno dictis corporibus ſubtrahitur proportio reſi
ſtentiarum extrinſecarum à proportione ponderum, vt velocitatum proportio re-
maneat, quę nulla eſſet, ſi dictarum reſiſtentiarum proportio , ponderum propor-
tioni æqualis eſſet,& hanc ob cauſam diuerſam velociratum proportionem in va-
cuo haberent ab ea,quæ eſt in pleno.

### Quòd in uacuo corpora eiuſdem materiæ æquali uelocita-
### te mouerentur.

### CAP. X.

QVòd ſupradicta corpora in vacuo naturaliter pari velocitate mouerentur,
hac ratione aſſero.

Sint enim duo corpora.o. et.g.omogenea, et. g.ſit dimidia pars ipſius.o.ſint alia
quoque duo corpora.a.et.e.omogenea primis , quorum quodlibet æquale ſit ipſi.g.
& imaginatione compręhendamus ambo poſita in extremiratibus alicuius lineæ, cu
ius medium ſit.i.clarum erit,tantum pondus habiturum,punctum.i.quantum centrũ
ipſius.o.quod.i.virtute corporis.a.et.e. in vacuo ,
eadem velocitate moueretur, quacentrum ipſius.
o:cum autem diſiuncta eſſent dicta corpora.a.et.e.
à dicta linea,non ideo aliquo modo ſuam velocita



tem
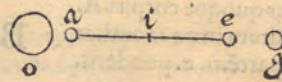
tem mutarent, quorum quodlibet esset quoque tam velox, quàm est. g : igitur. g. tam velox esset quam. o.

*Corpora licet inæqualia eiusdem materiæ & figuræ, si resisten-*
*tias habuerint ponderibus proportionales*
*æqualiter mouebuntur.*

## CAP. XI.

E Adem ratione, quam cap.antecedente præscripsimus, posset ostendi, si duo corpora.o.et.g.suas resistentias, ita ad inuicem proportionatas haberent, ut sunt eorum pondera, in pleno pari velocitate prædita esse, quod in fine capitis noni leuiter attigi, quia punctum.i.tam velox esset, ut centrum ipsius.o.cum à tanto pondere i.motum esset; quanto centrum ipsius.o.atque tan
tam resistentiam duo corpora.a.et.e. quáta ipsum
o.solum haberet ex hypothesi, dicta tamen corpo
ra.a.et.e.tam separata, quam coniuncta, eandem
velocitatem retinerent. g. igitur tam velox esset,
quam.o.
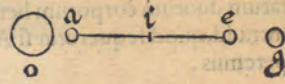
*Maior hic demonstratur esse proportio ponderis corporis den-*
*sioris ad pondus minus densi in medijs desioribus, quam*
*sit eorundem corporum in medio minus denso, nec*
*corporum pondera seruare proportionem*
*densitatis mediorum.*

## CAP. XII.

P Roposita nobis cum fuerint duo corpora. A. et. B. area corporea æqualia, quorum. A.densius sit ipso. B.probabo in medio magis denso, maiorem proportionem futuram ponderis ipsius. A.ad pondus. B.quàm in medio minus denso.

Sit igitur.p.g.pondus totale ipsius corporis. A.et.q.k.ipsius corporis. B.vnde. p.g. maius erit ipso.q.k. Sit quoque.o.g.pondus, quod medium magis densum subtrahit à pondere.p.g.et.n.k.sit pondus, quod idem medium subtrahit à pondere.q.k. et f.g.sit pondus, quod medium minus densum subtrahit à.p.g.et.i.k.illud, quod idem mediū subtrahit ab.q.k.vnde.o.g.æquale erit.n.k.et.f.g.ipsi.i.k. quia quod ad areā attinet, corpora supponuntur æqualia, vnde proportio.p.f.ad.q.i.maior erit ea, quæ

est.o.f.ad.n.i. communi
scientiæ notione, quia si
scinderet aliqs.p.f.in pun
cto.c.ita.vt.c.f.æquale es-
set ipsi.q.i.proportio.c.f.
ad.q.i.esset vt ea, quæ est.
o.f.ad.n.i. (hoc est nulla)
sed
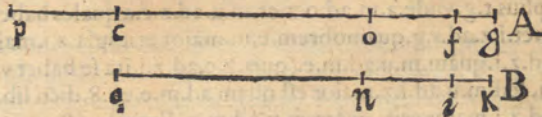
sed proportio.p.f.ad.q.i.maior esset ea, quæ est.c.f.ad.q.i.ex.octaua lib.quinti, vn-
de ex.12.eiusdem lib.maior esset.p.f.ad.q.i.quàm.o.f.ad.n.i.ex.33.igitur eiusdem,
maior erit proportio.p.o.ad.q n.quàm.p.f.ad.q.i. Sic quoque se habebunt ad inui
cem velocitates,quod est propositum. Cum autem proportio.p.o.ad.q.n.maior sit,
quàm.p.f.ad.q.i.permutando igitur maior erit propor.tio.p.o.ad. p.f.quam.q.n. ad.
q.i.aut euersim maior erit proportio.q.i.ad.q.n.quam p f.ad.p. o.vnde si proportio
p.f.ad.p.o. esset ac ea, quæ est.o.g.ad.f.g.non esset.q.i.ad.q.n.ut est.o.g.ad. f. g. aut
vt.n.k.ad.i. k. quod idem
est, de quibus quidem re-
bus, exemplis propositis
quinto capite mētionem
feci.

Velocitatibus autem se-
quentibus pondera, sequi
tur proportionem veloci-



citatum duorum corporum hetereogeneorum eandem non esse per diuersa media,
contra id,quod sequeretur si Aristotelis opinionem.8.cap. lib. 4. physicorum re-
ciperemus.

## *Longe aliter ueritatem se habere quam Aristoteles doceat in fine libri septimi physicorum.*

## CAP. XII.

NOn tam facile est assignare proportionem velocitatum duorum corporum na
turalium,quam Aristoteles vltimo cap.lib.7.physicorum putauit.

Quamobrem sint duo corpora.B.et.D.materia magnitudineq; diuersa,pondere
tamen,& figura similia,& proportio resistentiarum,quas recipiunt à medio dū mo-
uentur,sit.ut.o.i.ad.a .e.denotentur deinde velocitates totales absque vlla resisten-
tia ab.a.u.et.o.c.quæ æquales erunt ad inuicem per communem scientiam ex sup-
posito,sint alia deinde duo corpora.V.et.M.eodem modo se habentia ut prima.B.
et.D.in eodem medio,sed ex diuersa materia ab ea, quæ est illorum duorum corpo
rum, magnitudine tamen & figura ijsdem similia : significentur quoque eo-
rundem resistentiæ per. t.s.et.n.r.& eorundem velocitates à nulla ex resistentijs di-
minutæ,per.n.x. et.t.g.vnde.n.r.æqualis erit.a.e.et.t.s.ipsi.o.i.et.n.x.ipsi.t.g:n.x. ta-
men et.t.g non erunt equalia.a.u.et.o.c. Sed exempli gratia,ponamus ea esse mi-
nora. Supponamus nunc.e.u.velocitatem esse quæ remanet ipsi. B. cum applicata
erit resistentia.a.e.dicto corpori.B.quæ diminutam facit totam.a.u.per.a.e.sitq;.i.c.
ea,que remanet ipsi.o.c.corporis.D.et.r.x.ea,quæ remanet.n. x.corporis.V. et. s. g.
ea,quæ est ex.t.g.corporis.M.Vnde communi omnium cōsensu assequemur.e.u. ma
iorem futuram.r.x.et.i.c.ipsa.s.g.Scindatur deinde.a.m.ad equalitatem.n.x.et. o.z.
ipsius.t.g.vnde.a.m.ad.o.z et.m.u.ad.z.c.æquales habebimus,ut quoque.e. m.ad.r.
x.et.i.z.ad.s.g.quamobrem.e.m.maior erit ipsa.z.i.maior igitur erit proportio. z.c.
ad.z.i.quàm.m.u.ad.m.e.(quia.z.c,ad.z.i.ita se habet vt.m.u.ad.i.z.ex.7. lib. quin-
ti, sed.m.u.ad.i.z.maior est quam ad.m.e.ex.8.dicti lib.vnde ex.12.eiusdem.z.c.ad
ad.z.i.maior erit, quàm.m.u.ad.m.e. Ergo ex.28. maior proportio erit. c.i. ad. z.i.

quam

quam.u.ad.m.e.& ex.27.maior erit proportio.c.i.ad.u.e.quam.z.i.ad.e.m.ideſt.s.g
ad.r.x.quod Ariſtoteli in mentem non venerat. Alijs quoque modis idem proba-
ri poteſt, vt ſi diceret aliquis, maiorem proportionem eſſe.e.m.ad.m.u.quam.i.z.ad
z.c.(quia.e.m.ad.m.u.eadem eſt ratio vt ad.z.c.ex.7.quinti, ſed proportio.e.m.ad.
z.c.maior eſt quam.i.z.ad.z.c.ex.8.eiuſdem, ergo ea, quæ eſt.e.m.ad.m.u.ex.12.ma
ior erit, quam.i.z.ad.z.c.)vnde componendo, ea quæ eſt.e.u.ad.m.u.maior erit illa,
quæ eſt.i.c.ad.z.c.& permutãdo, quam ea, quæ eſt.e.u.ad.i.c.ea, quæ eſt.m.u.ad.z.c.
& ex.33.quinti, ea, quæ eſt.e.m.ad.i.z.maior erit ea, quæ eſt.e.u.ad.i.c.
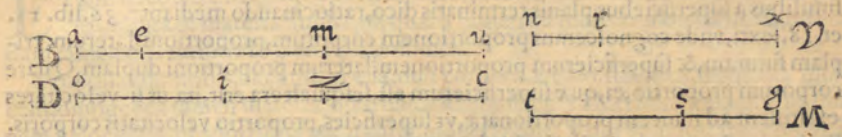


## Quid ſequatur ex ſupradictis.

### CAP. XIIII.

EX præcedenti capite manifeſtè depræhenditur, in vniuerſum Ariſtotelis opi-
nionem veram non eſſe in prima parte vltimi capitis.lib.7.phyſicorum; quia
in eo loco ſupponens ipſe corpus.B.præcedentis capitis eſſe dimidiam partem ipſius
D.quantum ad aream corpoream ſpectat(ſunt tamen pondere ad inuicem æqualia)
ait.B.futurum duplo velocius ipſo.D.Ego verò præcedenti capite accepi. e. u. pro
velocitate reſidua corporis.B.(ſubtracta ea tamen parte, quam ei reſiſtentia adimit,
quæ erat.e.a.)et.i.c.pro ea, quæ eſt corporis.D.et.r.x.pro ea, quæ eſt corporis. V.et.
s.g.pro ea, quæ eſt corporis.M.Dicat nunc Ariſtoteles, quę nam harum duarum pro
portionum dupla erit? quia ſi earum aliqua talis erit, alia nullo modo eſſe poterit,
vt iam oſtendi, etiamſi duo corpora.V.et.M.eaſdem conditiones habeant, quas.B.
et. D.Ratio autem, quæ Ariſtotelem induxerit ad illud credendum, nulla alia eſſe
potuit, quàm quod putarit reſiſtentias proportionatas eſſe magnitudinibus corpo-
reis, ideſt quemadmodum.B.erat corporaliter dimidia pars ipſius.D. ſic etiam habe
ret medietatem eius reſiſtentiæ, quam habuiſſet corpus.D. Quod etſi verum eſſet,
non tamen ſequeretur neceſſariò in quibuſlibet corporibus futuram velocitatum
proportionem eandem, quæ reſiſtentiarum eſt, vt ſuperiore capite oſtendimus.

## Num rectè ſenſerit Philoſophus reſiſtentias proportionales
eſſe cum corporibus mobilibus.

### CAP. XV.

QVòd Ariſtoteles crediderit reſiſtentias proportionatas eſſe corporibus, erra-
uit. Si ſuperficies ijſdem proportionatæ eſſent, dubium non eſt, quin
reſiſtentiæ quoque ipſæ, ijſdem proportionatæ exiſterent, ſupponendo eas ſimiles
ſitu, dum eadem corpora mouerentur. Sed eadem proportio non eſt inter ſuperfi-

Z      cies,

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cies,& quæ inter corpor a reperiũ: Ariſtoteles igitur in eo defecit. Quòd autē inter ſuperficies non eadem ſit proportio, quæ inter corpora extat, ſi primo ad ſphæricas mentem verterimus, intelligemus proportionem eam,quæ inter duas ſphæras reperitur triplam ſemper exiſtere ei, quæ eſt inter ipſarum diametros ex vltima.12. libr. Euclid. Eſt autem proportio, quæ eſt inter ſuperficies ſphæricas æqualis ei, quæ eſt ipſorum circulorum maiorum ex.16.lib.quinti,cum ex. 31. primi de ſphæra & cylindro Archimedis, omnis ſphærica ſuperficies quadrupla, ſit maiori circulo ipſius ſphæræ, ſed proportio, quæ eſt inter dictos circulos, eſt dupla ei,quæ eſt inter eorũdē diametros ex.2.lib.12.Euc.ergo ,pportio,quæ eſt inter corpora,ſeſquialtera erit ei,quæ eſt ſuperficierum,& non æqualis,ut Ariſtoteles putauit.Idem de corporibus ſimilibus à ſuperficiebus planis terminatis dico,ratiocinando mediante. 36.lib. 11. et.18.ſexti,vnde cognoſcemus proportionem corporum,proportioni laterum,triplam futuram,& ſuperficierum proportionem,laterum proportioni duplam.Quare corporum proportio,ei,quæ ſuperficierum eſt,ſeſquialtera erit,ita ut ſi velocitates extitiſſent ad inuicem proportionatæ,vt ſuperficies,proportio velocitatis corporis. B.ei,quæ eſt corporis.C.fuiſſet ſubſeſquialtera proportioni corporum,& non æqualis eidem.

*Idipſum aliter demonſtratur.*

## CAP. XVI.

ALio quoque modo probari poteſt non eſſe in vniuerſum verum id,quod Ariſtoteles in prima parte capitis vltimi lib.7.phyſicorum ait, ſic ſcribens. Si.A.quidem ſit id quod mouet.B.verò id quod mouetur,et.C.ſit longitudo per quam, et.D.tempus in quo eſt motum, in tempore nimirum æquali, potentia æqualis.A.dimidium ipſius.B.per duplum mouebit ipſius.C.per ipſum autem.C. in dimidio temporis.D.ſic enim erit rationis ſimilitudo.

Sit ergo corpus.o.ſeptimi capitis pondere æquali corpori.u.eiuſdem capitis, ſed area corporea minusipſo.u. pro medietate. Simile tamen figura. Imaginemur nũc tertium aliud corpus omogeneum ipſi.u.quod ſit.i.magnitudine & figura ſimile ipſi o.vnde minor erit ipſo.u.pro media parte,& hanc ob cauſam ipſum.u.erit duplo magis graue, quàm ipſum.i.& per conſequens ipſum quoque.o.duplo grauius erit quã ſit ipſum.i. ex. 7. libr. quinti Euclidis. Ipſum ergo corpus. o. duplo velocius erit, quam ipſum.i.ex primo ſuppoſito cap.2.huius lib.Vnde ex.9.quinti,velocitas ipſius i.æqualis eſſet ei,quæ eſt ipſius u. cum Ariſtoteles ſcribat.o. quoque futurum duplo velocius ipſo.u.φ cap.7.huius lib.falſum eſſe demonſtraui.

*De alio Aristo. lapſu.*

## CAP. XVII.

SCribit Ariſtoteles in ultimo cap.lib.7.phyſicorum in hunc modum. Si duo quædam ſeorſum per tantum ſpatium tanto tempore duo ſeorſum pondera mouent,& compoſita per longitudinem æqualem, æqualiuè in tempore,compoſitum ex ponderibus vtriſq; mouebunt,eſt enim in eis eadem ratio.

Quod

Quod in vniuersum nec etiam poteſt eſſe verum in pleno, quia cap. 14. iam pro-
baui, non eandem proportionem eſſe inter ſuperficies corporum, & ipſa corpora.

## Quomodo dignoſcatur proportio uelocitatis duorum ſimilium corporum omogeniorum inæqualium.

### CAP. XVIII.

ETiam ſi reperire in qua proportione motus naturaliter moueantur duo corpo-
ra, figura & materia ſimilia, inæqualia tamen ad inuicem, non facile ſit, oſten-
dam tamen qua ratione id conſequi poſſimus.

Proponantur nobis, exempli gratia, duo corpora. a. et. o. ſphærica, inæqualia inui-
cem, omogenea tamen materia, quorum. a. maius ſit; ſi voluerimus inuenire in qua
nam velocitatis proportione naturaliter mouerentur. Volo vt inquiratur corpus. i.
ſphæricum, alia tamen & diuerſa materia conſtans, ſed pondere æquale corpori. o. &
ſuperficie tam proportionata ſuperficiei corporis. a. quàm eſt ea, quæ eſt ſui ponde-
ris ad pondus ipſius. a. Hoc facto, indagetur, quænam erit proportio inter ſu-
perficies corporum. i. et. o. quæ ſemper dupla eſt, vel ſubdupla ei quæ eſt diametro-
rum; ut iam cap. 15. dixi, & hæc proportio ſuperficierum ſphæricarum ipſi⁹. o. et. i. ſub
trahatur ab æqualitate, quod igitur remanebit, erit proportio velocitatū inter duo
corpora. o. et. i. ideſt inter. o. et. a. vt exempli gratia, ſi proportio ſuperficiei. o. ſuperfi

ciei ipſius. i. ſeſquitertia eſſet, ſub
trahendo eam ab æqualitate, rema-
neret proportio ſubſeſquitertia, vnde
velocitas corporis maioris ( quod in
præſenti loco ſupponitur eſſe. o. ) ei,
quæ eſt corporis minoris, quale eſt
corpus. i. ſubſeſquitertia eſſet ; aut
dicamus quòd. i. eſſet velocius ipſo
o. in proportione ſeſquitertia ex ſe



cundo ſuppoſito ſecundi capitis huius libri. Sed. i. tam velox eſt quàm ipſum. a. ex.
11. cap. ergo proportio velocitatis ipſius. a. ſeſquitertia erit ei. quæ eſt ipſius. o.

## Quam ſit inanis ab Ariſtotele ſuſcepta demonſtratio quod uacuum non detur.

### CAP. XIX.

EX ijs, quæ ſuperius demōſtrauimus facilè cognoſci poteſt irritam eſſe eam ratio
nem, quam Ariſtoteles. 8. cap. lib. 4. phyſicorum ad deſtruendum vacuum, cō
finxit. Vt igitur idem facilius oſtendamus, compræhendamus imaginatione infini-
ta media corporea, quorum vnum altero rarius ſit, in qua placuerit nobis ex propor
tionibus, incipiendo ab uno, imaginemur etiam corpus. Q. denſius primo medio, cu-
ius corporis, totalis grauitas ſit. a. b. & poſitum in ipſo medio, amittat partem. e. b. ip-
ſius grauitatis, & in ſecundo medio amittat. i. b. & ſic per gradus vnde nobis patebit

dicto corpori. Q. Nunquam remansuram suam totalem grauitatem. a. b. in quolibet ex dictis medijs. Nunc si quærat à me Aristoteles proportionem velocitatis corporis. Q. per vacuum ad velocitatem dicti corporis per plenum, ego ei proponam proportionem ipsius. a. b. ad. a. e. exempli gratia, dicens, ꝗ quéadmodum. a. b. maius est ipso. a. e. sic etiam corpus. Q. velocius erit in vacuo, quàm in pleno, dicti autem pleni densitatem appellabimus. e. b. Aristoteles dicet nunc, ꝗ aliud quoddam medium in eadem proportione subtilius ipso. e. b. desumatur; quemadmodum. a. e. minus est ipso. a. b. sit ergo istud. i. b. in quo Aristoteles credit corpus Q. futurum tam velox ut in vacuo, in quo aberrat, ꝗ a proportio velocitatis corporis. Q. in medio. i. b. ad velocitatem eiusdem in medio e. b. ita se habebit, ut. i. a. ad e. a. ex ultimo supposito capit. 2. huius libr. quæ minor esset ea, quæ est. a. b. ad. a. e. ex. 8. lib. quinti Eucli.

$$Q \overset{a}{\vert}\rule{8cm}{0.4pt}\overset{e}{\vert}\ \overset{i}{\vert}\overset{b}{\vert}$$
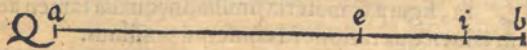
## Non satis dilucidè Aristotelem de loco ratiocinatum fuisse.

### CAP. XX.

QVæ Aristoteles de loco scribit multas in se continent difficultates. Primum, cap. 4. lib. 4. physicorum ait, omne corpus esse in suo proprio loco, supponendo vnum centrum pro loco grauium, et vnam circunferentiam pro loco leuium corporum. Sed quomodo punctum potest esse locus ipsius corporis, cum omni dimensione capacitateꝗ; sit denudatum? vnde si centrū locus esset corporum grauium, omnia dicta corpora grauia, extra proprium locum existerent, quia nullum ex iis est, ꝗ sit in centro. Adde quod neque hoc cum loci definitione ab ipso posita consentiret cum ipse dicat in eodem cap. locum esse superficiem quandam, & non interuallum, licet huiusmodi definitio falsa appareat primo ex incōuenienti falso, quod ipse hinc sequuturum dicit, idest, quod si locus interuallum esset, infinita loca existerent, quod reuera nec ob hanc causam inconueniens existit, quia eodem planè modo quo aliquod corpus potest esse infinita corpora, (quod ipse diceret in potentia) sic etiam interuallum aliquod posset esse infinita interualla. Cum autem dicat superficies corporis ambientis esse locum eius corporis, quod continetur, cogitur dicere lineam, quæ circundat superficiem, superficiei locum esse, & puncta ipsius lineæ, quod reuera absurdum est. Locus corporis est interuallum illud eadem magnitudine & figura, qua corpus ipsum præditum est, quod si non esset, sed esset superficies, quemadmodum Aristoteles voluit, maximum inconueniens sequeretur, scilicet æquales locos capere inæqualia corpora, aut corpora æqualia, locos inæquales occupare, quod scitu facillimum est, cum Theon super Ptolomęi Almagestum iam probarit sphæricam superficiem maius interuallum corporeum continere, quàm aliam quâuis superficiem dictæ sphęricæ æqualem, vnde possent facilè reperiri duo loci, quorum alter millies altero maior esset, capaces tamen corporum æqualium, aut reperiri duo corpora, quorum alterum millies maius esset altero, quę tamen corpora apta essent ad occupandos locos ęquales, quamuis Aristoteles dicat, locum, neque maiorem neque minorem esse debere locato. Sed interualla corporea ęqualia à quauis figura terminata, continebunt semper corpora ęqualia. Corporeum igitur interuallum est

reuera

reuera locus corpori adęquatus, cum corpus in interuallum superficiale non intret,
quamuis interuallum corporeum ingrediatur. Et hoc modo nullũ est corpus, quod
in mundo aut extra mundum ( dicat autem Aristoteles quicquid voluerit ) locum
suum non habeat.

## Utrum bene Aristoteles senserit de infinito.

### CAP. XXI.

TRactans Aristoteles in fine quinti cap. lib. 3. physicorum de infinito ait, impos
sibile cum sit inuenire locum infinitum, & omne corpus in loco cum sit, impos
sibile quoque esse in rerum natura aliquod infinitum corpus reperiri. Omittamus
quòd cum Aristoteles debuerit beneficio loci destruere infinitum, ordine peruerso
de infinito prius, quàm de loco disputationem instituat; sed dicamus ipsum intelli-
gere de infinito corporeo, & cum probauerimus corporis locum esse corporeum in
teruallum, non autem superficiem, neque opus sit in definitione interualli mentio
nem aliquam facere terminorum, vnde ipsum infinitum esse potest, neque aliqua ra
tione de hac re dubitari potest; hoc modo nullum inconueniens sequeretur, quòd
extra cęlum reperiri possit corpus aliquod infinitum, quamuis, id ipse nulla euiden-
ti ratione inductus perneget. Sensit quoque, absque eo, ꝗ aliquam rationem propo
nat, aliquid extra cœlum reperiri quemadmodum apparet ex fine cap. 9. lib. primi
de cœlo, cum etiam ait cap. 8. lib. 8. physicorum, infinitas partes alicuius continui es-
se solum in potentia, non item in actu, hoc non est illico concedendum, quia si omne
totum continuum, & re ipsa existens, in actu est, omnis quoque eius pars erit in actu,
quia stultum esset credere, ea quæ actu sunt, ex ijs, quæ potentia existunt, componi.
Neque etiam dicendum est continuationem earundem partium efficere, vt poten-
tia sint ipsæ partes, & omni actu priuatæ; Sit exempli gratia linea recta. a. u. continua
quæ deinde diuidatur in puncto. e. per æqualia, dubium non est, quin ante diuisioné,
medietas. a. e. tam in actu(licet coniuncta cum alia. e. u.) reperiretur, quàm totum. a.
u. licet à sensu distincta non esset. Idem affirmo de medietate. a. e. idest de quarta
parte totius. a. u. & pariter de octaua, de millesima, & de quauis, ita vt essentia actua
lis infiniti hoc modo tutò concedi possit, cũ ita sit in natura. Sed peius etiam sensit
Aristoteles eodem loco capitis quinti lib. 3. physicorum, negando infinitum posse
connumerari inter quantitates, dicens vnam aliquam quantitatem intelligi vt cubi
tum, tricubitum, & cætera; vbi non considerat eadem etiam ratione intelligi posse
aliquam quantitatem infinitorũ cubitorum, & in quantitatis definitione nullam es-
se necessitatem terminorum, vt exempli gratia in definitione numeri, non est neces
sitas alicuius determinati numeri, quia multitudo, non minus infinita, quàm finita,
intelligi potest. Vbi posteà cap. 8. libr. 4. physicorum ait nullam esse differentiam
inter infinitum, & vacuum, reuera nihil absurdius hoc dicere fingereue poterat.

*Exagitatur ab Aristotele adducta temporis definitio.*

## CAP. XXII.

CVM senserit Aristoteles tēpus absque motu esse nő posse, ea tamen ab inuicē separans, volés definire tēpus ait, ipsū esse mot⁹ mensurā numerūq̃;.Quæ quidem definitio, natura sua non est bona, quia tempus, neque numerus est, neque etiā est mēsūra motus p se,sed tm̃ p accidēs, quia nihil est, qď numeret aut mensuret aliud,quod non sit eiusdē speciei cū illo quod mēsuratur, aut numero circunscribiť, vt exēpli gratia,nulla vnquā superficies p se numerabit aut mēsurabit lineā,aut corpus;nec linea superficiem aliquā, aut corpus : nec corpus lineā aliquā aut superficiem ; Sed linea lineam mensurabit; superficies superficiem; & corpus corpus ; etiāsi tam vna ex iis quantitatibus quàm altera sit continua. Cum verò motus non sit tempus,neque tempus sit motus,sed inter se maximè differant,sequetur ex iis , alterum nullo modo per se esse mēnsuram alterius,nisi per accidens. Et si alicui videtur , ꝙ ad significandam aliquam quantitatem motus,dicere huiusmodi operationem duarum horarum,aut duorum dierum, aut duorum annorum spatio completam esse,sit ponere tantum tempus: animaduertere debet hoc simpliciter non esse verum , quia horarum,dierum. & annorum interualla,imaginatione concipiūtur vt motus corporum cęlestium,sine quibus,neque anni,neque dies,neque horę existerent, etiā si omnis motus sit(vt ita dicam)locatus in tempore,ut corpus in loco , vnde motus motu, & tempus tempore , non autem aliud ab alio mensuratur . Tempus ex necessitate ( phylosophicè tamen loquendo)res est æterna,motus non item, quia diuersis modis terminari potest & cessare , & interim dum cessabit quiescet corpus , quod primo mouebatur. nihilominus tamen,tempus continuabit cursum suum. Tempus igitur potius locus motus erit dicendum, quàm numerus aut mensura eius,& tale est, vt consumatum uideatur à continuò quodam fluxu vnius instantis , quemadmodum iam dixi in.38.capite meę gnomonicæ,& cum dico ab vno instanti,vnum in specie , & non in numero intelligo , quod à sensibus nostris percipi non potest , neq; etiam notari,quia nouum semper instans nobis occurrit. & si aliquis aliquod exemplū (largo modo ) incompræhensibilitatis ipsius instantis desideraret,imaginetur rotam aliquam albam, in qua sit nigrum aliquod punctum sensibile, aut è contra rotam nigrā imaginetur , in qua sit punctum album, quæ rota velocissimè moueatur ; huiusmodi punctum,nullo modo assignari poterit,magis ab una parte quàm ab altera; immo se se nobis offeret semper in forma lineæ circularis. possumus aliquo modo etiam sumere exemplum à sono,quia omnis chorda cuiuslibet instrumenti musici , dum sonus editur,tremit, unde huiusmodi sonus,appellari potest aggregatum aliquod ex innumerabilibus sonis.eodem modo se habet sonus,quem ędunt campanę, & omnia instrumenta tam naturalia,quàm artificialia,quæ quantò velocius tremūt,tantò acutiorē generant sonum,& quantò tardius,tantò grauiorem. Neque est quòd in admirationem ducamur,quòd sensui unum aliquod continuum appareat id, quod discretorum est multitudo ( non putet tamen aliquis me negare continuitatem succesfiuam ipsius temporis) quod clare cognosci potest à niue, aut à chrystallo , aut à vitro,aut à saccaro in minutissimas partes redacto, quæ continuam aliquam albedinē nobis ad inspiciendum offerunt,quod nihil aliud est,quàm innumerabilis quædam multitudo minutorum reflexorum. Idē dico de sputo, & qualibet spuma,& quanto

to

to minutiora funt corpufcula à quibus vt à fpeculis reflectitur lumen,tantò magis ag
gregatū illud albū apparet. Hæc autê exempla cū fint,nec non largo modo fumpta,
mirū non erit fi claudicare videbūt.Sed ut ad motū , & têpus reuertamur ( quæ funt
cõtinua fucceffiua)Ariftoteles in definiendo tempore,non reduxit in mentem,quod
fcribit decimo metaphyficę et.4.cap. fecundo.libr.de cęlo,omnia videlicet, ab eo ,
quod minimum eft in fuo genere,menfurari,& ex fcipfo in phyficorum libris , tem-
pus non eft de genere motus;ergo eius ipfius rationum ui,tempus non erit menfura
motus,fed motus quidem poteft menfurare motum, videlicet velocior minus velo-
cem,& breuior longiorem;& numer⁹ menfurat numero,& tempus tempore in quan
tum longum eft,aut breue,non in quantum velox , aut tardum; Nullum autem in-
conueniens fequetur fumendo tempus tam proportionale motui, quam locus cor-
pori,quia motus decem milliarium,quæ aliquis vnius horæ fpatio conficiat, erit pro
portionalis corpori denfo,& motus vnius milliaris eadem hora peracti, proportiona
lis erit corpori raro; & quemadmodum corpus denfum occupat minus interuallum
loci,contra quam fiat in corpore raro:fic etiam motus velox breuiori temporis fpa-
tio peragetur,quam tardus.

## *Motum rectum effe continuum,uel diffentiente Ariftotele.*

### CAP. XXIII.

ARiftoteles.8.capi.8. phyficorum ait impoffibile effe aliquid per lineā rectam
nunc vno modo,nunc altero,ideft eundo, & redeundo per dictam lineam in
extremis abfque quiete moueri. Id quod contrà poffibile effe dico. Pro fpecula-
tione cuius rei imaginemur circulum.u.a.n. motu continuo circa centrum.o.in quá
libet partem,aut dextrā,aut finiftrā ferri; & imaginemur pūctum.b.extra ipfum,ubi
magis nobis videbitur, à quo ducantur duæ lineæ recte.b.u.et.b.n.contiguæ ipfi cir-
culo in punctis.u.et.n. Imaginatione quoque inter has duas lineas, alteram quæ fit.
u.n.aut.c.d.aut.e.f.aut.g.h.conftituamus in quali
bet parte , fumemus etiam punctum . a . circun-
ferentiæ dicti circuli , à quo vfque ad.b. lineam.
b.a.imaginemur fixā in.b. fed quod remanat mo
bile,fecundum quod mouebitur punctum.a. vn-
de aliquádo hæc linea erit eadem cum.b.u. & ali
quando cum.b.n.& aliquando ab.b.u.verfus. b.
n.proficifcetur, & aliquando ab.b.n.verfus. b.u.
vt accidit lineæ directionis , & retrogradationis
planetarum,vnde circulus.u.a.n.erit,vt epiciclus
et. b.vt terræ centrum. Clarum nunc erit,quòd
quando linea.b. a.eadem erit cum. b. u. aut cum
b.n.non quiefcet,quia in inftanti reuertetur,quia
b.u.et.b.n.in puncto, dictū circulum tangunt , &
dicta.b. a. interfecabit femper aliquam ex dictis
u.n.aut.c.d.aut.e.f.aut. g. h. quod interfectionis
punctum fit,t. Imaginemur nunc quod fecudū
punctum. t. aliquid per aliquam ex dictis lineis
                                   mouea-

moueatur,clarum erit quod tale aliquid,nunquam quiescet,etiam si sit in quouis ex
tremo.Aristotelis igitur opinio, tuta non est.
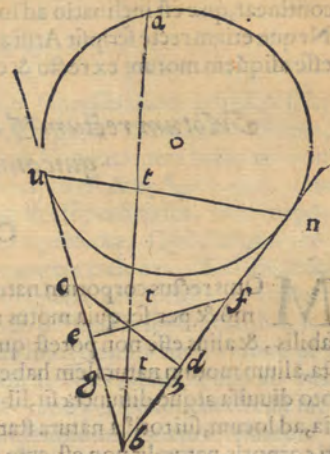
## *Idem uir grauissimus an bene senserit de motibus corporum uiolentis & naturalibus.*

### CAP. XXIIII.

ARistoteles in fine.8.physicorum sentit corpus per vim motum, & separatum à
primo mouente,moueri,aut motum esse per aliquod tempus ab aere, aut ab
aqua,quæ ipsum sequûtur.quod fieri non potest ; quia imo aër , qui in locum deser-
tum à corpore subintrat ad fugandum vacuum,non solum hoc corpus non impellit,
sed potius id cohibet à motu,quia aer per vim à corpore ducitur retrò , & diuisus à
parte anteriori à dicto corpore,resistit similiter,& quantum dictus aer in dicta parte
condensatur,tantum in posteriori rarefit,vnde per vim sese rarefaciens non permit-
tit,vt dictum corpus cum ea velocitate fugiat,cum qua aufugeret, quia omne agens
in agendo patitur.Quamobrem cum aer à dicto corpore rapiatur, corpus quoque
ipsum ab aere rapitur. Huiusmodi autem rarefactio aeris,naturalis non est, sed vio
lenta;& hanc ob causam resistit,& ad se trahit,sed non sufferente natura,vt inter vnũ
& aliud ex dictis corporibus reperiatur vacuum;iccirco sunt hæc semper contigua ,
& mobile corpus aerem deserere cum nequeat,eius velocitas impeditur. Huiusmo
di igitur corporis separatim à primo mouente velocitas oritur à quadam naturali im
pressione,ex impetuositate recepta à dicto mobili, quæ impressio & impetuositas,
in motibus rectis naturalibus continuò crescit, cùm perpetuò in se causam mouétê,
idest propensionem eundi ad locum ei à natura assignatum habeat. Aristo. 8. cap.
primi lib.de cœlo,dicere non deberet ꝗ quantò propius accedit corpus ad terminũ
ad quem,tantò magis sit velox;sed potius, ꝗ quantò longius distat à termino à quò
tantò velocius existit.quia tantò maior sit semper impræssio, quantò magis moue-
tur naturaliter corpus,& continuò nouum impetum recipit,cum in se motus causam
contineat,quæ est inclinatio ad locum suum eundi, extra quem per vim consistit.
Neque etiam rectè scripsit Aristo.9.cap.lib.8.physicorum et. 2.lib. primi de cœlo
esse aliquem motum ex recto & circulari mixtum,ꝗ omninò impossibile est.

## *Motum rectum & naturalem non esse primo & per se quicquid Aristoteli uisum sit.*

### CAP. XXV.

MOtus rectus corporum naturalium sursum, aut deorsum, non est naturalis pri
mò & per se, quia motus naturalis perpetuus est,aut vt melius dicam,inces-
sabilis , & alius esse non potest quàm circularis, nullaꝗ pars cum suo toto coniun-
cta,alium motum naturalem habere potest, quàm eum,qui est totius. si autem à suo
toto diuulsa atque disiuncta sit,liberéꝗ vagetur,spontè , & quàm breuissima potest
via,ad locum,sui totiꝰ à natura statutum proficiscitur. hic motus primò, & per se di-
cti corporis,naturalis non est,cum à causa naturæ suæ contraria, sit generatus, idest,

ab eo,

## DISPVTATIONES. 185

ab eo quod sit extra suum locum, vbi contra naturam suam reperitur. Vnde huiusmodi motus, partim & non omninò, naturalis est. Is autem proprius est & naturalis motus, qui dicti corporis essentiam conseruat. hoc autem non præstat hic rectus, cum destruat, ergò hic motus primò & per se naturalis non est.

*Omne corpus esse in loco proprio graue, ut Aristoteli placuit, non est admittendum.*

### CAP. XXVI.

A Rist.4.cap.lib.4.de cęlo sic scribit.
Suo enim in loco grauitatem habent omnia præter ignem, signum cuius est vtrem inflatum plus ponderis, quam vacuum habere,&c.

Quo in loco, manifestè indicat se causam néc grauitatis, nec leuitatis corporum naturalium nosce, quæ est densitas aut raritas corporis grauis, aut leuis, maior densitate, aut raritate medij permeabilis, in quo reperitur.

Exemplum ꝙ ipse de vtre inflato proponit, debuisset saltem ei oculos ad veritatem, quæ clarissimè fulget, inspiciendum aperire. Verissimum est, vtrem inflatum plus ponderis habere quàm vacuum, aut quando aer in eo non est per vim inclusus.

Ratio autem huius rei est, quia quando inflatus est, ea quantitas aeris, in eum per vim iniecti, minorem occupat locum, quàm si eidem liberè vagari permitteretur, vnde violenter, quodam modo, condensata est, & quia corpus densum in minus denso, semper descendit, & minus densum in magis denso ascendit. Hanc ob causam vter inflatus plenus corpore magis denso, quàm est medium quod eum circundat, descendit, non quia aer in aere, aut aqua in aqua sit grauis.

*Haud admittendam opinionem Principis Peripateticorum de circulo, & sphæra.*

### CAP. XXVII.

C Vm Aristoteles senserit circulum esse figurarum superficialium primã, & sphę rã esse primã corporearũ ꝓ earũ periferias, decipitur. Sunt enim vltimæ, non primæ. Sunt quidem (in quò rectè sentit) perfectè, licet rationem huius rei non nouerit. Nam centrũ cuiuslibet rei, eiusdem rei principiũ est, & eę figurę, quæ ipsum æqualiter circundant, possunt appellari perfectę, siue sint superficiales, siue corporeæ, & è contrà illæ, quæ contrario modo se habent, imperfectæ. Quòd autem perfectum est, licet natura sit primum, est tamen vltimum generatione. Sed quando Aristoteles duas dictas figuras pronuntiauit primas, vt perfectas, prioritate scilicet ea, quæ oritur à perfectione, verum dixit; sed quando de figuris superficialibus loquens, vult circulum esse primum, quia ab vna tãtum linea terminetur; nó minus pro circulo, quam pro oxigonia seu elipsi, aut cucurbitali, aut aliis multis figuris ab vna tantum linea terminatis concludit. Neque etiam hæc ratio perfectionem circuli mó strat, quia aliæ figuræ, à lineis curuis terminatę, eandem conditionem sortiuntur. Circulus sphęraꝗ;, non ex vno solo angulo recto constant, vt idem Aristoteles putat

Aa cap.

cap.4.lib.4.de cęlo,etiam si triangulus ex duobus angulis rectis consurgat, sed sunt figurę infinitorum angulorum rectorum,& hanc ob causam à me dicuntur vltimæ & perfectę, quia infinito nihil addi potest. Numerus angulorum rectorum circuli, est minor duplo infinito per duo infinita angulorum contingentiæ, quæ duo infinita minora sunt quouis angulo acuto rectilineo,& numerus angulorum rectorum solidorū sphęræ,minor est quadruplo infinito per.4.infinita angulorum solidorum cõtingentiæ,quæ.4.infinita,minora sunt quouis angulo solido acuto terminato à tribus planis. Triangulus inter figuras planas superficiales est primus,& circulus vltimus; & pyramis quadrilatera, inter corpora est prima,& sphęra vltima.

## Occultam fuiße grauissimo Stagirita causam scintillationis stellarum.

## CAP. XXVIII.

VBi Aristoteles ait scintillationem stellarum fieri ratione aspectus nostri ob,maximam distantiam,maximum errorem committit, vt etiam facit quum putat visionem fieri extramittendo;contra id,quod alio loco , immo contra veritatem ipsam asseruit. Scintillatio ergo stellarum , neque aspectus nostri ratione, neque alicuius mutationis earundem stellarum,sed ab inæqualitate motus corporum diaphanorum mediorum nascitur,quēadmodum clarè cernitur,quòd si inter aliquod obiectum,& nos,aliquis fumus,qui ascendat,intercesserit,videbimus obiectum illud quasi tremere. Hoc autem tantò magis fiet, quantò magis distabit obiectum ab ipso fumo; vnde admirationi locus non erit,si stellas fixas magis scintillare,quam errantes cernamus. Lumen stellæ ad oculum nostrum accedens,perpetuò per diuersas diaphaneitates penetrat, medio continuorum motuum corporum mediorum, vnde continuò eorum lumen variatur,& hoc in lõginquis magis, quàm in propinquis stellis apparet, quemadmodum ab exemplo de fumo allato , & etiam ab aliquibus vitris ex superficie non plana,sed irregulari constantibus,quilibet cognoscere potest.

## Dari continuum infinitum motum super rectam atque finitam lineam.

## CAP. XXIX.

OMnes hactenus senserunt impossibile esse dari per imaginationē motum continuum & perpetuū super vnam lineam rectam finit:in quo tñ decipiuntur. Imaginemur iõ duas lineas parallelas. a.b. et.t.x.quarū b.a.sit īfinita à qualibet parte,& in ea imaginemur punctum.a.moueri continuò ad quam voluerimus partem, &
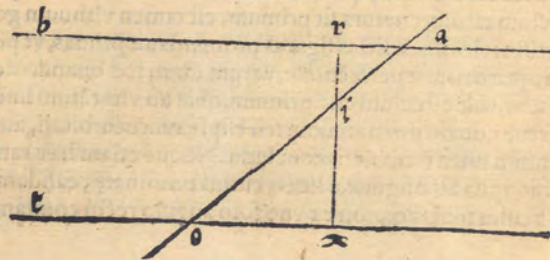
& in linea.t.x.imaginemur punctum fixum, quod sit.c.imaginemur etiam inter.c.⟨et⟩ a.vnam lineam rectam.c.a.& inter duas parallelas dictas.r.x.fixam, & motus puncti sit ab.b.versus.a.ita ut.c.a.secet.r.x.in puncto. i. quod intersectionis punctum mouebitur ab.r.versus.x.continuò, in tempore infinito, neque vnquam idem erit cum puncto. x.

## Non esse solis calorem à motu locali ipsius corporis solaris, ut Aristoteli placuit.

### CAP. XXX.

ID nullo planè modo est admittendum quod Aristoteles credidit calorem solis à motu locali ipsiusmet corporis solaris, & non à lumine, prouenire, quemadmodum manifestè asserit primo metheororum cap.3.circa finem sic scribens.

Vt igitur tepor gignatur atque calor, solis latio duntaxat, satis est efficere, &c. sed cap.7.lib.2.de cœlo sic scribit, Caliditas autem ab ipsis, lumenq́; ideo fit, quia aer ab illorum motione fricatur.

Vbi non solum ostendit se opinari, quòd motus corporum cœlestium sit causa caloris, sed etiam luminis, paulò autem post dicit, superiorum autem corporum vnum quodque fertur in sphæra, vt ipsa quidem non igniantur. Opinio profecto absurda. Nam cùm corpus solare fixum sit in spissitudine sui orbis deferentis, secundum communem opinionem, non mouetur per se, sed accidentaliter, cum scilicet fertur à dicto suo orbe, vnde fieri potest, vt in motu sui orbis, nullum ex orbibus suorum deferentium augis fricet, sed si fricaret, id faceret mediante vno solo puncto, vt cuilibet, aliquantulum in mathematicis versato patet. Quam ob causam, rationi côsentaneum non esset credere, quòd tantùm caloris gigneretur. Quod tamen si possibile esset, quid ergo fricatio superficierum orbis sui, cum iis, quæ sunt deferentium augis efficeret? Quãdo tamen hoc fieret, vt scilicet à fricatione superficierum procederet calor, nil planè discriminis inter hyemen, & æstatem intercederet, nec inter calorem diei, & noctis, nec inter vnam horam, aut alteram; sed secundum Aristotelis rationes, Venus, Mercuriusq́;, magis calefacere quam sol de berẽt, cum ita sint veloces vt ipse Sol, & eodem magis propinqua terræ. Verum Aristotelis tẽporibus, nullũ aliũ planetã quam solem putabãt philosophi supra Lunã esse. Atque etiam côtigeret mense Decembri, quam Iunio, magis inualesceret calor, cum huiusmodi mense sol ad nos propius accedat, quam mense Iunii. per differentiam maiorem diametro regionis elementaris, (nam solaris eccentricitas maior est semidiametro elemẽtaris regionis) non considerans Aristoteles differentiam caloris, quæ nascitur ex eo, q̃ Sol aut altius supra orizontem excurrat, aut infra eundẽ feratur; neque eam, quę à longitudine, aut breuitate diei proficiscitur. Sed quia Aristoteles eodem cap.tertio Metheororum intelligit de motu rapto, idest diurno, siue dicamus vniuersali, hinc sequi deberet, q̃ Sol maiorem caloris uim mense Martij & Septembris, quàm aliis mensibus, profunderet. quia in iisdem temporibus, sol virtute huiusmodi motus velocior existat, quàm alio quolibet tempore anni, cum tunc per æquatorem circuõoluatur. Multa quoque alia incommoda sequerentur si Aristotelis rationes admitteremus. Sed clarè uidemus, mediante reflexione aut refractione radiorum solarium, q̃ vniente sese lumine, vnitur quoque, & augetur calor, atque omnis res ad comburendum apta accenditur, & inflammatur. In lumine igi-

Aa 2    tur

tur continetur calor,& non in motu ipſius ſolis,& ita in lumine ſedem habet,ut ſi ſol
quieſceret,neque in orbe ſuo circumageretur, infeliciſſima eſſet ea regio, in cuius
Zenith ipſe reperiretur.

## Vnde caloris ſolis prodeat incrementum æſtate,et hyeme decrementum.

### CAP. XXXI.

CVm capite ſuperiore oſtenderim calorem ſolis non aliunde, quàm à lumine
prouenire, oſtendam nunc ex ordine, ex quot,quibusq̗; cauſis oriatur magna
differentia eius caloris æſtatis ad hyemem, quarum nonnullæ ab antiquis obſerua-
tæ fuerunt,aliæ autem à nemine, quod ſciam. Sunt autem quinque ad minus eæ cau
ſæ,quarum vna eſt diuturna ſolis mora,tempore æſtatis ſupra orizontem, quæ cau-
ſa ab antiquis poſita,& citata fuit. Aliam quoque huius rei cauſam iidem antiqui
dicebant eſſe propinquitatem ſolis noſtro Zenith, ſed hæc cauſa immediata non
eſt, quia ab ea tres cauſæ immediatæ naſcuntur;quarum vna eſt maior vnio radij re
flexi cum radio incidenti; ſecunda maior quantitas luminis in ſuperficie terræ ;
tertia,minor reſiſtétia vaporum ipſi radio luminoſo facta;quarta verò eſt impreſſio
caloris facta in terra,quæ cum aliis cauſis coniuncta calorem adauget. quæ quidem
cauſæ nemini adhuc,quod ſciam, in mente venerunt. Quòd autem attinet ad ma-
iorem coniunctionem radii reflexi cum incidente,quiſque, uel ſaltem mediocriter
in cathoptricę cognitione verſatus hoc verum eſſe cognoſcet. Vt hoc tamen in-
poteſcat facilius. Imaginemur. q.p.et.b.d.eſſe duas particulas æquales ſuperficiei
ipſius terræ,ſuper quas cadant duo radii luminoſi ſolis.e.q.et.n.d.quorum.e.q.ſit ad
modum obliquus,et.n.d.quaſi perpendicularis, vnde radii reflexi.p.a.et.b.u. aſcen
dent cum angulis æqualibus eis, qui ſunt ſuorum cadentium,cum omnis angulus re-
flexionis ſemper æqualis ſit angulo ſuæ incidentię,vt cuilibet in cathoptrica,vel me
diocriter verſato patet.Mixtio autem primorum obliquorum erit.q.o.p.& ea , quæ
eſt minus obliquorum.b.i.d.quorum duorum triangulorum nullus unquam erit,qui
dubitare poſſit.q.o.p.non eſſe minorem.b.i.d.cum anguli.q.et.p.trianguli.q.o.p.a-
cutiores ſint angulis.b.et.d.trianguli.b.i.d.ex ſuppoſito. Quòd uero attinet ad ma-
iorem quantitatem luminis ſuper terræ ſuperficiem; Imaginemur radium.a.q.cuius
reſpectu etiam imaginemur duos ſuperficiei terræ ſitus, quorum vnus ſit.q.o:cui di-
ctus radius ſit perpendicularis,& alter.q.p.cui radius.a.q.ex obliquo incidat. Ima-
ginemur ergo triangulum.q.o.p.cuius angulus.o.rectus eſt ex ſuppoſito,unde. q. o.
minor erit.q.p.ex.18.primi Euclidis.hinc ſit,vt ſuper. q.o.cadat vniuerſum lumen,
quod ſuper.q.p.diffunditur. Sit.q.u.æqualis.q.o.& ſit imaginatione protracta.u.n.
æquidiſtans.p.o.a.vnde.q.u.illuminata erit à radio.n.q.minore radio.a. q. ergo mi-
nus calida erit ſuperficies.q.u.ipſius terræ,quàm.q.o. quia maius lumen in ſe maio-
rem calorem includit: quod manifeſte apparet in radiorum vnione mediante refle-
xione, aut refractione. Sed quod attinet ad minorem reſiſtentiam vaporum ad ip-
ſum radium luminoſum, etſi primo capite meæ Gnomonicæ leuiter id attigerim, ni
hilominus tamen,& idem ipſum hoc loco proponam. Denotetur,exempli gratia,
ſuperficies terræ ab.o.g.et ea,quæ eſt vaporum ab.n. a. ſupponatur etiam ſol in ſitu.

q.qui

q. qui sit Zenith pūcti.o.& etiã in.p.ipsi orizóti propinquus, aut extra Zenith, cuius duos radios.q.o.et.p.o.Imaginemur, quorum duæ partes.a.o. et. n. o. erunt aliquo modo ab ipsis vaporibus offuscatæ, sed.o.n.breuior est.o.a.ex.7.lib.3. Eucli. minorem ergo resistentiam habebit à vaporibus sol in Zenith, quàm extra eundem commorans, & quantò longius erit idem ab ipso Zenith, tanto maiorem resistentiam à dictis vaporibus inferri ex eadem.7. lib.3.Eucli. dicemus.
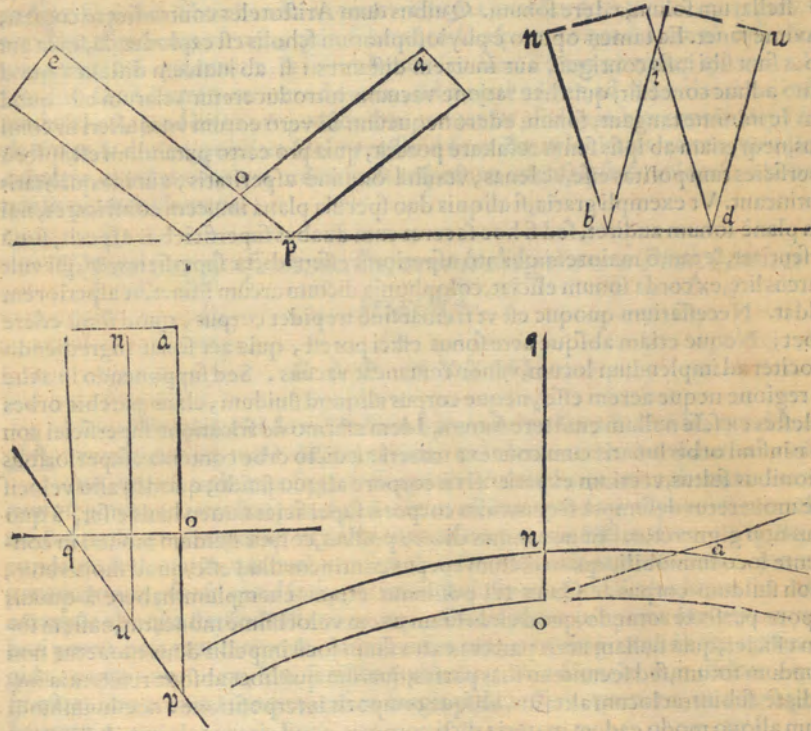


*Nullum corpus sensus expers à sono offendi, præterquam Aristoteles crediderit.*

## CAP. XXXII.

POsse sonum corpus aliquod, quod sensu sit destitutum , vt Aristoteles.9.cap.libr.2.de cęlo putauit, offendere, est falsum.

Corpus enim non nisi à corpore potest lædi, non ergo à sono, cum sonus corpus non sit. Sed aer,& ignis, cum è contra sint corpora, hoc facilè præstare possunt implendo aliquem locum velociter ad excludendum vacuum; vnde generatur sonus. Quod hucusque à nemine animaduersum fuisse comperio.

Pyta-

*Pytagoreorum opinionem de sonitu corporum cœlestium non fuisse ab Aristotele sublatam.*

## CAP. XXXXIII.

SEnserunt Pythagorici orbes cęlestes dum circunuoluuntur, non autem corpora stellarum solum, ædere sonum. Quibus dum Aristoteles contradicere cogitat, maximè fauet. Ea tamen opinio è phylosophorum scholis est explodenda, quia aut orbes sunt sibi ipsis contigui, aut inuicem distantes : si ab inuicem distant ( quod nemo adhuc concessit, quia hac ratione vacuum introduceretur ) clarum est, quod cum se minime tangant, sonum edere nequeunt: Si verò eorum vnus alteri sit conti guus, neq; etiam ab ipsis sonus resultare poterit, quia pro certo putandum est, ipsorū superficies tam politas esse, ac lenas, vt nihil omnino asperitatis, aut inæqualitatis contineant. Vt exempli gratia, si aliquis duo specula plana inuicem confricaret, nul lum planè sonum audiret, sed si hoc faceret cum duabus superficiebus asperis, sonū persentiret, & tanto maiorem; quantò asperiores essent dictæ superficies, & qui vult vt arcus lirę, ex corda sonum eliciat, cōlophonia dictum arcum illinet, vt asperiorem reddat. Necessarium quoque est vt tremat siue trepidet corpus, quod sonū edere debet; Neque etiam absque aere sonus effici potest, quia aer sonat ingrediendo velociter ad implendum locum, vt non remaneat vacuus. Sed supponendo in æthe rea regione neque aerem esse, neque corpus aliquod fluidum, clarè patebit orbes cœlestes ex sese nullum emittere sonum. Idem affirmo de fricatione superficiei con cauæ infimi orbis lunaris cum conuexa materiæ à dicto orbe contentæ; superioribus rationibus fultus, vt etiam experientia à corpore aliquo fluido, quod in alio velocis simè moueretur desumpta fretus, cuius corporis superficies tamen lenis esset, à quo sonus non gigneretur. Et non minus dicere possum, corpus fluidum moueri in con tinente loco immobili, quam dictum corpus continens illud esse, quod moueretur, & non fluidum corpus. Cuius rei possumus etiam exemplum habere à quouis corpore perfectè rotundo, quod circa suum axem velocissimè moueatur, nullum so num efficiet, quia nullam aeris partem extra suum locū impellit dum mouetur non secundum totum, sed secundum suas partes, quarum quælibet absque resistentia im mediatè subintrat locum alterius, absque temporis interpositione. nec huiusmodi locum aliquo modo eadem materia dicti corporis, quod circunuoluitur: destitutum dimittat. Sed si Pythagorici de alia quadam harmoniæ specie ab ea, quæ est sono rum, vt à diuersis velocitatibus motuum, aut à diuersis magnitudinibus aut distantiis, aut stellarum influxibus intellexissent, rectè sensissent ex parte, non autem omnino, quia ea harmoniam efficere nequeunt, quæ ad inuicè secundum interualla harmoni ca proportionata non sunt, vt sunt dupla, sesquialtera, sesquitertia, sesquiquarta, ses quiquinta, supertripartientia quintas, superbipartiētia tertias, & quę ab ijs dependēt idest coniuncta sunt cum duplis; de consonantijs loquendo. de dissonantiis idem di co, quæ harmonicis inseruiunt modulationibus, vt sesquioctauū, sesquinonū, sesqui quindecimū, sesquiuigesimūquartū, sesquioctuogesimū, & superbipartiens vigesi masquintas. Verū quidem est nonnulla harmonica interualla in aspectibus cōperta fuisse, vt Ptolomeus ostendit, & alii quoque asserunt. inest tamen huic rei nonnihil difficultatis. vt exempli gratia, si subtrahamus diatessaron extra diapason, remanet diapente, & si à diapente subtrahamus semiditonum, remanet ditonum ( quæ duæ

conso-

consonantiæ,eum habent respectum ad inuicem , quem habent diatessaron,& dia-
pente , quia quemadmodum semiditonum  , & ditonum simul coniuncta,compo-
nunt diapente, sic diatessaron,& diapente simul vnita componunt diapason;& qué-
admodum terminus,qui diuidit diapason in diatessaró,& diapente,est mediator har
monicus inter extrema diapason diuisi,sic etiam terminus,qui diuidit diapéte in se-
miditonum,& ditonum,mediator est harmonicus inter extrema ipsius diapente diui
si) subtrahendo deinde à diapason semiditonum remanet exachordum maius, & ab
eodem diapason subtrahendo ditonum remanet exachordum minus , quę quidé nó
accidunt aspectuum circulo, quia subtrahendo aspectum quadratum ab opposito ,
remanet aliud quadratum, & subtrahendo sextilem à trino remanet quoque alius
sextilis.  Quòd autem attinet ad motus, ad magnitudines,ad distantias,& ad influ-
xus,nihil est,quod hisce proportionibus conueniat , sed quia hæc omnia depédent
ab ísinita,& diuina ,puidétia Dei,necessariò fit vt istæ velocitates,eæ magnitudines,
distantiæ,& influxus,talem ordinem,& respectum inter seipsa,& vniuersum habeãt,
qualis perfectissimus sit.

## *De raro et denso nonnulla, minus diligenter à Peripateticis*
*perpensa.*

### CAP. XXXIIII.

ANtiqui Peripatetici de videndo in hyeme animalium halitu.Id, quod in æsta
te non euenit, malè disputauerunt, quia hoc nascitur à condensatione hali
tus, quę ab ambiente frigore fit.quia halitus is ab ore,aut naso animalis exiés
non est purus aer attractus primò,sed mixtus est cum quodam vapore excrementi-
tio,& subtili,quo semper ab ea parte euacuat corpus,qui statim ab aere frigido cir-
cundatur,& densatur, quam ob causam ab ipso ea luminis pars reflectitur, quæ eum
penetrare non potest, quod in hypocaustis, huiusmodiq; calidis locis non fit.  Idem
exemplo ab aqua statim à cisternis,aut profundis puteis in hyeme extracta compro
bari potest,quia tunc temporis, huiusmodi aqua , cum magis calida sit, quàm fri-
gida, emittit vaporem, qui facillimè videtur , ob rationem iam dictam, quod
in æstate non cernitur in aqua , etsi ea magis calida esset , quam ea , quæ in hyeme
hauritur.

Ratio autem,quàm ab antiperistasi desumptam citarunt iidem ad inquirendum,
cur aqua subterranea magis calida,aut minus frigida,hyberno tempore , quàm ea,
quæ est supra terram sit,vana est,quia hoc non aliunde fit,quàm ab eo,ǫ terræ por-
ri à frigoris siccitate sint clausi,vnde vapores & exalationes non tam facilè exire pos
sunt. quamobrem calefiunt subterraneæ partes.  Fimum,foenum,frumentum hac in
re sunt nobis exemplo,in quibus sępissimè visum est ignem accendi.

Priore illa quoque ratione de antiperistasi dicta,volunt philosophi maiorem ca-
liditatem hyęme,quàm ęstate in animalium stomacho contineri, non animaduerten
tes siccitatem, frigiditatis partes superficiales corporis,restringétem,sanguinem ver
sus originem suam impellere, qui in eo loco copiosior cum sit,eas partes tunc tem-
poris calefacit magis.

Neque etiam ijdem nouerunt causam,vnde fiat,ut in æstate impleto vase vitréo,
aut argenteo , aut ex materia non porosa constante,aqua frigida,vas sudet, quod
tempo-

tempore hyemis,non nifi in calidis locis euenit,quem fudorem , dicebant ipfi , effe
eandem aquam,quæ per porros vafis exiret, quod falfiffimum eft,quia fi per porros
aqua frigida exiret multò magis exiret calida , cum fubtilior fit,& ad penetrandum
aptior. Sed hoc non aliunde oritur,quàm à condenfatione aeris vas circundantis ,
caufata à frigiditate vafis refrigerati ab aqua , quemadmodum tempore hyberno
clarè videmus mane fuperficies interiores vitri feneftrarum fudare,quia extrinfecû
frigus refrigerando vitrum,intrinfecum aerem fibi contiguum congelat .
    Neque filentio inuoluendum eft,nec Ariftotelem,neque alium ex fuis fautoribus
animaduertiffe denfum,& rarum effe caufam ventorum.Rarû autem & denfum, me
diante calore & frigore fit , & fi à partibus , in omogeneis, licet argumétari; de toto
deducat confequentiam qui velit,obferuans in calidis æftatis diebus,dum aliqua nu
becula ad Solem cooperiendum incedit,ibi ftatim agitationem aeris fentiri;ea verò
nubécula prætergreffa cum fuerit,& in ea parte , aer ad priftinam raritatem caufa-
tam à calore Solis redierit, quiefcit;huiufmodi autem aeris agitatio,à nulla certè ex
halatione proficifcitur,fed à motu folum locali, quem dum condenfatur, facit. Om
ne denfum natura fua frigidum eft; omne rarum calidum, & è conuerfo. Et frigida
aura,quæ à flabellis caufatur,non folum à nouo aere qui nos tangit, fed etiam a denfo,
fo, quod in agitatione eiufdem aeris fit,nafcitur.

    Cum autem de raritate & denfitate difputationem fufceperim , non fine ratione
mihi videt illorû opinioné explodédâ effe,qui Lunę maculas nó aliud effe dixerunt,
quàm aliquas partes rariores aliis eiufdem Lunæ partibus,non obferuantes rarum,&
denfum,proportionabilia lumini,quod ab huiufmodi corporibus reflectitur, non ef
fe. quia corpus aliquod rarum aliquando aptum erit ad reflectendum maius lumen ,
quàm corpus minus rarum ut manifeftè apparet à nubibus reflecti lumen : quod
ab aere non fit. Non defuerunt qui contrarium dixerunt,ideft,eas Lunę partes, den
fiores effe;neque unquam aliquis fuit qui de diaphano,aut opaco mentionem fece
rit,quia melius eft credere, eas partes diaphanas,fiue perfpicuas magis effe, quàm a-
lias,quę per aliquod fpatium, folis radio ingreffum permittant, & alię partes cû fint
opacæ ipfum à fuperficie reflectant. diuerfa tamen ratione à fpeculo , cum in pleni-
lunio tota ferè Lunę pars illuminata cernatur,quamuis dictum lumen extenfiuè & in
tenfiuè fit minus eo, quod ipfa in nouilunio recipit. Indignum autem mihi videtur
ijs refpondere,qui dixerunt huiufmodi maculas,terræ vmbras exiftere,cum craffiffi-
mæ ignorantiæ tenebris fint circunfufi,vt etiâ fuit Cornelius Agrippa,qui primo de
occulta philofophia dicens fe noffe modum quendâ naturalem à Pythagora inuen-
tum,quo in Luna id totum,quod ipfe fuper fpeculum fcripfiffet,videretur . oftendit
manifeftè fe ignorare luminum vmbrarumq́; naturam.quia nulla vnquam vmbra ge
nerari poteft à corpore,quod aut opacum non fit , aut officio opaci non fungatur ,
vt nunc dicemus de diaphaneitate aquæ. Neque corpus opacum illuminatum adû-
brare poteft,nifi opacum illud in linea recta fitum obtineat, quæ inter lucidum & il
luminatum extenditur. Neque etiam refpondebimus ijs,qui fentiunt quotiefcun-
que nulla effet terra,fed totus hic globus effet aqua,toties non futuram eclipfim lu-
narem,ratione diaphaneitatis aquæ. Quod falfiffimû eft, quia omne corpus fphę-
ricum quantumuis diaphanum fit,dummodo fit denfius aere , luminofos radios re-
frangit,& eos ad inuicem interfecare facit,qui deinde vltra interfectionis punctû di-
fgregantur, ita vt amittant illuminationis actum. Adde ꝗ etfi huiufmodi corpus
aqueum,fphęricum non effet,fed cubicum,illud fuper aliquâ ex eius fuperficiebus
ad angulos rectos radius folaris percuteret,non eum tamen penetraret, quia dictus
radius perpetuò debilitatur,& eò magis,quo maiorem profunditatem in diaphano
                                                                         eius

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eius corporis, quod sit densius aere acquirit, nec totus radius vnquam dictum corpus
ingreditur, cum ab eius superficie magna pars reflectatur. Resistit ergo huiusmo-
di corpus lumini, & quantò magis spissum aut profundum existit, tantò validius resi
stit. Habemus huius rei testes, piscatores vnionum, in ijs mundi partibus, quæ pau-
cis ab hinc annis Hispanorum opera nobis innotuerunt, qui affirmant ad maris vsq;
fundum lumen Solis non peruenire.

Immediata ratio, cur nebulæ in ijs locis in quibus cóspiciuntur permaneant, & uúc
altiores, nunc verò depressiores cernantur, nòn ea est, quam Aristoteles cap.3.lib.1.
metheororum proponit, sed inde oritur, quòd sint eædem densiores ea parte aeris,
quæ ipsis supereminet, & rariores e a, quæ ijsdem subiacet. Quòd autem alicuius cor
poris densitas maior ea, quæ est medij, in quo reperitur, causa sit, vt ipsum corpus de
scendat, & maior raritas eiusdem corporis, ea, quæ est quoque me dij, efficiat, vt di-
ctum corpus ascendat, iam Archimedes in lib.de insidentibus aquæ docuit.

Rectissimé instituit natura, vt corpora densiora versus loca angustiora, & minora
(intelligendo ea loca orbicularis figuræ) quæ ad centrum propius accedunt, & rario
ra ad ampliora loca, & maius spatium occupantia, sese reciperent. tum quia eadem
quantitas materiæ condensatæ, eget minori loco quam rarefacta, cú etiam, quia cú
corpus densum non ita ad velocitatem motus localis, vt rarum, idoneum sit, ad eas
partes accedat, quæ motibus tardioribus magis sunt aptæ, corpora autè rara ad eas,
quæ velocioribus motibus sunt aptiores sese transferant. præterquam ꝙ reuera ap-
pareat pro maiori parte, corpus magis densum, minus diaphanum; aut magis opacú
futurum, quàm rarum, licet sæpissimè videamus contrarium, vt superius innuimus.
est tamen naturale proportionatumq́; magis opacum denso, & diaphanum raro,
quàm è contra. Quamobrem summa ratione inducta natura voluit, vt corpora ma
gis opaca, aut minus diaphana, magis vicina centro colligantur, vt spatium, quòd re
manet, absque vllo impedimento à radijs solaribus penetrari possit. Tres autem eæ
causæ, quas hoc loco posui, propriæ sunt, immediatæ, & per se, ex quibus fit, vt corpo
ra densiora descendant, & rariora ascendant in medijs minus densis, aut minus raris
dictorum corporum, quæ à nemine, ꝙ sciam, hucusque propositæ fuerunt.

Qui autem asserunt cucurbitæ, quam apponunt chirurgi, effectum ex eo nasci, ꝙ
calidi sit attrahere, valdè aberrant à vero quia hoc, non nisi à raro, & à denso imme-
diatè, à calido & frigido causatis efficitur, quia aer in cucurbita rarefactus à calore
& per consequens dilatatus, statim vt à dicto calore deseritur, iterum condensatur &
tantò citius, quantò aer ambiens frigidior existet, & quia eadem materia cum con-
densata fuerit minorem semper occupat locum, restringens igitur sese in cucurbi-
ta aer dum condensatur, necessariò fit, ne ulla, scilicet pars vacua remaneat, ꝙ cum
alius aer ingredi cucurbitam nequeat aliud corpus ingrediatur. Idem cum amphora
in qua nullum aliud, quam aëreum sit corpus experiri possumus, si eã ad ignem pri-
mò calefactam, deinde cú ore in amplo aliquo cyatho, aut alio vase. vino, aut aqua
pleno vbi videbimus huiusmodi liquorem statim sursum ferri, quia dum calefit am-
phora, rarefit quoquè aer qui in ea continetur, & quia rarescit dilatatur, & quia
dilatatur, eget maiore loco; & ideo magna pars eius foras exit; Cum verò ea aeris
portio, quæ intus remanserit, iterum condensatur ob defectum caloris, restringitur,
minoriq́; indiget loco; Quod cum ita se habeat, necessarium est, ne aliquis locus va
cuus remaneat, vt aliud quoddam corpus ingrediatur, cum ad ingrediédum aeri non
patuerit aditus. quòd si corpus admodum non erit fluxile, aut humidum, ita vt ingre
di amphoram possit ita amphorę hærebit, vt non cito diuelli possit, & eo modo sępe

                                                        B b        cum

cũ admiratione videm⁹ fragile vas vitreũ magnũ, & graue lapideũ corpus eleuare. Sed vt ad densũ & ad rarum redeamus, mihi videtur frigidum esse consequentem qualitatem densi,& calidum rari, quia quæuis res dum calefit, rarefit, & quælibet materia dum refrigeratur, simul condensatur. Qua ratione fit, vt terra frigidior sit aqua, & ignis calidior sit aere.

Nec propriè locutus est Aristoteles.9.&.10.capite primi lib.& secundo secundi metheororum cum dixerit caloré Solis eum esse,qui sursum humores,vaporesꝗ; eue hat,quia Sol nil aliud facit,quam calefacere,cuius caloris ratione,ea materia rarefit, & ob rarefactionem leuior facta ascendit,non quia sursum à Sole feratur.

Quę subsequuntur,cum raró ac denso simbolum habere videntur.cum raro, scili- cet calidum,humidum,leue,sublime,diaphanum,lumen,clarum,lux,albũ,dies, mo- tus, velox, simplex, disgregatum, molle, lene, acutum,subtile,coctum,spaciosũ, dulce,voluptas,audacia,lætitia,liberalitas,veritas,industria,amor, misericordia,hu- manitas,sanitas,vita,& iis similia. Cum denso verò frigidum,siccum,graue, imum, opacum,vmbra,obscurum, tenebræ,nigrum,nox,quies, tardum,mixtum,congrega tum,durum, asperum,ob tusum,crassum,crudum,angustum,amarum,dolor, timor, melãcholia,auaritia,mendacium,inertia,odium,crudelitas,feritas,infirmitas,mors, & ijs similia.

Verum est quod ea ratio,qua Aristoteles ait aerem humidum esse, parui est mo- menti,quia similiter de igne inferri posset,qui facilius à termino alieno, quã aer,aut aqua terminari potest.

## Motum rectum curuo posse comparari etiam disentiente Aristotele.

### CAP. XXXV.

SEd vt ad Aristotelé redeamus,rectè dicere non potest motum rectum ad curuũ comparabilem non esse.4.cap.lib.7.physicorum,vbi errat quoque dicens repe riri non posse lineam aliquam rectam alicuius circuli circunferentiæ æqualem. quia Archimedes iam probauit in lib.de quadratura circuli, triangulum illum orthogo- nium,cuius vnum ex lateribus circundantibus angulum rectum æquale esset semi- piametro alicuius circuli,& aliud circunferentiæ,æqualem futurum dicto circulo. Il lud igitur triangulum orthogonium,quod æquale erit alicui circulo, & habebit ali- quod ex suis lateribus circundantibus angulum rectum æquale semidiametro dicti circuli, aliud quoque latus ipsum angulum rectum circundans,ex necessitate,circũ- ferentiæ dicti circuli habebit æquale. Potest igitur dari vna quædam recta linea ę- qualis circulari contra Aristotelis opinionem,qui non benè reuocauit in mentem, quod scripsit de relatiuis,cum dixit quadraturam circuli posse quidem dari,etsi tũc t̄pis de ea nó haber et̄ scientia. Si igit̄ dicta quadratura dari pót,potest etiã dari vna recta linea æqualis circunferentiæ eiusdé circuli,ob rationes iã dictas. Sed si Arist. dixisset,circularem corporum cęlestium motum, comparabilem non esse recto cor- porum elementarium, verum dixisset, non quia eorum alter circularis, alter ve- rò sit rectus, sed quia cœlestis regularis sit, neque modò tardus, modò velox, sed vnam semper & eandem velocitatem retinens,mot⁹ aũt,qui est corporũ elemen
tarium

tarium è contrà se habeat, præter id, q̃ nunquam fuit neque sit futurus aliquis horũ
rectorum, qui naturales dicuntur, qui tam velociter moueatur, ut motus cœli, quia si
voluerimus considerare motum diurnum. 24. horarum, secundum opinionem com-
munem, reperiemus calculando, Lunam in quadraturis cum Sole, dum inuenitur in
æquatore, singulis horarum minutis moueri per. 5 00. milliaria Italica vel circa, & in
coniunctionibus, & oppositionibus ipsius Solis. 1000. vel circa, & Solem tempore æ-
quinoctiorũ. 18000. & Saturnũ circa æquatoris situ. 260000. & ampli⁹ de stellis aũt
fixis circa æquatorem positis quiuis cogitet; quod reuera difficillimum quibusdam
videbitur, quod quidem non occurrit secũdum pulcherrimam Aristarchi samij opi-
nionem, diuinitus à Nicolao Copernico expressam, contra quam nil planè valent
rationes ab Aristotele; neque etiam à Ptolomeo propositę. Motu verò proprio, quo
libet horę minuto, Sol mouet per milliaria circa. 48. Luna quãdo cõiuncta est, aut op
posita reperitur Soli. 36. milliaria, & in quadraturis. 18. Saturnus. 24. Iupiter. 40:
Mars. 100: Venus. 26: Mercur. 5. Sed Saturnus motu rapido, vno horæ minuto mo-
uet circa. 260000. milliaria, vt diximus Iupiter circa. 170000. Mars. 75000. Venus.
10000. Mercurius. 2000. corpus aũt elementare, & si moueret motu recto hoc mó,
& velocius etiam corpore cęlesti, non obseruans tamẽ uniformitatem, ut dictum cœ
leste facit, cum eodem nullo modo comparari posset, quia rectus dictus naturalis,
suam semper velocitatem adauget, ob continuam impressionem, quam recipit à cau
sa perpetuò coniuncta cum ipso corpore, quę est propensio illa naturalis eundi bre-
uiori quadam via ad locum suum, ita vt etiam si dictum corpus elementare à motu
tardiore ad velociorem, superare posset motũ alicuius corporis cęlestis, ij duo motus
intersecarent sese in vno solo pũcto, quod diuidi distribuiq; in partes nequiret, idest
non nisi in vno solo temporis instanti redderentur æquales, vt ita dicam. Neq; solũ
loquor de circulari cœlesti cum recto elementari, sed de qualibet alia motuum spe-
cie, siue sint ambo recti, siue ambo curui, quando aliquis eorum irregularis erit.

*Minus sufficienter explosam fuisse ab Aristotele opinionem cre-
dentium plures mundos existere.*

## CAP. XXXVI.

MAior ratio, qua Aristoteles eorum opinionem, qui plures esse mundos dixe
runt, refutare nititur, in eo consistit, quod is credat partes terræ, quæ alijs
mundis assignarentur, ad huius mundi centrum inclinationem habere, & sic ignem
illorum, propensionem habiturum ad circunferentiam huius.

Quæ certè ratio tam debilis est, vt per se cadat, non considerans ipse, quòd si
essent dicti mundi, eorum quilibet suum proprium centrum, suamq; propriam cir-
cunferentiam haberet, terrasq; & ignes haberent inclinationem ad centra circunfe-
rentiasq; suorum mundorum, absque eo, q̃ vna terra, alterius centrum appeteret; vt
exempli gratia, si doctissimi Aristarchi opinio est vera, rationi quoq; consentaneum
erit maximè, vt quod Lunæ contingit, cuilibet etiã ex aliis quinque planetis eue-
niat, idest, vt quemadmodum Luna suorum epiclorum ope circũ terram voluitur,
quasi per circunferentiam alterius cuiusdam epicycli, in quo terra sit instar centri
naturalis (idest sit in medio) delati ab orbe annuo circa Solem; Sic etiam Saturnus,
Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio sui epici-

cli maioris,fitum habens, voluantur; quod quidem corpus, & aliquem quoque ha-
beat motum circa fuum axem, fit opacum, ijs conditionibus, quæ terræ funt fimi-
les,præditum exiftat,& in dicto epyciclo fint res fimiles iftis lunaribus.

### *An recte loquutus fit Phylofophus de extenfione luminis per uacuum.*

## CAP. XXXVII.

ARriftoteles fecundo lib.de anima fentit φ per vacuum non extenderetur lu-
mé, quod procederet à corpore lucido.Quod verifimile nó eft; qa quéadmo
dum quantò rarius eft aliquod corpus, tanto aptius eft vt diaphanum exiftat;& quã-
tò rarius eft dictum corpus,tantò minorem quantitatem materiæ contineat; fic quã
tò magis diaphanum eft, cum ex perexigua materia conftet,tantò magis liber tran-
fitus luminis patet ; Vnde quantò minor quantitas materiæ erit in dicto fpatio,tan
tò nitidius pertranfibit lumen. Sequitur ergo, quòd vbi nulla effet materia , totum
lumen libere tranfiret. Color cęruleus quem videmus in profunditate aquæ,& ae-
ris,color eft aquæ & aeris,qui denotat refiftentiam factam ab aere & ab aqua ipfi lu
mini ; Quod quidem lumen ubi corpus aliquod non effet, minime reflecteretur,fed
abfque vllo impedimento recta tranfiret.

### *An recte phylofophiæ penus Ariftoteles fenferit de loco im- pellendo à pyramide.*

## CAP. XXXVIII.

ARiftoteles.8.cap.lib.3. de cœlo,difputans contra antiquos de elementorum
figuris, ait pyramidem implere poffe locum corporeum . quod verum non
eft . Cubus quidem id facit ab . 8. enim cubis perfecte impletur locus , fed non
item. 12 .pyramides, ut Ariftoteles fenfit ( ideft fex fuper aliquam exagonam figu-
ram fuperficialem & fex fub eadem) id præftant,cum potius maius vacuum rema-
neat ad quamlibet partium fupra , & infra, quam plenum . Rectius Ariftoteles
egiffet, fi probaffet ratione immobilitatis conuenire pyramidem terræ, quam cu-
bum. quamuis,de horum corporum altero , fit ftultum hoc credere. decepti tamen
fuerunt antiqui, credentes cubum ad motum minus idoneum effe, quam reliqua
quatuor corpora regularia (loquor autem habita volubilitatis ratione ) quia pyra-
midale eft illud, quod ita fe habet, vt multis rationibus probari poteft, quarum vna
hæc nobis fufficiet . Scimus iam ex communi conceptu corpus fphęricum effe ma-
gis volubile, inftabileq;,quàm alia fint. Illud ergo corpus,cuius figura ad fphæri-
cam magis accedet,ad uoluendum,& ad mouendum facilius erit quouis alio, quod
æqualis fit quantitatis,& fibi omogeneum materia,vt exempli gratia corpus. 20.ba
fium ad voluendum, & ad mouendum promptius erit eo, quod ex.12.conftat,& id,
quod eft. 12. eo, quod eft.8.& id, quod eft. 8.eo, quod eft.6.& id , quod eft. 6. vt
cubus eft, eo, quod eft.4. cuiufmodi eft pyramidale . Huc accedit , quòd pyrami-
dale corpus aliam conditionem habet, quàm cubicum , cum in quauis facie inalte-

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rabile fit, cubicum autem econtrà fit alterabile vndequaque, fuaq; quadrata in rhú-
bos mutare poffit, iifdem exiftentibus lateribus.

### *Examinatur quam ualida fit ratio Ariftotelis de inalterabilitate Cœli.*

### CAP. XXXIX.

ARriftoteles textu. 22. primi lib. de Cœlo ita inquit.
Accidit autem, & hoc per fenfum fufficienter, quo ad humanam dixiffe fi-
dem, & omni prȩterito tempore fecundum traditam inuicem memoriam, nihil vi-
detur tranfmutatum neque fecundum totum vltimum cȩlum, neque fecundum par-
tem ipfius propriam vllam.

Hoc autem in loco Arifto. non confiderauit, ꝗ fimiliter de terra dici poffet, quan
do ipfa ita eminus profpiceretur, imo abfque dubio putandum eft, ꝗ fi terra lucè So
lis prædita effet, & aliquis ipfam ab octauo orbe vellet videre, nullo pacto cerne-
ret, cum fidera illa quæ primæ magnitudinis vocantur, & quæ plufquam centies ma
iora ipfa terra putantur non nifi vt puncta videantur.

                                                                IN

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# IN QVINTVM
## EVCLIDIS LIBRVM.

*Vamuis omnia libri quinti Euclid.uerißma ſint.
Animaduertimus tamen permultos ſumma cũ
difficultate eorũ demonſtrationes percipere. Præ-
cipuè ubi quinta, aut ſeptima deffinitiones eiuſ-
dem libri neceſſariæ ſunt. Illæ enim adeo obſcuræ
uidentur, ut longè facilius admiſſuri ſint hæc no-*
*ſtra poſtulata tanquam clariora. Atque etiam tanquam intelleɛtui
commodiora, quam ſit illud quintum idemẹ́, ultimum poſtulatum
eiuſdem in primo libro poſitum, de linea duas alias ſecante. Quan-
doquidem ijs noſtris poſtulatis admiſſis, ſequentia Theoremata per
facillima reddentur.*

### Horum autem primum eſt.

Qᴠᴏᴅ tota compoſita ex æquali numero partium æqualium , ſunt inuicem
æqualia.

Vt ſi quis diceret omnes proportiones quæ cópoſitæ ſunt ex æquali numero alia-
rum proportionum inuicem æqualium, ſunt etiam inuicem æquales, quod Eucli-
des conatur demonſtrare in.2 2.et.2 3.quinti libri.

### SECVNDVM.

Qᴠᴏᴅ ſi à totis æqualibus detraɛtæ fuerint æquales partes, quæ remanent erunt
partes inuicem æquales.

Et è conuerſo ſi æqualibus æqualia addas compoſita erunt inuicem æqualia.
Quod in ipſis proportionibus hoc loco ſemper intelligendum eſt.

### TERTIVM.

### Quẹ eſt Euclidis ſeptima propoſitio.

Qᴠᴏᴅ ſi fuerint plures termini æquales inuicem, ratio ſeu proportio vnius ip-
ſorum ad alium tertium terminum maiorem,minoremúe,ſed eiuſdem generis, erit
eadem quæ cuiuſuis alterius termini ad eundem tertium.  Et è conuerſo, quæ fuerit
proportio tertij termini ad vnum prædiɛtorum æqualium, eadem erit , ſpecie , cum
alio eorundem terminorum.

QVAR-

## IN QVINT. LIB. EVCLI. 199

## QVARTVM.

### *Euclidis verò nona propositio.*

QVOTIESCVNQVE proportio vnius plurium terminorum collatorum cum ali
quo tertio eiusdem generis, eadem fuerit cum ea quæ est cuiusuis alterius dictorum
terminorum cum eodem tertio, aut proportio dicti tertij, cum aliquo dictorum, eadem fuerit cum ea quæ ipsius est ad aliquem alium eorundem terminorū, tunc eiusmodi termini, æquales erunt inter se.

## QVINTVM.

### *Euclidis verò octaua propositio.*

QVOTIES plures erunt termini, quorum vnus fuerit maior altero, si comparentur alicui tertio eiusdem generis, proportio maioris ad tertium illum, maior erit
ea, quæ est minoris ad prædictum tertium, & proportio illius tertij ad maiorem, minor erit ea, quæ eiusdem tertij ad minorem terminum comparati.

## SEXTVM.

### *Euclidis verò decima propositio.*

QVOTIES proportio vnius, ex pluribus terminis comparatis ad aliquem tertium, maior fuerit proportione alicuius alterius dictorum cum eodem tertio, primus
ille terminus, altero maior erit. Et quoties proportio tertij termini ad vnum quā
ad alterum terminum maior fuerit, eiusmodi terminus altero minor erit.

## SEPTIMVM.

### *Euclidis verò undecima propositio.*

PROPORTIONES, quarum vnaquęque cum aliqua tertia æqualis est, ipsæ quoque inter se sunt æquales. Vt illud, Quæ vni & eidem sunt æqualia, sibi inuicem
sunt æqualia.

## OCTAVVM.

### *Euclidis verò duodecima propositio.*

QVOTIESCVNQVE proportio vnius ex pluribus antecedentibus cum suo ex
pluribus consequentibus, æqualis fuerit ei cuiusuis alterius dictorum antecedentiū,
cum suo plurium cōsequentium, proportio totius aggregati antecedentium cum toto aggregato consequentium, dictæ primę proportioni ęqualis erit, nempe illius an
tecedentis ad suum consequens.

NONVM

## NONVM.

### *Euclidis uero tertiadecima propositio.*

QVOTIESCVNQVE aliqua proportio plurium proportionum inuicem æqualium, tertia aliqua proportione, maior aut minor fuerit, quælibet prædictarum æqualium inter se, tertia illa proportione maior aut minor pariter erit.

### DECIMVM.

QVOTIESCVNQVE fuerint ex vna parte plures termini (siue coniuncti siue disiuncti sint) æquales singuli vni tertio termino; ex altera verò parte totidem fuerint alteri tertio termino æquales, proportio aggregati priorum terminorum ad suũ tertium, æqualis erit proportioni aggregati reliquorum terminorum ad suum tertium, & è conuerso, ita se habebit primus tertius terminus ad suos multos terminos, sicut se habet secundus tertius terminus ad suos simul sumptos.

### VNDECIMVM.

Aggregatum ex partibus proportionalitatis continuæ, quod inter maximum, & minimum terminum omnium terminorum proportionalium compræhenditur, sem per multiplex est ad singulas partiales proportiones, ex quibus ipsum componitur.

### DVODECIMVM.

Quæuis proportio quocunque modo diuisa fuerit, ex iis partibus componitur, in quas diuiditur.

*Cum enim hæ præpositiones sint ita conspicuæ ipsi intellectui, ut absq; dubio inter obie ct a ipsius intellectus connumerari possint, nullus sanæ mentis eas negabit.*

### THEOR. I. II. ET III.

PRimum, secundum, & tertium theorema quinti Euclidis ab ipso satis exactè demonstratur, studiosus itaque autorem consulat.

### THEOREM. IIII.

QVartum vero Theorema Euclidis ego sic demonstrarē. sit, verbi gratia, proportio . a. ad. b. quæ est. c. ad. d. sumptis multiplicibus. e. et. f. ad. a. et. c. æqualiter, item multiplicibus. g. et. h. ad. b. et. d. dico proportionem. e. ad. g. esse eandem quæ est. f. ad. h. Habemus enim ex. 10 postulato præmisso, eandem futuram proportionem. e. ad. a. quæ est. f. ad. c. & ita. b. ad. g. quæ est. d. ad. h. ex præsupposito verò cũ sic se habeat. a. ad b. sicut. c. ad. d. erit ex primo postulato eadé proportio. e. ad. g. quæ est. f. ad. h. Nam proportio. e. ad. g. componitur ex eis quæ sunt. e. ad. a: et. a. ad. b.
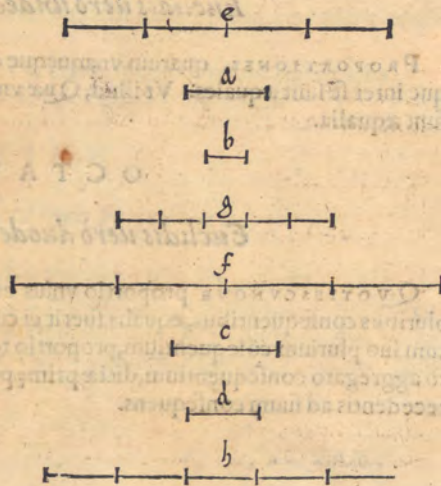
b.et.b.ad.g.& similiter proportio.f.ad.h.cóponitur ex eis quæ sunt.f.ad.c.et.c.ad. d.
et.d.ad.h.

### THEOR. V. ET VI.

CIRCA 5.et.6.theorema nihil notandum occurrit.

### THEOR. VII. VIII. IX. X. XI. XII. XIII.

THeoremata à. 6.in.13. cum sint de obiectis intelligibilibus, sine ullo medio,
ab intellectu cognitis, inter axiomata à me relata fuerunt. 7. inquam quinti
Euclid.fecimus tertium Postulatum,.8.quintum,.9.quartum,.10.sextum, .11.septi
mum,.12. octauum,.13.nonum.

### THEOREM. XIIII.

QVartumdecimum Theorema ex Euclide demonstrabitur, mutatis tantum
theorematibus ab interprete notatis, ita vt loco. 7.8.noni, & decimi citetur
tertium.5.4.et.6.postulatum à me propositum.

### THEOR. XV.

QVintumdecimum Theorema sic demonstrabo; Sit,exempli gratia,a.termi-
nus antecedens.et.b.consequens, qui-
bus duo multiplices sumantur. c. et. d. Dico
eandem proportionem habiturum. c. ad. d.
quam.a.ad.b. habet. In primis enim manife-
stè patet quamlibet partem ipsius. c. habitu-
ram eandem proportionem cum qualibet par
te.d.quam habet. a.ad. b.quare ex. 7.et.8.po
stulato propositum elucescet.
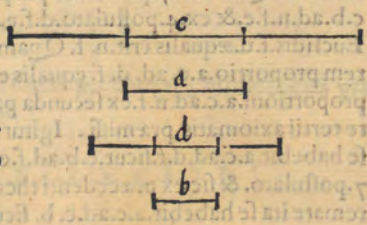
### THEOREM. XVI.

SExtumdecimum theorema sic demonstrabitur. Sit,exempli causa,eadem pro
portio.a.ad.b.quæ est.c.ad.d. Dico ͷ ita se habebit.a.ad.c.sicut.b.ad.d.Cogi-
temus itaque alterum istorum terminorum.c.aut.b.medium inter.a. et.d.quare
primum intelligamus.b.inter.a.et.d.proportio ipsius.a.ad.d.componetur ex ea quę
est.a. ad.b.& ea quæ est.b.ad.d.ex.12. postulato. Et ex eodem, illa ipsa proportio.
a.ad.d.pariter componetur ex ea quæ est.a.ad.c.& ea quæ est.c.ad.d.sumpto. c.pro
medio termino. Ex quo sequitur,aggregatum duarum proportionum, videlicet.a.
ad.b.et.b.ad.d.æquale esse aggregato.a.ad.c.et.c.ad.d.ex quibus aggregatis æqua-
libus si duas proportiones æquales subtraxerimus,eam videlicet quæ est.a.ad.b.& il
lam quæ est.c.ad.d.supererunt duæ proportiones
inter se æquales.erit enim proportio.a.ad.c.æqua
lis proportioni.b.ad.d.ex prima parte secundi po
stulati diuisim.
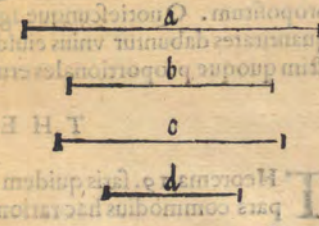
Alia etiam ratione idipsum demóstrari potest,
sumpto. b. pro medio termino inter.a. et. c. et. c.
pro termino medio inter. b. et. d. quare propor-
tio.a.ad.c.componetur ex.a.ad.b. et.b. ad. c. illa
verò quæ est.b.ad.d.ex.b.ad.c.et.c. ad. d. ex. 12.

Cc        postu-

poſtulato . Sed cum proportio.a.ad.b.æqualis ſit
proportioni.c.ad.d.communis autem.b.c:propor
tio. itaque.a.ad.c.æqualis erit.b.ad.d. ex ſecunda
parte.2.poſtulati compoſitè,& ſic habebimus pro
poſitum,ita quòd quotieſcunque dabûtur.4. quã
titates ex una parte proportionales, illæ ipſæ ex
altera proportionales erunt.
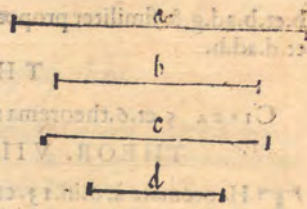
## THEOR. XVII.

DEcimiſeptimi theorematis hæc eſt demonſtratio . Ita ſe habeat.a.c. b.ad. c.
b.ſicut ſe habet.d.f.e.ad.f.e.Probo ita ſe habere.a.c. ad.c.b.ſicut ſe habet. d.
f.ad.f.e. Cogitemus itaque alterum terminum ſcilicet.n.f.qui ſic ſe habeat.ad.f.c.
ſicut ſe habet.a.c.ad.c.b.Quare ex præcedenti theoremate ita ſe habebit.a.c.ad. n.
f.ſicut ſe habet.c.b.ad.f.e.& ex.8 poſtulato ita ſe habebit.a.c.b.ad.n.f.e.ſicut ſe ha-
bet.c.b.ad.f.e. Sed cum ex præſuppoſito ita ſe habeat.a.c.b.ad.c. b.ſicut ſe habet.
d.f.e.ad.f.c.ideo ex præcedenti theoremate ita ſe habebit.a.c.b.ad.d.f.e.ſicut ſe ha-
bet.c.b.ad.f.e.demonſtratum autem eſt ita ſe habere.c.b.ad.f.e.ſicut ſe habet.a.c.b.
ad.n.f.e.Quare ex.7.poſtulato proportio.a.c.b.ad.d.f.e,æqualis erit proportioni.a.
c.b.ad.n.f.e.& ex.4.poſtulato.d.f.c.æqualis erit.n.f.e. Itaque ex 3. poſtulato primi
Euclidis.f.d.æqualis erit.n. f. Quamob
rem proportio.a.c. ad. d. f. equalis erit
proportioni.a.c.ad.n.f.ex ſecunda par-
te tertij axiomatis præmiſſi . Igitur ita
ſe habebit.a.c.ad.d.f.ſicut.c.b.ad.f.e.ex
7.poſtulato. & ſic ex præcedenti theo-
remate ita ſe habebit.a.c.ad.c. b. ſicut.d.f.ad.f.e. quod erat propoſitum : Quotieſ-
cunque igitur dabuntur.4.quantitates coniunctim proportionales, diuiſim quoque
proportionales erunt.
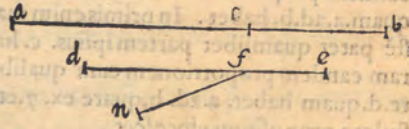
## THEOREM. XVIII.

THeorema.18.hac ratione demonſtrari poteſt . Detur proportio.a.c.ad.c.b. ſi-
milis ei quæ eſt.d.f.ad.f.e.probo ita ſe habere.a.c.b.ad.c.b.ſicut ſe habet. d.f.
e.ad.f.e. In primis notum eſt ex.16.theoremate ita ſe habiturum,a.c.ad.d.f.ſi
cut.c.b.ad.f. e. Quare ex.8.poſtulato ita
ſe habebit.a.c.b.ad.d.f.e.ſicut.c.b.ad.f.e.
Itaque ex.16.theoremate ita ſe habebit.
a.c.b.ad.c.b.ſicut.d.f.e.ad.f.e.Quod erat
propoſitum . Quotieſcunque igitur . 4.
quantitates dabuntur vnius eiuſdemq́; generis diſiunctim proportionales, coniun-
ctim quoque proportionales erunt.
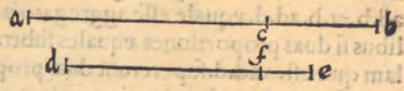
## THEOREM. XIX.

THeorema.19.ſatis quidem apud Euclidem demonſtratur : eius tamen tertia
pars commodius hac ratione demonſtrari poterit (nempe) quod cum ſit pro-
portio

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portio.a.ad.b.quæ eſt.c.ad.d.probabo ita ſe habituram proportionem. b.ad.a.ſicut
ſe habet.d.ad.c.hoc argumento: ſi.a.ad. b. ita ſe
habet ſicut.c.ad.d.ex.16. theoremate ita ſe ha
bebit.a.ad.c,ſicut.b.ad.d. Quare ſic ſe habebit
b.ad.d.ſicut.a.ad.c.Itaque ex eodem. 16. ita ſe
ſe habebit.b.ad.a.ſicut.d.ad.c.

## THEOREM. XX.

Q Vamuis.20.theorema apud Eucli.perfectè demonſtratum fuerit, poteſt ni-
hilominus & hac via demonſtrari. Sic ſe habeat proportio.a. ad. b. ſicut ſe
habet.c.ad.d.& proportio.b.ad.e.ſicut. d. ad.
f. Dico φ ſi. a. maius fuerit. e. pariter.c.maius
erit.f.& ſi.a.minus fuerit.e:c.quoq; minus erit
f.ſin verò equale,ét æquale erit. Nam ex pri
mo poſtulato certi ſumus ita ſe habere pro
portioné.a.ad.e.ſicut ſe habet proportio.c.ad
p. Quare ex.12.theor ρpoſitū manifeſtū erit.

## THEOREM. XXI.

V Igeſimumprimum theorema,ſatis apud Eucli. probatum , nihilominus præ-
ſcripto nunc modo demonſtrari poterit.

## THEOREM. XXII.XXIII.

D VO hæc theoremata in primum poſtulatum collegimus.
Sequentia verò cum exactè apud Eucli.demonſtrentur non eſt cur nos in
ijs immoremur.

Cc 2

# PHYSICA,
# ET MATHEMATICA
## RESPONSA.
### *JO. BAPTISTAE BENEDICTI PATRITII*
### *Veneti , Philosophi Mathematici.*

## Ad Lectorem.

*T Nil magis virtutis est proprium , quàm agitari,& inceßabili motu prodeße. Ac velu ti fulgidum sydus ante oculos spectantiũ com micare . Ita mihi mathematicis ijsq; maxi mè philosophicis speculationibus dedito, sapiß sime,ut in principium summorum aulis , & amplißimis ciuitatibus degenti,ubi multa sem per Nobilium mira curiositate , sciendi desiderio , & conferendi cu piditate referta, uersantur ingenia,contigit,modo ab his , modo ab illis, aut uerbis tentari, aut literis prouocari ad disserendum , de his, in quorum studijs uersamur . Quarum concertationum & re sponsionum, quoniam non omnino indigna existimaui , quæ memoriæ comendarentur , partem aliquam apud me conseruaui. Vbi uerò per ocium licuit,collegi , relegi, ac tandem de manu mittere decreui . Tum ut scientia ipsa quo magis diffundetur, crescat ; & quicquid ualeo,sine inuidia in communem utilitatem conferam. Tum ut ui- rorum præstantißimorum , qui me suis interrogationibus excitaue runt, quantum in me erit, gratitudinis ergo , nomina reddam im- mortalia,& eorum exemplo alios,ocio sordidiore abiecto, quod solet ourialium præcipuè excelsa ingenia corrumpere , ad sciscitandum conferendum, & disserendum, de rebus serys,& quæ usui aliquan- do eße poßint, & quandoq; euulgari mereantur,alliciam. Tu inte- rim nostris laboribus fruere, & nostram diligentiam boni, & æqui consule , & Vale .*

DE

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# DE TEMPORVM
## EMENDATIONE
## IO. BAPTISTAE BENEDICTI
### Patritij Veneti , Philofophi
### Mathematici.

*AD SERENISS. EMANVELEM PHILIB.*
*Allobrogum & fubalpinarum gentium Ducem*
*Inuictißimum.*

### EPISTOLA.

Irvm, Quàm lectione epiftolæ feu ( vt vocant) Breuis. S. D. N. Gregorij XIII. Pont. Max. quod ad me nuper tua Celfitudo mifit ex Nicea, vt meam de ea re fententiam proferrem, delectatus fim; ex quo, non tantum recta illius mens ac verè fancta cogitatio, fed etiam aperta maximaq; , fi ad exitum per ducat̄ , imo fummè neceffaria vniuerfo orbi vtilitas percipi poteft; qua de re memini cum Celfitudine tua aliquando fermonem habuiffe . Vidi præterea cum ipfo breui tranf- miffum compendium Domini Aloifij Lilij: cuius mihi fententia perplacet, de correctione eius diei, qui 134. quoque anno præter, neceffitatem, gignitur . qui fanè dies perpetuæ retrogradationis ingreffus Solis in Zodiaci figna, caufa fuit. quod ita perfpicuè patebit. Cum Numa Pompilius anni curfum correxit emendauitq; , ea fanè mente id videtur præftitiffe, vt principium Ianuarij primi menfis anni , præcisè in ipfo hyemalis foltitij puncto collocaretur. quod hac tempeftate , dictam ob caufam adeò retroceffit, vt circa vndecimam diem Decembris effe reperiatur. quod fi centefimo trigefimo quarto quoque anno detractus dies vnus fuiffet, nihil erroris prorfus accidiffet. Atq; dies hic (vt alias Celfit. tuæ fignificaui) inde generatur, quod quarto quoque anno addentes nos ad quarti anni dies. 365. diem horarum . 24. ob errorem annuum horarum quinque minutorum. 49. fecundorum ferè. 16. ( anni æqualis fiue medij ) fallimur quarto quoque anno in minutis. 42. fecūdis propè. 56. amplius quàm par fit minutis fcilicet. 10. fecundis ferè. 44. fingulis annis ; qui numerus. 134. multiplicatus, diem penè horarum. 24. conftituit; penè inquam , quia minutum vnū deeffet tm̄modo, & fecunda. 44. fi decē illa minuta, &. 44. fecunda annua, exquifita effent atque perfecta; quæ tamen differentia nullius adeo effet momenti, aut certè perexigui, vt vix exactis. 111086. annis, diem vnum afferret . Itaq; planè neceffaria eiufmodi effet emendatio, aptaq; eius ratio à D. Lilio oftenditur , prout etiam Petrus Pitatus Veronenfis tradidit, in eo, quem de vera anni quantitate tractatu confcripfit, nempe vt tribus primis centefimis annis, centefimus quifque annus communis fit, quartus fubfequens centefimus intercalaris: quod fanè fieri neceffe eft . Nam

cùm

cùm tribus centesimis cōmunibus,tres quartas diei partes plus æquo detraxerimus,
non enim centesimo quoque anno, sed centesimo trigesimo quarto, dictus dies de-
trahi debet, postquam tres integros dies, qui quadringentis detrahendi erant, tre-
centorum annorum spacio detraxerimus; sitque 134. penè tertia pars. 400. quarto
annorum centenario,tres quartæ diei partes recuperabuntur; atque ita in fine qua-
dringentorum annorum omnia exactè suo loco restituta erunt. Idcirco dictus iam
quadringentesimus annus intercalaris & non communis constituendus erit, non alia
de causa, quam vt bissexti ordinem sequamur.

Is verò modus, qui à D. Lilio traditus est, de ratione inueniendi singulis mensibus
Nouilunij diem, interdum fallere nos posset vno die; prout Ianuario proximè
lapso accidit; quo ex præscripto modo nouilunij, dies nonus illius mensis esse debuis-
set, qui fuit tamen dies septimus, sexta decima hora cum dimidia post meridiem. Ne
que etiam tutum est, via integrorum dierum, nulla habita horarum aut minutorum
ratione, nec minus ea, quæ à Pitato tradita est, mediorum seu æqualium motuū pro
gredi : At censerem potius veros motus sequendos esse ex calculis exactarum tabu-
larum, quales Prutenicas esse iudico; Et cum solius Paschæ causa laboremus hac in
re, pleniluniorum verorum, in multos annos tabulas formarem, quæ æquinoctia ver
nalia sequuntur, cum assignatione diei Paschatis præcisè, prout fecit Pitatus; non
via tamen æqualium pleniluniorum sed verorum. Porrò quod ad Paschatis cele-
brationem attinet, rationi consentaneum est, concilij Niceni decretum ea de re ser
uari, prima scilicet dominica die post primum plenilunium, quod æquinoctium ver-
nale sequitur; hoc tamen animaduerso, si dictum plenilunium primum post æquino-
ctium contingens, diē dominicum sortiretur; nulla ratione tali die Pascha celebran-
dum esse; verum subsequenti, ne cum Hebreis consentiat Ecclesia Christi : quæ fuit
causa, vt in decreto concilij Niceni statutum sit, à quartadecima, in vigesimam pri-
mam celebrari debere : Quod mihi Petrus Pitatus non animaduertisse videtur, cum
ex ei° sentétia in suis tabulis die Paschate declarata, huiusce anni Pasca celebrandū
fuerit. 23. Martij, ipsomet de plenilunij non tantum æqualis, sed veri.

Dies autem Paschatum elapsorum, quos hactenus examinaui, reperi omnes con
cordare cum ea regula, quam nonnulli de die carnis priuij tradiderunt. nempe pri-
mum diem martis post nouilunium Februarij, carnis priuij diem esse; non autem cū
sáctione Patrum concilij Niceni, qua statuerunt à vigesima prima Martii dirigen-
dum esse Paschatis diem, vt porè qui sibi persuaserunt, circa eum diem æquinoctiū
perpetuò esse debere; prout tunc temporis erat. Non itaq; error accidit, quod Pa
scha ex huiusmodi suppositione concilij, post vigesimam primam lunę celebretur,
cum seruata regula concilij non fuerit. Prout manifestū est de Paschate anni. 1566.
celebrato. 14. Aprilis(quę fuit. 24. lunę)quod. 7. dicti mensis celebrādum erat. Tum
anno. 1569. 10. Aprilis solenne fuit Pasca, quod tertia eiusdem esse debuerat. Anno
deinde 1572. 6. Aprilis, dies fuit Paschatis, quæ. 30. Martij futura erat, anno vero
1575. in tertiam Aprilis Pascha incidit, casurum in. 27. Martii.

Cum igirur(vt ex diplomate ad Celsit. tuam misso patet)S. D. N. mens sit atq; vò
luntas, ut quisque liberè in medium proferat quid hac de re sentiat: quædam mihi
non omnino præmittenda occurrunt, quæ tantis cœptis non nihil adiumenti for-
tasse adferre queant.

Atque illud in primis non tantum ut corrigatur Calendarium ōb Pascha cætera-
que festa mobilia ab illo manantia, vt decreto concilij Niceni sancitum est, scilicet
vt ipsum Pascha celebretur prima dominica post primum plenilunium, quod æqui-
noctium

noctium vernale proximè sequitur; verum etiam quò anni principium emendetur, scilicet vt ad suum verum principium reuocetur annus. nempè ad diem hyemalis solstitij, quæ prima Ianuarij dies esse debet.

Deinde, tot dierum menses constituantur, quot hac nostra tempestate, sol in ipsis Zodiaci signis versatur. Postremò, quædam festa immobilia in alios dies transferā tur, celebrenturq; aptis temporibus: quod à. S. D. N. mente dissentire minimè videtur. cum non magis de festis mobilibus quam immobilibus agat, imo etiam planè æquum sit, vt habeatur vtrorunque ratio, quò statutis temporibus celebrentur.

Vt autem ad primam Ianuarij diē verum principium anni reuocetur; censerem ex eo anno, quem corrigere voluerimus, non modò dies. 10. esse detrahendos, verū etiam vnum & uiginti, illo ipso anno; idq; duplici via; aut partiendo menses, atque ex illis demendo eos dies, qui minus ad rem hanc facere videbuntur, ac tum remaneat annus trecentorum quadraginta quatuor dierum ita vt decem menses sint dierum duorum spatio solito breuiores, alter mensis vno deficiat: aut constituto Decébri dicti anni dierum decem, dies autem ille, qui decimum proximè sequitur, sit & primus Ianuarij, & dies solstitij. ob quam causam existimarem consultissimum eiusmodi annum esse milesimum quingentesimum septuagesimum nonum. Quò quam primum. S. D. N. Pontifex max. suis temporibus huius correctionis manifestos effectus experiri & perpendere, atque dispositionem anni non solum principio, sed cæteris partibus suis in vniuersum tam concinnè apteq; respondere, & astrorum motibus, & Ecclesiæ sacrosanctæ sanctionibus, se authore lætari possit.

Omnino itaq; iudico detrahendos esse vnum & viginti dies elapsi erroris: non decem tantum, quo hyemmalis conuersio ad initium Ianuarij reuocetur; idque ne à communi opinione de ipso anni principio veritas discrepet, quæ principium Ianuarij, anni principium arbitratur. etenim cum credant omnes annū à Ianuario inchoari, veritas autem ipsa sic se habeat, vt nobis septentrionalibus tunc inchoet annus, cum ad nos Sol accedere incipit, aut dies augetur; non conuenit principia eiusmodi separata & discrepantia esse. Et hanc fuisse Numæ Pompilio mentem credibile est, qui ad annum Romuli decem mensium, Ianuarium & Februarium addidit, vt principium Ianuarij principium esset anni: cuius rei argumentum esse potest, quod C. Iulij Cæsaris temporibus ( qui multis annis post Numam fuit) atq; vti Pont. Max. corrigendorum festorum curam suscepit hyemale solstitium per aliquot dies retrocesserat; nec mirum tamen esset, si Numæ temporibus, exactè prima Ianuarij die non fuisset hyemale solstitium, adhuc pubescente in Italia Astronomia.

Huiusmodi autem correctio dierum. 21. post. 2300. annos à Numa, quæ sit perpetuo seruitura, media emendatione ea, quæ de tribus centesimis annis communibus, & quarto intercalari, superius proposita fuit, non repudianda ei videatur, qui sciet, qua ratione Numæ Pompilij annus corrigeretur, octauo quoque anno, intercalando annum vltimum medijs diebus. 90. quo prima dies Ianuarij ad verum principium anni, hoc est hyemale solstitium, reduceretur.

Alio item argumento cuique patere potest, priscos Romanos statuisse annum ab hyemali solstitio initium sumere, vt inquit Ouidius primo Fastorum.

Bruma noui prima est, veterisq; nouissima Solis.

Principium capiunt Phębus, & annus idem.

eo quod diem naturalem à medio noctis inchoarent, ab eo puncto scilicet, quo Sol ad nostrum hemispherium accedere incipit.

Tribuebant igitur veteres diei, atque anno principium ab eo puncto, quo Sol

ad

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ad nos accedit:cum punctum Zodiaci, quod tropicum hyemalem Capricorni nobis producit,respondeat puncto meridiani sub terra, in quo Sol semel in die reperitur: Quòd apertè norunt hi, qui sub polo boreali constituti sunt. Atq; facilè discerne re possumus, diem scilicet & annum, quasi sibi ad inuicem medio suarum partium respondere; solstitium inquam hyemale, mediæ nocti, æstiuum meridiei, æquino-ctium vernale ortui Solis,autumnale occasui. Quam tamen similitudinem, multò quam nos manifestius deprehendunt,hi qui(ut diximus)sub polo boreali versantur.

Quod si quis dubitet hac ratione correcto anno, quo nam pacto ad calculos cœ-lestes motus medijs tabulis astronomicis hactenus in lucem æditis redigi possint, id facilimum sanè erit,exempli gratia;aliquis planetę situm,aut alicuius stellę fixæ,quo cunque die mensis anni correcti inuenire cupit,detrahat ex huiusmodi tępore dies. 21.ab Aera Christi, cum residuo supputet stellam, cuius situm scire desiderat;sum-pta quacunque tabula, supputatio erit exacta: Cuius ratio cuilibet manifesta erit, qui sciet annum vt potè. 1579.dierum.344.tantummodo constitutum fuisse. Nam in ijsdem locis cœli prima die Ianuarij correcti,erunt stellæ quibus esse solebant.11 Decembris præcedentis anni ex supputatione tabularum: atque ita deinceps. Alia præterea via idem perfici posset inuentione omnium motuum cęlestium ipso princi pio anni.1580.correcti: hoc statuto, vt hi motus radices essent Aeræ S.D.N.Grego rij x 111. quod si alio tępore quispiam motus cęlestes ad calculos redigere voluerit, supputabit ab Aera huiusmodi, quæ anno.1580.principium habuerit: Quæ vt nobi lius nomen sortiatur,idq; merito ex nomine Gregorij.XIII.Pont. Max.appelletur; exemplo antiquarum, quæ ex Principum nominibus sunt appellate:vt tanto Pontifi ci, cùm ex alijs multis,tum etiam ex hac non infima re, inter mortales immortale no men comparetur. Ei verò summæ, quæ ex huiusmodi Aera Gregoriana ex tabu-lis colligetur, ipsiusmet Aeræ radices addantur, vt exactus calculus habeatur. Et hæc sit primæ sententiæ nostræ explicatio.

Altera erit numerum dierum mensium anni alia ratione quàm nunc se habeat,or dinandum esse:nempe vt Ianuarius, Nouember atq; December dies.29.singuli con tineant, Februarius,Martius,& October.30.Aprilis,Maius,Augustus,& September dies.31.Iunius,ac Iulius. 32.atque id hac potissimum de causa, vt Sol unum quodq; signum calendis mensium ingredi possit. Nam detractis (ut dictum est) diebus.21. & reuocato ingressu Solis in principium Capricorni ad principium Ianuarij, in quo signo hac nostra tempestate,Sol, dies propè.29.& quartam vnam versatur: si Ianua rius.29.dies continebit, exactis hisce diebus, ingredietur Aquarium circa princi-pium Februarii; hæret autem hoc nostro sæculo in Aquario Sol dies propè.29.cum dimidio;quare si Februarius erit.30.dierum,elapsis ipsis diebus, Sol ingredietur pi-sces circa principium Martii:& sic de cæteris.

Quamobrem si generali correctione annus emendandus erit, pulcherrimè acci-det,si menses anni cum duodecim partibus cœlestibus, itineris annui Solis,concor-dauerint;eisq; aptè responderint.Qua ex re,varię vtilitates promanabunt,presertim Nautis, Agricolis, Medicis, & alijs qui vera principia, & interualla temporum per specta habebunt:terminos item & interualla incrementi & diminutionis dierum & noctium, & eorundem æqualitatis. Exempli causa, scient omnes principium Ianua-rij,esse non modo anni principium, verum etiam hyemis,esse minimam anni diem, & eius noctem maximam; principium incrementi diei,& diminutionis noctis;atque etiam omnia illa,quæ ex huiusmodi conuersione Solis ad nos dependent. pariter scient omnes primam diem Iulij, non tantum æqualiter annum diuidere, sed prin cipium

cipium quoque esse æstatis, maximam diem, noctem minimam totius anni; princi-
pium diminutiónis diei & incrementi noctis, vnà etiá ea, quę Solis conuersionem
ad auſtráles ſequuntur.

Necnon intelliget vnuſquiſque primam diem Aprilis, primamq́; Octobr. æqui-
noctiorum dies eſſe; primam autem diem Aprilis, initium veris; Octobris Autumni;
Item Aprilis diem eſſe eum, quo dies noctis prolixitatem vincere incipit: Octobris,
quo nox diei longitudinem ſuperat, & alia huiuſmodi, quæ ab æquinoctijs depḗdḗt.

Si vero quiſpiam obijciat, modum hunc noſtrum & ordinem perpetuum eſſe non
poſſe, ob motum augis Solis; quod punctum cum fuerit in principio Capricorni, tũc
Sol hærebit in ſigno Sagittarij. 3 2. diebus, totidem in Capricórno, in Géminis vero
2 9. totidem in Cancro; ex quo ſequetur prioribus cótrarius effectus; huic ego reſpon
debo, tale quidpiã non euenturum, niſi exactis ab hoc anno annis. 2 4000. quod ſi
mundus poſthac totidem annis, quot fuit antehac, perdurauerit, punctus augis non
amplius à ſitu præſenti, quàm. 4 5. gradibus diſtabit. Verum demus modũ noſtrum
& regulam in annos ter, aut quater mille ſubſeruire poſſe, nec amplius, certè hoc
toto tempore nullius momenti penè erit, quæ accidere poterit mutatio, tametſi ela-
pſis quatuor millibus annorum Februarius eſſe debebit. 2 9. dierum. Aprilis & No-
uember. 3 0. Iunius & October. 3 1. Auguſtus. 3 2. in aliis verò menſibus nihil mutan-
dum erit. Ecce quam ſit nullius momenti mutatio.

Quæ ſi Iulij Cæſaris temporibus fuiſſent animaduerſa nunquam omiſſa fuiſſent,
ſed ſcientiæ Aſtronomicæ nondum (vt ita dicam) confirmata ætas, cum alibi, tũ maxi
mè in Italia, quo minus hæc aut ſcirentur aut ſtatuerentur impediebat.

Tertia ratio eſt, vt non ſolũ feſta mobilia, verum ét immobilia ad meliorem regu
lam (vt dictum eſt) reuocentur, ſi ſuis temporibus celebranda erunt. Quorum primũ
eſt Natiuitas Domini, & quæ ab ea pendent; nempe Circunciſio, Epiphania, Purifi-
catio, Annunciatio; & Natiuitas Io. Baptiſtæ. ita vt dies Natalis Domini celebretur
prima die anni, cum Dei filius naſci voluerit circa verum principium anni, quod à
ſolſtitio hyemali initíum ducit, & in ipſo principio diei naturalis ex Romanorũ ſen
tentia, media ſcilicet nocte, tanquam qui ſummæ lætitiæ principium, poſt longos &
graues filiorum Adæ mærores, eſſet allaturus. Nec forſan Ianuarij nomini, à vete-
ribus Iano bifronti dicati hæc mutatio non conueniret, cum in ipſo ſeruatore, duæ
veluti frontes & formæ vnitæ ſint, duæ ſcilicet naturæ diuina & humana. Hac ratio
ne abuſus tolletur, natus ex diuerſis moribus Tabulariorum, quorum alij monumen-
ta, ſeu quæ uocant Inſtrumenta, à die Natiuitatis Domini incóhant, alij à Circunci-
ſione, alij à Calendis Martij, nonnulli à Paſchate; quæ varietas innumerabiles lites
affert & abuſus propè infinitos, ob dubiam & ancipitem ſcripturam. Indictionum
præterea ordini, hic noſter modus nihil officiet; celebrato Natali celebrabitur Cir-
cunciſio octaua Ianuarij. Epiphania. 1 3. ciuſdem. Purificatio. 1 1. Februarij quæ erit
4 0: dies à Natiuitate ſeruatoris. Prima Aprilis Annunciatio Virginis ſolennis erit,
ipſo nempè die æquinoctij, natiuitas Diui Io: Baptiſtę celebrabitur Prima Iulij die
quæ erit ſolſtitij æſtiui, cum illa diminutionem capit. vt rectè Diuus Auguſtinus il-
la verba Io: Baptiſtę interpretatus fuerit. Illum opportet creſcere, me autem minui:
in quibus ſić tantus Doctor philoſophatur, vt tempus etiam natiuitatis ſerui & do-
mini præclare notet dicens, natus eſt ſeruus cum decreſcunt dies, natus eſt Dominus
cùm creſcere incipiunt.

Inſignes etiam Theologi admonuerunt habendam rationem eſſe nonnullorum
feſtorum, vt Diui Antonij, diuorum Fabiani & Sebaſtiani, & aliorum ſanctorum,

                                                            D d        ſi forte

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si forte in octauam Epiphaniæ inciderint: Verum hęc.S.D.N.curæ erunt, ut in aptiſ
sima tempora transferantur.

Admonuerunt præterea transferendos esse dies festos Beati Stephani, Ioannis,
& Innocentium, vt quemadmodum factum est hactenus, diem natalis proximè se
quantur, ob multorum Doctorum, non recentium modo, sed etiam antiquorum ob
seruantiam; qui suis omelijs & concionibus multa piè, de mysteriis successionis Festo
rum huiusmodi tradiderunt.

Cuperent etiam præclari Theologi diem Assumptionis Beatæ virginis incidere
in primam Septembris, Natiuitatem autem in. 2 5. vt quemadmodum toto illo men
se in signo Virginis sol versabitur, ita Ecclesia Dei in cęlebrandi tantæ Virginis ma
tris Dei laudibus occupetur.

Atque hęc sunt Serenissime Princeps, quę longa & attenta cogitatione à me exa
minata, atque perpensa fuerunt; quæ si tam diligenter & accuratè expendentur ab
his, quorum interest, quam mihi apta & rationi consentanea, ac vera penitus, imo
(quod me magis afficit) etiam tibi visa fuerunt; non dubito quin placitura sint; & vo
tis summi Pont. aliqua ex parte satisfactura. eò magis quòd te iubente, & cogitata à
me, & scripta fuerint.  Vale Princeps Serenissime, & qua soles hylaritate cętera no
stra, etiam has breues vigilias suscipe & foue.  Dat. Augustæ Taurinorum Kal.
Aprilis.  M D L X X V I I I.

T. Celsitudinis.

Deditissimus Mathematicus.

Io. Bap. Benedictus.

D B

# DE CIRCVLO
## AMBIENTE QVADRILATERVM.
### AD SERENISS. CAROLVM EMANVELEM
#### Pedemontis Principem.

PRoblema quod à celfitudine tua nobis proponitur non folum poffibile eft, fed facile etiam ad foluendum, hoc eft quod circulus talis inueniatur, qui poffit circunfcribere, feu capere quadrilaterum ex quatuor datis rectis lineis terminatum, vel fic, datis quatuor rectis lineis ex quibus quadrilaterum poffit effici, tale efficiatur vt circa ipfum, circulus poffit circunfcribi.

Sint igitur. 4. lineæ propofitæ. b.d:q.b:a.q:et.a.d.ex quib⁹ poffibile fit quadrilaterum conftitui, tale verò conftituatur, vt aliquis circulus poffit ipfum circunfcribere. imaginemur autem hoc factum effe, quod quidem quadrilaterum fit.a.d.q. b. cuius



Dd 2
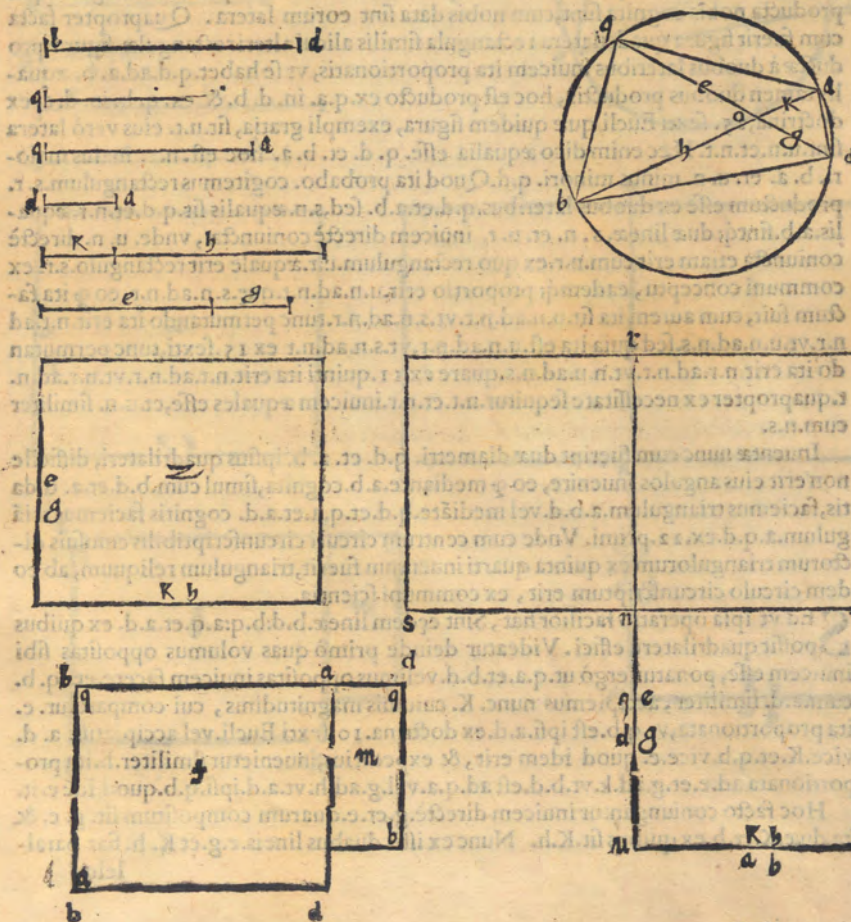
diametri sint.q.d.et.a.b.quæ se inuicem interfecent in puncto. o. vnde cum anguli
contra se positi circa.o.æquales inuicem sint ex.15.primi Eucli.& angulus.a.q.d.æ-
qualis angulo.a.b.d.& angulus.q.b.a.æqualis angulo.q.d.a.et.b.q.d.angulo. b.a. d.
ex.20.tertij tunc triangulus.a.o.q.similis erit triangulo.d.o.b.et.q.o.b.similis trian-
gulo.a.o.d.ex definitione. Vnde eadem proportio erit ipsius.q.o.ad.b.o.quæ ipsius
q.a.ad.b.d.& ipsius.b.o.ad.o.d.eadem quæ.q.b.ad.a.d.& ipsius.q.o.ad. o. a. eadem
quæ.q.b.ad.a.d.proportio igitur.q.o.ad. o. d. cognita nobis erit, vt composita ex
ea quæ est. q. o. ad.o.b. ex.o.b. ad.o.d. quæ nobis cognitę sunt, mediante
proportione ipsius. q. a. ad. b. d. & ipsius.q.b.ad.a.d.proportio similiter ipsius.b.o.
ad. o. a. nobis cognita erit, vt composita ex proportione ipsius. b. o. ad. o. q. &
ipsius.o.q.ad.o.a.cognitis,mediante proportione ipsius.b.d.ad. q.a.& ipsius.q.b.ad
a.d.cum autē proportio ipsius.q.o.ad.o.b.nobis cognita sit, tunc nobis cognita erit
proportio ipsius.q.d.ad.a.b.Nam ut.q.o.ad.o.b.est vt.a.o.ad.o. d. ex similitudine,
quare proportio compositi ex primo,& quarto terminorum ad compositum ex.2.&
tertio,cognita erit.sed quod sit ex.q.d.in.a.b.cognitum nobis est, vt æquale duobus
productis,hoc est ex.q.a.in.d.b.& ex.q.b.in.d. a. ex secunda primi Almagesti.quæ
producta nobis cognita sunt,cum nobis data sint eorum latera. Quapropter facta
cum fuerit figura quadrilatera rectangula similis alicui alteri rectangulæ figuræ pro
ductæ à duobus lateribus inuicem ita proportionatis, vt se habet.q.d.ad.a. b. æqua-
lis tamen duobus productis, hoc est producto ex.q.a. in. d. b.& ex. q. b. in. d. a. ex
doctrina,25. sexti Eucli.quæ quidem figura, exempli gratia, sit.u.t. eius verò latera
sint.u.n.et.n.t. Hæc enim dico æqualia esse. q. d. et. b. a. hoc est. n. t. maius maio-
ri. b. a. et. u. n. minus minori. q.d.Quod ita probabo. cogitemus rectangulum.s. r.
productum esse ex duobus lateribus.q.d.et.a.b. sed,s.n.æqualis sit.q.d.et.n.r.æqua-
lis.a.b.sintq; duæ lineæ. s. n. et. n. t. inuicem directè coniunctæ, vnde. u. n. directè
coniuncta etiam erit cum.n.r.ex quo rectangulum.u.t.æquale erit rectangulo.s.r. ex
communi conceptu, eademq; proportio erit.u.n.ad.n.t.quę.s.n.ad.n.r. eo ợ ita fa-
ctum fuit, cum autem ita sit.u.n.ad.n.t.vt.s.n.ad.n.r.tunc permutando ita erit.n.t.ad
n.r.vt.u.n.ad.n.s.sed quia ita est.u.n.ad.n.r.vt.s.n.ad.n.t ex 15.sexti,tunc permutan
do ita erit.n.r.ad.n.t.vt.n.u.ad.n.s.quare ex 11.quinti ita erit.n.t.ad.n.r.vt.n.r.ad.n.
t.quapropter ex necessitate sequitur.n.t.et.n.r.inuicem æquales esse,et.u.n. similiter
cum.n.s.

Inuentæ nunc cum fuerint duæ diametri. q.d. et. a. b. ipsius quadrilateri, difficile
non erit eius angulos inuenire, eo ợ mediante.a.b.cognita,simul cum.b.d.et.a. d.da
tis,faciemus triangulum.a.b.d. vel mediāte.q.d.et.q.a.et.a.d. cognitis faciemus triā
gulum.a.q.d.ex.22.primi. Vnde cum centrum circuli circunscriptibilis cuiusuis di-
ctorum triangulorum ex quinta quarti inuentum fuerit,triangulum reliquum, ab eo
dem circulo circunscriptum erit, ex communi scientia.

SEd vt ipsa operatio facilior fiat, Sint eędem lineæ.b.d:b.q:a.q.et.a.d. ex quibus
possit quadrilaterū effici. Videatur deinde primò quas volumus oppositas sibi
inuicem esse, ponatur ergò ut.q.a.et.b.d.velimus oppositas inuicem facere,et. q. b.
cum.a.d. similiter, accipiemus nunc. K. cuiusuis magnitudinis, cui comparetur. e.
ita proportionata,vt.q.b.est ipsi.a.d.ex doctrina. 10.sexti Eucli.vel accipiatur. a. d.
vice.K.et.q.b.vice.e. quod idem erit, & expeditius,inuenietur similiter.h. ita pro-
portionata ad.e.et.g.ad.k.vt.b.d.est ad.q.a.vel.g.ad.h.vt.a.d.ipsi.q.b.quod idé erit.

Hoc facto coniungantur inuicem directè.g.et.e.quarum compositum sit. g. e. &
ita duæ.K.et.h.ex quibus sit.K.h. Nunc ex istis duabus lineis.e.g.et K. h. fiat paral-
                                                                                                  lelo-

Ielogrammum.Z.deinde fiant alia duo parallelogramma rectangula quorum vnum
sit ex.q.a.in.b.d.reliquum verò sit ex.q.b.in.a.d.quæ quidem sint.f.m.

Quo facto designetur rectangulum.u.t.ex.25.sexti, quod æquale sit duobus re-
ctangulis.f.et.m.simile tamen.Z.cuius rectanguli vnum latus correspondet.e.g.reli-
quum verò.K.h.in proportione,sed in æqualitate,vnum correspondet.q.d. reliquũ
vero.a.b.diametris ipsius quadrilateri.

Accipiatur nunc latus illud quod correspondet.K.h.hoc est ipsi. a. b. maius scili-
cet,& simul cum.b.d.et.a.d.formetur triãgulũ. a.b.d.ex.22.primi Eucli. circa quod
circunscribatur circulus ex.5.quarti.& inuentum erit quod quærebamus.

PER
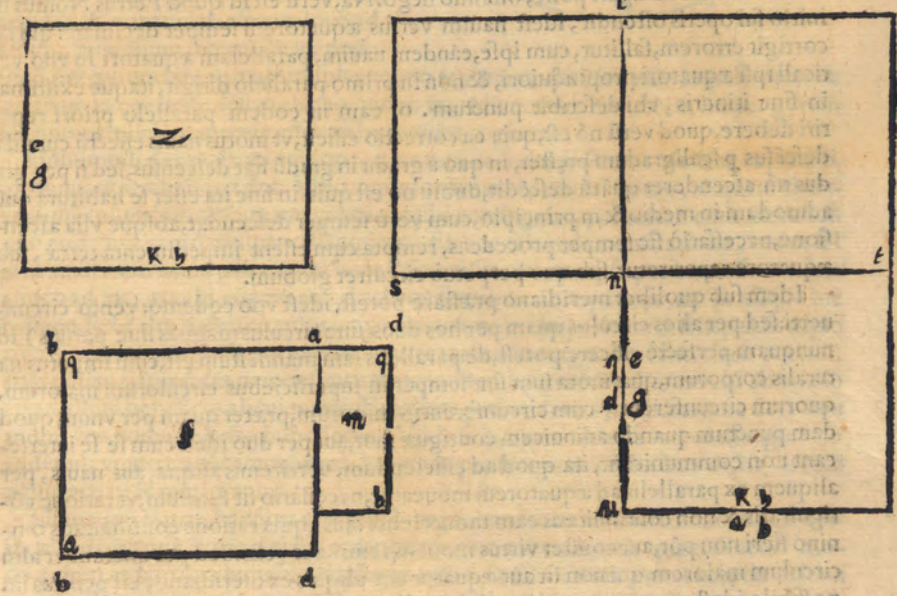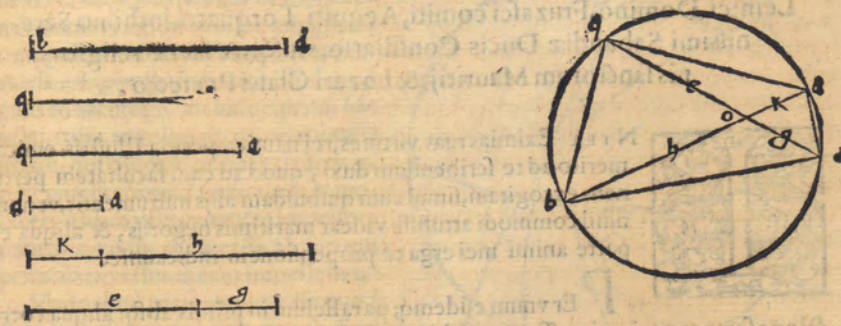
# PER EVNDEM PARALLELVM
## absque correctione semper nauigari non posse.

*Vbi notantur Petri Nonij lapsus in correctione erroris nauis.*
*Et alij Petri Medinæ errores.*

ILLVSTRISSIMO ANDREAE PROVANAE
Leinici Dominò, Fruzasci comiti, Aequiti Torquato, inthimo Sere-
nissimi Sabaudiæ Ducis Consiliario, eiusq̃;, & sacræ religio-
nis sanctorum Mauritij, & Lazari Claisi Præfecto.

NTER Eximias tuas virtutes, rei nauticæ peritia Illustris emicat
merito ad te scribendum duxi, quod ad eam facultatem perti-
nens excogitaui, simul cum quibusdam alijs instrumentis, vt non-
nihil commodi attulisse videar maritimis negotijs, & aliqua ex
parte animi mei erga te propensionem indicauisse.

Er vnum eũdemq̃; parallelum in primis absq̃; aliqua corre
ctione semper nauigari posse, omninò nego. Nã, verũ est id quod Petrus Nonius in
initio sui operis ostendit, idest nauim versus æquatorem semper declinare: qui cũ
corrigit errorem, fallitur, cum ipse, eandem nauim, parallelam æquatori in vno ver
ticali ipsi æquatori propinquiori, & non in primo parallelo dirigit, itaque existimat
in fine itineris, vbi describit punctum. o. eam in eodem parallelo priori repe-
riri debere, quod verũ nó est, quia ea correctio efficit, vt motus nauis effectũ cuiusdã
descésus p scaligradum prestet, in quo à gradu in gradũ fiat descensus, sed si per gra
dus tm̃ ascenderet quãtũ descédit, dubiũ nó est quin in fine ita esset se habitura quẽ
admodum in medio & in principio, cum verò semper descendat, absque vlla ascen-
sione, necessariò sic semper procedens, remota cum essent impedimenta terræ, sub
æquatore reperiretur, sub quo perpetuò circuiret globum.

Idem sub quolibet meridiano præstare potest, idest vno eodemq̃; vento circun-
uerti: sed per alios circulos quam per hos duos (siue circulus magnus siue paruus) id
nunquam perfectè efficere potest, de parallelis iam manifestum est, cum impetus na
turalis corporum, quæ mota sunt sint semper in superficiebus circulorum maiorum,
quorum circunferentię cum circunferentijs minorum, præter quam per vnum quod
dam punctum quando adinuicem contiguæ sunt, aut per duo idest cum se se interse-
cant non communicant, ita quod ad efficiendum, vt triremis aliqua, aut nauis, per
aliquem ex parallelis ad æquatorem moueatur, necessario sit futurum, vt ratione có-
tiguitatis & non continuitatis eam moueri curemus. quia ratione continuitatis om-
ninò fieri non pót, aut constet virtus mouens remis, aut velis. Sed per quemlibet aliũ
circulum maiorem, qui non sit aut æquator aut aliquis ex meridianis, est penitus im
possibile. idest vt vnius venti vi nauis impellatur. Quod vt clarè pateat, sit orizon.
a.c.b.d. & æquator.c.q.t.d. & vnus meridianorum sit. a.r.n.t.b. in quo. n. sit Zenit sub
quo primum nauis reperiatur et.r. sit polus septentrionalis. Ponamus etiam quod
azimut.

azimut.f.q.u.n.p.conftituat angulum.a.n.p.feu.f.n.b.cum meridiano graduum.45.
vnde tot graduum erunt arcus.a.p.et.b.f.orizontis, quapropter punctum.f.commu-
ne ipfius orizontis cum azimut,erit medio in loco inter. b.et.c.& ideo quarta.n.f.ip
fius azimut fecabit quartam.c.t.ipfius æquatoris in puncto.q.& habebimus triangu-
lum.q.t.n.cuius angulus.t.rectus erit,& angulus.n.cognitus fimul cum latere,n.t. la-
titudinis loci,quibus rebus mediantibus deueniemus in cognitionem lateris.q.n.la-
teris.q.t.& anguli.q.ex.4.primi Copernici fi voluerimus.

Ponamus nunc nauem à puncto.n.difcedere feu iter facere verfus.u.punctum, &
in ipfo.u.reperiri, iam in hoc fitu ha-
bebimus angulum huius fecundi me-
ridiani.r.u.p.qui quidem in hoc cafu
minor effet angulo. r. n. p. extrinfeco
trianguli.r. u. n. ex conuerfo fecundæ
partis.48.ppofitionis tertij lib.de triã
gulis Monteregij,feu ex.13.primi Me
nelai,cuius anguli.u.arcus orizontalis
fit.x.e. qui quidem minor erit arcu. a.
p.vt patet ratione anguli.r.u.e. mino-
ris,ergo alius ventus nauem impellet à
puncto.u.verfus.q.diuerfus ab illo qui
prius ab.n.verfus.u.eam impellebat.
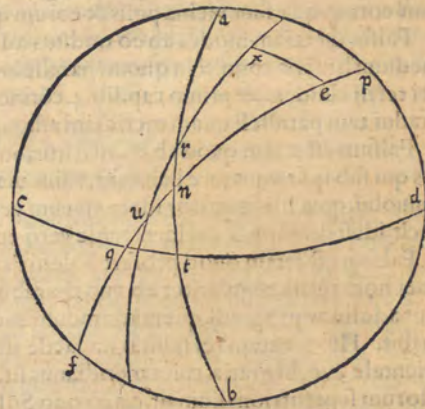


Vnde clarè patet verum effe quod
dico,hoc eft quod aliquo modo fieri
non poteft,vt nauis ab aliquo loco ad
alium, breuiffimo interuallo ire poffit
ideft per gyrum circuli maioris fphæræ vno tantummodo vēto eam impellente,præ
ter quam in equatore,feu in aliquo quouis meridianorum, ños autem ire per gyrum
alicuius paralleli dementia effet,nifi neceffitas cogeret.

Huiufmodi demonftrationis ope, quantum decipiatur Petrus Medina cap.6.lib.
3.cognofcitur,vbi fic fcribit; Vbicunque locorum reperiatur homo, aliquem circu-
lum qui vniuerfum ambiat imaginatione fibi confingens, per totum eum circulum
vno eodemq́; vento nauigatio fufcipitur. Ex hac etiam demonftratione, quàm fal
fa fit charta maritima patet,cuius beneficio exiftimant nautę fe per breuiffimum iter
a loco ad locum vehi etiamfi dicti loci non fint ambo in æquatore, aut in aliquo me
ridiano,fed extra dictos circulos vnico tantum vento impellente & fi in paruis æquo
ribus hic error parum depræhenditur,forte tamen in magno Oceano clarè pateret.
In fuperius igitur dicta demonftratione iam oftendi, quod fi velimus vehi ab vno lo
co ad alium beneficio alicuius circuli maioris,præter duos iam dictos,hoc fieri non
poteft vno eodemq́; vēto impellente. Vnde fequitur,omnia ea interualla quæ vno
eodemq́; vento tranfibimus futura longiora,præterquam in duobus dictis circulis
æquinoctiali & meridiano.

Cum verò Petrus Medina cap.7.volens probare chartam maritimam bonam ef-
fe, planifphęrium Ptolomei & Iordani citat, non animaduertit quàm diuerfo mo-
do à charta maritima huiufmodi inftrumentum fit fabricatum,cum exceptis orizon
te recto,& meridiano in dicto inftrumento quilibet alius circulus fit circulus, fiue fit
almicantarat,fiue azimut,fiue æquator,fiue tropicus,fiue zodiacus, fiue alius quiuis
circulus,cum in charta maritima ne vna quidem fit linea, quę non fit recta, quolibet
nomine vocetur.                                                          Superius

Superius positæ meæ demonstrationis ope, deuenimus in cognitionem magnitu
dinis arcus.n.q.cognoscimus etiam angulum.n.q.t.vnde nobis manifestũ esset quo
vento oporteret iter facere.cum à puncto.q.nauis aliqua discessura esset, in eodem
azimut proposito. Idem etiam dico de puncto.u.cum cogniti essent arcus.n.u.et.n.
r.vt supponitur,simul cum angulo.r.n.u.vnde cognitus esset nobis angulus.n. u. r.ex
11.primi lib.Copernici, ex quo ventus nobis cognitus foret.

Modus autem quem idem Medina cap.9.lib.tertij ad cognoscendam distantiam
vnius meridiani ab alio præscribit,in genere est falsus,etiam si is ab antiquis eum de
sumat,qui,hic non viderunt quam magna inter meridianos differentia sit interuallo
rum eorum quæ sunt vicina polis & eorum quæ sunt circa æquatorem.

Falsus est etiam modus ab eo traditus ad cognoscendos gradus longitudinis per
medium itineris cogniti in quouis parallelo extra æquatorem facti,& hoc cap. 14.li
bri tertij eiusdem,& primo cap.lib.4.cõtinetur,vbi.17.leucas cum dimidia cuilibet
gradui tam paralleli quàm meridiani assignat.

Falsum est etiam quod ab eo asseritur,Solem,cum reperiretur in æquatore, circa
eos qui sub ipso æquatore habitant,vnius diei noctisq́; spatio per omnes uentos cir-
cunuolui.quia illis æquator idem est cum verticali, qui duos tantum rhumbos pro-
ducit,idest orientis,& occidentis:hic verò error,in secundo cap.lib.6.habetur.

Falsum est etiam quod profert Solem ijs qui habitant sphæram obliquam, qua-
libet hora tertia,regulariter ab vno rhumbo ad alium ex præcipuis idest ab vno azi
mut ad alium progredi,quemadmodum eadem cap.2.lib.6.et.7. cap. septimi libr,
scribit. Huius autem rei falsitas ita facile depræhendetur, ponamus hemisphęrium
orientale, verbi gratia,cuius meridianus sit.p.z.b.æquator aũt.e.m.vnus verò paral
lelorum septentrionalium sit.c.a.in quo Solem existere ponamus,orizon autem sit.
b.m.zenit vero.z.polus arcticus.p.sit postea azimut.z.q.à meridiano distans per gra
dus.45.qui quidem azimut in hoc hemisphęrio erit rhumbus illius venti,quem uul-
go Itali Sirocum dicunt,et.z.m.sit azimut verticalis qui in hoc hemisphærio erit ró
bus venti orientalis, ita ꝗ secundum Medinam à rhumbo.z.m.ad.z. q. Sol absoluet

spatium tẽporis trium horarum , & aliud
æquale temporis spatium absoluet à rhũ
bo.z.q.ad.z.b.ex ipso Medina, vnde ar-
cus.a. o. paralleli esset graduum. 45.&
item arcus.o.c.

Ponamus nũc Solem reperiri in ęqua
tore,vbi per ipsum Medinam arcus.u.m.
similiter esset graduum.45.& sic.u.e:pro
tracto ergo arcu. p. o. f. palam erit ar-
cum.f. e.fore graduum.45. sed cum
arcus. e. u. sit graduum.45. ex supposito
ipsius Medinæ, sequeretur arcum. e. f.
æqualem esse arcui.e.u.pars igitur æqua
lis erit suo toto .
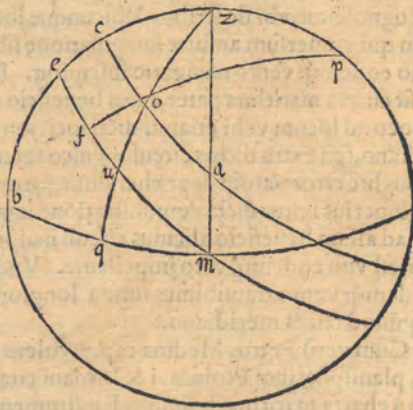


Id etiam quod Petrus Nonius pagina
124.et.125.lib.de arte nauigandi con-
tra nautas de distantijs Solis à meridiano scribit,hanc opinionem Petri à Medina &
eorum qui idem ei persuaserunt falsam esse demonstrat.

Fal-

Falsum est etiam id quod cap.3.lib.6. pronuntiat,ita dicens.

Quod cum verum esset à parte oriétali insularum quæ azore dicuntur, pyxidem versus eum ventum qui vulgò Græcus dicitur,& ab occidentali versus eum qui Magister dicitur,vergere , huius rei nulla est ratio.

Ego enim huiusmodi rationem reperiri posse contendo , quæ talis est, quia pars rosæ(ut vocant) à magnete tacta, ad aliquod punctum,aut situm globi terrę , in eodem meridiano insularum,quæ Azore dicuntur,vltra situm poli arctici in terra dirigeretur ,ita vt situs dicti poli in terra esset in dicto meridiano, inter locum qui ab indice rosæ aut pyxidis respiceretur, & dictas insulas,id quod superius scripto meridiano facile cognosci potest,sumendo pro insulis situm.e.in meridiano et. z. pro polo , et.p.pro loco qui à pyxide sit visus , imaginãdo deinde pyxidem in.f. magis orientali quam est.e.clarum est lineam quæ respicit(ponamus)f.p.versus Græcum & ab alia parte versus Magistrum declinare.

## De Armilla Nautica.

### AD EVNDEM.

CVm sæpe viderim quam in magnis æquoribus nos fallant,atq; decipiant maritimæ,seu nauigatoriæ chartę,quemadmodum aliquoties inter nos sermonem habuimus : in id totus incubui vt aliquam machinam excogitarem,quæ difficilis non esset , efficeretq̃; vt nauis super aquę globum, beneficio circulorum maiorum,quam optimè posset , idest breuissimo itinere ab uno loco ad alium ferretur. Id q̃ mihi ex animi voto successurum putaui,beneficio quinq; circulorum circundantium aliquẽ globum terrestrẽ & maritimum,quales ij sunt qui in inferiori Germania à Gerardo Mercatore struuntur,qui vno pede cum dimidio diametri constet,idest sesquipedẽ.

Sit ergo,exempli gratia,huiusmodi globus.a.b.d.circa quem duo circuli, aut cir
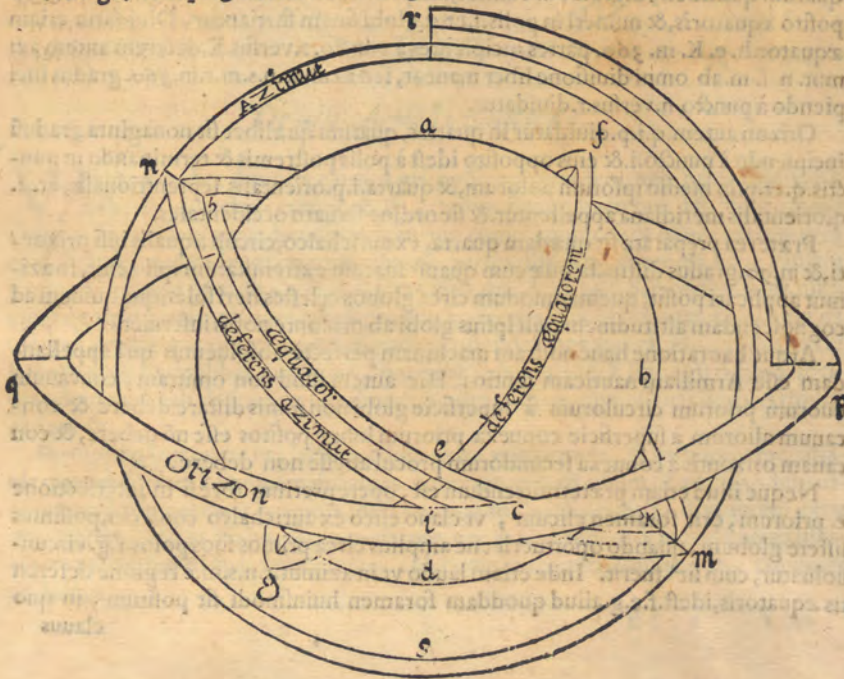
culares lineæ ex aurichalco applicentur inuicem coniuncti per medium ad angulos
rectos,quorum prior.f.e.g.in se globi polos mediantibus extremitatibus axis mun-
di contineat, qui quidem poli à punctis suarum intersectionum per quartã ex æquo
in punctis.f.et.g.ita distent, vt globus circa eosdem, in situ longitudinis mundi vol-
ui possit . Huiusmodi autem circulus,æquatoris deferens appelletur.

Secundus autem circulus sit.h.e.K. cum primo ad angulos rectos in puncto. e . &
in suo opposito connexus,& is appellabitur æquator,& poli.f.g.primi poli dicentur.

Circa huiusmodi duos circulos, alios etiã duos existere vellé simul cõiũctos medio
ad angulos rectos. In quibus quidé intersectionis punctis sint duo poli,qui hos duos
circulos cum secundo priorum idest cum æquatore in duobus punctis inuicem op-
positis connectant; quæ æquatoris puncta à punctis intersectionis eiusdem cum suo
deferente , ratione vna quarta distent, quorum duorum circulorum primus sit. n.i.
m.quem deferentem azimut appellabimus; secundus. r. n.s. m . azimut locorum no
minabimus.eorundem intersectionis rectæ,puncta sint.n.et.m. à quibus duo poli ex
aurichalco confecti similes primis.n.h.et.m.K.vsque ad puncta. h. et. K. æquatoris
perueniant, qui spissitudinem æquatoris distantem à puncto.e.vna quarta penetrẽt,
ita vt æquator circum circa.n.h. et. m.K. in situ latitudinis mundi verti queat. Et
hos , secundos polos nominabimus.

Alius deinde circulus.q.i.p. duos posteriores circulos ambiat, cum deferente ta-
men azimut mediantibus duobus polis in puncto.i.& in suo opposito ex equo distan
tibus à secundis polis vnius quartæ spatio iungatur. Ita vt dictum deferens azimut
circa hos tertios polos volui possit , atque hunc circulum.q.i. p.orizontem vniuersa
lem vocabimus. Hic vero orizon super quatuor quartas circuli , aut super quatuor
paruis columnis,ut fieri solet innixis suæ basi , ita ponatur, vt moueri non possit.

Primus autem circulus.f.e.g.deferens æquatoris in. 4. partes æquales diuidatur,
quarum quælibet.90.gradibus constet,incipiendo ab intersectionibus. e. & eius op
posito æquatoris,& numeri in polis.f.et.g.globi finem sortiantur . Diuidatur etiam
æquator.h. e. K. in. 360. partes incipientes à puncto.e.versus.K.deferens autem azi
mut. n. i. m.ab omni diuisione liber maneat, sed azimut.n.s.m.r.in.360. gradus inci
piendo à puncto.n.versus.r.diuidatur.

Orizon autem.q.i.p.diuidatur in quartas, quarum quælibet sit nonaginta graduũ
incipiendo à puncto.i.& eius opposito idest à polis postremis & terminando in pun-
ctis.q.et.p.in medio ipsorum polorum,& quarta.i.p.orientalis septentrionalis, et. i.
q.orientalis meridiana appellentur.& sic ordine seruato occidentales.

Præterea preparata sit quædam quarta, ex aurichalco,circuli æqualis ipsi orizon-
ti,& in.90.gradus distincta quæ cum quauis suarum extremitatùm ipsi zenit, in azi-
mut applicari possit, quemadmodum circa globos celestes fieri solet;quę quidem ad
cognoscendam altitudinem poli ipsius globi ab orizonte nobis inseruiet.

Atque hac ratione hanc nostram machinam perfectè absoluemus quã appellan-
dam esse Armillam nauticam sentio . Hic autem illud non omittam , concauum
duorum priorum circulorum à superficie globi non nimis distare debere & con-
cauum aliorum à superficie conuexa priorum longe positos esse nõ debere, & con
cauam orizontis à conuexa secundorum procul abesse non debere .

Neque illud etiam prætermittendum est , operepretium fore si in intersectione
e. priorum, erit foramen elicum , vt clauo elico ex aurichalco confecto,possimus
sistere globum, quando oportuerit , ne amplius circa primos suos polos.f.g. circun-
uoluatur, cum sit⁹ fuerit. Inde etiam laudo vt in azimut.r.n.s.m.è regione deferen
tis æquatoris,idest.f.e.g.aliud quoddam foramen huiusmodi sit positum , in quo
                                                                         clauus

clauus elicus vſque ad circulum. f. e. g. perueniens, æquatorem ſiſtere poſſit, ne circa ſecundos polos. h. et. K. amplius moueatur quum noluerimus eum mutare ſitum.
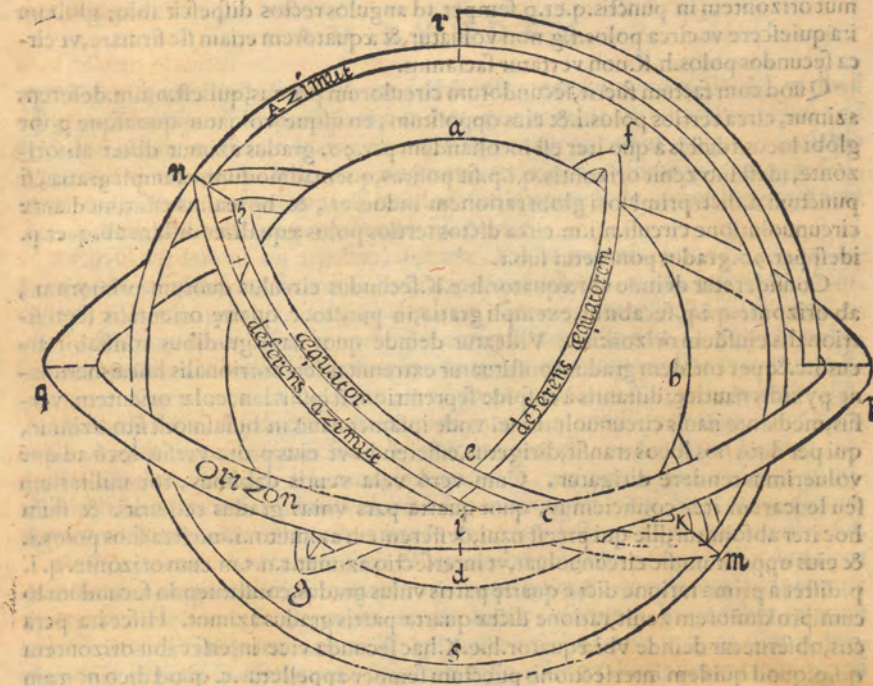


## De uſu Armillæ nautica.

VT autem noſtra Armilla nautica vti poſſimus pyxidem nos prius oportebit habere, diuerſam tamen ab ijs, quibus nautæ hactenus vſi fuere : nolo enim vt tâ craſſa minerua beneficio vétorum communium circa hanc rem nos gera mus, ſed ratione graduum orizontis in. 360. partes diſtincti, atque ob hanc cauſam ſentio, vt ima pars pyxidis penitus detecta videatur, & in. 360. partes diuidatur, nilç; aliud quam quandam lanceolam ſupra eius acum eſſe volo, quæ dum mouebitur na uis, per gradus quamlibet orizontis partem oſtendet; hos autem. 360. gradus, ita ſe habere volo, vt quçlibet quarta. 90. contineat, ſupputatioç; à linea meridiana inci- piat, & in verticali deſinat, vt huiuſmodi diuiſio cum ea, quæ eſt orizontis Armillæ eadem ſit.

Prçſupponâtur nunc in globo duo loci extra æquatorem, & in diuerſis meridia- nis quomodolibet adinuicem diſtantes, à quorum vno ad alium ſit nauigandum iti- nere quo ad fieri poterit breuiori, ideſt per gyrum circuli maioris, dixi autem extra æquatorem, ideſt vt ambo, nec in æquatore, nec in uno eodemç; meridiano exiſtât, quia vt aliàs dixi in huiuſmodi locis, vnico tantum vento comite, iter conficere poſſumus.

<div align="right">Ec   2      Volo</div>

Volo primum vt mediante circūuolutione globi circa primos polos.f.g. & æqua
toris circa secundos.h.K.hoc est per longitudinem , & latitudinem, hi duo loci in
globo propositi sub azimut.r.n.s.m.secundorum circulorum situ sortiantur,qui azi-
mut orizontem in punctis.q.et.p.semper ad angulos rectos dispescit ibiq; globum
ita quiescere vt circa polos.f.g.non voluatur,& æquatorem etiam sic firmare,vt cir-
ca secundos polos.h.K.non vertatur faciamus.

Quod cum factum fuerit,secundorum circulorum primus,qui est.n.i.m.deferens
azimut, circa tertios polos.i.& eius oppositum , eo usque voluatur quousque prior
globi locus,idest is à quo iter est incohandum per.90. gradus azimut distet ab ori-
zonte, idest sub zenit orizontis.q.i.p.sit positus,quemadmodum,exempli gratia , si
punctum.a.dicti primi loci globi rationem induceret , & borealius esset,mediante
circunuolutione circuli.n.i.m.circa dictos tertios polos æqualiter distans ab.q. et.p.
idest per.90.gradus poneretur sub.r.

Consideretur deinde vbi æquator.h.e.K.secundus circulus duorum primorum ,
ab orizonte.q.i.p.secabitur,exempli gratia,in puncto. c. quartę orientalis septen-
trionalis eiusdem orizontis . Videatur deinde quot nam gradibus constabit ar-
cus.i.c.& per totidem gradus constituatur extremitas septentrionalis lineæ meridia-
nę pyxidis nauticę, distantis à cuspide septentrionali ipsius lanceolæ orientem ver-
sus,mediante nauis circunuolutione. vnde ipsamet nauis in huiusmodi situ azimut,
qui per duos hos locos transit,dirigetur,efficiendo vt eius prora versus locū ad quē
voluerimus tendere dirigatur . Cum verò vela ventis dabimus , tot milliarium
seu leucarum iter conficiemus , quot quarta pars vnius gradus requirit . & dum
hoc iter absoluitur,ille qui pręest naui,defferentem azimut.n.i.m.circa suos polos.i.
& eius oppositum,sic circunuoluat,vt intersectio azimut.r.n.s.m.cum orizonte. q. i.
p.distet à prima ratione dictæ quartę partis vnius gradus,constituendo secundum lo
cum,proximiorem zenit,ratione dictæ quartæ partis gradus azimut. Hisce ita pera
ctis,obseruetur deinde vbi æquator.h.e.K.hac secunda vice intersecabit orizontem
q.i.p.quod quidem intersectionis punctum semper appelletur.c. quod dico non am
plius in eadem distantia mansurum,ut prius à puncto.i.sed aut longius distabit , aut
propius accedet, vt in præsenti exemplo.quemadmodum ex se manifestum est, cū
poli globi,idest equatoris sint extra azimut,vt presupponitur,quia loci sunt in diuer
sis meridianis .

Pro huiusmodi autem distantiæ ratione denuo dirigatur nauis prout æquator.h.
e.K.in orizonte.q.i.p.nobis ostendet, atque hoc modo omnium iter quasi breuissi-
mum fiet. dico autem,quasi,quia omnibus modis necessariò conficitur iter contor-
tum & in formam serpigineæ lineæ. Applicantes deinde per vices extremitatem
quartæ appositæ(de qua superius mentionem fecimus)ipsi zenit.r.efficientes ut per
situm poli globi pertranseat, deueniemus in cognitionem altitudinis eiusdem ab
orizonte,& per consequens quantum itineris per latitudinem eiusdem globi pere-
gerit. mediante deinde intersectione orizontis.q.i.p.cum æquatore, cognoscemus
quātum itineris per longitudinem eiusdem globi,in ipso equatore fuerit peractum,

In-

### *Instrumentum ad ortum, & occasum Lunæ cognoscendum*
### *qualibet anni die.*

## AD EVNDEM.

ECce tibi vir Illuftriff. modũ conficiédi inftrumenti nuper à me inuéti, vt tibi fi
gnificaui, quo,fcire poffis fermè in dies, qua hora(de aftronomicis loquor) ad
determinatum parallelum & abfque multa fupputatione,etiam abfque Aftrolabio
Luna oriatur occidatq; . In quo inftrumento poteris etiam videre quo in figno Sol,
& fæpius itidem Luna permeat, & huiufce afpectus cum Sole , atque longitudinem
diei noctisq; toto anni tempore exactè difcernere.

Circularis lamina ex argento, aut ære, aliaue materia paranda eft , in cu-
ius fuperficie ambarum facierum Zodiacus delineabitur, modo inferius depicto,
deinde pro anno quinque circuli fibi inuicem côcentrici, at refpectu Zodiaci excen
trici celabuntur in ea, adeo vt vtriufque centri diftantia fit pro. 32. parte femidia-
metri concauitatis Zodiaci è regione locis augis,temporis qui noftra ætate circa fi-
nem fecundi gradus cancri inuenitur, eandem viam , in hoc , fequuti, quam Stofle-
rus in dorfo Aftrolabij docet . At nomina menfium media ponantur inter duos
maiores circulos, poftea inter fecundum, & tertium ab vna facierum laminæ, ar-
cus femidiurni , ab altera vero arcus feminocturni, per quinos quofque dies collo-
centur,ita exactè , vt hic fubtus videbis.adeo vt numeri dierum & ipforum dierum
figna fint in interuallis vicinioribus centro communi dictorum quinque circulorum.

Pofteaquam ab vna & altera facierũ laminæ hæc infculpta fuerint,aliæ duæ circu
lares laminæ,magnitudinis femidiametri minimi quinque circulorum accipiantur:
quarum vna pro ortu, & altera pro occafu Lunæ deferuiet . In qualibet ipfarum
conftituétur circuli quatuor, eo modo qui paulo inferius cernitur, quos omnes diui-
demus in triginta fpacia æqualia: & in interuallo qđ inter duos primos circulos pofi
tum eft,triginta dies annotabim° qui ipfos Lunæ triginta dies prefcribét,vt in figura.

Poftmodum in lamina quæ ortus Lunæ indicabit, ac duorum maiorum circulo-
rũ interuallo è regione numeri.1.videlicet primi diei,ponemus horas.12.& minuta.
48.ex aduerfo diei fecundi ho.13.et min.36.ex oppofito tertij ho.14.min.24.& fic
fucceffiuè augendo per min. 48.& indicem è diuerfo diei. 30.ftatuédo, qui coitus
Lunæ cum Sole fignificabit: atque lineas afpectuum,vt inferius videre eft facilè in
ueniemus.

Altera in lamina quæ occafum Lunæ indicabit,poftquam diftincta fuerit,vt alte-
ra. 30. dies ac cęteræ lineæ, eo modo quo in fuperiori collocabuntur, at numeri in-
terualli maioris, aliter difponentur, vt potè ex aduerfo diei primi folum. 48.minu-
ta defcribi debent, è directo fecundi diei ponenda erit hora vna cum minutis.36.&
è regione tertij infcribentur.2.horæ, & min.24.& fic ex ordine per. 48. minuta au-
gendo.

Nunc lamina ortus Lunę,cum°anno arcuum feminocturnorum, & illa occafus cum
anno arcuum femidiurnorum concētrari debet, & ita noftrum inftrumentum perfe-
ctum erit & abfolutum.

Quoties igitur voluerimus medio inftrumento dignofcere fermè in tali orizonte
qua hora Luna oriatur, ita neceffe erit volubilem rotam ortus flectere,ut index ve
niat è regione diei menfis in quo talis operatio fit & tali rota firma manente perfpi-
cere,

cere ex aduerſo diei Lunæ, numerum horarum & minutorum in maiori interuallo
ipſius rotæ notatorum, qui cum arcu ſeminocturno anni, quo cum in ipſa rectitudi-
ne centri conueniet colligetur, & ſumma quæ ex tali ſupputatione proueniet aper-
tas faciet horas aſtronomicas, quibus ferè etſi non exactè in die propoſito Luna
orietur. Idipſum fiet pro occaſu Lunæ.

DIEM ætatis Lunæ iam totus orbis ſcit inuenire , media ſupputatione nu-
meri Epactę currentis cum numero menſium, ſumpto principio à Martio,
adiunctis diebus menſis currentis, & detracto numero. 30. à ſumma prędicta, ſi ab ip
ſa dictus numerns. 30. ſuperatur.

Sed ne aliquis putet ſufficere tantummodo additionem quatuor quintarum ho-
rę qualibet die. à nouilunio inchoando , ſciendum eſt huiuſmodi receſſum Lunæ
(quamuis non ita exactæ fiat) non computandũ eſſe ab orizonte aliquo, ſed à recto,
ſeu à meridiano quod idem eſt, quemadmodum vnicuiq; mediocriter erudito pa-
tere poteſt. At propoſitum nobis non eſt ſcire qua hora Luna in meridiano repe-
riatur, ſed in noſtro obliquo orizonte, in parte orientali ſeu occidentali, propterea
igitur addendus eſt, ei ſummæ temporis, qua Luna diſtat à meridiano, arcus ſemi-
diurnus, vel ſeminocturnus illius loci Zodiaci, in quo Luna reperitur illa die in pro
poſito parallelo, vt ſciatur proximę, qua hora (ex aſtronomicis) Luna erit in ori-
zonte oriètali, vel occidentali dicti paralleli. ſupra dicta enim additio quatuor quin
tarum horæ tantummodo, ſufficiens erit temporibus æquinoctij, ſed aliis anni tem-
poribus falli ratione iam dicta.

Pro Lunæ ortu.

Ad lati. 45.

Pro Luna occafu.

Ad lati. 45.
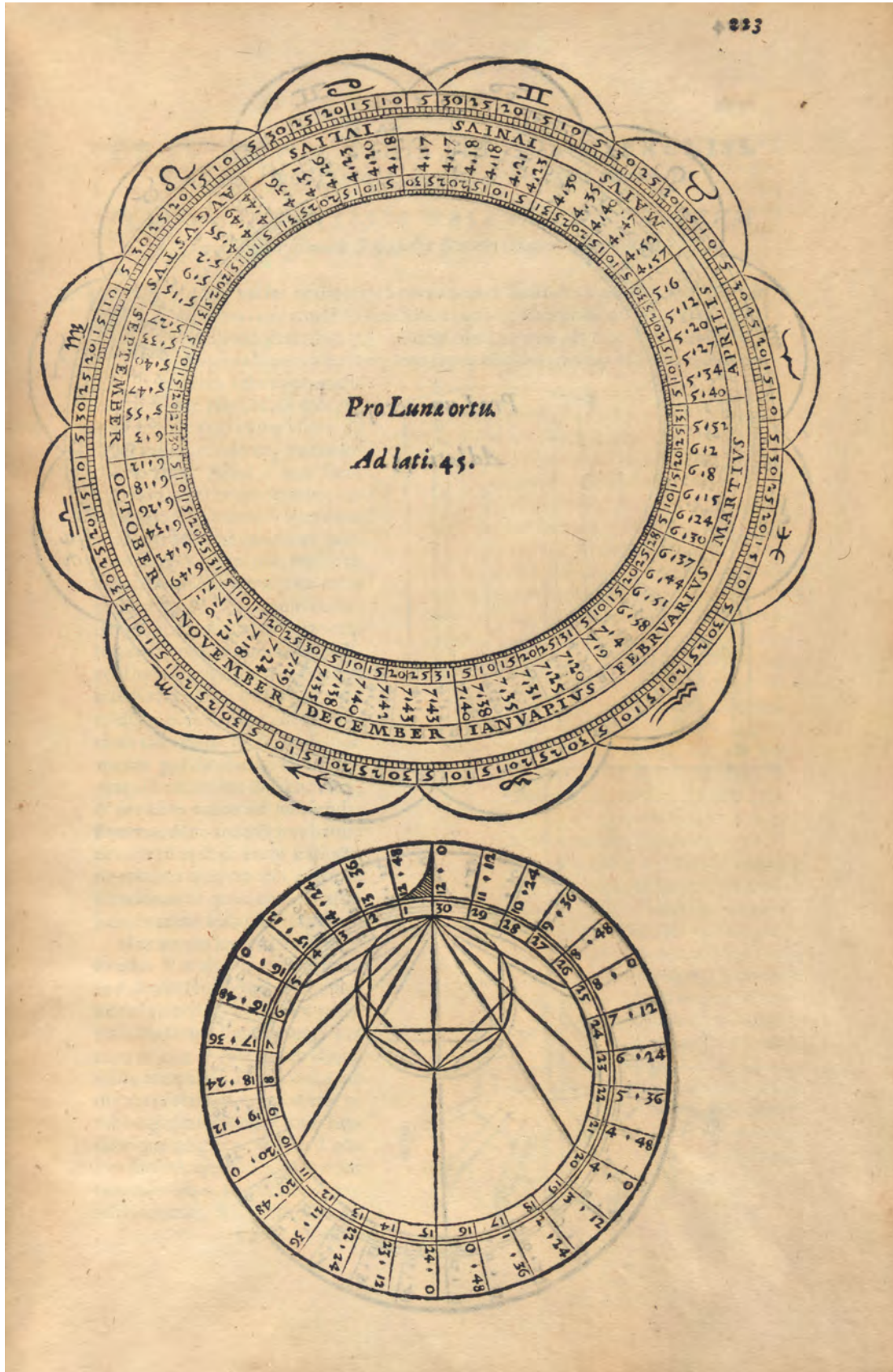
## DE LVCERNA SPIRITALI QVAM SERENISS.
### Sabaudiæ Duce.D.meo collendiss.anno.1570.conftruxi.

*CLARISS. FRANCISCO BARBARO VENETORVM*
*apud Sereniſſimum Sabaudiæ Ducem Oratori Illuftriſſimo.*

Eron varias ac diuerſas hydraulicas,& ſpiritales machinas propoſuit,in ter quas tamen nullam ſimilem, ei quâ ego Sereniſſimo Sabaudiæ Duci conſtruxi, deſcribit, quę quidem fuit Lucerna , & erat huiuſmodi , vt à magno aliquo vaſe oleo pleno ſupra alicuius triclinij tabulatum poſito,

ſubtilis quidam tubus perpendiculariter per tabulatum exiret , & in dictum triclinium vſque ad medium deſcenderet , ita tamen vt hic ſolus tubus , non item vas oleo plenum cerneretur , cuius quidé tubi inferiori extremitati iunctum eſſet quoddam paruum receptaculum olei,ſimile co operculo alicuius pyxidis,è cuius ambitu prope baſim multi diuerſi quæ tubi æquales & orizontales, cuiusuis longitudinis proſilirent,quorum quilibet in extremitate ſua,exiguam quandam pyramidé,appéſa haberet, in qua elli chnú eſſet cú mixo. oleú deinde medio ppédicularis tubi ad rece ptaculú extrinſece deſcendebat, & per alios tubos ad nutriendas flammas dum arderét ferebatur: at vero cú eędem erant extinctæ ne minima quidem olei gutta de ſcendebat: id quod eos qui aſtabant in admirationem trahebat.

Hæc autem lucerna ſic erat có ſtructa. Vas oleariū cylindricum vt in ſubſcripta figura patet,cuiuſ uis magnitudinis , omni ex parte clauſum faciendum curaui , ita ta men vt eius coopereulum aliquā tulū concauum eſſet , in cuius me dio erat foramen. e. quod erat os tubi.e.g.qui ſub eiuſdé vaſis fun do vſque ad.g.tranſibat, ſed poſtea ſurſum, quaſi vſq; ad cooper culum in ſitu.c. ab inferiori parte reflectebatur, & ibi terminabat.

Ff Vnde
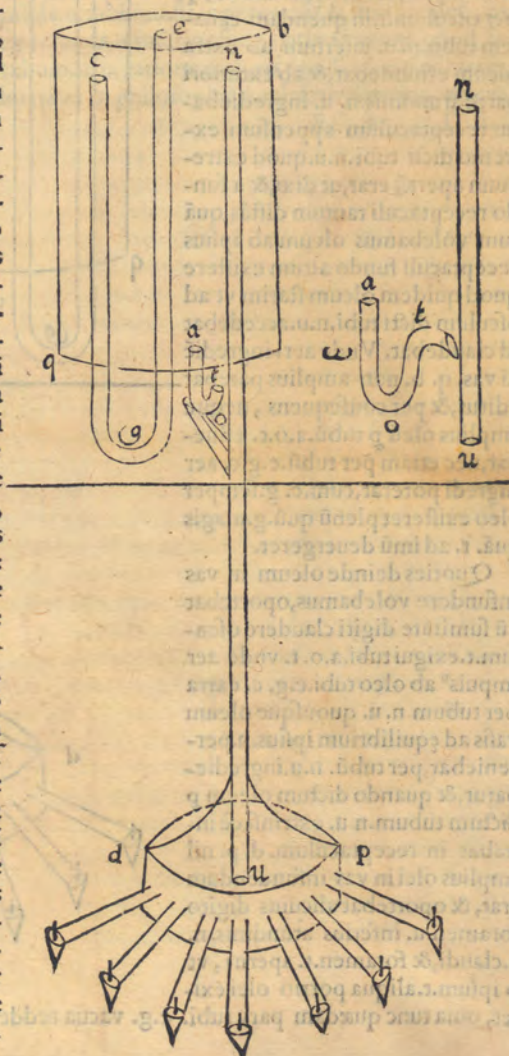
Vnde oleum quod in vas infundebatur per foramen.e. dictum vas poftea ingrediebatur per foramen.c. Habebat deinde tubum.n.u.rectum, qui à fitu.n. propinquo co operculo ad libellam extremi.c.incipiebat, & per fundum contignationemǿ; vfque ad centrum fupradicti receptaculi(circa quod tuborū ope appenfę erant ellychniorum pyramides)tranfibat, atque huiuiufmodi tubi. n. u. extremitates tam fupérius quam inferius erant apertæ,& hic tubus aeris erat. Præterea aliud quoddam fora-

men in vafis fundo feceram, cui
paruum tubū.a.o.t. reflexum, ita
tamen, vt. o. altius éffet quam. g.
aptáuerā,atq; p hunc reflexum tu
bum.a.o.t.oleum vafis exibat, ǫ
per ofculum.t.in quendam cana-
lem tubo.n. u. infertum, ab extra
oleum effundebat,& ab exteriori
parte arundinis.n. u. ingredieba-
tur receptaculum appenfum ex-
tremo dicti tubi.n.u.quod extre-
mum apertū erat,ut dixi,& à fun-
do receptaculi tantum diftās,quā
tum volebamus oleum ab ipfius
receptaculi fundo altum exiftere
quod quidem oleum ftatim vt ad
ofculum dicti tubi.n.u.accedebat
id claudebat. Vnde aeri ingredié
di vas. q. b. non amplius patebat
aditus,& per confequens , neque
amplius oleū p tubū.a.o.t. eflue-
bat,nec etiam per tubū.e.g.c. aer
ingredi poterat,cum.c. g.femper
oleo exifteret plenū quū.g.magis
quā. t. ad imū deuergeret.

Quoties deinde oleum in vas
infundere volebamus,oportebat
cū fumitate digiti claudere ofcu-
lum.t.exigui tubi.a.o. t. vnde aer
impulsꝰ ab oleo tubi.e.g. c. extra
per tubum n. u. quoufque oleum
vafis ad ęquilibrium ipfius.n.per-
ueniebat,per tubū. n.u.ingredie-
batur.& quando dictum oleum p
dictum tubum.n. u. extrinfecè in-
trabat in receptaculum. d. p. nil
amplius olei in vas infundendum
erat, & oportebat alicuius digito
foramen.u. inferius arundinis. n.
u.claudi,& foramen.t. aperiri, vt
p ipfum.t.aliqua portio olei exi-
ret, quia tunc quædam pars tubi. e.g. vacua reddebatur , &cum per. t.nil amplius
olei
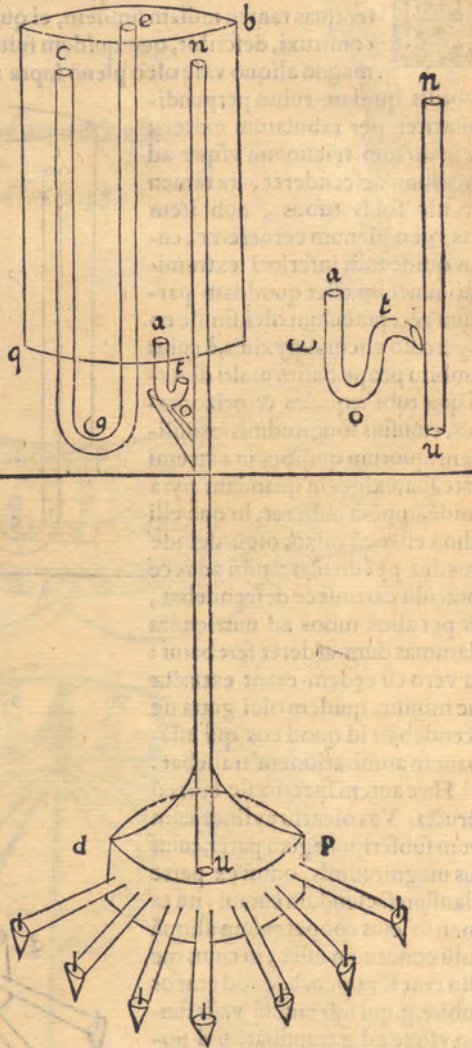
olei egrediebatur, aperiebatur.u.& per ipfum.t.denuo tantum olei exire permitte-
bamus, quantum in réceptaculo ad claudendum foramen.u.idoneum exifteret.Ra-
tio veró,quę me mouit,ut punctum.g.inferius ipfius.t.conftituam, eft, quia cũ clau-
fum erit.u.per dictum.t.oleum non amplius egredietur,quia pondus olei in tubo. c.
g.maius euadet oleo quod vfque ad.t.progrederetur,tubum autem.e.g.c. reflexum
facio,ne cogamur claudere foramen.e.quia hoc difficile præftaretur, tubum etiam
a.o.t. furfum verfus reflexum conftitui,vt aerem ab ingreffu per foramen.t. arceré,
quia huiufmodi aer nunquam defcendit fi corpus magis denfum non defcendat.

Verum eft, ɋ melius erit, vt maiores difficultates euitemus, ftatuere dictum tubũ
a.o.t.ita curuum vt eft.ɤ. qui cum fuo extremo inferiori ipfi.n.u.fit contiguus ita ta-
men ut dictum extremum inferius nõ fit inferius quam.o.quia totum oleum exiret.

Volui etiam vt fuperior extremitas.n.tubi.n.u.fit in aere vafis & non in oleo, ne
per eam oleum exeat,quia cum extremitas.u.inferior fit. g. totum oleum quod fu-
peraret ofculum . n. per dictum tubum.n. u. ratione maioris ponderis egrederetur,
quéadmodum cuilibet , vel mediocriter in philofophicis rebus verfato innotefcet.

Ff 2 DE

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# DEFENSIO EPHEMERIDVM.

## AD ILLVST. D. BERNARDVM
### Trottum.

DITA erant scripta quædam, quorum titulus ANIMADVERSIONES IN EPHEMERIDAS. & breuis alia disputatio de erroribus calculorum Astronomicorum.ac demū Theses quædam typis datæ.11.Augusti.1581.quę omnia cum ad manus meas peruenissent,non potui,non eis animum admouere, cū ibi de his studijs ageret, in quibus partem non exiguam annorum meorum consumpsi.nec tamen scribere aliquid statueram; tum quod existimarem viros Astronomiæ peritos facile,quanti facienda essent ea quæ edita erant iudicaturos,alijs verò haud gratam futuram harum rerum tractationem.Tum quod si ingenue meam sententiam proferre voluissem, non poteram,sine maxima authoris molestia ferè omnia reprobare . Quandoquidem solet vnusquisque indignationem concipere ex his, quæ suæ opinioni repugnant, id omne maleuolentiæ potius quam veritatis studio tribuens. Qui nimo cum nec deessent qui dicerent in meipsum directa ea tela fuisse,nullam fidem eis adhibendam duxi, nec enim qui in ephemeridas inuehitur, me arguere potest, qui nullas ephemeridas scripsi,nec tabulas composui. Nec si author quidpiam ex his quæ à nobis edita fuissent impugnare voluisset,ægrè ferre debuissem,modo à veritate nusquam deuiasset . Liberum enim est cuique scribere quodlibet . nec Aristotelem afficit iniuria, quicunque illi fidem suam non accommodat,& si valdè iniquus sit, quisquis maiorum opiniones veras,& ab omnibus merito comprobatas nó admittit.Hine mihi satis oībus fecisse videbar, cum his qui de scriptis illis me interrogauerāt respondissem,ea non satis firmis esse innixa fundamētis , & quod ad talia tractanda opus fuisset exercitatiore iudicio. Verumtamen cum tu vnus maxime omnium desideres tibi clarius,quæ nam de his mea sit sententia explicari, non tam tuis precibus deuictus quam mea ipsius cupiditate de te benemerendi impulsus, non sustineo diutius animum tuum hęsitantem relinquere. Atque vt tibi adeo honesta cupienti morē geram paucis hisce scriptis incultis quidem, vt ab homine omni prorsus facundia destituto exaratis,sed ex quibus nihilominus facile, atque perspicue,vt spero,cōceptum animi nostri percipere possis , si tamen eam præstantis ingenij tui aciem adhibueris, qua soles intima quæque scientiarum penetrare , nostræ opinionis summam perstrinxi,quę ad te mittere decreui.& quamuis ipsa res de qua agitur, quæ exactiorem desiderat expositionem,prolixiorem me esse cogerit quam voluissem . multa tamen me obmisisse intelliges , non admodum necessaria his quibus Astrologiæ noti sunt termini, vt tuarum occupationum rationem mē etiam habere intelligeres, atque vt summam oblectationem cōcipiam animo si me tibi aliqua ex parte satisfecisse intellexero ita humanitati tuæ gratiam habebo, quę mihi occasio nē pbuit, imò verò me impulit,ad ea proferēda:quæ grata esse possint tui similibus, idest pclaro & cādido ingenio præditis,atq; ad euellēdā ex eorū animis falsam opinionem

nionem, si quam fortasse ex illorũ scriptorum lectione cõceperunt circa ea, de qui-
bus nunc sum acturus.

Quemadmodũ igitur ab hoc authore ter scriptum fuit de cõtradictionibus, siue
erroribus Ephemeridum, & earum calculos sequentium, & de ratione qua cognosci
potest situs & locus alicuius superioris planetæ, diuersus ab eo, qui ab ipsis Epheme
ridibus assignatus est, ita disputationem hanc meam diuidam in tres partes, quo sci
licet minus confusè, & magis distinctè à me scribatur, psupponendo, vt animaduer-
tere potes, huius scriptoris intentionem, aliam non fuisse, quã ostédere, quod scripto
res Ephemeridum diuersimode eiusdem temporis locum planetæ assignauere ; &
quod cum faciant modo nimium velociter currere, modo nimium in vno signo mo-
rari, vt(exempli gratia)Martem interdum faciunt morari sex, aut septem mensibus
in vno signo. Idq; postea in causa esse ait, vt Astrologi iudiciarij fallãtur, & simul
careant certis fundamentis rationum quibus futura iudicent, & prædicant. Primum
ergo videndum est, quam rectè hic vsus sit arte, & scientia, vt aliorum opiniones, &
scripta redarguere posset. Deinde videbimus quomodo verum sit, & possibile id
quod ab Astrologis hactenus creditum, atque traditum est, & qua ratione possint fie
ri veri calculi à peritis regularum scientiæ.

In primo igitur tractatu inscripto Animadnersiones, præsupponit Author pro-
fessores huius scientiæ nescire inuenire vera loca planetarum, quia vtuntur Ephe-
meridibus, in quibus eorum loca non rectè sunt notata. Quod secundum ipsum ori
tur, ex errore calculatorum, seu computistarum, potius quam ex varietate tabula-
rum, à quibus Ephemerides sumptæ sunt, hoc tamen verum non est, Ephemeridas,
scilicet, ita inter se differre, ratione errorum computistarum tantummodo, sed po-
tius ratione ipsarum tabularum, & si interdum contingere possit error aliquorum mi
nutorum, nec non graduum, non propterea Ephemerides ita spernendæ sunt. In
multis enim calculis, tales errores excusabiles sunt, cum ab innumerabilibus propè
accidentibus oriri possint, præsertim in calculis prutenicis.

Videatur deinde vbi is profert quinquagesimum enuntiatum centiloquij Ptole-
mæi, satis mendose. Ptolemæus enim ibi sic ait.

» Non obliuiscaris esse centum viginti coniunctiones, quæ sunt in stellis erraticis,
» in illis enim est maior scientia eorum quæ fiunt in hunc mũdum suscipiendi incre-
» mentum, & decrementum.

» Nam, neque eo in loco, neque alibi, Ptolemæus quidquam eius dicere voluit
quod ab hoc profertur.

Pergatur postea in pag. 2. & videbitur hunc existimare absurdum quod Saturni,
& Iouis coniunctio vera anni. 1563. potuerit esse in Leone signo igneæ triplicitatis
cum eorum coniunctio vera anni. 1544. fuerit in Scorpione, signo triplicitatis
aqueæ, & cum coitus eorum anni. 1583. futurus sit in Piscibus, signo pariter tripli-
citatis aqueæ. Ita enim ait.

» Nam postquam duæ stellæ coiuerint, non prius sub alio alterius triplicitatis signo
» inter se sunt conuenturæ, quam per omnia signa quæ eiusdem ternarij cum primo ex
» titerint prius coniungãtur. Ita sentit Ptolemæus, cæteriq; non aspernendi nominis
» Astronomi.

» Et tamé Ptolemæus nunquam quidquã huius rei attigit, & quamuis Albumasar
& Alchibitius de eo loquãtur, is tamen eos non intellexit, cum illi ibi nó agant de
pe-

periodis apparentibus, aut veris, fed de mediocribus aut æqualibus, & quidem re-
ctè dicunt, quia lineæ eorum mediorum motuum non coeunt in aliquo figno alte-
rius triplicitatis, priùs quam pertranfiuerint omnia figna illius, in qua inceperunt.
Itaq; nullum inconueniens fequitur, fi in veris côiunctionibus non reperitur hæc re-
gula. Fieri enim poteft, vt lineæ mediorum motuum coniungantur in vno figno, cor
pora verò eorum planetarum coeant in alio, cum rarò eueniat, vt linea medij mo-
tus, eadem fit cum linea veri.

Nunc quidem tamé non affirmauerim, nec negauerim eorum coniunctionem an
ni. 1563. fuiffe potius in Cancro, quam in Leone. Sed tantum dicam vanû effe cre
dere id eueniffe propter fimilem naturam, aut qualitatem fignorum. Hunc enim
refpectum non habent illi planetæ in veris fuis coniunctionibus. Exempli autem
caufa ponamus, quod rectè fupputatæ fuerint coniunctiones annorum. 1484. 1504.
& 1524. quod attinet ad differentiam duodecatemorij, fcilicet prima in. 24. gradu
Scorpij, fecunda in. 20. Cancri, tertia in. 10. Pifcium. Cum fecunda anticipauerit
trigonum perfectum cum prima, gradibus. 4. & tertia anticipauerit trigonum perfe-
ctum cû fecunda gradibus. 10. fi forte prima vt facta fuit in. 24. gradu Scorpij facta
fuiffet in. 2. gradu eiufdem, planû eft φ fecûda facta fuiffet in. 28. gradu geminorum
& tertia in. 18. Aquarij, quæ figna funt diuerfæ triplicitatis ab illa Cancri. Infuper
fi coniunctio anni. 1544. quæ fuit in. 28. gradu Scorpij fuerit recta correfpondés præ
cedenti, anni. 1524. per gra. 18. fine dubio fi coniunctio anni. 1524. facta fuiffet in
18. gradu Aquarij, illa anni. 1544. fuiffet in. 6. Scorpij figni alterius triplicitatis quã
fint Gemini, Præterea, vt anno. 1544. côiunctio facta eft in. 28. gra. Scorpij, & 1563
in. 29. Cancri, ponendo eas effe rectas, quod attinet ad fuperandum trigonum vno
gradu, fi anno. 1544. facta fuiffet in. 30. Scorpij, anno. 1563. proculdubio facta fuif-
fet in primo gradu Leonis. Et fuppofitis ijs interuallis, quæ fuperfunt, aut defunt per
fectis trigonis, fi coniunctio anni. 1524. fuiffet in. 20. gradu Pifcium, anno. 1544.
fuiffet in. 8. Sagittarij. Quæ quidem omnia aduerfantur opinioni huius fcriptoris.

Quod autem opinatur coniunctionem anni. 1583. fore in Ariete, fic dicens pagi
na fecunda.

„ Non erit ab re fi & eandem Saturni, & Iouis coniunctionem in primo igneæ tri-
„ plicitatis figno, quod eft Aries futuram afferamus anno. 1583. fi ab accidentibus no
„ bis licet, vt ab omnibus paffim conceditur, planetarum loca difcernere.

In eo fallitur, nã neq; Saturnus, neq; Iupiter, errát à vero per. 9: nec. 8. gra. ac ne p
4. quidé in qbufuis Ephemeridibus aut tabulis. Itaq; videbit eiufmodi côiunctioné
contra fententiã fuã fieri in Pifcibus, nó aût in Ariete. (vt poftea res ipfa nos docuit
fub méfe Aprili. poft qdé fcriptã hãc epiftolã, vulgariq; fermone trãfmiffã, fed an-
tequã in latinû trãflata, & huic volumini inferta cû alijs Typographo cômitteretur.)

Vbi poftea meminit magnæ periodi annorum. 960. non tantum ei cogitandum
erat hãc fuiffe opinionem antiquorû, vt videri pôt apud Albumafaré & Alchibiciû,
fed etiam perpendendum an effet vera, priufquã ei adhæreret. Hic enim fuit vnus
ex erroribus illius ætatis, quæ nondum penetrauerat intima huius fcientiæ. Sunt
tamen illi antiqui excufatione aliqua digni. Ponebant enim vigefimo quoque an-
no præcisè fieri mediam coniunctionem Saturni cû Ioue, & in quolibet figno eiuf-
dem triplicitatis côiungi quater. Itaque in qualibet triplicitate dicebant eos coire
duodecies.

                                                                        Quod

Quod secundum primum suppositum finiebatur spacio annorum.240. qui nume rus fit. ex.20.duodecies multiplicatis. Et quia triplicitates sunt.4. ideo credebant in spacio annorum.960.qui numerus fit ex.240.quater multiplicato,perfici.48.cõ iunctiones,priusquam redirent ad se coniungendos in eodem loco, ubi prius iuncti fuissent. Primum autem suppositum,quod vigesimo quoque anno iũgerentur, colligebant sic ratiocinantes. Si Saturnus annis.30.peragit suum cursum per omnia signa Zodiaci, Iupiter autem peragit eum annis.12. Saturnus ambulauerit.4.signa,et.4.quintas partes signi,siue gra.24. dum Iupiter peragit integrum ambitum idest annis.12.Itaque desunt ei anni.8.ad perueniendum ad.20.quibus.8.annis Saturnus pambulat signa tria & quintâ parte unius signi.i.gradus.6.qui iuncti dictis signis.4.& gra.24.faciunt signa.8.quæ Iupiter item percurrit in annis.8.atque ita in annis.20.Iupiter percurrit.20.signa antequã perueniat ad Saturnũ,cum Saturnus eo dem tempore perfecerit cursum signorum.8. Eandem conclusionem etiam fortas se collegerant ex dictis suppositis, dicentes, si Saturnus annis.30.ambulat.12.signa proculdubio annis.20.ambulat.8.signa,quo tempore Iupiter perambulat.20.ad ra tionem.12.signorum in annis.12.

Verum hoc suppositum non est bonum,quoniam,si ita esset, coniunctiones horũ duorum planetarum nunquam exirent ex vna triplicitate , & non modo.960.quoq; anno,sed etiam sexagesimo rursus coniungerentur in eodem puncto.nec coniunctio nes eorum(semper autem intelligo de medijs) unquam egrederentur ex illis tribus signis Zodiaci.

Sed periodus æqualis Saturni,est dierum circiter.10740.atque ita minor an.30. atque etiam.29.cum dimidio.periodus autem æqualis Iouis, est circiter.4328. vt ego eam comperio,quidquid alij dicãt, vtq; planius alias ostendam. Itaq; hæc per iodus Iouis,etiam minor est ann.12.prætermittendo in supputatione tã Saturni quã Iouis quasdam minutias horarum & earum partium, quæ hac in re pro nihilo haberi possunt. Atque his duabus periodis eccentricorum duorum planetarum possumus cognoscere interuallum quod erit inter vtramque mediam coniunctionem,hoc modo agendo,& ratiocinando.

Si Saturnus diebus.10740.circuit gradus.360.diebus.4328. qui sunt periodus Iouis,conficiet gradus.145.& min.4.idest min.8704.& eadem regula inueniemus q Saturnus. 30.quibusq; diebus,conficiet min.60.& secunda.20. Iupiter autem singulis.30.diebus,conficiet min.149.& secunda.43.vnde subtrahendo minuta Saturni à minutis Iouis,supererunt min.89.cum secun.23. Itaq; Iupiter.30.quibusq; diebus velocitate cursus,superabit Saturnum minutis.89.cum secundis.23.Atq; dicendo,si minuta.89.cum secundis.23 dant nobis dies.30.supradicta,minuta.8704.dabunt nobis dies.2921.quibus iunctis cum diebus.4328.periodi Iouis,efficientur dies 7249.idest anni Aegiptij.19.cum diebus.314.& hæc erit æqua periodus temporis inter vtranque coniunctionem horum duorum Altiorum planetarum.Vt autem pla nius ostendatur hanc operationem rectam esse (nam demonstrationem speculatiuã huius operationis in.113.Theoremate nostræ Arithmeticę cuiq; videre licet) fieri potest his alijs calculis.

Si Saturnus diebus.10740.transit per gra.360.in spacio dierum.2921. transibit per gradus.97.min.54.quibus iunctis cum gra.145.min.4.supra notatis, efficientur gra.242.min.58.Deinde,si Iupiter spatio dierum.4328.transit per gra.360.igitur spatio.2921.per eandem regulam inueniemus eum transire gradus.242.mi.58.qui numerus **par** est illi Saturni. Cum ergo Iupiter confecerit vnum ambitum post con
iun-

iunctionem cum Saturno, vt rursus assequatur Saturnum, transeundum ei erit p gra.
242.min. 58.iter confectum à Saturno toto tempore annorum.19.& dierum. 314.
ad rationem graduum.360.diebus. 10740.(possumus etiã dicere gra.243.quia præ
termisimus quasdam exiguas particulas periodorum perfectorum cuiusque planetę
in superioribus supputationibus.) Illos verò gradus. 243.Iupiter conficiet diebus.
2921.ad rationem graduum.360.diebus.4328.Atq; ita vt diximus,ab vna coniun-
ctione ad aliam intererunt anni.19.Aegiptij cum diebus.314. vel circa.

Nunc autem vt videatur an tabulæ Alfonsi conueniant cum hoc nr̃o calculo, cõ-
siderabimus, ꝗ Era(vt vocant)dicti temporis annorum.19.cum diebus.314.est dua-
rum tertiarum sexagenarum,secundæ nullius,& 53.primarũ siue dierum. Et per hãc
Eram colligendo motum mediocrem,tum Saturni,tum Iouis,omissis radicibus,& in
cipiendo ab Ariete,comperiemus ꝗ vtriusꝗ; planetæ lineæ eiusmodi motus transi
bunt per min.56.tertij gradus Sagittarij, idest coniunctæ erunt.

In fine postea secundæ periodi, cuius era erit.4.tertiarum,secundæ.1.et. 47.pri-
marum sexagenarum,locus mediocris vtriusꝗ; erit in min.56.gra.sexti Leonis. In
fine verò tertiæ periodi,cuius era erit.6.tertiarum.2.secundarum,et.41.primæ, lo-
cus eorum mediocris inuenietur in.56.minuto gradus.9.Arietis. Atq; ita deinceps
in fine cuiusꝗ; periodi,locus eorũ mediocris coniunctim semper distabit à loco me
diocri præcedentis coniunctionis gradibus.117.idest in trigono antecedenti, minus
gra.3.Vnde apparet has coniunctiones procedere in contrariam partem respectu or
dinis signorũ Zodiaci,sed respectu ordinis graduum signorũ,semper progrediunt or
dine per ternos gradus nunquam retrogradientes. Hinc sequitur,vt non duodecies
in omni triplicitate coniungantur hi duo planetę, vt antiqui putauerunt, sed decies
tantum.& ad summum ter in singulo signo,spatio annorum.198.& dierum.220.aut
circiter,non autem.240.nec.242. Atque decem vices comprehendunt gra. 27.&
vltima vice inueniuntur in signo sequenti alterius triplicitatis. Exempli gratia, po-
namus ꝗ prima vice cõiungant in gra.2.Arietis,secunda coniunctio erit in. 5.Sagit-
tarij,tertia.in.8.Leonis.quarta in.11.Arietis,quinta in. 14. Sagit. 6. in. 17. Leonis.
septima in. 20.Arietis,octaua in.23.Sagittarij,nona in, 26. Leonis, decima in. 29.
Arietis,et vndecima erit in gra.2.Capricorni signi sequétis triplicitatis.Decem igi
tur interualla singula annorum. 19.& dierum.314. faciunt annos.198.& dies. 220.
Immo per tabulas Alfonsi,eiusmodi periodus non modo non reperitur annorũ.242
nec.240.vt antiqui credidere,sed tribus diebus minor annis.198.& diebus.220.id-
est per dictas tabulas inuenitur esse annorum.198.& dierum.217.tantum,qui nume
rus multiplicatus per.4.triplicitates,efficiet periodum maiorem, quæ erit annorum
794.& dierum.138.quo tempore dicti planetæ redeunt ad eundem locum vbi pri-
mum se coniunxere.

Vt exempli gratia,locus mediocris Saturni & Iouis in fine annorum.198.dierum
217.reperitur in gradu.30.Sagittarij. Si quæsiuerimus hunc locum per aggregatũ
annorum.794.& dierum.138.cum annis.198.& diebus.217. quorum summa est.
992.& dies.355.inuenietur locus mediocris ipsorum planetarũ in dicto vltimo gra
du Sagittarij. Sed si quęsiuerimus eorum locum mediocrem per aggregatum anno
rum.198.& dierum.217.cum annis.960.quod erit summa annorum.1158.& dierũ
217.reperiemus Iouem in gradu.18.Sagittarij & Saturnum in.16. Leonis distanti-
bus inter se duabus eorum lineis motuum mediocrium gra.circiter.122.Atq; Iupi-
ter præcedet,& oportebit ꝗ coniunctio eorum mediocris fuerit multis annis ante
omittendo(vt dixi)radices,quia satis est inuenire interuallum inter lineas eorum me
diorum motuum.

Debe-

Debebat igitur author animaduerſionum non quaſi cæcus eæcos ſequi, ſed prius laborare, vt certior fieret, an interuallum annorum. 960. Verum eſſet.

Sed peius eſt, ꝙ idem author paulo inferius citat coniunctiones horum duorum planetarum anni. 1493. et. 1512. quas neſcio vnde ſumpſerit.

Nam, etſi inter hos annos eſt interuallum annorum. 19. tamen tantum abeſt, vt coiuerint dictis annis, vt Saturnus anno. 1493. ante finem Auguſti fuerit in. 28. gradu Aquarij, Iupiter verò in. 28. Leonis ex diametro oppoſiti. Et anno. 1512. per totum menſem Iunium & Auguſtum, Saturnus fuerit in Libra, Iupiter verò in Ariete, itaque inter ſe ſimiliter oppoſiti, & ſi perfecta oppoſitio non fuit poſtea niſi ad finé Iunij ann. 1513. & locus Monteregij ab eo citatus, vbi ait eum ponere coniunctionem anni. 1484. in gra. 23. min. 4. Scorpij, eſt mendoſus. Nam ipſe Monteregius ponit dictam coniunctionem in mi. 42. gra. 24. non autem in min. 4. ipſius gradus. Sed hic error nullius eſt momenti, fortaſſe qui impræſſorum incuria irrepſit.

Pergatur poſtea obſecro ad paginam. 3. ipſarum Animaduerſionum, vbi hic conatur oſtendere calculatores non obſeruaſſe verum modum, ſic dicens.

» Anno. 1484. Nouembris. 25. Saturno locum conſtituit Monteregius in grad. 23.
» min. 4. Scorpij. Anno poſtmodum ſubſequenti qui eſt. 1485. eundem in min. 7. Sa-
» gitarij collocat. 21. Februarij die. Interꝗ; tempora duo interſunt menſes dies. 26.
» At cum ex motus ſui natura Saturnus hoc temporis ſpacio gradus. 4. non debeat trã-
» ſcendere, ſit tamen inter vtrunq; tempus differentia graduum. 3. minutorum. 3. quæ
» ratione ſui motus requirunt menſes. 6. vt eos perficiat, conſtat pluſquam tribus menſ
» ſibus fallere nos Saturnum.
» Hic videre licet quam veram viam hic ſecutus ſit ad aperiendos errores Epheme
ridum, & miſeri Monteregij, qui Saturnum claudum facit tantum itineris conficere tribus méſibus, quãtũ vix confeciſſet méſib. ſex. Sed fortaſſe ratiocinat hoc modo.

Si motus naturalis Saturni facit vt circumeat totum cęlum annis. 30. igitur menſibus. 30. conficiet duodecimam partem circuitus, cum menſes. 30. ſint duodecima pars annorum. 30. & quia duodecima pars circuitus cęli intelligitur conſtare ex. 30. gradibus, igitur quilibet menſis poſtulabit gradum vnum. Ideo illi. 6. aut. 7. gradus poſtulant tempus, amplius menſium ſex.

Atque eiuſmodi mira ratiocinatio poteſt in. 2. exemplo eius, inſcripto.
» De eodem ex eodem

Vbi miratur, ꝙ Monteregius faciat Saturnum ambulare gra. 9. min. 10. in menſibus. 7. & diebus. 6. Ad quod iter Saturno ſeni opus eſſet ſaltem menſibus. 9. eius iudicio.

Sed ſi hoc miratur, quid dicturus fuiſſet, ſi animaduertiſſet, quod idem calculator Monteregius facit Saturnum ambulare immo volare gra. 9. min. 48. non in 7. ſed in 2. menſibus cum dimidio, videlicet à. 10. die Iunij vſque ad. 26. Auguſti eiuſdé anni. 1504.

Quid ſi etiam animaduertiſſet ꝙ à. 10. die Iunij ſupradicti vſque ad. 16. Ianuarij anni ſequentis, faciunt Saturnum, ſurſum, deorſum curſitare amplius gra. 17. mi. 54. Immo ſi animaduertiſſet, quod anno. 1524. Stoflerinus ab initio anni, vſque ad medium Maium, ideſt menſib. 4. cum dimidio, facit Saturnum ambulare gra. 15. Profectò ob has velocitates, eius iudicio, tam abſurdas, obſtupuiſſet.

Vbi autem in tergo eiuſdem paginę ait, quod gradibus. 13. min. 42. reſpondent menſes. 19. errauit in calculo, nam ex eiuſmodi tempore ſecundum eius regulam ef-

G g      fice-

ficerentnr singuli ambitus Saturni ad rationem annorum amplius. 40.

Videamus nunc vbi agit de Ioue,& reperiemus ꝗ in primo exemplo circa annū. 1484. reprehendit Monteregium, quia facit Iouem ambulare gradus.14.cum min. 6.in mensibus.2.diebus.4.ad quod iter,vt ipse ait,opus esset saltem mésibus.11. atq; ita secundum ipsum,Ioui opus esset anno vno pro singulo medio signo. Vbi bonus hic vir pariter cæcutit.

Idem in secundo exemplo sumpto à Stoslerino ait, ꝗ Ioui ad cursum vnius gradus,& min.5.opus est diebus.30.non autem mensibus.7.& diebus.28. vbi ostendit, se paruum discrimen facere inter Iouem, & Saturnum.

Miratur postea ꝗ Stoslerinus faciat laborare generosum Iouem ferè mensibus sex in vno gradu. Sed multo magis, vt puto, miratus esset, si vidisset, quod idem Stoslerus in eodem ánno facit, quod Iupiter die.4.Ianuarij sit in eodem puncto, in quo postea reperitur die vltima Augusti. At fortasse dici posset,quod Iupiter propter prudentiam,& bonitatem suam factus est Rex omnium Deorum, vt ait Homerus, & ideo expulit è sede Saturnum,& ascendit in altiori cœlo. Vnde euenit vt factus fuerit lentior in cursu, Saturnus autem velocior. Aut iam tot annos esse natum Iouem, vt iure credi possit eum iam factum,esse senem, & pariter tardiorem in se mouédo. aut túc temporis illum detentum fuisse in sibi dilecta Arcadia cū Calisto. Aut fortasse erat in alta specula intentus audiendo ingenti certamini Timoclis & Damidis, vnde pendebat exitium aut gloria familiæ suæ, nam alioquin Stoslerus non depræhendisset eum tam otiosum & morantem. Sed iam relinquamus Saturnum & Iouem,& ad Martem veniamus.

Ferox & inquietus Mars,qui semper bella & ignes spirare solet,etiam, & ipse ab Astrologis factus est piger, & languidus, vt velint eum nonnunquam commorari in vno signo sex aut septem mensibus; quod nullo pacto placet authori Animaduersionum,cum pag.4.ita scribat.

„ Quod citra notam,ab omnibus creditur posse obseruari,quamuis à nobis non ac„ cipiatur.

Itaque ei videtur impossibile. Quia Mars peragit suum circuitum minus.2. annis. Sed audacior fuisse videtur, qui voluerit arguere tot egregios viros antiquos, & recentiores, qui vti diligentes rerum cœlestium obseruatores, ipsis oculis certi facti sunt tam de his effectibus Martis, quam aliorum,vnde coacti sunt fingere tantam magnitudinem eius epicycli,cum ipse nunquam obseruauerit motum,nec huius nec alterius planetæ, sed tantum viderit eius moram in Ephemeride scriptam. Si enim saltem diceret,se aliquo tempore obseruasse iterMartis,& comperuisse aliorum opinionem falsam,attulisset aliquem colorem sententiæ suæ. Sed si obseruasset, non scripsisset postea contra, vt puto. Res enim ita se habet,quod Mars in omni circuitu sui epicycli transiens per inferiorem partem ipsius epicycli, semper commoratur multis mensibus in vno duodecatemorio Zodiaci,scilicet.6.et.7. mensibus, atque etiam amplius,quod quidem ego sæpe obseruaui, præsertim anno. 1565.et. 1566. hoc ordine. Primum inspiciens Ephemeridas stadij, reperi ꝗ Mars secundum eum egrediebatur retrogradationem circa diem. 12. Ianuarij anni 1566.in. 16. grad. Geminorum. Et similiter quod anno.1565.die vltima Augusti Mars futurus erat in eodem supradicto loco,priusquam retrogradi inciperet. Postea inueni,quod post retrogradationem die.11.Aprilis.1566.Idem Mars futurus erat in gra.16. Cancri, itaq; in his.30.gradibus à.16. Geminorum ad.16. Cancri consumebatur spatium mensium.7.& dierum.11.

Quo

Quo fupputato,fumpfi inftrumenta,& ad experimentum me paraui , & vltima
nocte menfis Augufti anni.1565.reperi Martem effe in dicto gradu geminorum vt
fcribebat Stadius. Deinde fingulis ebdomadibus obferuans retrogradationem;vidi
circa finem Octobris quod retrogradi incipiebat , & ea retrogradatio perfeuerauit
vfque ad medium menfem Ianuarium , aut circiter, anni. 1566. obferuaui poftea
etiam fitum eiufdem planetæ die.11.Aprilis fequentis eumq́; inueni in gradu. 16.
Cancri, vti eum pofuerat Stadius. Atque ita experimentum meum conuenit cum
calculo Stadij,comperiq́; eum non erraffe: Et fic quifque binis quibusq́; annis pote-
rit certior fieri de veritate . Si autem delectationis caufa id experiri volueris, expe-
ctato primam retrogradationem Martis, cuius initium fecundum Stadium futurum
eft circa diem. 20. Nouembris anni. 1582. & finis circa diem. 10. Februar.1583.
circa grad.9.Cancri , & animaduerte quando Mars erit circa dictum gra. 9. Can-
cri prius quam retrogradi incipiat, quod erit circa diem.19.Septem. 1582. Dein-
de afpice quum erit in grad.9. Leonis , quod erit circa diem. 7. Mai.1583.& vide-
bis φ ipfe Mars in his gra.30.morabit p menfes.7.& dies. 18.atq; vt eius rei pericu-
lum facias,obferua noctem præcedentem diei.19. Septem. 1582. locum lógitudinis
eius ftellæ,& idem poftea obferua nocte præcedente diei.7.Mai,aut nocte fequen-
ti. 1583.& inter duos hofce terminos obferua aliqua alia nocte ftatum eius.Mani-
feftoq́; videbis Martem confumere totum dictum tempus in hoc duodecatemorio .
Et quicunque aliquid intelligit in hac facultate quamuis non viderit Ptolomæi
Almageftum , minori labore poffet per calculos fcientificos colligere verita-
tem , fuppofitis tamen terminis fcriptis in theoricis planetarum . Qui enim vidit
Almageftum vel reuolutiones orbium cœleftium Nicolai Copernici,non poteft de
hoc vllo pacto dubitare . Sed qui nondum tantopere progreffus eft,falté capiat hui⁹
rei notitiam vniuerfalem,hoc modo. Supponat primum eccentricitatem deferen-
tis epicycli Martis,effe.6.partium taliũ, quales funt fexagefimæ femidiametri ipfius
deferentis,& femidiametrum epicycli effe, partium fupradictarũ.39.cum dimidia,
& quod argumenta vera, in temporibus primarum ftationum ( cum epicyclus eft in
auge,aut in eius oppofito,aut in lũgitudinib.mediocribus)iã ab antiquis recté fuppu
tata fint,ficuti funt. Et præfupponat motum diurnum centri epicycli.min.31.cum di
midio,quamuis reuera fit min.31.& fecundorum.27.aut circiter,nunc quidé præter
mittens,quod vnus habeat refpectum ad augem mediam epicycli, & alter ad cen-
trum æquantis. Atque his præfuppofitis fingat ( exempli gratia ) quod centrum
epicycli fit in quauis longitudinum mediarum , & Mars in prima maxima æqua-
tione argumenti , fcilicet in prima linea, quæ attingens epicyclum , à centro mundi
pergat ad circunferentiam Zodiaci, quæ erit illa linea cótingentiæ à qua proficifcés
Mars perget ad lineam primæ ftationis,vt poftea retrogradiatur, veluti fi in infrapo
fita figura maiori,cétrũ mũdi effet.o.& vnus arcus eccétrici effet.a.b.c.d.& vnã ex li
neis mediocribus longitudinum effet.o.c.f.& centrum epicycli.c.qui notabitur per
a.f.e.g.& lineæ contingentes epicyclum in punctis.i.et.t.fint notatæ.o.i.et.o.t.& li-
nea primæ ftationis.o.n.b.& linea fecundæ.o.u.d.fi igitur Mars effet in puncto.i.an-
gulus.i.o.e.maximæ æquationis argumenti effet gra.40.minut. 55.quãuis talis maxi
ma æquatio argumenti in longitudinibus mediocribus Alfonfi ponatur effe gra.41.
minut. 10. quod euenit quia calculatores ipfarum tabularum interuallum.o.c.quod
in eo fitu epicycli interponitur inter centrum mundi , & centrum dicti epicycli,ac-
ceperunt partium fexaginta præcifè,nihili facientes minuta illa. 18. aut circiter,quę
verè funt præter dictas partes.60. quandoquidem euenit vt dictum interuallum in

Gg  2  tali

tali fitu epicycli fit bafis vnius trianguli orthogonij,cuius vnum ex illis duobus late-
ribus eft femidiameter eccentrici partium.60.pręcisè,aliud eft interual lum eccen-
tricitatis partium.6.eiufmodi . Angulus ergo.i.o.c.vt dixi, erit partium. 40. minu.
55. qui angulus continuò variatur fecundum fitum epicycli . & cum centrum
eius eft in auge eccentrici. eft minimus quã effe poffit. eftą; tantum grad.36. min.
46.& in oppofito ipfius augis eft grad.47. min.1. maximus quam alibi vnquam fit,
& fi c continuò variatur,fecundum fitum,quem habet epicyclus in eccentrico. Qui
qui idem angulus inuenitur per doctrinam.27. et.28.libri primi Monteregij de trian
guli s. Nam triangulus.c.i.o.eft femper rectangulus in puncto.i.& latus.c.i. refpectu
femidiametri eft datum . Quod.c.i.erit veluti partium.39.cum dimidia , et dictum
interuallum.o.c.veluti partium.60.min.18.& quia datur nobis etiam eccentricitas
veluti partium.60.talium, & cum.c.o.fit linea veri motus epicycli , & latus fimiliter
vnius trianguli, cuius duo latera funt fupradicta,fcilicet femidiameter eccentrici, &
eccentricitas, inter fe compræhendentes angulum datum. Nam femper præfuppo
nitur datus locus centri ipfius epicycli, cum ipfe eft extra augem aut oppofitum eius
quia in auge linea.o.c.conftat ex femidiametro eccentrici & interualli eccentricita-
tis. & in eius oppofito, ipfa linea.o.c.eft minor dicto femidiametro eccentrici per in
teruallum dictæ eccentricitatis. Vnde etiam poffumus extra augem, vel oppofitum
eius cognofcere.o. c. tanquam latus dicti trianguli duorum laterum cũ angulo cogni
torum. Idą; per.49.propofitionem libri primi eiufdé Monteregij cum fcilicet dictus
angulus nõ fuerit rectus. Nam fi fuerit rectus videbitur per.27.et.28.fupra citatas.
  Cum igitur habeamus angulum.c.o.i.gra.40.mi.55.angulus.o.c.i.tanquam reli-
quus ex recto,erit grad.49.mi.5.cui refpondet arcus.i.g.epicycli confectus à Marte
in diebus circiter.105.ad rationem min.28.aut circiter in fingulos dies, prætermif-
fis nunc quidem minutijs cum exigui momenti fit error.15.aut.20.dierum ad verifi
cationem longæ morę Martis in vno duodecatemorio, atque per hoc tempus cen-
trum epicycli conficit gradus.55.min.7.aut circiter,ad rationem minutorum.31. cũ
dimidio in fingulos dies. qui numerus graduum.55.min.7:differt à numero graduũ.
40.min.55.maximæ æquationis argumenti gradibus.14.mi.12.nec refert quod gra.
55.min.7.habeant refpectum ad centrum æquantis, magis quam ad centrum mũdi,
quia differentia non eft tanta , vt poffit inducere errorem menfium . Hinc fequitur
quod in fine dictorum dierum.105.Mars erit in linea.o. c. veri motus epicycli , fed
gradibus.14.min. 12.vlterius quam in primo loco,in quo erat in Zodiaco,& erit in
medio fuæ retrogradationis. Sed quoniam Mars manifeftè retrogradi non incipit
in puncto.i.contingentiæ,imo ab illo puncto vfque ad terminum primæ ftationis li
neæ.o.n. interponitur arcus.i.n.epicycli,qui eft graduum.32.mınu.14.Idą; cogno-
fcitur fubtrahendo arcum.f.i.n.graduum.163.mi.9.qui eft inter augem , & primam
ftationem,à gradibus. 180. ( qui arcus. f. i. n. erit verum argumentum , quod fi-
militer variatur fecundum fitum epicycli,etfi eiufmodi varietas, nobis nõ eft magni
momenti,vnde poffumus præfupponere , quod.c.centrum epicycli non alteret in-
teruallũ.c.o.à centro mũdi, cũ ncn poffit intercedere,error mẽfiũ reliquum verò.g.
n.graduum.16.min.51.fubtrahendo ex arcu.g.i.graduum.49.minuti. 5. vnde reli-
quus nobis erit arcus.n.i.graduum.32.min.14.in eiufmodi tamen fitu mediocrium
longitudinum. Nunc hic arcus epicycli graduum.32.mi.14.fit à ftella Martis die-
bus.69.ad rationem fupradictam, omittendo quod ipfa ftella habeat refpectum ad
augem mediocrem epicycli, & quod dicta aux mediocris mutet diftantiam à vera
propter motum epicycli,quod nunc quidem parui refert, in quibus diebus.69.cen-
trum

trum epicycli conficit gra.36.min.13.ad rationem supradictam. Restat nunc no-
bis inuenire angulum.b.o.c.in centro mūdi,inter duas lineas,b.o.et.c.o.quarū prior
est primæ stationis,altera est veri motus epicycli, quod facilè intelligemus per di-
ctam.49.lib.1.Monteregij,cum duo latera.n.c.et.c.o.& angulus.n.c.o.sint nobis no-
ta. Hoc autem fiet fingendo lineam.n.h.perpendiculerem ad.o.c.quæ tanquam si-
nus anguli.n.c.h.erit partium.28986.talium qualium.n.c.esset partium.100000.&
c.h.tanquam sinus anguli.c.n.h.restantis ex vno recto,erit partium.95706.dicendo
postea si.n.e.tanquam sinus totalis partium.100000.dat nobis.n.h.partium.28986
quid dabit nobis diameter.n.c.tanquam partium.39.mi.30.inueniemus.n.h.venire
nobis ex partibus 11.mi.27.& idem faciendo de.c.h.inueniemus quod veniet no-
bis partium.37.mi.48.quibus subtractis ex tota.c.o.quę est partiū.60.mi.18.reliqua
erit nobis.h.o.partiuni.22.min.30.capiendo postea radicem quadratā summæ qua
drati.n.h.cum quadrato.h.o.veniet nobis.n.o.partium.25.min.12.talium qualis.n.
h.est partium.11.min.27.si igitur ad.o.n.tanquam partium.25.min.12.respondet.
n.h. partium.11.minuti.27.linea. n. h. ad. o. n.tanquam partium 100000.respon
debit part.45436.tanquam sinus anguli.n.o.h.qui angulus erit gra.27.minut. 1.sub
tracto postea hoc angulo ab angulo. c. o. i. graduum . 40. minut.55.remanebit an
gulus.n.o.i.graduum. 13 . minut. 54. inter lineam contingentiæ , & lineam primæ
stationis in eiusmodi situ. Et ideo Mars accesserit ad lineam.o. c. veri motus epi-
cycli. Sed quia linea.o.i.contingentiæ,propter motum centri epicycli,in dictis die-
bus.69. confecerit gradus. 36. minut. 13.( præsupposita semper eadem di-
stantia.o c.quamuis nonnulla sit differentia,quam nunc prætermittemus)& Mars in
dicto tempore retrogressus fuerit per dictum angulum gra.13.mi.54.quibus dedu-
ctis,ex.36.& min.13.restabunt gra.22.min.19.itaque in diebus.69. Mars promo-
tus fuerit à primo situ gra.22.min.19.aut circiter,prius quam retrogradatio eius in-
cipiat esse appa-
rens.

Nunc à prima
statione vsque ad
lineâ veri motus
epicycli sunt gra.
16.min. 51.ipsius
epicycli, vt supra
vidim⁹ quos Mars
transit in diebus
36.aut circiter ad
rationem min.28.
in singulos dies,
quo tempore cen
trum epicycli, in
tali distantia à cê-
tro mundi confice
ret gra.18.mi.54.
ad rationem min.
31.cum dimidio
in singulos dies ,
quib⁹ deductis ex
gra.27.min.1.an-
guli
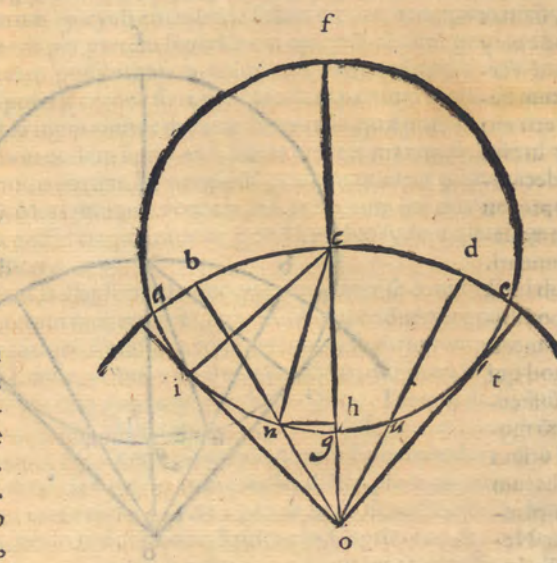
guli.c.o.n.remanebunt gra.8. min. 7. pro numero dimidiæ retrogradationis quum
Mars erit in linea.o.c.veri motus epicycli.  Quibus gradibus.8. min. 7. subtractis à
grad. 22. minu. 19. per quos Mars progressus erat, supererunt grad.14.minut.12.
quibus ipse in media retrogradatione existens in linea.o.c.veri motus epicycli pro-
motus erit à principio primi situs. Quod cum eo concordat quod supra diximus cir-
ca hos gradus.14.min.12.vltra primum situm in spatio dierum.105.vt supra, ad tā-
tum enim ascendunt.69.et.36.nunc fingendo ꝙ Mars pergat in suo motu compo-
sito qui constat ex his duobus circulis, vi eccentrici, & epicycli ( quanquam
vt dixi omittimus illam summam subtilitatem seu scrupulositatem cōtinuæ inæqua-
litatis distantiæ centri epicycli à centro mundi, & præterimus etiam irregularitatē
eius circa centrum mundi, propter regularitatem eius circa centrum æquantis,
atque etiam missum facimus motum epicycli recti à sua media auge ) fingendo
inquam, quod Mars dicto motu suo pergat, vsque ad punctum secundæ sta-
tionis, præteribunt alij dies .36. vt prius, quibus iunctis cum. 105. fiet sum-
ma dierum. 141. & Mars retrogressus erit per alios grad.8. minut. 7. quibus sub-
tractis a gradibus.14.m in.12.per quos progressus erat supererunt.gra.6.min.5. qui-
bus ipse Mars in fine suæ retrogradationis promotus erit à primo loco vnde moueri
cœpit. Inter hanc igitur secundam stationem lineæ.o.u.d.& lineam.o.t.secundæ con-
tingentiæ,Mars diebus.69.vt prius,confecerit gradus.32.min.14.sui epicycli,& eo-
dem tempore linea contingens.o.t.ambulauerit gra.36.min.13.vt prius,à quo itine-
re subtracto angulo secundo.d.o.t.graduum. 13.min.54.supererunt gra.22.min.19
vti prius,quos Mars ambulauerit directè & apparenter, quibus additis ad gra.6.mi-
nut. 5. quibus Mars progressus erat à principio motus, fient gra.28.min.circiter.24.
quibus processerit à priori loco in diebus.177.idest in.141.et.36.qui sunt ferè men-
ses.6.Itaque Mars partim sursum partim, deorsum ambulans detentus erit mēsibus
6. in grad.28.Zo-
diaci, atq; si finxe-
rimus quod epicy-
clus moueat ver-
sus oppositum au-
gis,lōgior erit mo-
ra planetæ in eius-
modi duodecate-
morio,propter au-
gumentum æqua-
tionis argumenti.
Itaque probata à
nobis est possibi-
litas huius moræ
Martis. Quod qui-
dem mihi sufficere
videtur nó mo-
do tibi, sed etiam
cuiuis, qui harum
scientiarum prin-
cipia teneat. Ne-
que enim nūc do-
cere volo eos qui
in
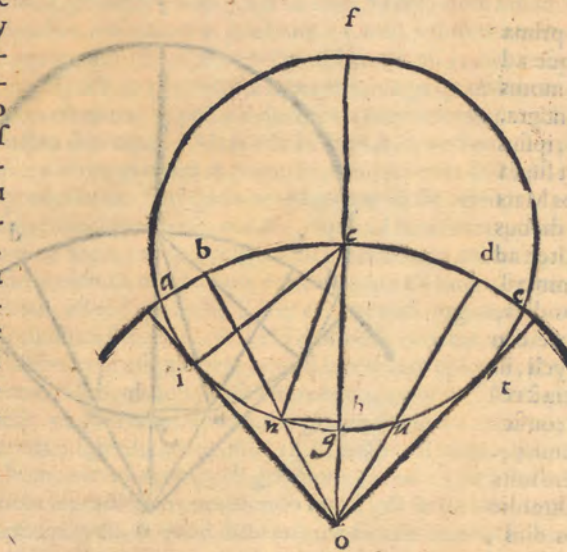
in ijs funt confumati, nec curam mihi fufcipere erudiendi imperitos. Satis igitur fit oftendiffe, quod qui fcripfit Martem commorari poffe tam multos menfes in vno figno, non impoffibilem rem tradidit. Immo per obferuationes huius veritatis millies factas, Aftrologi fecere fupradictas fuppofitiones neceffarias ad reducédum in fuas caufas, & ad regulam, eiufmodi veriffimos effectus.

Non oportebat autem fcriptorem harum animaduerfionum tantopere eiufmodi mora commoueri, fed cogitare ꝙ fortaffe calculi facti fuerunt eo tempore quo mifer Mars à Vulcano rete vinctus erat. Vnde cum non ita celeriter fe expedire poffet iter eius fegnius peractum fuit. Aut ꝙ quũ vulneratus fuit in bello Troiano, vis eius & agilitas per aliquantulum temporis imminuta fuit. Atque fi hic etiam intellexiffet eum aliquãdo fuiffe in poteftate Othi, & Ephialtis vinctũ & carceri inclufum ménfes tredecim, dum ab Eribea folutus fuit, vt tu, antiquos fequens, eleganter fcribis in illis tuis pulcherimis dialogis. non exiftimaffet, credo, tam abfurdum quod alius eũ detinuiffet fex aut feptem menfibus, fed operam dediffet vt a te intelligeret quid fibi vellet tam longa captiuitas.

Sed vt ad rem redeamus. Idem pag. 4. ait, quod verus motus Martis diftat à medio circiter dies. 8. fupponens medium motum effe dierum. 683. ꝙ etiam falfum eft. Sed vtcunque fit, fallitur. Solet enim periodus veri motus Martis effe dierum circiter. 708. modo paulo plus, modo paulo minus, & interdum poteft etiam effe multo breuior, ficuti erit à die. 3. Decembris anni. 1593. vfq; ad initium Iunij. 1595. Tunc enim erit tantum dierum. 545. & non quidem fine ratione, nam dicto initio Decembris Mars paulo ante cæperit effe directus, cum centrum epicycli erit circa medium Tauri, & eius ftella in principio Arietis & initio Iunij. 1595. Mars parum diftabit ab initio retrogradationis, regreffus tamen ad initium ipfius Arietis, & centrum epicycli erit circa medium Aquarij, in cuius figni medio, hac ætate reperitur oppofitum augis, & in quo fitu, æquationes arguméti funt, quam maximę effe poffint, quum centrum epicycli circuiuerit folum circiter tres quartas totius ambitus, & Mars circuiuerit per partem fuperiorem epicycli circiter gradus. 252. Hoc aũ tem dico, vt oftendam poffibilitatem huius eius extraordinariæ velocitatis. Nam quicunq; voluerit poterit certior fieri, per calculum partium motus Martis.

Vbi autem poftea idem author miratur interualla, quæ ponũtur inter coniunctiones Iouis, & Martis in eodem figno, eaꝗ; vocat errores maximos, oftendit fe non recte confideraffe motus eorum. Et præcipuè primum miratur ꝙ inter annum. 1528. et. 1553. Iupiter & Mars nunquam coeant in Leone, cum hæ duæ coniunctiones inter fe diftent ann. 25. afferens pro ratione, quod hæc duo fydera, altero quoque anno coniunguntur, fic dicens.

,, Qui fciet has duas ftellas fecundo quoque anno inter fe coniungendas, mirabitur
,, quomodo non poterunt numeratores, huiufmodi animaduertere errores.

Et præter hanc rationé fortaffe ét confiderauit, ꝙ in dicto temporis interuallo Iupiter séper fuit in Leone, vt ann. 1540. et. 1541. Mars aũt in eo fæpe fuit. Vnde impoffibile ei videtur eos non conueniffe in dicto figno. Idemꝗ; dici poteft de alijs coniunctionibus eorundem planetarum, atque has differentias temporum inter dictas coniunctiones ipfe tribuit erroribus calculorum Ephemeridum, non autem tabularum, vt fupra dixit. fed nefcio quare vellet dictos planetas coire in Leone, fi quum Iupiter in eo erat anno. 1540. et. 1541. & in eo deambulabaᵼ, Mars interea erat mõ in Libra, modo in Scorpione, Sagittario, Capricorno, & alijs fignis vfq; ad Cancrum, in quo cum repertus fuit anno. 1541. cogitans congredi cum Ioue in Leone,

ne,comperit eum inde aufugiſſe . Idq́; fortaſſe, Iupiter data opera fecit , vt huiuſ-
modi Aſtrologos in admirationem induceret.

Idem dico de alijs coniunctionibus horum duorum.

Quod poſtea ait, eos ſecundo quoque anno coniungi, animaduertendum eſt,qa
( vt iã dixi) duæ ſunt ſpecies coniunctionum, quarum vna eſt linearum eorum me-
diorum motuum,altera corporum eorum,ſaltem in longitudine, cum ambo inue-
niũtur in eodem circulo,qui tranſit per polos ecclipticæ,nam eos inueniri in eadé
linea recta trãſcũte per centrum mundi,rariſſimum eſt. Atque coniunctio ſupra di-
ctarum linearum vocatur media,& inter Iouem & Martem fieri ſolet ſpatio dierũ.
816.cum dimidio, aut circiter. Altera dicĩ vera,ſiue apparens , & irregulatiſſima,
quæ quidem non ſeruat tempus determinatum . Quare quamuis altero quoq; an-
no coniungantur,& Iupiter duodenis annis tranſeat per totum Zodiacum,non ideo
neceſſe eſt,vt in ſpatio.24.annorum coniungantur in ſingulis ſignis, nunquam in eo
deficientes,vt ipſe credit loquens de veris coniunctionibus apparentibus , eo quod
ſint irregulatiſſimæ, vt dixi.

Atque ſi quis velit inuenire periodum coniunctionum mediocrium horum duo-
rum planetarum,ita faciendum erit. Sumat periodum motus mediocris Iouis,quę
eſt dierum.4328.& Martis,quæ eſt dierum.687.in quo tempore Martis, Iupiter am
bulat gra.57.min.8.& diebus.30.conficit.grad.2.minut. 29. & ſecun. 23. ad ratio-
nem gra.360.in diebus.4328.Mars vero ad rationem graduũ.360.in diebus . 687.
ſingulis.30.diebus conficit.gra.15.mi.43.ſecũ.14.vnde differentia inter eos eſt gra
duum.13.mi.15.ſecũ.51.per quam diuidendo productum graduum. 57.min. 8. in
dies.30.obueniẽt dies. 129. & duæ tertiæ.quibus addendo periodum Martis fient.
816.cum dimidio, aut circiter. Atque hęc eſt periodus infallibilis mediarum con
iunctionum Iouis cum Marte.

Nunc venientes ad tabulas Animaduerſionum, videbimus hæc mirabilia eius,in
quo conſiſtant & vbi ſint tam multi inſignes errores.

Primum igitur neminem latet quod calculus Saturni,à Leouitio editus, difert à
calculo Stadij circiter gra.2.aut.3. cum Leouitius faciat eum progredi per tãtum in
teruallum, moco plus, modo minus, & ſimiliter Iouem. ſed longe minori diffe-
rentia,& ſępe gra.1.minus,atque in alijs planetis differunt,modo plus,modo minus.
Huic igitur mirũ videtur,quod vnus ex his calculatoribus detineat Saturnum plu-
ribus menſibus in vno ſigno,& alter in alio, non animaduertens dictam differentiã
eſſe eius rei cauſam. Miratur item,quod vnus ex is faciat Saturnum morari paucis
menſibus in vno ſigno,alter vero eum ibi detineat integris annis. Vt exempli gra-
tia, verſus finem ſuæ tabulæ Saturni,dicit quod Leouitius eum carceri includit in
geminis annis. 2.menſe vno,& diebus. 9.Stadius vero clementior eum liberat intra
menſes.3.& dies.14. Sed hic non cogitat,quod Stadius facit eum ingredi in gemi-
nos anno.1559.die.10. Iunii, & ambulare directum vſque ad diem.6.Septembris,
eiuſdem anni gra.6.min.34.eumq́; poſtea retrogradum inde exire die. 22.Decem.
eiuſdem anni,cum ingreditur in Taurum, vbi partim retrogradus , & partim dire-
ctus manet vſque ad diem.20.Februa.1560.rediens poſtea in geminos , in quibus
manet vſque ad diem Iunii.1561.& inde ingreditur in Cancrum, ambulatq́; dire-
ctus.gra.4.min.59.vſque ad diem.4.Octob.Vnde retrogradiens rurſum intrat in Ge
minos die.28.Decemb.eiuſdem anni,atque ibi partim retrogradus partim directus
manet vſque ad diem.12.Apr.1562.itaque in pluribus vicibus facit eum morari in
Geminis dies circiter.816.ideſt circiter menſes.27.ſumpſit autem hic ſcriptor bre
uiſſimam

uiſſimam moram cauſa comparationis cum calculo Leouitij, vt faceret differentiam apparere maiorem. Tamen in quouis dictorum temporum nunquam inuenietur Leouitius differre à Stadio plus gradibus tribus integris. Idem fecit in multis alijs lo cis dictorum virorum eos conferens tum in Saturno tum in Ioue, & Marte, putás ma gnum eſſe errorem, φ planeta non perambulet totum ſignum, in quod eſt ingreſſus vel directus vel totum retrogradus. Atque hæc opinio ſimilis eſt ſuperiori de con iunctionibus veris Saturni, & Iouis, vbi dicit quod nunquam coniunguntur in vno ſi gno alterius triplicitatis, niſi perfecerit coniunctionem in omnibus ſignis primæ tri plicitatis. Verum vt ſuperſedeam vlterius diſputare, mihi videtur, quod hactenus dixi, poſſe tibi ſatisfacere, quod attinet ad ſciendam ſententiam meam ſuper dictis Animaduerſionibus latinè ſcriptis. Hoc tamen non prætermittam, φ hic non ani maduertit, népe φ differétiæ locorum planetarũ quæ ſũt inter ephemeridas Leouitij &Stadij, euenere, quia vnus ſupputat cũ radicibus, & fundamétis Alfonſi alter verò Reinoldi ex Copernico recentius obſeruatis, ita idem euenire poterit futuris tem poribus, ſi ſupputati fuerint dicti motus, & loci cum recentioribus obſeruationibus cum impoſſibile ſit tam ſubtiliter, tanque perfectè ſupputare loca & mot⁹ eorum, vt lungo interuallo temporis non comperiantur in eis aliquæ differentiȩ, cuius rei re medium eſt ſemper ſequi recentiores obſeruationes & tabulas.

Atque vt tibi ſatisfaciam etiam circa alia ſcripta vulgari lingua edita menſibus.4 poſt latina, etſi intelligere potes, qualia poſſint eſſe alia eius ſcripta, ex ijs quæ ſupra dicta ſunt, atque etiam ex eo, quod dicit ſe miſiſſe multa exempla ſuarum Animad uerſionum in varias terras, illis qui profitentur has ſcientias, aut earum ſtudioſi ſunt, nec quenquã inueniſſe qui ad tã laudabilem prouinciam motus ſit, nec vidiſſe, φ ali quis reſponderit eius rationibus; laudabilem prouinciam, autem puto, φ intelligat correctionem ephemeridum, verens, ne culpa calculatorum, qui eas ſumpſere e ta bulis, tam differentes ſint, vt quibuſdã locis cap. 1. Videtur, & præcipuè vbi ſic ait.

    » Perche eſſendo impoſſibile alli ſtudioſi di dette ſcientiæ di non ſeruirſi delle
    » ephemeridi, maggiormente a quelli che non ſanno ſeruirſi delle tauole, e cono-
    » ſcendo d'incorrere in errori ſenza hauerui altro rimedio, ſarebbono forzati di ab
    » bandonare i ſtudij loro.

Quanquam circa finem dicti capitis redeat in meliorem viam & aduerſetur ſi bijpſi vbi ſic ait.

    »    Che poi eſſi poſſeſſori della ſcienza, &c.

Etiam aperiam tibi, quæ mea ſit de ijs ſententia.

Hic igitur in ſcriptis Italicis, vt morderet aliquem ex ijs, qui eius ſuperiora ſcripta non laudauerant, occaſionem capit aperiendi aliquos illius errores, per editionem collationis quorundam calculorum a ſe collectorum illius, atque etiam aliorum, cu ius calculi ſunt in ſecunda, & ſeptima figura. Sed prius quam veniamus ad defenſio nem harum duarum figurarum vide obſecro quam alienum ei videatur, quod alij dixerint differentiam ephemeridum non eſſe magni momenti, non afferens reſpe ctum vllum, qui enim dixerunt eiuſmodi differentiam non eſſe magni momen ti id dixerunt habito reſpectu ad ſignum in quo eſt planeta, vt (exempli gra tia) quamuis in ponendo loco Saturni Leouitius interdum differat à Stadio gra dibus. 3. quum vterque eum ponat in eodem ſigno, tunc id nullius momenti eſt, & ſic in coniunctionibus aut alijs aſpectibus duo, aut. 3. gradus non faciunt alteratio nem ſenſibilem, cum virtus coniunctionum, & aſpectuum inſit, & duret per mul tos gradus ante aut poſt ipſum punctum. Nec quicquam tamé eſt qui dubitet, quin præſtaret ſcire ſubtiliter ipſum punctum. Nec vnquam fuit aliquis qui negauerit re

                                                    H h            fer-

ferre vt anni directionum correspondeant gradibus æquatoris. Et præterea in ephe
meridibus videntur certè motus & aspectus luminarium, quamuis insit differétia mi
nutorum. Nam non differunt gradibus, præter situm parum distantem à vero om-
nium planetarum, quorum cognitio in cœlo, quamuis circa eorum locum error es-
set gra. 10. tamen in hoc prodesset, & tempus aspectus eorum, etsi non diei præcisè,
quia influentia eiusmodi a spectuum, præterquam Lunæ durat multis diebus, & non
vno tantum. præterquam quod ipsæ ephemerides ostendút nobis tempus ecclipsiú,
in quo certè non differunt nec diebus nec multis horis, &itidem multa alia.

Non sunt igitur contemnendæ ephemerides, nec habendæ pro re nullius pretij,
vt hic ait.

Quod attinet ad illa alia, quæ hic vocat errores ephemeridum, tam de apparenti
coniunctione Saturni cum Ioue in signis alterius triplicitatis prius quam peregerit
præcedentem, quam de faciendo currere Saturnú, & de retinendo Ioue, de detiné
do Marte. 6. aut. 7. mensibus in vno signo, de Marte, & Ioue non coeuntibus singu-
lis. 24. annis in quolibet signo, & eius generis alia, minime verum est quod sint er-
rores, quamuis huic præbuerint occasionem toties errandi.

Comparatio postea inter eius calculos sumptos partim ex tabulis Iunctini, & par
tim ex ephemeridibus Stadij tanquam calculis Copernici, & calculos figurarum su-
per eis positarum supputatarum à diuersis per ephemeridas Alfonsinas, etiam pro-
posita ab eo est ad ostendendum magnam & monstruosam differentiam, vt ait cap.
2. vbi miratur, quod cum ex communi sententia calculi Copernici meliores sint, cal
culatores dictarum figurarum potius eos sumpserint à tabulis Alfonsi, quam Coper
nici. Quæ admiratio quam aliena sit, considerandum permittam cuiuis intelligen-
ti harum facultatum, cum sæpe accidere possit. vt cum aliquis velit scire solum vni
uersalia alicuius genesis, siue natiuitatis, cum non inueniantur ephemerides Coper-
nici, sed tantum Alfonsi, calculator vtatur tantum ephemeridibus, quas inuenit, tú
causa vitandi tædij calculi tabularum, qui magni laboris est, præcipuè in tabulis Pru
tenicis Reinoldi. tum quia superflua ei est summa subtilitas, cum non curet laborare
circa directiones vt factú est pro secúda figura ab hoc proposita, quæ erat anni. 1551
quo non inueniebátur ephemerides Copernicæ, quæ non editæ sunt ante annum
1554. præter quam quod ille nobilis vir pro quo supputata fuit dicta secunda natiui
tas dubitabat de anno, vt hic similiter scit. quare potuisset perdi tempus, & labor, si
supputata fuisset per tabulas Reinoldi, nam Iunctini tabulæ nondum editæ fuerant.
Calculus postea septimæ figuræ, qui erat reuolutio dictæ secundæ natiuitatis, duab⁹
de causis non factus est per tabulas prutenicas, primum, quia eius anni. 1580. non
inueniebantur amplius ephemerides Copernicæ. Ephemerides enim Stadij in-
cipiétes ab ann. 1554. desinút ann. 1576. & cótinuatæ postea quæ perueniunt vsque
ad annum. 1600. non peruenere ad manus calculatoris ante hunc annum. 1581. Al-
tera ratio est, quia in reuolutionibus, quoniam in eis non fiunt directiones, non po-
nuntur à doctis, ne minuta quidem. quare non solum non curant eas supputare per
tabulas, sed nec exquisitè quidem per ephemeridas. Calculi postea ab hoc sumpti
ex tabulis Iunctini, & positi sub dicta secunda figura, adeò rectè facti sunt, vt cum se
cundum ipsas tabulas oporteat Saturnum esse circa. 32. minutum gradus. 23. Aqua-
rij, ipse eum scribat in gra. 11. mi. 3. dicti signi. Iupiter similiter qui secundum dictas
tabulas inuenitur circa finem gradus. 5. Cancri, ab eo ponitur in min. 28. gra. 19. eius
dem. ex quibus planetis Saturnus in figura positus est in min. 27. grad. 23. Arietis, Iu
piter autem in min. 3. gra. 6. Cancri, Vnde secundum verum, inter calculum Alfon

si, &

fi & Iunctini in Saturno non erat differentia plus quàm minu.5.& in Ioue min.4.sed
secundum calculum huius in Saturno fuisset differentia gra.11.minu. 54. & in Ioue
grad.13.min.35. Atque hæ sunt quidem differentiæ magnæ, & monstruosæ vt ipse
eas vocat, vt etiam est illa Veneris,& Mercurij inter tertiam figuram, & eius calcu-
lum sumptum,non quidem à tabulis laboriosis,sed à simplicibus ephemeridibus Sta
dij, quæ differentia nõ est quidem paucorum graduum,cum sit tertiæ partis cœli in
quolibet dictorum planetarum . Huiusmodiq́; monstra certè non sunt orta à ta-
bulis siue ephemeridibus diuersis,sed sunt partus huius authoris.

Pergens postea assiduè bonus hic vir hominibus dare specimen doctrinæ suæ ape
riendo ( vt conatur) aliorum errores, proponit duas differentias inter primam fi-
guram,& suum calculum suppositum Saturni,& Iouis. Primum de Saturno ait , ꝗ
cum differentia sit gra.1.min.30.ostendit in directione , ꝗ accidens fit euenturum
anno vno,& mensibus sex ante,aut post, quasi eiusmodi differentia esset partium æ-
quatoris,sicuti est partium Zodiaci. Idem dico de differentia Iouis.Quod quidem,
manifestum est inditium scientiæ suæ , & quantum ea intelligat de quibus loquitur.

Quod postea attinet ad differentiam inter Copernicum & Alfonsum, circa Solé,
nullus est harum scientiarum peritus,qui id nesciat,& similiter de differentia situs cę
li in reuolutionibus annuis.

Quod verò ait septimam figuram malè supputatã fuisse,si non est maximus cer-
tè non est minimus monstruosorum eius errorum. Vbi itidem videri potest, quàm
alienus hic sit ab hac scientia. Nam si saltem curasset sibi ab aliquo supputandum
locum Solis per tabulas Alfonsi in instanti minutorum.36.pomeridianorum,certior
factus esset quod in illo puncto Sol inueniebatur in minu 54.grad.11.Geminorum,
idest præterierat gra.10.cum min.54. vel si curasset sibi inueniendum tempus, per
dictas tabulas cum grad.10.min. 54. Geminorum vt faciendum est, sequendo tamé
Alfonsum , & non per calculum Solis positum in ephemeridibus, vt parum periti fa
cere solent,vidisset ꝗ inuenta essent min.36.pomeridiana. Leuis tamen occasio hu
ic fuit suspicandi eiusmodi tempus esse falsum, quod viderit in illa figura Solé po
situm esse cum gra.11.& non cum gra.10.min.54.non animaduertens ita notatum
fuisse Solem vt omnes alios planetas, scilicet sine minutis, quum, vt dixi,in reuolu
tionibus non adhibeatur tanta scruplositas.

Quod deinde ait, in illa figura Solem positum esse in decima domo,& non in. 9.
id relinquam iudicio eorum qui sciunt numerare domos,saltem posuisset authori-
tate sua Solem in dicta decima diuersè ab exemplo ei dato ab amico, vt ostenderet
se dicere verum,vt in secunda figura discrepat ab ipso exemplo in collocando Leo-
nè, Virgine,& Libra,& Scorpio, quos malè locauit, & si alii benè se habent.

Atque quod hactenus à me dictum est , satis fit ad intelligêdum quale fit reliquũ
dictæ eius disputationis. Si enim velim pergere notare omnia eius errorum loca,es
set mihi inanis labor,& tibi nimia molestia. Et quamuis non defuerint præstantissi-
mi viri,qui visis eius scriptis familiariter eum monuere,& tu ipse,vt audiui,cum in-
strumêto theoricę in manib' ei ostéderis quo mó Mars possit morari amplius sex mé
sibus in vno signo.& præterea cum iam ab initio Taurinum aduenit , mecum com-
municauerit illa sua prima scripta, egoq́; eum monuerim,quod in varijs reb' falleba
tur,dissuaserimq́; ne ea imprimenda curaret, quia nullum honorem inde referret ,
eum hortans,vt potius alijs rebus operam daret,atque ei dixerim quod ad animad-
uersiones differentiarum ephemeridum attinet,quod id iam oês animaduerterant.
Mihi respondit se decreuisse illa edere , vt postea fecit, & tot admonitionibus non

H h     2 .     acquie-

acq uiescens, die.11.Augusti edidit chartam illam impressam inuitans ad disputationem quotquot adhærerent contrariæ sententiæ, volens sustinere Marté non posse commorari in vno signo amplius duobus mensibus, supponens partem principiorum ab omnibus admissorum,& in fine paginæ exponens modum, quo vtitur ad probationem suæ intentionis. Puto autem quod secum ratiocinabatur de Marte, vt fecit de Saturno in scripto latino, hoc modo. Si Mars in duobus annis ambulat per omnia.12.signa,necesse est igitur,vt in mensibus duobus ambulet per vnum sigum, cum menses.2.sint duodecima pars annorum duorum. Sed ibi statim in ipso initio committit errorem graduum ferè.7.dicens, quod medius motus Martis inueniebatur signorum.4.& gra.17.cum eo tempore dictus medius motus non esset reuera plus quam sign.4.grad.10.mi.36.verum hoc ad ea,quæ sequuntur exigui est momenti.Is postea particulatim colligit medium motum Martis ad diem. 29. Mai anni.1514.quem ait esse signorum.9.gra.27.min.53.& tamen reuera erat tantū signorum.9.gra.21.mi.29.sed missum faciamus etiam hunc errorem tanquā à primo pendentem. Cum deinde ibidem ponit centrum epicycli, similiter errat, nam centrum epicycli nunquam poni debet vbi est linea medij motus,nisi sit in auge,aut in opposito augis eccentrici,quia debebat collocare ipsum centrum tāto post lineā medij motus,quanta erat æquatio centri,quia medium centrum Martis tunc erat minus signis sex,& aux eccentrici eius erat in sexto minuto grad. 16. Leonis. Tamen hoc etiam leue est. Præsupponamus igitur quod centrum epicycli esset in grad.28 Capricorni, vt ipse credidit, idest gradibus.7.vlterius quam erat reuera.Ait postea se comperisse Martem ambulasse signa.4.& grad.22.eius epicycli,sed non explicat an intelligat de argumento medio, an de vero,quod vocatur æquatum,nam si intelligatur de medio,hoc esse non potest,cum mediū esset signorum.4.gra.24. mi.35. sed si intelligatur de vero, vt iure credendum est (alioquin etiam errasset) certè falsum est. Nā, verum, erat signorum.4.grad.29.minu.39. Itaque Mars non distabat à linea veri motus epicycli amplius gradibus. 30.& minu.21.ipsius epicycli, & æquatio argumenti secundo correcta erat gra.44.minu.2.à quo subtracta æquatione centri, quæ erat gr.5.minu.4.(cum centrum epicycli deberet tanto spacio esse post lineam medij motus quantum supra dixi)supererant gra.38. minu.58.addendi gradibus, & minu.medij motus, qui cum reuera essent grad.21.& minu.29.Capricorni, perueniebant ad minu.27. grad.1. Piscium. Sed præsupposito secundū ipsum, quod medius motus esset grad.28. Capricorni, & quod Mars esset non solū vbi hic ait, sed etiam in prima linea contingentiæ epicycli,idest in prima linea maximæ æquationis argumenti,& præsupposito etiam quod dicta æquatio esset æqualis illi, quam haberet ad medium Aquarij scilicet grad.47.quum centrum epicycli est in opposito augis,manifestum est, quod eiusmodi linea contingentiæ non transiret vltra grad.15.Piscium, & tamen hic ait, quod linea veri motus Martis vadit ad grad.16. Arietis. vnde oporteret, quod æquatio argumenti esset plus quam grad. 78. Quod si verum esset, &.o.c.etiam esset partium.54.secundum distantiam proximiorem centro mundi, semidiameter epicycli esset eiusmodi partium. 52. minut. 49.& quum Mars esset in. g. idest in opposito veræ augis epicycli,dum centrum epicycli esset in eiusmodi distantia à terra, distantia. o.g.idest à terra ad Martem non esset plus, quam vna sola pars ex dictis, cum minut.11. cum partes. 52. minu. 49. ad.54.sint vt sinus anguli gra.78.qui est partium.97814. ad sinum totalem partium 100000. Nam iam supra dixi, quod triangulus.o.c.i.est rectangulus. Hinc sequeretur

retur, quod in in-
teruallo.o.g. vni⁹
partis, & min. II.
respectu.o.c. par-
tium.54. Colloca
retur semidiame-
ter terrę cum spis-
situdine aeris , i-
gnis, cęlorum Lu-
næ, Mercurij, Ve-
neris, & Solis, prę
terquam quod vt
inter Solem , &
terram sunt circa
605. diametri ip
sius terræ , inter
terram , & Mar-
tem cum esset in
auge sui epicycli,
& epicyclus in au
ge eccentrici , in-
uenirentur cir-
ca . 60000. dia-
metri eiusdem ter
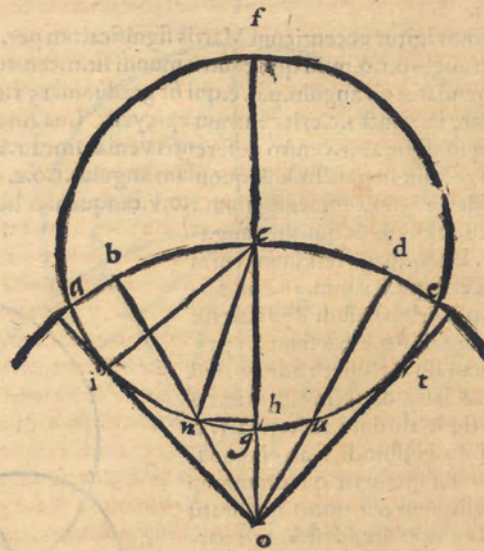
ræ, & tamen ea distantia siue interuallum non potest continere. 5000. diametri ter-
ræ. Et quod plus est, hic tam vastum facit hunc suum epicyclum, vt ambiente Mar-
te per inferiorem eius partem, necesse ei esset manere in vno duodecatemorio mul-
to plus quam. 7. aut. 8. mens. vnde hic multo magis miraretur quam prius. Hinc cer-
nere licet quam rectè facti sint hi eius calculi.

Vt autem etiam hinc aliqua vtilitas capiatur (prætermissis inconuenientibus vna
cum falsis suppositis huius ) Videamus ordine scientifico vbi poterat esse verus lo-
cus Martis, aut vero proximus, die. 29. Mai anni. 1514. quem hic exempli causa su-
mit. Idq; tam ad defensionem tabularum Alfonsi , quam ephemeridum ex eis col
lectarum. quæ quidem exactæ sunt, vt quisque peritus facile videre poterit , non au-
tem calculatæ à tam stupidis hominibus, vt à vero aberrent etiam gradibus. 46. vt
hic ait se depræhendisse.

Primum igitur supponemus eosdem illos terminos, quos ipse nec debet , nec po
test negare, præter ea quæ supra supposita sunt, nempe quod semidiameter epicycli
sit partium.39. minu. 30. & eccentricitas partium.6. talium qualium est semidiame-
ter deferentis diuisus in. 60. & quod dicto tempore aux eccentrici Martis esset cir-
ca minutum. 5. grad. 16. Leonis , scilicet graduum. 135. min. 5. & quod linea motus
mediocris esset circa minu. 30. gradus. 22. Capricorni , & quod verum centrū Mar
tis esset grad. 151. minut. 20. & quod argumentum verum esset grad. 149. minu. 39.
atq; ita ostendam , neque tabulas, neque ephemerides errare, ne quidem vno gra-
du, ac ne quidem multis minutis, non modò tam monstruosa differentia , vt ipse
ait.

Quare primum nobis scientificè inueniendum est , quanta esset distantia. o. c.
præ-

præcise ideſt interuallum inter centrum mundi,& centrum epicycli Martis in huiuſ-
modi ſitu.

Fingemus igitur eccenticum Martis ſignificatum per.p.c. m. cuius centrum ſit. r.
& lineam augis.p.r.o.m.in qua centrũ mundi ſit.o.centrum autem verum epicycli,
comprehendatur ab angulo.p.o.c.qui ſit graduum.151.min.30.ſecundum ſuppoſi-
tum. Quare in puncto.c.erit centrum epicycli. Imaginemur ergo.c.o.productam à
parte.o.quouſque ab.r.centro deferentis veniat linea.r.k.perpendiculariter,faciens
angulum rectum in puncto.k.& quoniam angulus.r.o.c. datur nobis graduum.151.
min.30.ideo cognoſcemus angulum.r.o.k.tanquam reliquum ex duobus rectis,qui

erit gra.28.min.30. & ſimiliter angu-
lum. o. r . k. tanquam reſiduum vnius
recti , qui erit gra.61.min.30.cuius ſi-
nus ideſt.o.k.erit partium 87881.et.k
r.vt ſinus anguli.r.o k.partium.47715
talium qualium.o.r. eſſet 100000. ſed
vt.o.r.eſt. 6.latus.o.k.erit.5.& min.16
et.r k.partium.2.min. 52. & quia. r. c.
eſt partiũ.60.eiuſmodi, ſi ab eius qua-
drato ſubtractum fuerit quadratum ip
ſius.r.k.reliquum erit nobis quadratũ
ipſius.k.c.cuius radix,ideſt.k. erit par-
tium.59.min.56.à qua.c.k.ſubtrahen-
do poſtea.k.o.partium.5.minu.16.re-
manebit.o.c.partium.54.min.40. pro
diſtantia quæſita.
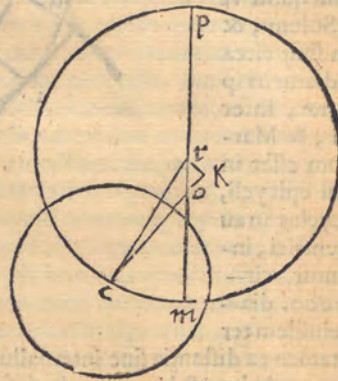


Fingamus poſtea epicyclum. f. n.g.
in quo argumentum verum graduum .
149.minu.39. ſit arcus.f.n.vbi Mars inueniatur in.n.per quem punctum tranſeat li-
nea.o.n.veri motus Martis. Deinde inueniamus angulum.c.o.n. æquationis argumé
ti,modo iam dicto,ideſt ducendo ſinum.n. h.arcus.n. g. qui arcus tanquam reliquus
argumenti veri, iam præſuppoſiti, ex dimidio circulo,erit graduum.30.minu.21.&
n.h.eius ſinus partium.50528.ſinus ſimiliter anguli.n.c.h.et.c. h. tanquam ſinus an-
guli.c.n.h.reſtantis ex uno recto grad.59. minu .39.erit partium.85295.taliũ qua-
lium.c.n.ſinus totus eſſet partium.100000.ſed vt partium.39.& min. 30. ſinus. c. h.
erit partium.34.min.5.et.n.h.partium.19.mi.57.reliquum poſtea.h.o.ex. o.c. par-
tium.20.min.35.quia iam ſupra inuenimus.o.c.eſſe partium eiuſmodi.54.minu.40.
vnde.o.n.vt radix quadrata ſummæ duorum.n.h.et.h.o.erit partium.28. minu. 41 .
talium qualium.n.h.inuenta fuit partium.19.min.57.quæ.n.h. erit poſtea partium ,
69552.talium qualium .n.o.partium.100000.& ſumpta dicta.n.h.vt ſinus dictarum
partium,dabit nobis angulum.n.o.h.quæſitum gra.44.min.4.qui per tabulas Alfon
ſi inuentus eſt gra.44.min.2.par huic, vt dici poteſt. Qui angulus gra.44. minu.4.
collectus cum angulo veri centri iam ſuppoſito graduum.151.minu. 20. & cum an-
gulo augis eccentrici Martis, ſimiliter ſuppoſitæ grad.135.min.5.dabit nobis ſum-
mam veræ diſtantiæ Martis à principio Arietis grad.330. min . 29. quod aliud non
ſignificat , niſi quod Mars inuenietur in minu.29.primi gradus Piſcium. Et Stofle-
rus in ſuis ephemeridibus ponit eum in.22. minuto dicti primi gradus , cuius diffe-
                                                                              ren-

rentia à tabulis
est minut. 5.tan
tū, & à meo cal-
culo min.7.vide
licet minima.

Nunc autem
nolui sumere ip-
sum angulum æ-
quationis à tabu
lis propter duas
rationes , primū
quia ne hic qui-
dem repræhen-
sor in hoc voluit
credere dictis ta
bulis. Sed id vo-
luit videre pro-
prijs oculis ī sua
theorica Martis.
Vbi īuenit quod
linea.o.n. transit
per gra.16.Arie
tis . Secunda ra-
tio est, vt videa-
tur quod dictæ tabulæ rectè supputatæ sunt,super dictis suppositis.
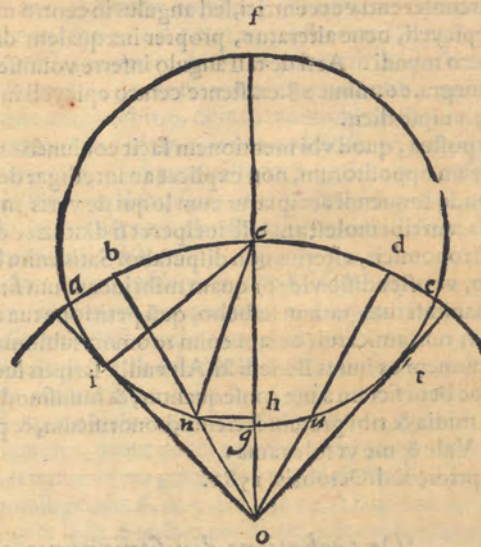
Sed vt videat quantus sit medius motus Martis die.29. Mai colligit frustatim , q̄
eleganter colligere poterat, vna opera in columellis ipsius medij motus eiusmodi
stellæ per eram eiusdem temporis,quæ erat.2.primarum sexagenarum.33. secunda
rum.32. tertiarum,et.52.quartarum.

Primum deinde suppositum quod scribit,scilicet , quod diameter epicycli sum-
ptus in longitudine media sit signorum.2.& grad.19.vti superfluum est,ita etiam fal
sum,nam eiusmodi diameter in dicto loco,non occupat ad centrum mundi plus quā
gra.66.min.28. idest signa.2.gra.6.min.28.quia proportio.o.c. ad semidiametrum
epicycli in eiusmodi loco est ut partium.60.minu.18.ad partes.39.min.30.quę duę
lineæ intellectę,vt latera vnius trianguli rectanguli, habebunt pro basi aliam lineam
partium similium.72.mi.5. Q̄uæ intellecta vt sinus totus dabit semidiametrum epi-
cycli partium.54798.tanquam sinum subiectum angulo gra.33.min.14. pro medie
tate illius, quod quæritur.

Nec prætermittenda mihi videtur ratio , qua credere possumus , hunc cogi-
tasse,quod diameter epicycli compleat spatium duorum signorum cum gradibus.
19.quæ quidem ratio alia esse non potest,nisi quod cum iste inuenerit, in cōmetarijs
Theoricarum,semidiametrum huiusmodi epicycli esse partium.39.min.30.talium
qualium sunt.60.illæ quæ sunt semidiametri huius eccentrici,dictas igitur partes.39.
mi.30.hic putauit esse gradus Zodiaci,& propterea dixit diametrū huiusmodi epicy
cli esse signorū duorū,& gra.19.qui numerus.79.duplus est numero. 39. cū dimidio
hoc autem dixit accidere in longitudinibus medijs, quia si hic intellexisset de pro-
portione horum duorum diametrorum, quæ est ut.120. ad.79. non specificasset lo-
cum

cum epicycli, cum ipsa proportio nullo modo alteratur existente epicyclo vbi volue
ris ipsius circunferentiæ eccentrici, sed angulus in centro mundi, cui subiacet dictus
diameter epicycli, bene alteratur, propter inæqualem distantiam centri epicycli
ab ipso centro mundi. At si de tali angulo inferre voluisset, iam probaui ipsum cō-
tinere solum gra. 66. minu. 28. existente centro epicycli in longitudinibus medijs &
non gra. 79. vt ipse dicit.

Omitto postea, quod vbi mentionem facit coniunctionum Solis cum Marte au-
gium & earum oppositorum, non explicat an intelligat de veris an de medijs. Nam
si ex eius modo loquendi accipiatur eum loqui de veris multum erraret.

Sed quia iam tibi molestum esse inciperet si diutius te detinerem in his conten-
tionibus astronomicis, vlterius non disputabo. Satis enim hactenus explicaui sentē-
tiam meam, vt ostendisse videor quam mihi iucundum sit tibi moré gerere. In quo
etiam hnmanitati tuæ gratiam habebo, quū petitione tua occasionem mihi dederis
efficiendi, vt tum amici tui ( amant enim te omnia sublimia ingenia) tum alij, si quā
falsam opinionem ex huius Benedicti Altæuillæ scriptis sumpsissent, eā relinquant,
& per te hoc beneficium à me consequantur, & huiusmodi occasionem, & iuuandi
hominum studia & tibi gratum faciendi, honorificum, & per gratum mihi fuisse in-
telligant. Vale & me vt soles ama.

Taurini pridie Kal. Octobris. 1581.

## De probatione diuisionis numerorum.

### AD EVNDEM.

Inter alia quæ à me scire cupis, vir doctissime, hoc vnum est, vt ex literis tuis ac-
cepi, vnde sit vt prisci nostri probatione numeri nouenarij potius quam septe-
narij vsi fuerint, & qua ratione non idem proueniat ex probatione numerorum
octonarij, senarij, vel quinarij, aut cuiuslibet alterius: Vnde pariter oriatur quod in
partitionis probatione necessum sit probationum euentus multiplicare cum proba-
tione diuisoris, ac eam quæ est producti postea cum probatione fractionis in sum-
mam colligere, &c. Ad hæc in primis respondeo, eum aliquoties accidere possit ta
les probationes nos fallere posse, idq; si in tali summa similis numerus, ut puta se-
ptem, aut nouem, plus vel minus æquo iustouè positus fuerit, attamen per raro eueni
re potest, vt quis per nouenarium potius quam per septenarium decipiatur. Exem-
pli gratia, ponamus summam esse. 100. quam numerus nouenarius vndecies solum
ingreditur, at septenarius quatuordecies, vnde quis sępiⁱ⁹ ex septenario, hac ratione,
quam ex nouenario numero se posse errare facile depræhendet, etsi ex probatione
nouenarij magis quam septenarij, vt practici scribunt, duabus de causis errare possi-
mus. Alia tamen ratio mihi suppetit, ob quam credibile est ipsos potius nouena-
rio adiutos fuisse, quam septenario, quæ est ob sui cum velocitatem tum facilitatē,
neq; enim in septenario est adeo facilis. Nā quamuis, tam vna quam altera aliud
non sit, quam numerorum ordines diuidere ( si de summis primo loquamur) aut è
summa superfluum ordinum colligere, & videre an idemmet superfluum ex eadem
summa emanet; attamen cum modus, qui in hoc adhiberi potest in nouenario quā
in septenario velocior sit, & ob id probationem nouenarij seligunt potius quam
septenarij.

Verum

Verum nolo te in ea,quæ falſa eſt, opinione conſiſtere,non idem, & cum octona-
rio,ſenario, vel quinario, aut quouis alio numero poſſe efficere, cum eademmet ra
tio,quæ in ſeptenario, aut nouenario,ēt in cæteris perhibeatur. Ponamus exemplū
hos tres ordinum numeros velle ſupputare, quorum primus ſit. 679. ſecundus. 846.
& tertius.935.& illorum ſummā.2460. nunc maiorem numerum primi ordinis ab
octonario menſi,proijciendo, remanebit.7. deinde maiorem numerum demendo à
ſecundo ordine,reſiduum erit. 6.ac ſi idem in tertio ordine fecerimus,erit nobis re-
liquum.7. Demum tria hæc reſidua in vnum collecta.20.efficient,à quibus ſi nume
rum maiorem ab octonario menſum dempſeris,ſupererunt.4. & totidem à ſumma.
2460.remanebunt,reiecto maiori numero ab octonario menſo. Atque idem me-
dio quouis alio numero, euenire poteſt.

Cuius ratio tam per ſe clara atque euidens eſt, quod ſi ſummam trium reliquorū,
quæ eſt.20. à ſumma.2460. ſubduxeris,remanebunt.2440.pro ſumma trium nume
rorum dictorum trium ordinum ab octonario menſorum,cui numero addito.16.pro
maiori numero ſummę reliquorū,qui ab octonario menſus ſit,ſupererunt.4.At ſi per
ſenariū experimētū feceris, remanebit.0.& ſic de reliquis per ordinem procedēdo.

Verum poſſes ſciſcitari, quare velocius, exceſſus ordinum, potius per noue-
nariū, quam per cæteros numeros, prout docet practici, inueniri queat, videlicet ag
gregando prius duas figuras numerorum primæ ſummæ, deinde alias duas. Exem-
plum ſit primus ordo.679.colligendo.6. et. 7.faciunt 13.& cum hæc ſumma ſit dua
rum figurarum, ſupputantur & ipſæ, è quibus prodeunt.4.& conſimilis erit proba-
tio numeri.67.facta per. 9. quod idem eſt, ac ſi quis diuidat.67.per. 9. ex quo reli-
qui erunt ſemper.4.

At quo ratio huiuſce rei perſpicuè dignoſci poſſit,in primis ſciendum eſt, cuique
ex ſe cognitum,atque exploratum eſſe, denarium numerum vnitate nouenarium ſu
perare, & ex hoc ſequitur, ſex denarios continere in ſe ſex nouenarios,& ſex vni-
tates.

At ſex vnitates,vna cum.7.faciunt.13.& quia in.13.eſt denarius,igitur in illo erit
vnitas ſupra.9.Quæ vnitas addita ternario,præbet nobis ſuperfluum, per quod.67.
ſuperat.54.iunctum cum.9.ſcilicet ſummam.63.

Idem dici non poteſt de octonario,ſeptenario, vel ſenario, & de reliquis, quo-
niam numerus denariorum,in cæteris minoribus nouenario non præbet illico nu-
merum exceſſus maioris numeri, qui à numero probationis menſus eſt. Et quod di
co de probatione aggregationis, idem intelligo de alijs operationibus, vt puta ſub-
tractionis,multiplicationis,& partitionis ſeu diuiſionis.

Vnde autem oriatur,vt in partitionis probatione opus ſit probationem euentus
cum diuiſionis probatione multiplicare, & productum cum fractionis probatione
ſupputare,ſeu aggregare, tibi non erit ignotum, quoties animaduerteris, quod
productum ipſius euentus cum diuiſore, adiunctum fractioni, perpetuo ſe æquat nu
mero diuiſibili. Et quoniam numeri probationum ſunt partes, quæ remanent ex
ipſis totis, detractis maioribus numeris ab eo dimenſis, quo pro communi men-
ſura vtimur (prout.7.vel.9.aut alium numerum,quem voluerimus) par eſt vt ex ip-
ſarum remanentibus partibus,velut ex ipſis totis idem fiat.

IO. BAPT. BENED.

## De falacia operationis triangulorum sphæricorum.

### AD EVNDEM.

QVod diebus præteris tibi significaui, idem nunc confirmo, scilicet sphærico-
rum triangulorum operationem sæpe nos fallere, vt exempli gratia, si pro
positus nobis fuisset triangulus. A.B.C.cuius angulus. A.nobis datus esset graduum.
114.mi.0.& eius latus. A.B,graduum.67.min.5.& latus. A.C.graduum.45. mi. 10.
si reliquos angulos cum tertio latere etiam cognoscere voluerimus, ex methodo. 11
primi Copernici propositum obtinebimus. vnde latus. B.C.esset graduum. 89.min.
30.angulus vero.C.graduum.57.min.14.angulus autem.B.grad.48.min. 38. Qua-
re vltimus hic angulus.B. falsus esset,eo quod operatio paruorum triangulorum in
causa est, quotiescunque eorum latera tam breuia sint, ut non eccedant vnum gra-
dum, quare ipsorum angulorum veram quantitatem non tribuunt. propterea igitur
cum voluerimus veram quätitatem ipsius anguli.B.oportet post quam inuenerimus
angulum.C.mediante arcu. D.E.supponere alium polum in.B.deinde producere.B
A.vsque ad.d.et.B.C.vsque ad.e.imaginando. B.d.et. B.e.duas quartas esse magno-
rum circulorum, extendendo postea. d.e. vsque ad intersectionem cum. A.C.& eü
dem ordinem prosequendo,tunc.e.d.nobis ostendet angulum.B.esse gra.40.mi.22
quæ erit eius vera quantitas. Cuius quidem rei experientiam possumus etiam fa-
cere,hoc modo,esto, exempli gratia, quod nobis datus sit angulus. C. graduum.57.
min.14.cum latere.A.C.gra.45.min.10.& latus.B.C.gra 89.min.30. Tunc si ordi-
nem.11.dicti lib.sequemur,obtinebimus intentum,hoc modo scilicet supponendo
in. A.polum , & non in.B. ducendo etiam. A.B.et. A.C.sed. A.B.viq; ad gra.90. du-
cendo postea. D.E. ita quod ab omni parte concurrat cum latere.B.C.producto,vn
de tam.f.C.B.F quam.f.D.E.F.erunt semicirculi magnorum circulorum. quare.C.
D.nobis cognitus erit gra.44.min.50.& sic etiam angulus.D.C.f.gra.57.min.14.ex
4.dicti lib.postea habebimus.F.l.gra.60.min.54.& angulum.f.gra.53.mi.24.aggre
gatum postea.f.C,cum.C.B.habebimus.f.B.gra 150.min.24.qui si a semicirculo dē
ptus fuerit,nobis remanebit.B.F.gra.29.mi.36.cum angulo.F.cognito cü sit æqua-



lis.f.eius opposito. Vnde ex dicta.4.co-
gnoscemus angulum.B.gra.40. min. 31.
qui ferè æqualis est superiori iam inuen-
to,nec ab ipso differt nisi per min.9. quæ
quidem differentia parua est respectu al
terius
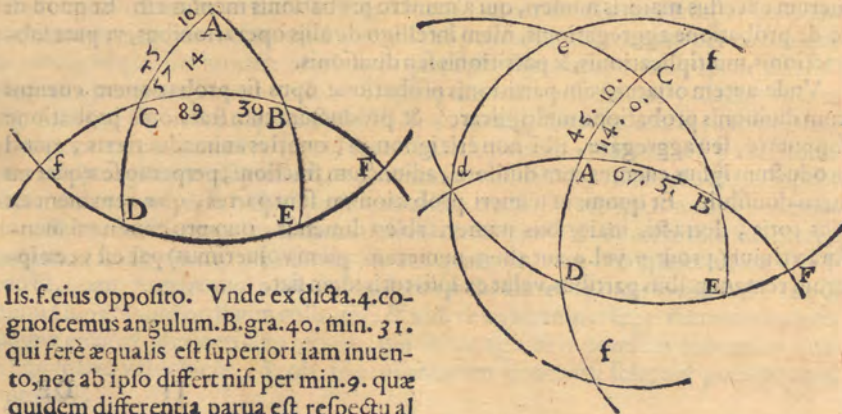
terius differentiæ quam supra inuenerimus.

Superius enim dixi non esse ponendum polum in.B.eo quod. B.C.sit gra.89. mi. 30.vnde nobis prodijsset triangulus.f.C.D.trium valde paruorum laterum, quorum latus.C.D.esset gra.0.mi.30.& latus.f.l.gra.0.mi.55.& latus.F.D.gra.0.mi.47.vnde angulus.f.gra.32.min.40.falsus esset, qui quidé postea nobis daret. D.E.gra.45 minu.16.falsum similiter.

## De passione circuli hactenus incognita.

## AD EVNDEM.

Vbitandum quidem nó est quin passiones circuli innumerabiles penè sint,quę quidem omnes ferè casu inueniuntur, vt mihi nunc accidit,quam tibi mitto, hæc autem est, quòd quadratum lineæ. a. g. in figura hic subscripta semper æquale est ei producto,quod fit ex.a.e.in diametro circuli.g.c.b.simul sumpto cum quadrato inscriptibili in dicto circulo,& simul cum quadrato lineæ.a. b. contingétis ipsum circulum,supponendo.a.g.per centrum ipsius circuli transire.

Pro cuius demonstratione à centro.e.duco semidiametrum.e. c. perpendicularé ipsi.g.a.& à puncto.c.ad.a.duco.c.a. quæ secabit circunferentiam ipsius circuli in pũ cto.d.eo, quod angulus.c.acutus est. Nunc ex.35.tertij,productum.c.a.in.a.d. æqua le est quadrato.a.b.productum autem.a.c.in.d.c.æquale est quadrato inscriptibili in circulo.g.c.b.ex.130.primi Vitellionis, ĩ qua propositione ipse Vitellio supplet pro eo,quod in quinta propositione libri de lineis spirabilibus Archimedis desideratur, sed quadratum.a.c.æquale est ijs duobus productis.per.2. secundi Eucli. ergo quadratum.a.c.æquale erit quadrato inscriptibili in circulo.d.c.g.& quadrato. a. b. sed quadratum lineæ.a.c.æquale est duobus quadratis,hoc est lineæ.a.e. & lineæ.e.c. ex pitagorica,quare ex communi conceptu duo quadrata lineæ.a.e.& lineę.e.c.hoc est lineæ. e. g. quod idem est, æqualia erunt duobus iam dictis, hoc est inscriptibili, & ei, quod fit ex.a.b.sed quadratum lineæ.a.g.æquale est quadrato lineæ.a.e.& qua drato quod fit ex.e.g.& duplo illius quod fit ex.a.c.in.e.g. hoc est producto. a. e. in diametrum. Quare quadratum lineæ a.g.æquale est quadrato circunscriptibili, & quadrato lineæ.a.b.& producto lineæ.a.e.in diametrum circuli.d.c.g.

Breuiori etiam methodo demónstrare possu mus quadrata lineæ . a. e. et. e. g. æqualia es se quadrato circunscriptibili,& quadrato lineæ. a.b.ducendo lineam.e.b.quæ æqualis est lineæ. e.g.tali methodo,hoc est, considerando , quod quadratum inscriptibile semper duplum est qua drato semidiametri , vel medietati circumscriptibili,quod quidem nihil aliud est, nisi æquale esse ijs duobus quadratis,hoc est lineæ.e.b.& lineæ.e. g.sed quadratum lineæ.a.e.æquale est iis duobus quadratis, hoc est lineæ.a.b.& lineæ. b. e. vnde quadrat um lineæ. a. e. cum quadrato lineæ.e.g.æquale est quadrato circunscriptibili,simul collecto cum quadrato lineæ. a.b.

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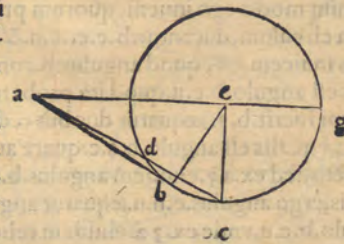
## Demonstrationes quarundam propositionum de quibus agit Cardanus capite primo.libro. 16. de subtilitate.

### AD EVNDEM.

EA quæ Cardanus in primo cap.lib.16.de subtilitate ita scribit,quod si diame-
tros producatur extra quantumlibet , alia verò diametro in centro secetur ad
rectos, ex huius fine &c. quæ quidem secundum illum est vndecima proprietas cir
culi, quoniam te id non intelligere scribis,idemq́; dicis etiam de duodecima , & si-
militer de tribus illis passionibus, quas ipsæ communes facit circulo , defectioni,seu
ellipsi , & hyperboli, tibi breuiter respondebo.

Circa vndecimam proprietatem circuli verum dicit . Imaginemur circulum.p.
d.q.à duabus diametris,inuicem ad angulos rectos coniunctis,diuisum.p.d.et.d.g.di
uidatur enim quarta.q.d.per quot partes æquales volueris,mediantibus punctis.b.a.
o.ducanturq́; ab ijsdem punctis tot perpendiculares diametro.d.g.quæ sint.b.m.a.n.
et.o.s.quæ quidem erunt parallelæ diametro.q.p.coniungatur deinde extremitas.d.
diametri.d.g.cum primo puncto.b.& protrahatur.d.b.vsque ad concursum cum dia
metro.p.q.protracto in puncto,h.Nunc dico.q.h.quæ adiacet diametro.q.p.æqua-
lem esse omnibus dictis perpendicularibus , quapropter coniungantur puncta. m.a:
n.o.et.s.q.& producantur vsque ad adiacentem diametro.q.p. in punctis.c.et. e. vn
de habebimus angulos. b.a.o.q. inuicem æquales ex.26.tertij,cum verò.o.s. a. n. et
b.m.parallelæ sint ipsi.p.h.tunc anguli.b.h.c:a.c.e:et.o.e.q.æquales erunt angulis.d⊃
b.m:m.a.n.et.n.o.s.ex.29.primi: quare anguli.h.c.e.q.erunt inuicem æquales, vnde
ex.28.eiusdem.b.h:m.c:n.e.et.s.q.erunt inuicê parallelẹ,& ex.34.e.q. æqualis erit.
o.s.et.e.c.æqualis.n.a.et.m.b.æqualis.c.h.verum est igitur propositum.

Duodecima vero ̗pprietas est,ut si fuerit circulus.a.b. e.q. cuius duo diametri ad
rectos coniuncti sint.a.e.et.q.b.& diameter.a.e.protractus indeterminatè ad partem
e.tunc si ab extremo.b.diametri.q.b.ducta fuerit.b.n.u.extra circulum,seu.b.u. n. in
tra circulum, vt in subiecta figura patet, ita vt secta sit à circunferentia circuli in pũ
cto.n.vel à diametro in puncto.u.semper id quod fit ex.u.b.in.b.n. æquale erit qua-
drato inscriptibili in dicto circulo , hoc autem diuersimodè cognosci potest, tribus
enim modis ego inueni, quorum primus ita se habet . Nam si punctus. u. fuerit ex-
tra circulum,ducantur.b.e.et.e.n.& habebimus duos triangulos.b.n.e:et.b. e.u.simi
les inuicem,eo , quod angulus.b.communis ambobus existit, & angulus.b.n.e.æqua
lis est angulo.b.c.u.quod ita probatur,nam angulus.b.n.e.cum angulo.b.a.e.(ducta
cum fuerit.b.a.)æquatur duobus rectis ex.21.tertij, sed ex quinta primi angulus.b.
e.a.æqualis est angulo.b.a.e:quare angulus.b.n.e.cum angulo.b.e.a.æquatur duobus
rectis,sed ex.13.eiusdem angulus.b.n.e.cum angulo etiam.e.n.u.æquatur duobus re
ctis,ergo angulus.e.n.u.æquatur angulo.b.e.a.quare angulus.b.n.e.æquatur etiã an-
gulo.b.e.u.vnde ex.32.eiusdem reliquus angulus.b.u.e. æqualis erit reliquo angulo
b.e. n. latera igitur erunt proportionalia ex.4. sexti, vnde ita se habebit.u.b.ad.b.
e.vt.b.e.ad.b.n.ex.16.sexti igitur verũ erit propositum.

Sed si punctus.u.intra circulum fuerit, triangulus.b.e.n.similis erit triangulo.b.u.
e.nam angulus.b.ambobus communis erit. Angulus vero.b.n.e. equalis est angulo.
b.e.u.ex.26.tertij, quare ex.32.primi reliquus angulus. b. e. n. æqualis erit reliquo
angulo

angulo.b.u.e. vnde ex.4. fexti eadem proportio erit ipfius.b.n. ad.b.e.quæ. b. e. ad
b.u.quare ex.16.eiufdem patebit propofitum.

Secundus autem modus ita fe habet,ducta.q.n.habebimus duo triangula ortho-
gonia fimilia inuicem . b.q.n.et.b.u.o.eo quod angulus.b.communis ambobus exi-
ftit, quare ex.4.fexti ita fe habebit.u.b.ad.b.o.vt.q.b.ad.b.n.vnde ex. 15. eiufdem
quod fit ex.u.b.in.b.n.æquale erit ei,quod fit ex.q.b.in.b.o.Sed ex. 16. eiufdem, φ
fit ex.q.b.in.b.o.equatur quadrato.b.e.quia.b.e.media proportionalis eft inter dia
metrum & femidiametrum eiufdem circuli.ex.4.eiufdem,quare quod fit ex.u. b. in
b.n.æquale erit quadrato ipfius.b.e.

Tertius modus adiungitur,& eft quod cum quadratum.u.b.exiftente.u.extra cir-
culum æquale fit ei, quod fit ex.u.b.in.b.n.fimul fumpto cum eo,φ fit ex.u.b.in.u.n.
ex fecunda fecundi,& idem quadratum.u.b.æquale duobus quadratis.u.o.et.o.b.ex
penultima primi, ideo duo dicta producta æqualia erunt dictis duobus quadratis.o.
u.fcilicet et.o.b.fed quadratum
o u.æquatur ei,quod fit ex. a. u.
in.e.u.& ei quod fit.ex.o.e.in fe
ipfam ex.6.fecundi , quare duo
iã dicta producta æqualia erunt
duobus dictis quadratis,o.b. fci
licet.et.o. e. & ei quod fit ex. a.
u.in.u.e.fed quod fit ex b.u.in.u
n.æquale eft ei quod fit ex. a. u.
in.u.e.ex. 35. 3. relinquit ergo
vt id qd̃ fit ex.u.b.in.b.n. æqua-
le fit duob° quadratis.o.b. et. o.
e.quare & quadrato ipfius. b. e.
ex Pitagorica.

Si autem puctũ.u.fuiffet intra
circulum idem eueniret . Nam
quadrato.b.e.æquãtur duo qua
drata.o.b.et. o. e. fed vice qua-
drati.o.e.dicemus quadratũ. o.
u.cum eo quod fit ex.a.u.in.u.e.
ex.5.fecundi,id eft quadratum.
o.u.cũ eo quod fit ex. b.u. in. u.
n.ex.34.tertij,vnde quadratum
b.e.æquale erit quadrato. o. b.
& quadrato.o.u. ideft quadrato
b.u.ex Pitagorica fimul cũ pro-
ducto.b.u.in.u.n.ideft producto
n.b.in.b.u. quod æquale eft qua
drato.b.u.cum producto. b. u.in
u.n.ex.3.fecundi.

Circa tres paffiones commu-
nes poftea circulo hyperboli,&
defectioni notandum eft primã
patere ex.36: primi Pergei , fe-
cun-

cundam verò ex.37.et.38.eiuſdem,propterea quod in.37.probat mediante maiori
diametro ipſius hyperbolis & defectionis.In.38.autem mediante minori diametro
ordinatè ad maiorem.
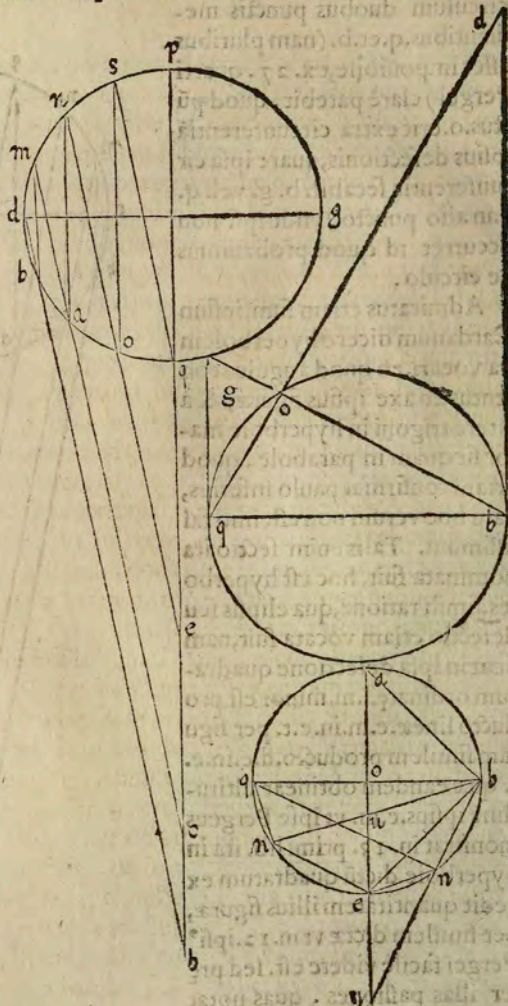
Tertia autem paſſio,non niſi circulo conuenit; pace ipſius Cardani dictum ſit.

Quapropter ſit circulus.q.o.b.cuius diameter ſit.q.b.contingentes vero ab extre
mitate diametri ſint.d.b.et.q.g.per punctum autem.o.quoduis,ipſius circūferentiæ,
tranſeant.b.o.g.et.q.o.d.tunc dico productum.q.o.in.q.d.vel.b.o.in.b.g.ęquale eſ-
ſe quadrato.q.b.quod ita probo.

Nam angulus.q.b.d.ſeu.b.q.g.rectus eſt ex.17.tertij Eucli.et.b.o. q. ſimiliter re-
ctus ex.30.ipſius lib.angulus verò.b.q.d.ſeu.q.b.g.communis eſt. quare.b.q. media
proportionalis erit inter dictas lineas.q.d.et. q.o.& inter.b.g.et.b.o.Vnde ſequetur
propoſitum ex.16.6.Eucli.

Sed ſi circa diametrum.q.b.mente fingamus aliquam elipſim, quæ tangat ipſum
circulum duobus punctis me-
diantibus.q.et.b. (nam pluribus
eſſet impoſſibile,ex 27 . quarti
Pergei ) clarè patebit , quod pū
ctus.o.erit extra circunferentiā
ipſius defectionis,quare ipſa cir
cunferentia ſecabit. b. g. vel. q.
d.in alio puncto , vnde ipſi non
occurret id quod probauimus
de circulo ,

Admiratus etiam ſum, ipſum
Cardanum dicere hyperbolem
ita vocari,eo quod angulus con
tentus ab axe ipſius figuræ, & à
latere trigoni in hyperbole ma-
ior ſit quam in parabole , quod
etiam confirmat paulo inferius,
nam hoc verum non eſt, imo fal
ſiſſimum. Talis enim ſectio ita
nominata fuit, hoc eſt hyperbo
les,ſimili ratione,qua elipſis ſeu
defectio etiam vocata fuit,nam
ſicut in ipſa defectione quadra-
tum ordinatę.l.m.minor eſt pro
ducto lineæ.e.m.in.e.t. per figu
ram ſimilem producto.d.e.in.e.
t. quæ eandem obtineat altitu-
diné ipſius.e.m. vt ipſe Pergeus
monſtrat in. 13. primi lib. ita in
hyperbole dictū quadratum ex
cedit quantitatem illius figuræ,
per ſimilem dictæ vt in.12.ipſi'
Pergei facilè videre eſt. ſed prę
ter illas paſſiones , quas notat
Car-
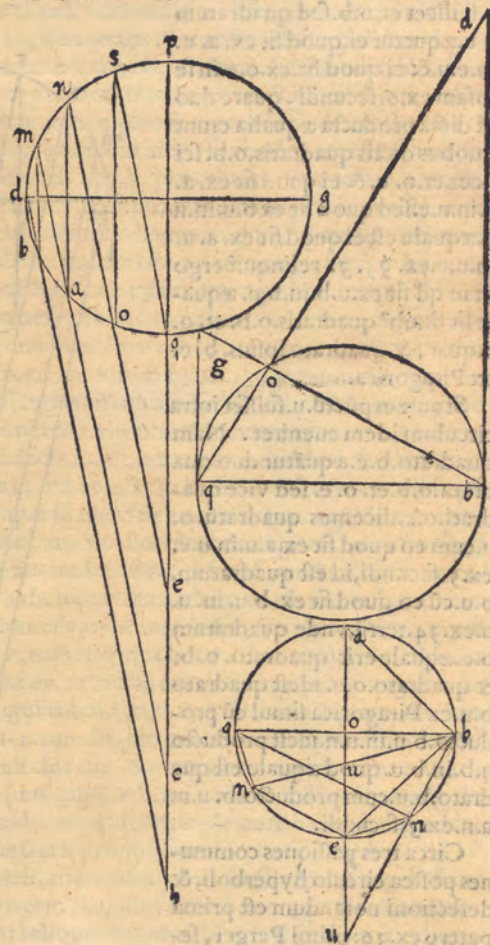
Cardanus in supradicto capite, multæ aliæ sunt, cum corollarium primæ tertij Eucli. sit passio propria ipsius circuli, & idem dico de propositione. 3. 4. 7. 8. 9. 11. 12. 13. 14. 15. 17. 18. 19. 20. 30. 31. ipsius tertij lib. nec non de. 8. 9. et. 10. tertijdecimi, & de prima. 3. 4. 5. 6. et. 7. quartidecimi eiusdem. Idem infero de ea quod scripsi Mario Nizzolio, Francisco Vimercato, Francisco Contareno, Angelo Agrimensori, & de alijs nonnullis à me excogitatis.

## DE FINE CORPORVM COELESTIVM,
### & eorum motu.

*Illustri viro, Philiberto Pingonio Sabaudo Cusiacensium Baroni.*

CVm antea meo nomine Sebastianus noster omnia ferè tibi retulisset, inter alia, quæ relinquebantur tibi dicēda, hoc vnum erat, quod si absque lumine superiori, in quem finem facta fuerint corpora cœlestia scire desideras, & humanam rationem sequi volueris, putandum tibi non erit ea solum effecta esse, vt tam vile corpus, vt est terra aquis irrigata, animalia, & plantas regant, cum ea corpora sint diuina, in numero incomprehensibilia, maximis magnitudinibus, & motibus velocissimis, prædita, id etiam minus putabunt hij, qui opinionem Aristarchi Samij, & Nicolai Copernici sequuntur, quorum ratione fieri non potest, vt credant. eius, quod ex vniuerso reliquum est, alium finem non habere, quam regimen huius centri epicycli Lunaris, vt illorum more loquar. Quā enim turpe esset si centra aliorum epicyclorum planetarum tali regimine priuarentur, id quod nullo modo cum ratione consentit, si tam vera est ea opinio, quemadmodum rationabiliorē eam existimāt. Neq; quidquam valet opinio Aristotelis, qui corpora cœlestia, ab ortu, & interitu libera esse sentit. dicens superioribus sęculis, à nostris antiquis nullam vnquam animaduersam fuisse alterationem in cœlo, cum non videat si quis esset in cœlo, neq; etiam obseruare posset alterationes quæ in terra, & circa terram fiunt, quæ in partibus, & non in toto spectantur: vnde etiam fieri potest, vt in cœlo sint particulares alterationes, quæ à nobis tamen, qui ab illis longè distamus, non comprehendantur, terra, mareq; (quamuis minimum respectu ipsius terræ) ratione totius ita se semper habuerūt quēadmodum sese habere corpora cœlestia videmus, sed alteratio, ratione tantum aliquarum minimarum partium quasi insensibilium, si cum toto comparentur sit. Quis enim scit, vt iam tibi dixi, quin, quemadmodum Luna circa terram voluitur, ipsaq; terra sit veluti centrum epicycli maioris eiusdem, vt Aristarchus Samius, & Nicolaus Copernicus censuerunt; sic etiam Saturnus, Iupiter, Mars, Venus, atq; Mercurius circa alia huiusmodi corpora, huic terræ similia, in orbem agantur, quasi specula, lumen Solis suo centro ex reflexione, deferentia (supposita dico vera illorum opinione) Nollé tamen tibi è mente excidere, vt aliàs te monui, ꝗ si communis opinio vera est, necessario fatendū sit corpus solare, dū in æquatore reperitur motu diurno quolibet horę minuto, magis quā decē & sepꝉ tē mille milliaria pagere, idest paulo minus quam. 18000. milliaria, Saturnum verò cum similiter est in æquatore, eodem tēporis spatio, quasi tercentamille milliaria Italica conficere, & sic per gradus alia corpora velociora alijs moueri; quæ quidem omnia, cū simplici gyro terræ circa suum

axem

axem(vt dicunt) tolluntur, quod sufficit ad recipiendum lumen, & influentias illo-
rum corporum. Et ita, veluti princeps corporum vniuersi, intra vnum an-
num circa eam vertitur. Ita etiam sufficeret, vt ipsa terra circa dictum diuinum
corpus solare, intersecando axem diurnum cum axe annuali(cum ab eo lumen, ca-
lorem, & influentiam suscipere debeat) circunuolueret. Rationes autem a Ptolo-
meo in contrarium adductæ apud ipsos, nullę sunt, quia quęlibet pars (vt inquiunt)
retinet naturam totius, præterquam ꝗ aer, & aqua, quæ ipsam terram circundāt, pla-
nè eundem naturalem impetum motus obtineant, qui tanto lentior est, quanto lon-
gius distat aer ab ipsa terra, secundum etiam talem opinionem, nulla necessitas fo-
ret, vt locus fixarum terminaretur aliquibus superficiebus, conuexa scilicet, & de-
uexa.

### De Luce, Lumine, & Colore, De obiectu oculi, De lumine Lunæ, & Rubedine nubium.

### A D E V N D E M.

Qvod proximè quærebas, an sit lux aliqua, quæ à corpore lucido non proue-
niat, mihi facilè ad considerandum videtur. hic enim oportet, vt nos ad id
quod perpetuò videmus referamus, existimo autem te velle dicere lumen, non lu-
cem, quia propriè lux, qualitas ea visibilis appellatur, quæ est in corpore lucido, à
quo quidem corpore lumen effunditur; lumen verò, ea qualitas esse dicitur, quæ ex
tra ipsum corpus reperitur, à luce, quæ in dicto corpore manet emanans. vnde pa-
tet, nullam lucem absque corpore subiecto esse posse, id quod cum fieri quîret, idē
de quolibet alio accidente dici posset, id est quod ex se, & absque aliquo subiecto
subsisteret.

Lumen deinde à luce proficisci patet, ꝗ penetrat diaphanum, neque aliquo mo-
do suum actum ostendit, nisi, aut per incidentiam, aut ratione opaci, ex reflexione,
cuius superficiei colorem induit. Atque hæc est causa, vt inter crepusculum matu-
tinum, aut vespertinum, nox etiam si sit serena, adeo obscura nobis appareat, quam-
uis totum vniuersum diaphanum, extra conum vmbræ, quæ ex terra prouenit sit vn
diq; radijs luminosis Solis collustratum; qui quidem radij, non nisi à suamet reflexio
ne à Luna, & ab alijs stellis ( vt corporibus opacis, quæ resistunt lumini, ne vlterius
penetrare possit, vnde retrò redit) comprehenduntur.

Ais etiam propria visus obiecta plura esse, nominans pro vno, colorem, & lucem
pro alio. Ego autem respondeo, obiectum oculi esse vnicum tantum, idest lumen.
Quod ad lucem spectat, iam tibi dixi, eam esse quandam qualitatem in corpore luci
do, & non extra ipsum positam, à quo quidem corpore, cum non exeat, oculi obie-
ctum esse nequit, sed lumen quidem ab ipsa luce productum. Color etiam, qui est in
corpore colorato, obiectum oculi esse non potest, cum dictum corpus non deserat,
sed lumen quidem ab eodem corpore reflexum, & huiusmodi corporis colore tin-
ctum: vnde tam lumen incidens, quam reflexum colore est semper imbutum.

Illud quidem coloratum est qualitate lucis corporis lucidi, a ut medij, per quod
transit, sed hoc colore corporis, à quo reflectitur.

Neque etiam te ignorare volo, lumina reflexa colorata, non reflecti à puris pro-
priisq́; superficiebus corporum coloratorum, eo qđ pauca corpora tam opaca repe-
riuntur, ut immediatè lumen à superficie propriè reflectāt, sed lumen penetrat ali-

<div align="right">quan-</div>

quantulum dicta corpora, & ita illorum colore afficitur, vbi verò non penetrat,non coloratur colore corporis illius.

Sed vt ad propofitum redeamus, dico lumen tantum effe vifus obiectum,quod fi colore eft imbutum, aut tale eft ratione colo ris lucis,quæ eum mittit,aut ratione medij per quem'tranfit, aut ratione corporis , vnde reflectitur . etfi fuperficies corporis vnde lumen reflectitur effet omnino priuata colore, fub afpectum non caderet , vt etiam cum huiufmodi fuperficies læuigata ,& polita eft fecundum continuitatè fuarum partium, videlicet, vt fpeculi radio tamen non profundante , & ideo perfectiffi morum quorundam fpeculorum fuperficies non cernuntur , fed lumen tantum reflexum', colore alicuius alterius fuperficiei , aut à luce , corporis lucidi , aut à medio per quem tranfit, confpicitur . Ego verò non affero colorem non effe qui d diuerfum à lumine, fed imagineris lumen effe veluti animam, aut fubftantiam & colo rem corporis formam accidentalem , cum nullum lumen à fenfu vifus percipi poffit, quod aliquo modo colore non fit'imbutum : & eundem refpectum quem fonus ad auditum,lumen ad oculum habet,quia vt fonus fecundum eam velocitatem,quæ à motione aeris,aut aquæ, ex collifione alioru corporum producitur ad euitandum vacuum,acutus,vel grauis fentitur, ita lumen originem ducens à corpore lucido per medium diaphanum aeris,aut aquæ,aut alterius huiufmodi corporis ad oculum tran fit colorem lucis,aut medij per quod tranfit,aut vnde reflectitur induit .

Quod verò Luna nullum ex fe habeat lumen , fufficiens inditium eft nos ipfam tantò magis obfcuram videre, quantò magis in cono vmbræ terræ immergitur , & fi eo tempore ipfam videmus rubeo colore affectam,hoc enim accidit, quia radij folares vndequaque refranguntur à vaporibus ipfam terram circundantibus, quæ quidem refractio fit verfus axem coni vmbræ terræ, & propterea vmbra dicti coni non eft æqualiter obfcura,feu tenebrofa,circa vero axè ipfius coni,magis quam circa eius circüferentiã, obfcura vr, & quia corpus lunare tale eft, vt facillimè recipiat qualecü que lumen , quod etiam manifeftè videtur dum ipfa Luna reperitur fecundum longitudinem inter Solem, & Venerem, quod pars Lunæ lumine Solis deftituta,à lumi ne Veneris aliquantulum illuftratur, quod ego fæpè vidi,& multis oftendi . Propterea dum ipfa Luna in cono vmbræ terræ reperitur adhuc videtur. Rubedo etiam illa nubium poft Solis occafum, vel ante ortum,aliunde non prouenit, nifi à qualitate vaporu, per quos folares radij tranfeunt, à quibus vaporibus, tali colore ipfi radij afficiuntur,eomet modo quo radius,cuiufuis corporis lucidit, trafiens per vitrum,feu aliud diaphanum coloratum.

258                 I O.  B A P T.  B E N E D.

# DE ICTV BOMBARDAE SECVNDVM
diuersas eleuationes. Et de quibusdam erroribus Nico-
lai Tartaleæ, circa idem.

*Illustri D. Iosepho Cambiano ex Ruffia Dominis, æquiti*
*strenuo, & tormentis bellicis Serenißimi Ducis*
*Sabaudiæ Præfecto.*

Xcogitaui quędam dum ocio frui licuit per absentiam Ducis Serenissimi,
quæ ad te scribere placuit, vt si probaueris in lucem quandoque profer-
re non dubitem, si despexeris, ocius supprimam, sunt autem huiusmodi.
Vnde fiat vt tormentum bellicum vehementi⁹ feriat ictu superius delato
quam orizontali, vt Tartalea scribit, quæsito secundo libr. primi quæsitorum, à ne-
mine adhuc ( quod sciam ) traditum est.

Rationes verò Tartaleæ nullius sunt momenti, quia si validæ essent, sequeretur
vt inclinata bombarda, adeo vt angulus sub orizonte factus æqualis esset ei, qui su
pra orizontem est, ictum bombardę in vtroque huiusmodi situ eundem esse futurũ.
& si aliqua differentia oriretur ratione grauitatis pilæ ab ipsa bombarda emissæ, hoc
fieret, vt scilicet velocior esset in motu inclinato quam in eleuato cum pondus, mo-
tui adeq non opponatur. Id quod non ita se habet, vera enim causa vnde fiat, vt bom
barda eleuata vehementius feriat, quàm ea quæ est minus alta, eadem est ferè, in ge-
nere, cum ea, qua aliquod corpus materia magis densa, sed simile & æquale alteri cor
pori materiæ minus densæ velocius mouetur ab vna eademque, aut æquali vi
compulsum. Est eadem etiam in specie ei, qua maiorem effectum producit
puluis, qui in locis subterraneis ponitur quum vasis optimè colligatis ferro in-
cluditur. Est etiam similis ei, qua longius impellitur pila, qua ludimus, ab ali-
quo instrumento ligneo, quando percutitur contra, quam cum secundum suum mo-
tum proijcitur. Id quod inde fit, quia virtus mouens maiori vi, & intensiori huius-
modi corpus percutit, quia corpus quod moueri debet, quanto magis resistit virtu-
ti mouenti ( certum tamen terminum præscribendo) in exiguo eo temporis spatio,
tanto maiorem virtutem colligit, quæ ipsum deinde tanto cum impetu mo-
uet, & tanto magis impellens concomitatur, vt maiorem effectum efficiat, quam si
ad mouendum sese facilè reddidisset. Atque hoc supradictis ictibus eleuatis acci-
dit, quia grauitas pilæ, ea est quæ resistens virtuti mouenti, dat ei commoditatem
colligendi dictam virtutem, multo magis quam esset ea, quæ ad depressiorem eleua
uationem eam impelleret. Et quia huiusmodi multiplicatio virtutis, nullam propor
tionem cum pondere pilæ gerit, volo inferre quod dum colligitur tanta virtus, col-
ligitur multo plus eo, quod ad impellendam dictam pilam sufficeret, ratione magnæ
velocitatis augumenti, quia quanto plus temporis ei conceditur ad commutandam
puluerem in ignem, tanto maior quantitas ignis progignitur, vnde fit, vt tanto ma-
iori loco indigeat, quamobrem tanto magis impellit, sed vt dixi, tanta cum veloci-
tate adaugeř, vt huiusmodi virtus longè superet resistētiā pōderis pilæ, & sic est cau
sa, vt effectus, quod experiētia innotescit producař. Sed ea ratio, qua sese idē author
in tertio quæsito ad aliquod impossibile, circa iter ipsius pilæ Legatum Hispanum
                                                                 redu-

reducere putat,nullo fundamento nititur,quia non eſt ſemper dicendum, quod quā
to velocior ſit quædam pila, tanto rectius moueatur, quia ei dici poſſet,vſque ad cer
tum quendam terminum velocitatis, per tantum ſpatij eam aptam eſſe, vt recta per
fectè moueatur, ſed ſi velocius iret, non tamen futurum, vt per idem ſpatium re-
ctius moueretur, ſed quod per longius ſpatium recta motum perageret, & ſic nihil
haberet quod replicaret, præter quam quod ipſe ſupponit id quod in 18. quæſito
negat, in quo ait pilam uicinam orificio,non adeo uelocem eſſe,quam cum aliquan-
tulum ab eodem eſt remota,ratione reſiſtentiæ ſui cyllindri aerei. Sed quod pila,
recta eat quanto altior,aut depreſſior bōbarda erit,iō fit, quia linea inclinationis na
turalis cum linea inclinationis uiolentæ angulum rectum non facit,unde quanto lon
gius diſtat à recto huiuſmodi angulᵒ,ſiue ſit acutus ſiue obtuſus,tanto minorem uim
habet, eodem planè ferè modo quem tertio capite mei tractatus de rebus mechani
cis deſcripſi. Quia in ictibus eleuatis, iter inclinationis violentæ ipſius pilæ verſus
terminum ad quem, incipiendo à loco ipſius pilæ cum itinere inclinationis natura-
lis,angulum obtuſum, & in ictibus inclinatis acutum conſtituit. Neque etiam hic
prætermittam notatu dignum errorem, quem Tartalea eodem loco commitcit, cū
putet indifferenter aliquod corpus impellere, aut percutere maiori cū impetu cum
eſt in itinere recto. Quia ſequeretur quod aliquod corpus graue perpendiculari-
ter ſurſum verſus proiectam, in qualibet parte ſui itineris, ſemper fortius percute-
ret, quam in qualibet parte itineris alterius cuiuſuis eleuationis obliquæ, quod quā
ſit falſum, tibi conſiderandum relinquo.

Eſt etiam falſa ea ratio, quam in quarto quæſito idem adducit, quia aer in motu
non tantum durat, quantum ipſe putat, imò huiuſmodi violenta agitatio,citò ceſſat
& citius etiam,quam ſi extra aliquam bombardam cum tanta violentia impulliſſet
ſaccum plumis plenum.

Ratio etiam quam in.18.quæſito de eo, quod pila pertranſeat illud corpus cyllin
dricum aereum adducit, eſt planè vana, quia ſtatim aer, qui prius in bōbarda erat
incluſus, extra ipſam erūpit,cedit,à pilaɋ diuiditur, vt ſi nunquam eam figuram in-
duiſſet,neque aer ambiens ei reſiſtit. Sed quod velocior ſit in certa quadam diſtan
tia, quam in principio erat,ſi hoc verū eſſet,ab alia cauſa dependeret,quæ partim ſi
milis eſſet ei,quæ efficit,vt corpora in motibus naturalibus,cum longius diſtant à ter
mino vnde naturaliter ſeſe mouerunt, ſint velociora, quia per aliquod ſpatium hu-
iuſmodi corpus moueretur quemadmodum motu naturali cietur.

Ratio autem eius quare pila,aut globus bombardæ ſibilet ab eodem in ſeptimo
quæſito nil valet,quia hoc fit cum pila aliquam paruam concauitatem habet.

In.27.autem quæſito ait, quod retrotrahendo ſignum,ictus altius tenderet,quod
poteſt etiam eſſe falſum, cum hoc non ſit neceſſarium, quia pila dum deſcendit,for-
taſſe tangeret ſcopum.

Kk 2 De

## De erroribus Ioannis Stadij.

### AD EVNDEM.

Figuram quam ponit Ioannes Stadius pag.147.in lib.suarum tabularum Prutenicarum, à Nicolao Copernico sumpsit pag.64.à tergo in libr. reuolutionum cœlestium, sed ipse Stadius eam non intellexit,omitto, quod mutauerit characteres ipsius figurę, vt illa sua videatur,quod nihil refert,alterat etiam demonstrationẽ, sed ipsum putare.i.K.perpendicularem à centro circuli semper dependere, est intol lerabilis error; nec vnquam verificatur hoc, nisi quando punctum. K. intersectionis diametrorum parallelorum, forte reperitur in axe mundi. Reliqua verò suæ demon strationis,si non intelligis, minimè miror, eo quod ipsemet Stadius seipsum confun dit. Veram autem demonstratio nem huiusmodi figuræ in dicto libr.Copernici cla rè videbis. Quod verò diuersè cogitaui nunc accipito.

Cum nobis cognita sit maxima ecclipticæ declinatio,vt puta. a. c. si latitudo etiã stellæ nobis data fuerit, vt puta.c.e.cognitus nobis erit totalis arcus.a.e.& eius sinus. e.m.& quia notus etiam nobis est sinus arcus.a.c.hoc est.c.n.& corda.e.f.medio eius arcus.e.p.f.minoris media circunferentia,per duplum latitudinis datæ,vnde.e.l.eius dimidium nobis cognitum erit,vel vt sinus arcus.e.p.cognitus etiam nobis est sinus. q.g.declinationis.a.g.datæ,cui æqualis est. m.t.ex.34.primi Euclid.vnde.e.t. nobis cognita remanet,cum verò duo trianguli.i.c.n.et.t.e.K.æquiãguli sint, propter duas parallelas.e.m.et.n.c.ex.28.primi, & propter duas.a.b.et.g.h.& propter duas. c. d. et.e.f.eo quod ex communi scientia anguli.c.et.e.sunt æquales,cum ex.29.dicti lib. vnusquisq; æqualis sit angulo.m.ꙍ.i.ita etiam infero de angulis.e.K.t.et.c.i.n. quorũ vnusquisque æqualis est angulo.ꙍ.x.t.& sic de alijs dico, eo quod vnusquisque eorũ æqualis est angulo.m.vnde cum cognitum nobis sit latus.n.c.et.c.i.et.t.e notũ etiam nobis erit.e.K.ex.19.septimi, eo q̃ ex.4.sexti sunt inuicem proportionalia, detrahendo postea.e.K. ab. e.l.cognito, vel ècontra, hoc ab illo,nobis innotescet.K.l.sinus longitudinis stellæ.

Valde etiam miror id,quod dictus Stadius pag.9.illius libr. scribit, hoc est, Solem maiorem esse Luna, solum. 1644. vicibus, propterea q̃ cum affirmet Solem maiorem esse terra ( vt etiam in Almagesto videre est ) 166.vicibus cum tribus quartis,terram vero maiorem Luna.39. vicibus cum quarta parte, tunc Solem oporteret maiorem esse Luna.6545.vicibus,& non.1644.

De
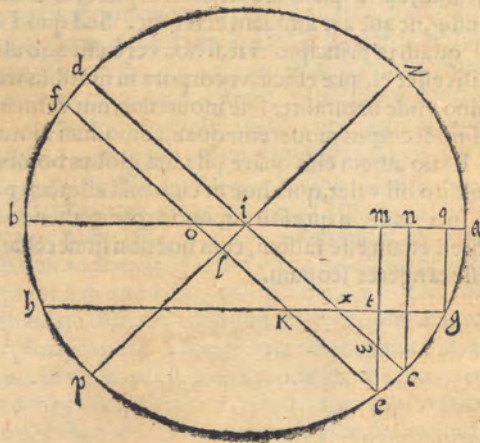
### De cognitione latitudinum ſtellarum.

## AD EVNDEM.

AD cognoſcendam latitudinem ſtellæ , eiusq; declinationem, Monteregius in 10.propoſitione.8.li. Almageſti methodū ſatis docuit,ſed ſi alia aliqua metho do hoc idem cognoſcere voluerimus,oportebit nos prius altitudinem poli cogno ſcere, deinde altitudinem meridianam ipſius ſtellæ,nec non horam , quādo ipſa ſtel la in meridiano ſupra terram reperitur,qua hora mediante,illicò cognoſcemus pun ctum eccliptiçæ à meridiano interſecto , eo tempore , quo ſtella cœlum mediat ſu pra terram. Et quia ex cognita altitudine poli, illico cognoſcitur altitudo æqua toris, cuius altitudinis differentia ab altitudine ſtellæ eſt declinatio ipſius ſtellæ, ha bebimus ideo eius declinationem cognitam; qua mediante ad cognoſcendū etiam latitudinem ita faciemus.

Sit exempli gratia.p.o.u.meridianus.u.a.verò æquator.e. a. autem eccliptica , & o.centrum aſtri.u.o.verò eius declinatio ab æquatore , et.e.a.arcus ecclipticæ inter æquatorem,& meridianum,hoc eſt minor quarta,et.a. u.aſcenſio recta ipſius arcus, et.u.e.ſit declinatio puncti.e.eccliptiçæ ab æquatore,reſiduū vero declinationis ſtel lç ſit.o .e .quæ oīa nobis cognita erunt, ſitq;.t.polus ecclipticus,à quo per.o.vſque ad ecclipticam tranſeat quarta.t.i.in qua querendus erit arcus.o.i.hoc modo.

Primum arcus.o.u:e.u:e.o:a.e:et.a.u.nobis cogniti ſunt,cum angulo.a.declinatio nis ecliptiçæ,& cum angulo.u.recto,vnde ex.4.primi Copernici,cognoſcemus angu lum.a.e.u.collateralem,& eius.o.e.i.quare in triangulo.o.e.i. cognoſcemus angulū e.et deinde.i. vt rectū,& latus.o.e.ergò ex eadē. 4.cognoſcemus arcū.o.i.quæſitum, & ſimiliter arcum.e.i.qui coniunctus vel dēptus ab.a.e.tribuet nobis longitudinem ſtellç,ſed quia huiuſmodi operatio in paruis triangulis valde fallit . Ideo tibi ſua deo alia methodo,hoc facere,hoc eſt inuenire angulum.o.trianguli.t.p.o.cuius duo latera.t.p.et.p.o.cognita nobis ſunt,cum angulo.p.Nam.o.p.eſt complementum de clinationis ſtellæ,et.p.t.eſt arcus coluri ſolſtitiorum inter duos polos , & angulus. p. reſiduum ex recto.t.p.a.duorum colurum dempto angulo.a,p.u.cognito aſcenſionis recte,vnde angulus.u.o.s.vt contrapoſitus cognitus remanet.angulus verò. u. rectus

eſt, & arcus.o.u.cognitus , quare cognitus nobis erit arcus.u.s.& angulus.u. s. o. vnde arcus.a.s.nobis cognitus remanebit cū an gulo.a.s.i.reſiduo ex duobus rectis.Et quia etiam angulus.s.a.i.cognitus eſt,cum ſit an gulus maximç declinationis Zodiaci ab æquatore. Ideo in triangulo. a. s. i. cuius duo anguli.a.·et.s.cum latere.a.s.dantur,fa cilè inueniemus arcum.s.i. cū arcu. a. i. ſed a.i.erit longitudinis ſtellæ dempto poſtea. s.i.ex.s.o.iam inuento habebimus arcum.i. o.latitudinis ipſius ſtellæ.

Hæc autem tibi ſcribo non vt ipſis vta ris,ſed potius vt tibi morem gerā, cum bre uiſſima methodus ſit illa,quā Monteregius ſcripſit ī.10.ppoſitione.8.li.in Almageſt.

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## IO. BAPT. BENED.
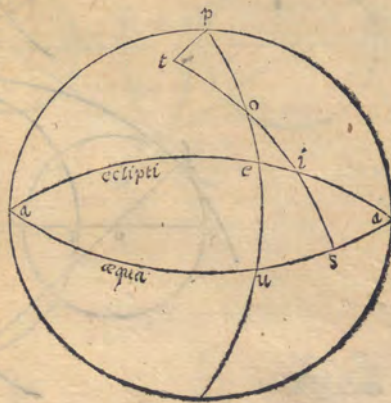
### *Qualiter circulus defignari poßit alios duos circulos propofitos includens.*

## CLARISS. PETRO PIZZAMANO.

SVperioribus diebus per tuas literas à me quæfiuifti, vt modum tibi fcribere vel-
lem, quo circulus defignari poffit circunfcribens alios duos propofitos circulos.
Qua in re vt tibi fatisfaciam quod maximè cupio ita rem accipe.

Propofiti circuli fint, aut inter fe contigui, aut interfecantes vel feparati. Efto pri-
mũ contiguos effe, qui fint.d.b.et.f.q.quorũ.d.b.maior fit et.f.q. minor, eorũ vero
centra fint.a et.o.punctũ autem cótingentię fit.i. Nũc ptrahaf.b.a.o.q.per cétra eo
rum ab vna circunferehtia ad aliam, quę quidem linea tranfibit per punctum. i. ex
11.tertij Eucli.deinde à diametro maiori abfcindatur. i. e. ad æqualitatem minoris
femidiametri, quo facto fumatur diftantia inter.e.et.b.circino mediante factoq; cen
tro.o.fcindatur, alio circini pede, circunferentia maioris circuli in puncto. u. à quo fi
mente concipiemus duas lineas.u.a.d.et.u.o.f.tranfeuntes per eorum centra.a. et. o.
vfque ad circunferentias in punctis.d.et.f. ipfę erũt inuicem equales, eo quod.e.i.fũ
pta fuit æqualis.o.f.et.o.u.æqualis.e.b. quare. u. f.æqualis erit.b.i.fed.u.d.etiã æqua
lis.b.i.ergo.u.d.æqualis erit. u.f.& circulus, cuius u.d. vel.u.f.erit femidiamerer, con
tiguus erit ipfis propofitis circulis ex conuerfo.11.iam dictæ. Idem dico pro circu-
lis fe inuicem fecantibus. Sed
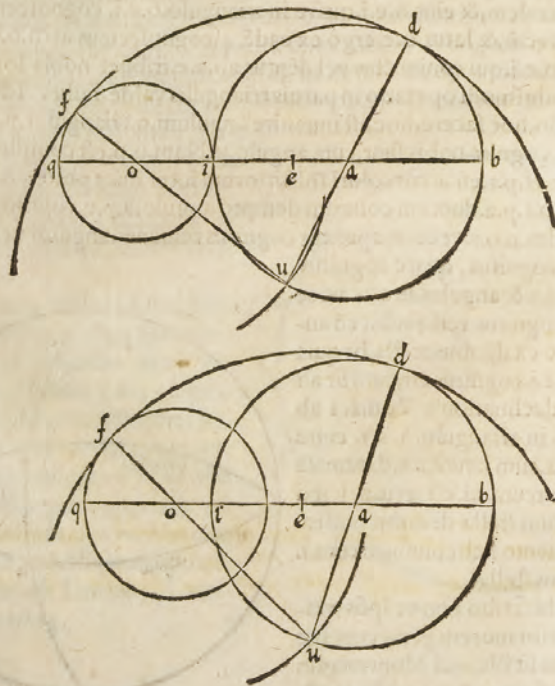
Sed si circuli propositi seiuncti fuerint, sumatur. b. i. diameter maioris, qui fiat se-
midiameter vnius circuli circa centrum. o. & hic circulus vocetur. h. x. coniunga-
tur deinde semidiameter. o. i. minoris circuli cum semidiametro. a. i. circuli maio-
ris, & ex huiusmodi composita linea, fiat vnus semidiameter. a. x. circuli. x. n. concen
trici cum maiori, & à puncto. x. intersectionis horum circulorum (posito quod se in-
uicem intersecent) ducantur per eorum centra. x. a. et. x. o. vsque ad ipsorum circun-
ferentias in punctis. d. et. f. duę
lineæ, vnde habebimus. x. d.
æqualem. x. f. eo quod tam in
x. d. quam in. x. f. reperiuntur
diametri, & semidiametri am-
borum circulorum, facto deni
que centro. x. vnius circuli, cu
ius semidiameter equalis sit
vni earum. x. d. vel. x. f. solu-
tum erit problema, dicta ra-
tione.
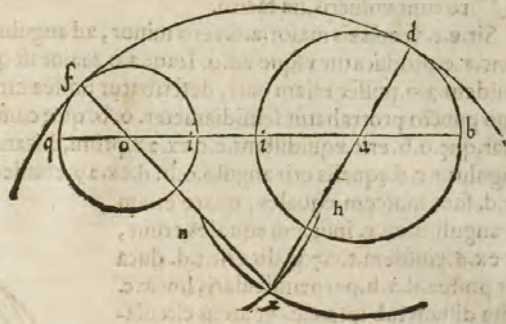
Si verò distantia duorum
propositorum circulorum tanta fuerit, quod secundi circuli nequeant se inuicem
tangere, vel secare, tunc alia via incedendum erit, quę talis est & generalis. Diuida-
tur tota. q. b. per æqualia in puncto. z. circa quod signétur duo puncta ab ipso equidi
stantia. K. et. p. distantia vero. a. K. facta sit semidiameter esse vnius circuli. K. x. circa
centrum. a. distantia autem. o. p. semidiameter alterius circuli. p. x. circa cen-
trum. o. qui quidem circuli se inuicem secent in puncto. x. à quo cum ductę fue-
rint. x. a. d. et. x. o. f. per centra dictorum circulorum, ipse erunt inuicé equales, eo qd̄
cum. b. K. æqualis sit. q. p. igitur. x. d. et. q. p. erunt inuicem equales, sed. f. x. æqualis est
q. p. quare. x. f. æqualis erit. x. d. tunc si. x. centrum fuerit vnius circuli, cuius semidia-
mer sit vna dictarum, problema solutum erit.

Talis etiam solutio commo-
da erit ad inueniendum dictum
circulum cuiusuis magnitudinis,
dato tamen ꝙ eius diameter, ma
ior sit. b. z. cum in nostra potesta
te sit accipere puncta. K. et. p. pro
xima vel remota ab ipso. z. ad li-
bitum. Vnde absque vlla diuisio
ne ipsius. q. b. per medium, satis
erit signare puncta. K. et. p. dua-
bus distantijs mediantibus. b. K.
et. q. p. inuicem æqualibus, &
etiam propositis.
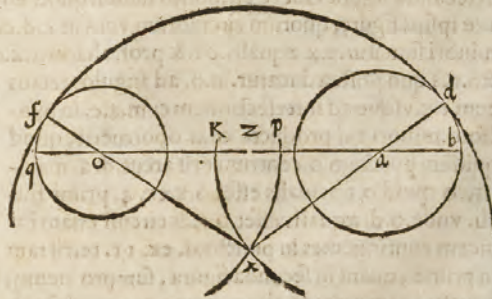
*In omnibus casibus, diuisa ꝫq bifariam, quod quidem punctū fiat centrū, circulus
descriptus ꝙ ꝫq transiens cosꝵ datos cir: in punctis ꝫq continget.*

Figuram

*Figuram superficialem ellipsi similem , ex datis axibus circino mediante delineari posse.*

## AD EVNDEM.

Figuram superficialem ellipsi similem, ex datis axibus, circino mediante delinea
re cum volueris, ita facito.

Sit.e.c. semiaxis maior.a.e. verò minor, ad angulum rectum inuicem coniuncti,
tunc.a.e. producatur vsque ad.o. Itaq;.a.o. maior sit quam distantia inter.o.et.c.quę
quidem.a.o. posset etiam dari, describatur postea circulus.a.d.b. circa centrum.o. à
quo puncto protrahatur semidiameter. o. b. quæ cum.a. o. angulum rectum consti-
tuat, quę.o.b. erit æquidistans.e.c.ex.28.primi, ducatur postea.b.c.d.et.o.t. d. vnde
angulus.t.c.d.ęqualis erit angulo.o.b. d.ex.29.eiusdem. ex quinta autem anguli. b.
et.d. sunt inuicem æquales, quare etiam
& anguli.d. et. c. inuicem ęquales erunt,
& ex.6.eiusdem.t.c.ęqualis erit.t.d. duca
tur postea.d.x.h. perpendicularis lineæ.c.
e. ita distans sub ipsa.c.e. vt arcus circula-
ris circa.t. delineatus ex semidiametro. t.
d. aptus sit eam secare, sumpto postea. r.
tam distante ab.e. vt. t. reperitur ab ipso
e.et.z.ab.e.vt.o.ab eodem, ducendo po-
stea duos alios arcus magnitudinis priorū
circa centra.r. et. z. habebimus proposi-
tum.
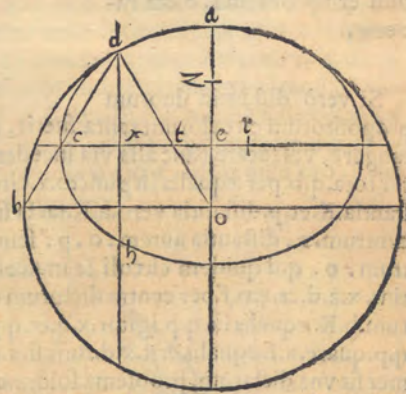
Sed cum quis voluerit prius arcus mi-
norum circulorum delineare circa maio-
rem axem, fiant cuiusuis magnitudinis, vt



in secunda figura videre est, posito tamen quod eorum diameter, minor sit minore
axe ipsius figurę, quorum circulorum vnus sit.c.d. circa.t. eius centrum, deinde in axe
minori sumatur.a.x. æqualis.c.t.& protrahatur.t.x.quę per ęqualia diuidatur in pun-
cto.n. à quo postea ducatur. n.o. ad angulos rectos
cum.t.x. vsque ad intersectionem cum.a.e. in pun-
cto.o. minori axi producta cum oportuerit, quod
quidem punctum.o. centrum erit arcus.d. a. maio-
ris, eo quod.o.t. æqualis esset.o.x.ex. 4. primi Eu-
cli. vnde.o.d. æqualis esset.o.a.& circuli etiam in-
uicem contingentes in puncto.d. ex. 11. tertij tam
in prima, quam in secunda figura, sumpto deniq;
puncto.s. tam remoto ab. e. quam. o. reperitur ab
eodem, ipsum, centrum erit alterius arcus opposi-
ti, possemus etiam absq; diuisione ipsius,t.x. consti
tuere angulum,x.t.o. æqualé angulo.t.x.o.vnde ex
6.primi haberemus.o.t. æqualem.o.x.
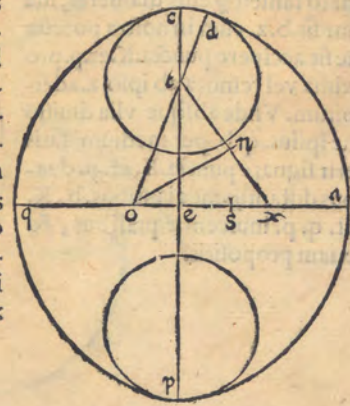


De

## De inuentione axis propositæ portionis datæ sphæræ.

## AD EVNDEM.

VT axem propositæ alicuius datæ sphæræ inuenire possis ita tibi operandum est vt gratia exempli . Proposita nobis est sphæra.c.i.e.t.diametri cognitæ. proposita etiam est nobis eius portio.n.e.u.axis.e.a.cognitæ minoris semidiametro, data etiam nobis est proportio alterius portionis minoris hemisphærio. i. e. t. ad portionem.n.e.u.quæritur nunc quantus sit axis.e.x.secundæ portionis hoc est desideramus cognoscere proportionem.e.x.ad.e.a.vel ad diametrum ipsius sphæræ.

Cuius gratia reperiatur primò proportio circūferentiæ maioris circuli ipsius sphę ræ ad c ius diametrum,quæ ferè est vt.2 2.ad.7.ex Archimede.

Quo facto,inueniatur quantitas superficialis huiusmodi maioris circuli,quæ semper æqualis est producto quod sit ex semidiametro in dimidium circunferentiæ ipsius circuli,ex eodem Archimede. Et sic cognoscemus quartam partem superficiei sphæricæ sphæræ proposite ex.3 1.primi lib.de sphæra,& cyllindro Archimedis.

Deinde sumatur tertia pars producti, quod sit ex semidiametro in superficiem maioris circuli, & habebimus conum,cuius basis erit circulus maior , altitudo verò semidiameter propositæ sphæræ ex.9. duodecimi Eucli.

Quadruplum postea huiusmodi coni,erit quantitas soliditatis,seu corporeitas totius sphærę ex.3 2.dicti lib.Archimedis.

Imaginemur postea ī sphærica portione.n.e.u.lineā.e.u.à sūmitate ad extremitatē basis,cuius.e.u.quantitatem cognoscemus , hoc modo scilicet,sumendo radicè quadratam producti.c.e.in.e.a. eo quod quadratum.e.u.æquale est quadrato a.u. & quadrato. a. e. ex penultima primi Eucli. hc, est producto quod fit ex.c.a.in.a.e.ex. 34. tertij eiusdē, & quadrato. a. e. hoc est producto , quod fit ex.c.e.in.e.a. ex. 3. secundi eiusdem.

Inuenta postea.e.u.ponamus eam vnius circuli semidiametrum esse,cuius superficialis quantitas etiam inue niatur , vt supra dictum est, quæ qui dē æqualis erit superficiei portionis n.e.u.ex.40.primi li.Archimedis de sphæra, & cyllindro.

Hæc autem quantitas vltimo inuē ta multiplicetur cum tertia parte semidiametri datæ sphæræ,& habebimus soliditatem vnius coni æqualis aggregato soliditatis portionis. n. e. u.simul sumptę,cū soliditate vnius co ni,cuius axis sit.a.o.residuū semidiametri nostræ sphæræ dempta.a.e.ba

LI sis
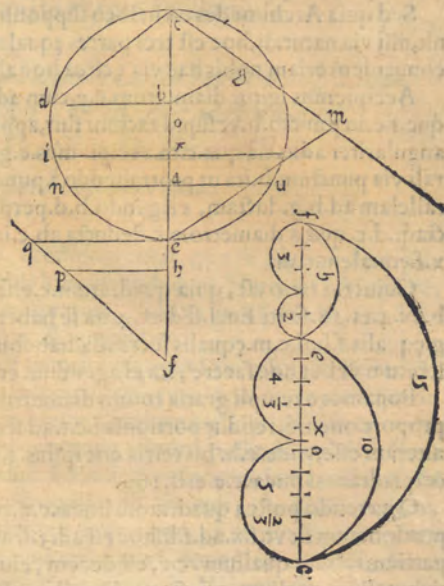
fis verô eadem quæ eſt portionis,cuius diameter eſt.n.u.ex.9.12.Eucli. & ex.42.id-
eſt vltima primi Archimedis de ſphæra, & cyllindro.

Nunc autem ex hoc aggregato iam vltimo dicto detrahatur conus,cuius.o. a. eſt
axis et.n.u.diameter baſis, qui quidem conus nobis cognitus eſt,cum.a. n. ſemidia-
meter eius baſis,nobis cognita ſit ex.34.3.Eucli.& ſic quantitas eius baſis, & ita ter-
tia pars.a.o.eius axis, quę multiplicata cum dicta baſi,cuius.n.u.eſt diameter, produ
cit dictum conum, qui quidem conus , vt diximus, demptus cum fuerit ex dicto ag-
gregato,relinquet nobis ſoliditatem portionis.n.e. u. vnde cognoſcemus proportio
nem iſtius portionis ad totam ſphæram propoſitam.

Sed cum nobis propoſita ſit proportio portionis.n.e.u.ad portionem.i.e.t.cogno
ſcemus etiam ſoliditatem huius ſecundę portionis.i.e.t.& ſimiliter proportionê hu-
ius ad totam ſphæram,& ad reſiduũ etiã ipſius ſphęrę hoc eſt portioni.i.c.t.

Protrahatur nunc diameter.c.e.à parte.e. vſq; quo.e.f.æqualis ſit.e.o.ſemidiame
tro ſphęrę, quæ quidem.f.e.diuidatur in puncto.h.ita vt proportio.f.h.ad.h.e.æqua-
lis ſit proportioni portionis.i.c.t.ad portionem.i.e.t.quod quidem hoc modo efficie
tur.applicabimus lineam.f.q.(indeterminatam)cum.f.e.ad quemuis angulum in pũ-
cto.f.in qua accipiemus duas lineas.f.p.et p.q.inuicem ita relatas,vt ſe habent in pro
portione duæ iam dictæ portiones,hoc eſt, vt.i.c.t.portio ad portionem.i.e.t. ducen
do poſtea.q.e.et.p.h.parallelam ad ipſam.q.e.diuiſam habebimus.f.e.in eadem pro
portione vt dictum eſt ex.2.ſexti,&.11 quinti Euclidis, vnde.c.e:e.f.et.f.h.nobis co
gnitę erunt.

Oportebit nos nunc cognoſcere quantitatem.c.x.hoc modo,videlicet, quęramus
quadratum,cuius.c.x.eius ſit radix,cui quadratum linęę.c.e.cognitum,ita ſit propor-
tionatum,vt eſt linea.x.f.ad lineam.f.h.quę nobis cognita eſt,quod rectè factum erit
ex eo,quod ſcripſit Archimedes in.4.ſecundi de ſphęra, & cyllindro.

Sed quia Archimedes eo in loco ſupponit id, quod nec ipſe, nec alius adhuc inue
nit,niſi via naturali,hoc eſt tres partes ęquales ex proportione data effici,non erit in
conueniens etiam nobis hac via , circa hoc aliquid dicere.

Accipiemus igitur diametrum.c.e.cum addita.e.f.eius ſemidiametro, diuidemus
que.f e.in puncto.h.vt ſupra factum fuit,applicabimus poſtea.c.m.indeterminatam
angulariter ad.c.e.à qua.c.m.accipiemus.c.g.æqualem. f. h. quęremus deinde natu-
rali via punctum.b.ita ut protrahendo à puncto.e.(altero extremo diametri)e.m.pa
rallelam ad.b.g.ductam, erigendo.b.d.perpendicularem ad.c.e.in puncto.b.protra
ctaq;.d.c. quæ à diametro.e.c.deducta ab.c.incohando vſque ad. x. relinquat nobis.
x.f.ęqualem.c.m:

Cuius rei ratio eſt , quia quadratum.c.e.ſe habet ad quadratum.c.d.vt.c. e. ad. c.
b.ex.4.et.18.ſexti Eucl.ſed ex.4.ita ſe habet.m.c.ad.c.g.vt.e.c.ad. b.c. & cum ſit.c.
g.ęqualis.f.h.ſi.c.m.ęqualis fuerit.f.x.habebimus propoſitum. Quod ſi quis per di-
ſcretum vellet hoc facere , ita ei agendum erit .

Ponamus exempli gratia totum diametrum.c.e.propoſitæ ſphæræ eſſe ut decem,
proportionemq; reſiduę portionis.i.c.t.ad ſecundam.i.e.t.hoc eſt.f.h.ad.h.e.ſeſqui-
alteram eſſe,vnde.e.h. bis tertia erit ipſius. f. h. totaq; linea.c.f.erit.15.et.f.h. erit.3.
& quadratum lineæ.c.e.erit.100.

Quærendo poſtea quadratum lineæ.c.x.cui quadratum.c.e.hoc eſt.100. ita pro-
portionatum ſit vt.f.x.ad.f.h.hoc eſt ad.3.ſi autem cogitauerimus. c. x. eſſe nouem
partium talium qualium . c.e. eſt decem , eius quadratum erit.81.et.x.f.erit. 6. par-
tium talium qualium.c.f.eſt. 15.dicendo poſtea ſi.100.dat.81.( ex regula de tribus)

<div align="right">x.f.</div>

x.f.hoc eſt.6.dabit.4. integra cum.
86. centeſimis, ſed nos vellemus no-
bis prouenire tria, eo q̃ ita eſt. f. h.
qua propter deſcendere nos oporte-
bit à nouem ad.8.& ab.8.ad.7. & à.
7.ad.6.tunc inueniemus.c.x.oporte-
re eſſe circiter quinque cum duabus
tertijs, operãdo poſtea ex regula de
tribus,ſi dixerimus quando.100.no-
bis dat.32.cum nona parte integri,
tunc nouem cum tertia parte integri
dabit.2.cũ.296.de. 300. hoc eſt.2.
cum circa.49. quinquageſimis, quæ
quidem quantitas, cum propinquiſſi
ma ſit lineæ.f.h.trium integrorum di
cemus.c.x. eſſe quinque integrorum
cum duabus tertijs partibus vnius in
tegri,et.e.x.reſiduum, hoc eſt axem
quæſitum portionis.i.e.t.eſſe circa.4
integra cum tertia parte vnius inte-
gri.
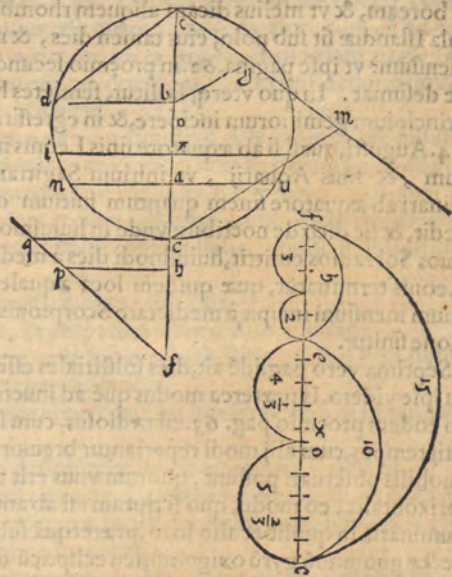
# DE ERRORIBVS THOMAE PORCACHII
## & Benedicti Bordonij in eorum inſularijs.

*Excellentiſſimo D.Io.Baptiſtæ Famello Ciui Decurioniq̃, Tau-*
*rinenſi Philoſopho, Medico , & in Accademia eius*
*Ciuitatis Medicinæ Practicæ Ordinario, Pri-*
*marioq̃, profeſſori celeberrimo.*

Ij perdant tuas adeo moleſtas, & aſſiduas curas, quæ te nimis à ſuauiori-
bus ſtudijs diſtrahunt, & à nobis longius abducunt. Nam, ut tibi quieté,
ita mihi ingentem adimunt voluptatem . Sed ne in aliquo erga te defi-
cere videar, quæ tibi olim promiſi,nunc mitto.

Negari quidem non poteſt,quin fuerit laborioſum opus Porcachij, & Benedicti
Bordonij,hoc eſt inſularium,qui rectè etiam feciſſent, cum loqui eos oportebat de
terminis ſphæræ ratione ſitus locorum,ſi ſeipſos alicuius excellentis Coſmographi
conſilio ſubmiſiſſent. Conſidera quæſo, quomodo admitti poſſit, id quod ait Por-
cachius initio ſui operis, ideſt Iſlandiam ſub Polo arctico iacere, inter auſtrum, &
boream:omittamus etiam quod idem in Proœmio lib. ſecundi, vbi ait Biarmiam,
(& non Iſlandiam ) eſſe ſub dicto polo arctico: in eodemq̃; principio repetit ipſam
Iſlandiam inter auſtrum, & boream per centum leucas Germanicas extendi, dein-
de verſus occidentem,ea duo ſtupenda miracula conſpici. Vide quæſo , quomodo
incolę ſub aliquo ex polis,habere poſſint occidentem,orientem,magiſtrum,auſtrũ,

Ll 2 &

& boream, & vt melius dicam aliquem rhombum. Sed quomodo fieri poteſt, vt inſula Iſlandiæ ſit ſub polo, eius tamen dies, & nox maior non ſit longior ſpatio triũ menſium? vt ipſe pagina.62.in proęmio ſecundi lib.affirmat, quamuis hoc à Bordone deſumat. In quo vterq; fallitur, ſentiétes huiuſmodi diem ab ingreſſu Solis, in principium geminorum incipere,& in egreſſu à Leone terminari,ideſt à.12.Maij ad 14.Auguſti,quaſi ſi ab æquatore finis Leonis ita declinaret, vt principium geminorum , & finis Aquarij , vt initium Sagittarij, nam ratio poſtulat, tantum declinari ab æquatore finem quantum initium diei, vbi maximus dies.24. horas excedit, & ſic dico de noctibus: vnde in huiuſmodi regione, vbi per tres menſes continuos Sol radios emittit,huiuſmodi dies à medietate Tauri incipit, & in medietate Leonis terminatur, quæ quidem loca æqualem declinationem habent, & ſic nox trium menſium incipit à medietate Scorpionis, & in medietate Aquarij, eadem ratione finitur.

Septima verò pag.idé ait,dies ſolſtitiales eſſe circa.24.Iunij.Qɗ,an tũc eſſet verũ, tu ipſe videto.Is præterea modus qué ad inueniédũ orienté,& occidétem præſcribit in eodem proęmio pag.63.eſt tædioſus, cum ſemper expectare nos cogat æquinoctij tempus,cum alij modi reperiantur breuiores, qui in qualibet reuolutione primi mobilis obſeruari poſſunt , quorum vnus erit mediante inuentione lineæ meridiane orizontalis, eo modo, quo ſcriptum eſt ab antiquis mediante Sole , aut Luna, quæ luminaria in quolibet alio loco,præterquã ſub polo efficiunt,vt extremitas vmbræ rectæ gnomonũ gyrũ oxigoniũ,ſeu eclipticũ ducat, ideſt in ijs locis,quorũ zenit eſt inter polum,& circulum arcticum,quemadmodum facit , vt alijs, exiſtentibus ipſis luminaribus extra æquatorem, & circulos arcticos gyrum hyperbolicum reddant. Sed id quod eidem Porcachio impoſſibile eſſe apud eos, qui habitant ſub polo videtur, ideſt vt multis rationibus, vt ipſe dicit,fieri non poſſit, ut fiat immediata quę dam, & ſubita mutatio à continuo die ad continuam noctem abſque eo quod ijs, ſaltem ſemel conceſſa ſint dies, & nöx terminata duodecim horarum, eſt magis ad mirandum impoſſibile, quod imaginari poſſimus, nam neceſſarium eſſet,ut orizonhabitatorum ſub polo ſecaret æquatorem contra id, quod ſuperius admiſerat, ideſt orizóté Biarmiæ, eſſe eũdé cũ circulo æquinoctiali. Vide etiam quid is ab antiquis colligat, loquens de iis, quæ in inſula Taprobana ad finem pag.186.admirabilia ſunt, ſcribens eiuſdem inſulę habitatoribus, Lunam ſuper terram non apparere ab octauo uſque ad decimumſextum diem : pręter quam , quod etiam ſcribit , in eadem inſula, tramuntanam non uideri, quod falſum eſt , quia hæc à polo arctico circiter quatuor gradibus diſtat noſtris temporibus . unde ab ijs qui ſunt ſub æquatore, cum ea ſupra orizontem eſt, conſpici poteſt, cúm ijſdem ſingulis diebus oriatur, & occidat. Idem etiam pro re admirabili ſcribit, uideri Canopum, qui à polo antarctico plus quam quadraginta gradibus diſtat.

## De erroribus Lucilli Philalthæi.

### AD EVNDEM.

Q Vod Lucillus Philalthęus tam eximius Mathematicus ſit , ut ipſum Antonius Berga facit , ego quidem non uideo. In ſuis enim commentariis de Cœlo, dicit primum,Pyramidem,quę inter corpora regularia primum locum tenet,

<div align="right">ſex</div>

sex basibus constare, pag. 15. 583. 632. et. 647. Omitto errorem ab eodem commissum in fine pag. 39. ubi oleum grauius esse quam aquam fatetur, cum id ad res mathematicas non spectet : Omitto etiam quod idem neget astrologiam pag. 74. 79. & quod etiam dicat pag. 89. Deum esse ad orientem, non considerans aliquibus populis nostrum orientem esse occidentem.

Quod idem ait pag. 241. Astrologiam esse antiquiorem Astronomia est falsissimum, quia iudiciaria semper præsupponit cognitionem situs stellarum, quæ ab Astronomia petitur. Mouebit tibi risum quod ait pag. 307. his verbis.

„ Verum propriè media dicitur illa, quæ rectam sphæram omninò habet, quæ eun„ dem polum orizontis & mundi obtinet, quæ orizontem habet diuidentem sphærā „ æquè secundum angulos rectè.

Paulo inferius continuans sermonem de sphæra recta, ait.

„ Et nisi tumor terræ, & gibum esset, ijs perpetuus esset dies sine nocte.

Linea verò. 56. ait habitatores spheræ rectè habere. 4. solstitia, sese ipsum huius rei planè ignarum prodens. 310. autem pag. sic scribit.

„ Quoniam repercutiuntur radij, & per idem centrum transeunt, ob id stupam ap„ positam centro radius accendit.

Quem quidem errorem ab Euclide desumit, et. 15. linea pag. 636. repetit.

Si vis ridere, legito. 16. primas lineas. 357. pag. Quod idem deinde dicat circa finem 396. pag. lucem esse substantiam corporis lucidi & corpoream, subijciam tuo iudicio, vt etiam quod ait. 397. pag. his verbis vtens.

„ Idcirco animalia illa, quæ nocte vagantur perpolita, dum volant, aerem terunt „ nocturnum, & fulgent.

Et pag. 398.

„ Multitudo radiorum non admodum facit ad excitandum calorem si solum inci„ dat sine repercussu, nec recta incidere iuuerit.

Quod falsum est cum radius incidens longè magis quam reflexus calefaciat. In fine autem. 405. sic scribit.

„ Sol in ortu & in occasu longius apparet, iccircò reuolui creditur. Hinc etiam in „ abside stare putatur, & in opposito absidis, vnde solstitia vocant, sed nobis in Can„ cro, antipodibus verò in Capricorno tum Sol abesse longius apparet vtrisque.

An hoc quid peius dici potest? Circa vero. 40. lineam pag. 459. sic scribit.

„ Si enim alij planetæ, & stellæ fixæ reciperent à Sole lumen, dum accederent ad So „ lem, vel recederent, aut contra, Sol ad eas appropinquaret, & abscederet, easdem „ lucis vicissitudinis subiret, quas Luna.

„ Hoc autem nondum deprehensum est, quin etiam Mercurius, Venus, suo interpo „ situ, Solem occultarent nobis, vt Luna.

„ Paulo inferius sic ait. Rursus æquè Saturnus, Iupiter, Mars, subire deliquium, „ more Lunæ, aut saltem obiectu terræ inter Solem & ipsos, quia tum ob interpositam „ terram non possent haurire lumen à Sole.

Hæc verò omnia, talia sunt, qualia ab ijs qui incipiunt intelligere sphæram non proferrentur. Omittamus, quod ait deinde.

„ Accedit quod si astra lumen à Sole acciperent eiusdem caloris essent. Itaque om „ nia siccarent, & nulla essent frigidæ constitutionis contra Astrologos.

Quia hac ratione, Luna, quæ negari non potest, quin ab ipso Sole lumé accipiat, eiusdem caloris esset cum eodem Sole. Sunt ea etiam ridenda, quæ idem ait pag. 460. lineis. 18. 19. 23. 26. 27. 29. quasi ea lux infinita ( vt ita dicam) magni Solis, non

in

in alium finem fit effecta quam ad illuminandam fuperficiem huius excrementi ipfius vniuerfi ad vtilitatem hominum, imò, vt rectius dicam, animaliū. vide etiam pag.632. et.633. vbi Ariftotelem de implendo loco non intellexit, cum citet fphæram, loco pyramidis, & inter.46. et.47. lineas dicat quadratū effe quid multiplex, cum fit vnicum tantum in fpecie, quia fpecies eft quadrilateri, & quadranguli, fed vbi in.6. linea pag.633. ait.

Item hexagonus.

Magnum errorem committit, vt etiam cum.12. linea.636. pag. fcribens.

Pyramis, fiue planum, fiue folidum, habet acutiffimum, & in. 2. libr. de anima pag.215. dicat de die poffe videri ftellas in fpeculo pofito in vafe aqua pleno, quod reuera eft valde abfurdum. Alios eiufdem errores tibi non patefacio, quia iam nihil amplius otij mihi eft, fed eos tu ipfe perfpicere, & cognofcere facilè poteris, & multò plures quidem, quam putas.

### Cur maius lumen extenuet minus.

### PIRRO DE ARZONIS.

EX tuis literis intellexi id, quod etiam fine ijs exploratum mihi erat. Sed concedo tantum effe dicere vbi eft maius lumen, minus non difcerni, quantum inter diu ftellas non videri: immo eft etiam magis vniuerfale, quia idem multis aliis luminibus, præter ea quæ funt ftellarum, ea ratione contingit, quia ingrediente per pupillam, tam lumine maiori, quam minori, reflexum ipfius maioris in oculo, in fitu niinoris, efficit, vt ipfum minus confundatur, & diftingui nequeat, quemadmodum aperte cognofci poteft in aliquo cubiculo, cuius parietes dealbati fint, in quo, vnicum tantum fit exiguum foramen, per quod aliqua lumina reflexa ab obiectis extrinfecis intra ipfum cubiculum ingredi poffint, vnde imagines obiectorū in parietibus confpiciuntur, fed fi per idem foramen ingrederetur etiam primarius radius Solis, reflexus huiufmodi radij efficeret, vt dictæ imagines, magis aut minus euanefcerent, prout dictus reflexus radij folaris, maiori, minoriue vi polleret.

Ad hoc tamen propofitum, nolo tibi filentio inuolui mirabilem quendam effectum eiufmodi rei. Hoc eft vt fiat foramen illud rotundum, magnitudinis tamen vnius fpecilli, quod foramen obturetur mediante vno illorum fpecillorum, quæ pro fenibus (non breuis vifionis) conficiuntur, hoc eft quorum ambæ fuperficies con uexæ funt, non autem concauæ. Deinde opponatur folium album papiri, adeo di ftans à foramine, vt extrinfeca obiecta in eo appareant. Quæ quidem obiecta fi à Sole illuftrata fuerint, tam clara, & diftincta videbuntur, vt nihil pulchrius delectabiliusq; videri poterit, inuerfa tamen. Sed fi ea directa videre voluerimus, hoc optimè faciemus, mediante reflexione alicuius fpeculi plani.

De

*Cur hyems valde frigida sequatur æstatem in qua calor viguerit.*

## NOBILISSIMO, NECNON INGENIOSISSIMO
Gabrieli Buschæ, Mediolanensi.

Vod dixi hyemem valde frigidam sequi æstaté, in qua calor viguerit, inde na scitur, quia calor terrę, aquæ, & aeris, non est naturalis horum corporum, vt est frigus, cum calor à Sole procedat, qui ea calefacit suo lumine, vnde quod æstate Sol præter modum calefaciat terrā, ideo cótingit, quod minora impediméta contra ria sortiatur, & cum eandem postea deserit, ad aliam partem æquatoris transmigrâs terra ad suam qualitatem reddit, maiori cum impetu, eo modo, quo res in motibus localibus naturalibus, qui etiam terminos sibi prefixos, & constitutos excedunt, hinc etiam hyeme fit glacies, ex calefacta prius aqua, quæ durior postea est atque frigidior alia. Aestas etiàm quæ sequitur hyemem valde frigidam, non erit admodum calida, quia Sol inueniens contrarium naturale valde potens, non tam facile illud pellere potest, vnde etiam si in Geminis, Cancro, & Leone, moram trahat, non sufficit tamen ut magnum calorem imprimere possit. Vnde sequitur duas æstates quarum una sequatur aliam, in eodem loco, uehementi calore præditas esse non posse, quemadmodum nec duas hyemes excessiuo frigore, remotis tamen accidentibus uentorum, pluuiarum, & niuium.

## QVOD MALE SENSERIT NICOLAVS TARTA-
lea circa attractionem machinæ tormentalis.

### AD EVNDEM.

Ffectus, quem scribit Tartalea quęsito quinto primi lib. necnon quæsito 21. et. 24. maxima cum ratione esse uidetur, non tamen ea quam ipse in quinto profert, quia uerum non est, vt quanto aliquid sit calidius, táto uehementius attrahat, eo quod si etiam huiusmodi res, in eodem calore, in quo semel reperitur, firma maneret; neque attraheret, neque aliquid impelleret. Nam dum aliquod corpus calesit, dilatatur, & per consequens circumcirca undiq; trudit, & partes uasis debiliores cedunt. dum uerò dictum corpus refrigeratur, restringitur, & dum in unum cogitur, si reperiatur in uase, quod aer, aqua, aut aliud aliquod corpus ingredi nequeat, dictum uas à quo circundatur frangit, ne aliqua pars loci uacua remaneat, sed si aliquod corpus ingredi potest, illud ipsum ad se attrahit, quemadmodum uidere licet in cucurbitulis. Vnde sequitur eam propositionem, qua dicitur, calidi est attrahere, ueram non esse, quia si hoc fieret, quanto aliquid calidus efficeretur, tanto magis attraheret, & è contra, cum tamen planè contrarium appareat, cum quanto magis aliquid calesit, tanto uehementius impellat, & quanto magis frigesit, tanto plus attrahat. Quapropter uerius dicemus, frigidi esse attrahere, calidi uerò expellere, quamuis per accidens. Ex quo sequitur, ut quanto calidior facta fuerit materia aliqua, aliquo loco determinata, redeundo postea ad suam priorem frigiditatem, tanto minori loco indigeat, similiter etiam è conuerso accidit, ut quanto frigidior repitur talis materia, tanto maiori loco, postea

272           I O.   B A P T.   B E N E D.

ſtea egeat ipſa ualde calefacta. Quod Tartalea in quinto queſito nõ animaduer-
terat.

### Solutiones aliqua, circa altimetriam.

### A D E V N D E M.

T Vas literas accepi, tuaſq́; dubitationes conſideraui, quas quidem non inutiles
inueni, quo ueró ad primam, dico te oportere illud Theorema ſpeculari or
dine huiuſmodi methodi, uidelicet quod quotieſcunq; habuerimus angulũ aliquẽ
cuiuſuis amplitudinis, puta. A.R.V. cuius duo latera. R. A. et. R. V. indeterminata
intelligantur, ſi ab aliquo puncto inter ipſas poſito, puta. u, quod etiam uocetur. i. du
ctę fuerint. 4. lineę ipſis dictis lateribus, hac ſcilicet cõditione, qd̃ duę ex dictis. 4. ſint

parallelę ipſis laterib⁹, puta
u.e. et. u. E. reliquę uero duę
ſeccent ipſa latera, ut V. u.
a. et. I. u. A. Dico nunc pro-
portionem. e. A. ad. e. a. ean
dem eſſe, quę. E.V. ad. E. I.
Nam ſcimus proportionem
E. i. ad. E. i. eandem eſſe quę
e. i. ad. e. A. ex ſimilitudine
triangulorũ, ſimiliter ‚ppor
tionẽ. E. u. ad. E.V. eãdẽ quę
e. a. ad. e. u. quare aggregata
ex iſtis erunt inuicem ęqua-
lia, uel ſi mauis ex ęqua pro
portionalitate, quod idem
eſt, ita ſe habebit. E. I. ad.
E.V. ut. e. a. ad. e. A.
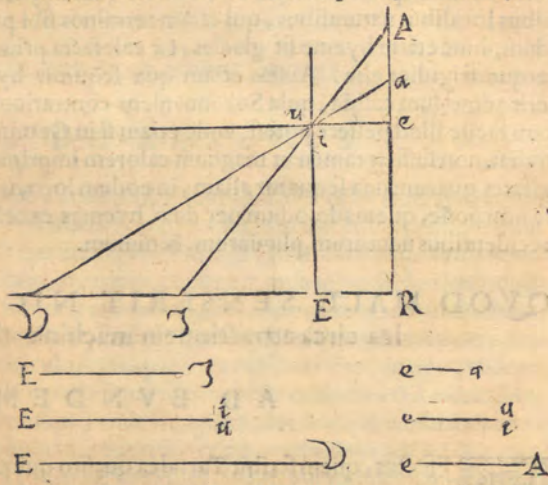


Suppoſito nunc plano orizontali. V.E. Altitudineq́; inacceſſibili. A. E. Duę ue-
ró ſtationes oculorum ſint. V. et. I. lineę autem uiſuales ſint. V. A. et. I. A. Et quadra-
tum geometricum ſit. b. e. Supponatur nunc pro prima dubitatione, quod in am-
babus ſtationibus filum perpendiculare ſeccet latus. e. c. non autem. b. c. (nam quan-
do in ambabus ſtationibus filum ſecat latus. b. c. nullum tibi dubium oritur, imo ma
nifeſtè patent partes lateris. b. c. terminatas à. b. & à filo proportionales eſſe. V. E. &
I. E. ſumpto. E. pro. b. et. I. V. pro punctis ſecatis à filo, ex euidẽti ſimilitudine trian-
gulorum quadrati cum triangulis. A.E.V. et. A.E.I.) Sed cum in preſenti caſu repe-
riatur triangulum. u. e. a. minus, in ſtatione remotiori, ſimile triangulo maiori. V. E.
A. & triangulum maius. i. e. a. proximioris ſtationis, ſimile triangulo minori. I.E.A.
( quod in alio iam dicto, caſu non accidit, ut unum triangulorum, minus ſcilicet, ſi-
mile ſit uno triangulorum, maiori ſcilicet & è conuerſo ) Non omnino abſque ratio
ne dubitas quo pacto fieri poſſit ut. a. e. remotioris ſtationis ad. a. e. propinquioris ita
ſe habeat quemadmodum. I. E. ad. E.V. Quapropter ſi precedentem figuram dili-
gen-

genter inspexeris, omnis tua dubitatio euanescet . in qua figura apertè vi debis cor-
respondentiam talium triangulorum inter se , nec magis , nec minus quam in infra-
scripta hic figura cernere licet, quamuis in hac, triangula quadrati, separata sint ab
imaginarijs. A.E.V. et. A.E.I. in supradicta vero coniuncta, & inuicem communican
tia in puncto.u.i.quod quidem nihil refert .  Dempta igitur.a.e. minori ex. a. e. ma-
iori, reliquum ita se habebit ad.a. e. minorem, vt, V . I. ad. I. E. quod nunc tibi
clarè patebit.  Vnde ex te poteris ordinem operationis prosequi, vt in cognitionem
peruenias ipsius.I.E.ipsius.A.E.& ipsius.I.A.vel.V.A.
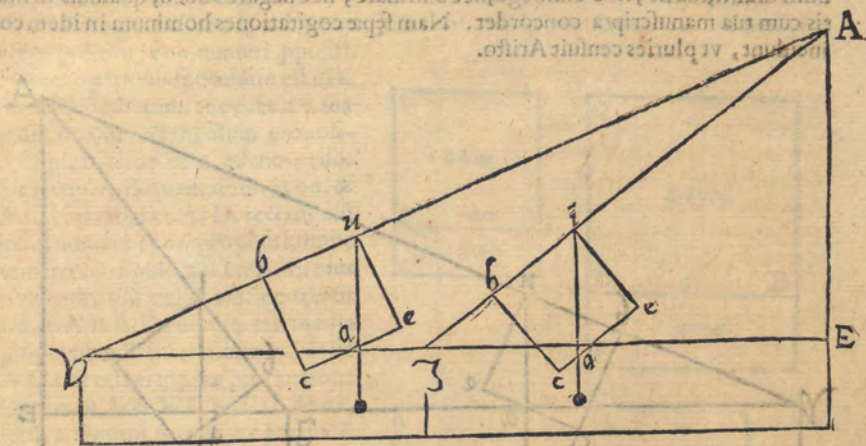


Sed quãdo in proximiori statione latus.b.c.in remotiori vero latus.c.e.secatur à fi
lo ( pro secunda dubitatione ) Tunc oportet imaginatione considerare latus. b. c. in
remotiori statione distentum esse vsque ad filum in puncto.n.vbi videbis triangulũ.
u.b.n.simile triangulo.A.E.V.ita vt.i.b.a.simile suo.A.E.I.reperitur,vbi tam in vno

AD EVNDEM.
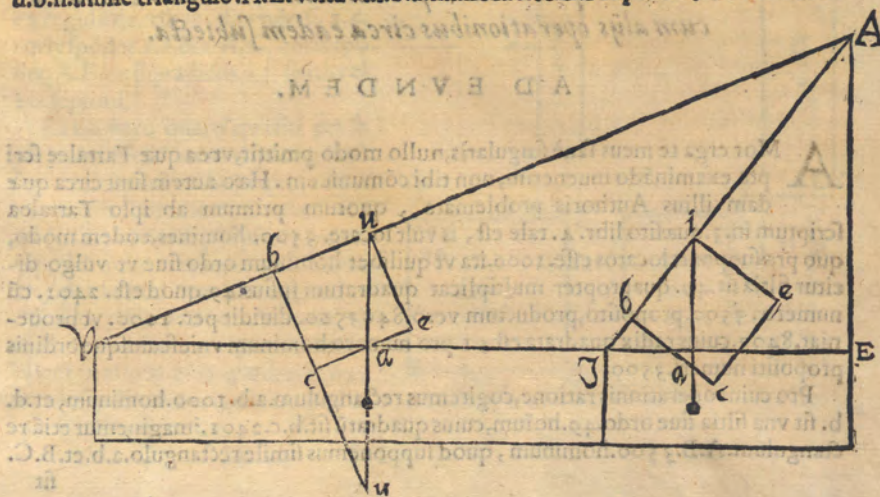


Mm          quam

quam in altero.i.b.et u.b.correspondebit ipsi.A.E.et.b.n.ipsi.E.V. et. b.a. ipsi. E. I.
quapropter iubeo, vt quæras quantum sit latus.b.n.ex regula de tribus, dicens si.a.e.
tribuit mihi.e.u.quid mihi dabit.u.b?eo quod.a.e.u.simile est.u.b.n. reperto autem
latere.b.n.ex quo dempto.b.a.breuioris distantiæ, residuum respondebit ipsi.I. V.vt
scis, vnde prosequendo operationem tibi cognitam, obtinebis intentum, hoc est co
gnosces reliqua interualla . Nihil enim miror demonstrationem Tartaleæ circa hu
iusmodi operationem te minime satisfecisse.

Quod autem quarta propositio illius scriptoris, de quo nuper mecū locutus es, vt
mihi dixisti, tua sit , hoc enimego, nec affirmare , nec negare audeo, quamuis in mul
tis cum tua manuscripta concordet. Nam sepæ cogitationes hominum in idem co-
incidunt, vt pluries censuit Aristo.
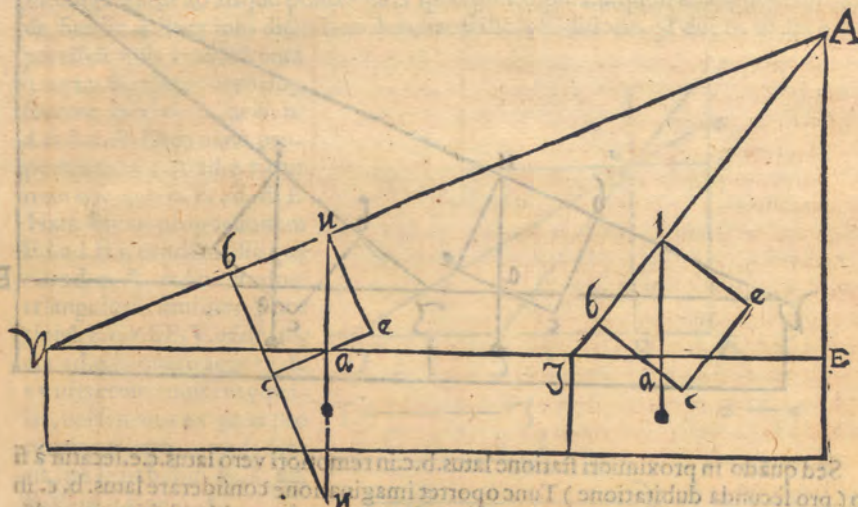


## Demonstrationes quorundam problematum Nicolai Tartaleæ
### cum alijs operationibus circa eadem subiecta.

## AD EVNDEM.

AMor erga te meus sanè singularis, nullo modo pmittit, vt ea quæ Tartaleę scri
pta examinādo inuenerim, non tibi cōmunicem . Hæc autem sunt circa quæ
dam illius Authoris problemata , quorum primum ab ipso Tartalea
scriptum in.3.quæsito libr. 4. tale est, is vult locare. 3500. homines, eodem modo,
quo præsupponit locatos esse. 1000.ita vt quilibet hominum ordo siue vt vulgo di-
citur filtia sit.49. quapropter multiplicat quadratum ipsius.49.quod est. 2401. cū
numero. 3500.proposito, productum verò.8403500. diuidit per. 1000. vt proue-
niat.8403.cuius radix quadrata est.91.pro numero hominum vniuscuiusque ordinis
propositi numeri.3500.

Pro cuius operationis ratione, cogitemus rectangulum.a.b.1000.hominum, et.d.
b. sit vna filtia siue ordo.49.hoīum, cuius quadratū sit.b.c.2401.imaginemur etiā re
ctangulum.A.B.3500.hominum , quod supponemus simile rectangulo.a.b.et.B.C.
<div style="text-align:right">sit</div>

ſit quadratum ipſius. D. B. Nunc ſupponendo. A. B. ſimile. a. b. clarum erit ex diffini-
tione ſimilium figurarum, quod eadem proportio erit. A. D. ad. D. B. quę. a. d. ad. d.
b. hoc eſt. A. D. ad. D. C. vt. a. d. ad. d. c. hoc eſt. A. B. ad. B. c. vt. a. b. ad. b. c. ex prima
ſexti, vel. 18. ſeu. 19. ſeptimi, tunc cum dixerimus ſi. a. b. ita reſpondet ad. b. c. ergo. A.
B. correſpondet etiam ita ad. B. C. quare ex regula de tribus rectè fit multiplicando.
A. B. per. b. c. productum verò diuidendo per. a. b. ex. 15. ſexti vel. 20. ſeptimi, cuius
prouentus radix quadrata erit quod quærebatur.

Sed aliter idem poſſe fieri ſpeculatus ſum, hoc eſt multiplicando numerum. 49.
ordinis. 1000. hominum cū radice quadrata numeri. 3500. propoſiti, productum ve-
rò diuidere per radicem quadratam ipſius. 1000. vnde prouentus. 91. erit numerus
vnius ordinis. 3500. numeri ꝓpoſiti.

Cuius oꝑationis ſpeculatio eſt iſta.
Sit. a. b. quadratum. 1000. et. a. c. ſua
radix et. a. d. rectangulum propoſi-
tum ipſius. 1000. et. a. e. vnus ordo.
Sit etiam. A. B. quadratum. 3500. &
A. C. eius radix et. A. D. rectangulū
ipſius numeri. 3500. propoſiti, ſimile
tamen rectangulo. a. d. et. A. E. eius
vnus ordo Cū enim. a. b. æquale ſit
a. d. et. A. B: A. D. tūc. a. c. erit media
proportionalis inter. a. e. et. e. d. & ſic
A. C. erit etiam media proportiona
lis inter. A. E. et. E. D. per. 16. ſexti,
ſeu. 20. ſeptimi, & quia proportio. A
E. ad. E. D. æqualis eſt proportioni.
a. e. ad. c. d. cum. A. D. ſupponatur ſi-
mile. a. d. ergo proportio. A. E. ad. A
C. equalis erit proportioni. a. e. ad. a.
c. quę medietates ſunt totorū æqua-
lium, rectè igitur fiet ſi procedamus
ex regula de tribus, dicendo ſi. a. c.
correſpōdet. a. e. tūc. A. C. correſpō
det. A. E. ex ſupradictis. 15. ſexti. vel
20. ſeptimi.

Ratio verò quarti quæſiti per ſe
patet, quod eſt inuenire pauimentū
ſeu aream quadratam, in qua poſſint
locari quot homines volueris, ita in
ter ſe ſiti, vt vnuſquiſque occupet.
7. pedes ipſius areę in longitudinem
et. 3. per latitudinem à lateribus.

Seu ex propoſito hominum nūme
ro inuenire numerum ipſorum loca-
bilem in aliqua area quadrata, ita,
vt vnuſquiſque occupet. 21. pedes
quadratos ipſius areæ.

M 2 Sed
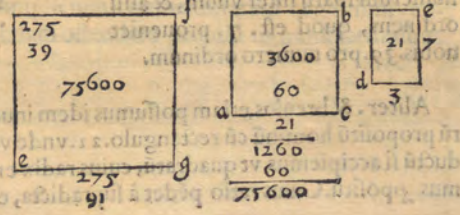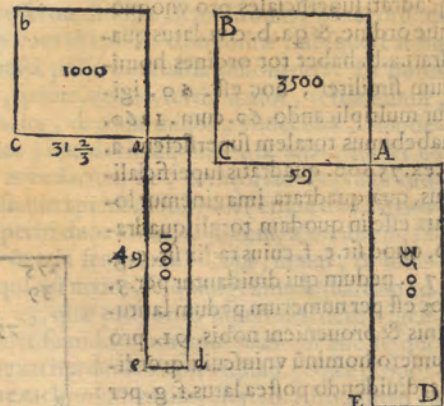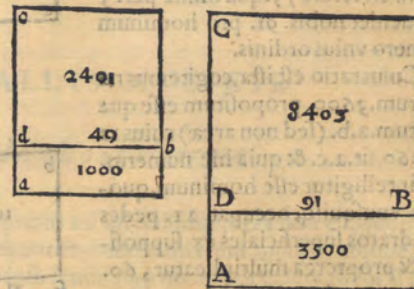
Sed aliter idem fieri poſſe inueni, hoc eſt mul tiplicãdo radicem quadratam pro-
poſiti numeri hominum per. 2 1.& productum item multiplicando per eandem radi
cem,& huiuſmodi producti radicem diuiden do per.3.vnde prouentus eſſet nume-
rus hominum vnius ordinis. Exẽpli gratia p roponuntur. 3600.homines,multiplica
bimus huiuſmodi numeri radicem
quadratam hoc eſt.60. per. 2 1. hoc
eſt per productum quod fit ex.7. cũ
3.& reſultabit nobis. 1260. quod ſi
multiplicabitur, per.60.hoc eſt per
eandem radicem , reſultabit nobis.
75600. cuius producti radix qua-
drata eſt ferè.275.qua diuiſa per. 3
proueniet nobis. 91. pro hominum
numero vnius ordinis.

Cuius ratio eſt iſta,cogitemus nu
merum.3600. propoſitum eſſe qua
dratum.a.b. (ſed non areæ) cuius ra
dix.60.fit.a.c. & quia hic numerus.
60.intelligitur eſſe hominum,quo-
rum vnuſquiſq; occupat. 2 1. pedes
quadratos ſuperficiales ex ſuppoſi-
to, & propterea multiplicatur, 60.
cum.2 1.vnde nobis veniat.1 2 6 0.
quadrati ſuperficiales pro vnoquo-
que ordine, & qa. b. c.vt.latus qua-
drati.a.b. habet tot ordines homi-
num ſimiliter , hoc eſt. 6 0. igi-
tur multiplicando. 60. cum . 1260.
habebimus totalem ſuperficiem. a.
b.ex.75600. quadratis ſuperficiali-
bus, quæ quadrata imaginemur lo-
cata eſſe in quodam totali quadra-
to, quod ſit.e. f. cuius radix ſit. e. g.
2 7 5. pedum qui diuidantur per.3.
hoc eſt per numerum pedum latitu-
dinis & prouenient nobis. 91. pro
numero hominũ vniuſcuiuſq; ordi-
nis, diuidendo poſtea latus.f. g. per
numerum ſpatij inter vnum, & aliũ
ordinem, quod eſt . 7 . proueniet
nobis.39.pro numero ordinum.
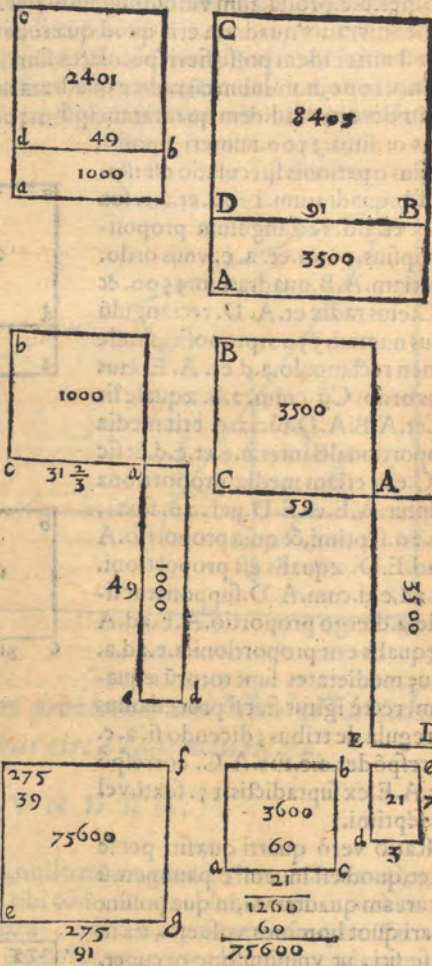
Aliter. & breuius etiam poſſumus idem inuenire, hoc eſt multiplicando nume-
rũ propoſitũ hominũ cũ rectangulo.2 1.vnde veniet nobis ,p ductũ.75600quod pro
ductũ ſi accipiemus vt quadratũ, cuius radix erit.275. quæ diuidatur p. 3. habebi-
mus ,ppoſitũ.Cuius ratio pẽdet à ſupradicta, eo qđ loco multiplicãdi.a.c. (hoc eſt.
60.)per

60.)per.21.deinde productū etiam multiplicare per.b.c. (hoc eſt. 60.) breuius erit multiplicare totum numerum.3600.per.21.cętera verò facere, vt diximus.

Sed vnaquæq; iſtarum operationum, aliquid imperfectionis patitur, eo quod cū aliquis cuperet quadratum perfectum ſuperficiale habere,abſq; aliquo defectu, vel exceſſu,aliquid aliud adhuc facere oporteret, hoc eſt, inuentum cum fuerit quadratum.e.f.cum ſuis radicibus.e.g.et.g.f.pedum.275.vnaquaque, vt in dicto exemplo factum eſt, oportebit numerū quærere minorem ipſo.275. ſed proximiorem menſurabilem ab.3.& ab.7.quod facilè fiet ſi diuiſerimus.275.per. 21. detrahendo fracta diuiſionis ab ipſo.275.quæ quidem fracta in hoc exemplo ſunt.2. vnde remanebit.273.pro numero laterum quadrati ſuperficialis, in quo poſſent locari.3549.homines,eo ordine quo ſupra dictum eſt, quorum ſcilicet vnuſquiſque obtineat. 21. pedes ſuperficiales.

# DE INTERVALLIS MVSICIS.

## Cypriano Rorè Muſico celeberrimo.

Pɪɴɪo Hectoris Euſonij Cypriane mi dilectiſſime,vera non eſt,quod aliquis rectè poſſit intelligere rationes conſonantiarum muſicæ, abſque cognitione illarum mediante ipſo ſenſu, imo nemo pōt calere theoriã muſices, niſi aliquo mō verſatus ſit in praxi. Qūo enim cognoſci poterūt quid nam ſint diapaſon, diapente, diateſſeron,ditonus, ſemiditonus, hexachordum maius, aut minus, & conſonantiæ ex ijs cum diapaſon compoſitæ, abſque earum praxi? vnde ſequetur neq; etiam cognoſci poſſe interualla diſſonantia. Et purus practicus non intelliget quid ſit octaua, quinta,quarta, tertia maior, tertia minor, ſexta maior, ſexta minor, decima maior, decima minor, vndecima, duodecima, decimatertia maior, aut minor, aut decimaquinta, & aliæ, ita vt ad comparandam perfectionem muſicæ neceſſarium ſit, & thęoriam & praxim addiſcere. Cum pręterea Ludouicus Folianus apertè monſtrarit (etiam ſi id a diatonico ſintono Ptolomei deſumpſerit) reperiri duos tonos, maiorem,& minorem, id eſt ſeſquioctauum, & ſeſquinonum, & tria ſemitonia, maius, minus, & minimum, ideſt ſeſquiquintumdecimum, qui eſt maius, ſeſquiuigeſimumquartum ideſt minimum, & mediocre, vt. 27. ad. 25. quæ proportio ſuperbipartiens vigeſimaſquintas appellatur, & cum cognouerit ſemiditonum conſonantem eſſe ſeſquiquintum, ditonum ſeſquiquartum, & hexachordum minus, vt.8.ad.5. quæ proportio dicitur ſupertripartiens quintas,& hexachordum maius, vt.5.ad.3.hęc autem vocatur ſuperbipartienstertias; omnium ſimplicium conſonantiarum cognitioni, extremam impoſuit manum. Et quia tibi etiam oſtendere promiſi in modulationibus

hæc

hæc omnia interualla feruari, ideo ad te mitto feptem hic fubſcripta exemplá, in
quorum primo, & ſecundo, inter diefim, et. b. in fuperiori, agnoſces interuallum mi
nimi femitonij, & ſi ibi ſit dieſis, tanquam terminus ad quem, et. b. tanquam termi-
nus à quo : quod autem inter diefim et. b. ſit femitonium minimum, facilè agnoſces
ſi ſubtraxeris decimã minorẽ à maiori, quã facit ſuperius cũ inferiori, ideſt cũ baſſu.

Qua quidem modulatione tu etiam vſus es in cantilena illa, quæ Galica lingua
incipit . Hellas comment. Eadem, ego quoque in meis cantilenis latino ſermo-
ne compoſitis, quæ Moreta vocantur aliquando vſus ſum.

Sed in tertio exemplo inuenies femitonium maius, neceſſariò genitum in fupe-
riori, ſi fextam maiorem cum baſſu efficere volueris, quia tenor, à ditono cum
fuperiori ad diapentem, & ad vniſonum cum baſſu procedit, vbi quieſcit, progre-
diendo poſtea baſſus ad ſemiditonum cum tenore, tunc ſi à proportione huius ſep-
timæ, quæ eſt vt. 9. ad. 5. hoc eſt ſuperquadripartienſquintas demptum fuerit hexa-
chordum maius, ſeu fexta maior, quæ eſt vt. 5. ad. 3. remanebit proportio. 27. ad. 25.
quæ maior eſt quam. 32. ad. 30.

In quarto exẽplo habebis femitonium minus in fuperiori, quod quidem remanet
ex ſubtractione ditoni cóſonãtis ab diateſſaron cóprehenſa à fuperiori cum tenore.

In quinto exemplo videbis tonum minorem, & tonum maiorem ſucceſſiuè vnum
poſt alium in tenore, detrahendo primo ſemiditonũ à diateſſaron, quod fuperius fa-
cit cum tenore, vel detrahendo diapente ab hexachordo maiori, quod facit tenor
cum baſſu, vnde remanet tonus minor ſeſquinonus, detrahendo poſtea diateſſaron
à diapente, quod ſuperius facit cum tenore, remanebit tonus maior feſquioctauus.

In fexto exemplo deinde videbis tenorem aſcendere per duos tonos minores ſuc-
ceſſiuè vnum poſt alium in tenore, ſi dẽpſeris ſemiditonũ à diateſſaron cũ fuperiori.

In. 7. exẽplo demum videbis fuperiorẽ aſcendere per duos tonos maiores ſuccef-
ſiuè vnũ poſt aliũ, ſi dempſeris diateſſaron à diapente, quod facit tenor cũ fuperiori.

De

## EPISTOLAE. 279

### *De eodem subiecto.*

### AD EVNDEM.

QVod aliàs tibi dixi, verum est, quod necessarium nullo modo sit, vt modulando, desinat cantilena in eodem tono (quod Græci phthongum appellant) à quo incœpit. immo necessariò semper ferè, altius, aut depræssius terminatur, per differentiam alicuius interualli æqualis, vel multiplicis ipsi commati sesquioctuagesimæ, quod quidem comma, quamuis cantabile non sit, insensibiliter tamen generatur, & toties ab aliqua parte ipsius cantilenæ posset dictū comma generari, versus acutum, vel graue, quod in fine ipsius cantilenę, vocis phtongus reperiatur distans à primo per interuallum alicuius toni sesquinoni, seu sesquioctaui plus, minúsue, vt in subscripto exemplo clarè videre potes in prima figura, vbi superius à.g.primę cellulæ ad.g.secundæ, interest vnum cōma, eo quod progrediens superius in prima cellula ipsius cantilenæ à quarta ad quintam cum tenore, ascendit per tonum sesquioctauum, à prima cellula deinde ad secundam, tenor ascendit similiter per tonum sesquioctauum cum transeat à quinta ad quartam, quod facit cum superiori, in secunda cellula postea, cum superius descendat à maiori sexta ad quintam, quod facit cum bassu, seu à quarta ad tertiam minorem, quod facit cum tenore, tunc descendit per tonum sesquinonum, ita quod non reuertitur ad eundē phthō gum, vbi prius erat in prima cellula, sed reperitur per vnū cōma altius, qđ quidē cōma est differentia inter tonū sesquioctauū & sesquinonū, vt alias tibi demōstraui.

Progrediendo igitur hoc modo, videbis quod cum tenor à secunda cellula ad tertiam transeat à tertia minori ad quartam, quod facit cum superiori, descendit per tonum sesquinonum, vnde in tertia cellula altius remanet quam in prima per vnū comma, in qua tertia cellula, cum iterum transeat superius à quarta ad quintam, φ facit cum tenore, eleuatur per tonum sesquioctauum, prosequendo deinde tali ordine, videbis in quarta cellula cantilenam auctam per duo commata, in sexta, aūt cellula per tria commata, in octaua verò per.4.commata, vnde hac methodo, si cantilena prolixior debito esset, vel si talia interualla frequentiora reperirentur, posset cantilena à principio ad finem differre per. 9. commata, & plus etiam, quæ quidem

inter-

interualla superant tonum sesquinonum, & si essent. 10.commata superarent tonum
sesquioctauum, eo quod aggregatum ex.9.commatibus continetur sub istis duobus
terminis hoc est. 1500946352969991121. et. 1342177280000000000. quæ qui-
dem proportio maior est proportione sesquinona, summa verò. 10.commatum con
tinetur sub. 1215766545905692888o1. et. 10737418240000000000. quæ pro-
portio maior est tono sesquioctauo, quod autem dico de ascensu cantilenæ, idem as-
fero de eiusdem descensu, & hoc non tantum per interuallum illius commatis, quod
est differentia toni maioris à minori, sed etiam per illud quod est differentia semito
nij maioris à minori, vt in secundo exemplo hic subscripto videre est in descensu
cantilenæ per comma & comma, vt differentia inter semitonia maiora & minora,
vbi in prima cellula discedens bassus à quintà cum superiori, & ab vnisono cum te-
nore descendens ad tertiam minorem cum ipso tenore, facit cum superiori septimà
maiorem, quæ est vt.9.ad. 5. superquadripartiensquintas scilicet, à qua discedens
postea superius, vt faciat cum bassu sextam maiorem, descendit per semitonium ma
ius, à qua sexta maiori descendens bassus, & ascendens per quartam, efficit cum di-
cto superiori tertiã maiorem, à qua discedens superius, vt efficiat quartam cum ipso
bassu (qui quidem bassus transit in tenorem) ascendit per semitonium minus, diffe-
rens à semitonio maiori per vnum comma, vnde cantilena remanet depressa per
vnum comma. cum deinde idem faciat inter tertiam, & quartam cellulam, per a-
liud comma descendit; & sic toties facere posset, vt postremo valde deprimatur
cantilena à primo phthongo.



Quod autem hic supradictum est, circa instrumenta artificialia non accidit, qua
propter organa, & clauicimbula concordantur certo quodam ordine, ita vt omnes
consonantiæ, excepta diapason, seu octaua, sint imperfectæ, hoc est, aut diminutę,
aut superantes à iusto, vt exempli gratia, omnes quintæ sunt diminutæ, quartæ verò
sunt excessiuę, quod quidem fit, vt tertiæ, & sextæ, non multum auribus dissonent,
eo quod si quintæ omnes, & quartæ, perfectæ essent, tunc omnes sextę, & tertiæ in-
tollerabiles essent, & à perfectis differrent per vnum comma, quod manifestum no-
bis erit hoc modo, accipiamus tres diapentes, seu quintas, consequenter successiuas
vnam post aliam, hoc est tres proportiones sesquialteras, quarum aggregatum erit
vt. 27.ad. 8. quæ proportio, dicitur tripla supertripartiensoctauas, & quæ à practicis
                                                                              appel-

appellaretur tertiadecima maior, vt exempli gratia, esset Gamaut cum secundo ela
mi, tunc talis tertiadecima valde odiosa esset sensui auditus, à qua, si dempta fuerit
diapason, seu octaua, remaneret quoddam hexachordum maius, seu sexta maior, au-
ribus valde inimica, sub proportione. 13. ad 8. sed hæc proportio differret à propor
tione superbipartientetertias perfecti hexachordi maioris, hoc est sextæ maioris
consonantis, per proportionem sesquioctuagesimam, hoc est per vnum comma,
quod quidem est etiam differentia aggregati trium sesquialterarum, à tertiadeci-
ma maiori consonanti, hoc est excessus proportionis triplæ supertripartientis octa-
uas, supra triplam sesquitertiam, quæ est summa ipsius duplæ cum superbi partien-
tetertias .

A tali summa igitur trium sesquialterarum efficitur tertiadecima maior dissonans
excedens consonantem per vnum comma (cuius proportio est. 81. ad. 80.) quæ con-
sonans continetur in proportione. 10. ad. 3. vt supra dixi.

Hæc igitur est vera ratio, propter quam debemus comma distribuere in organis
& clauicymbalis, cum ab aggregato trium quintarum producatur talis excessus su-
pra perfectam, seu consonantem tertiamdecimam maiorem, quod quidem aggre-
gatum, cum demptum fuerit à quintadecima, relinquet nobis tertiam minorem
dissonantem, & mancam, per eundem excessum à consonanti. quæ quidem tertia
minor dissonans subtracta à diapente seu quinta perfecta, relinquet nobis tertiam
maiorem dissonantem, quæ consonantem excedit per eundem excessum comma-
tis, & hæc demum tertia maior dissonans, dempta ex diapason, seu octaua, relin-
quet nobis hexachordum minus, hoc est sextam minorem dissonantem, & muti-
lam à consonanti per eundem excessum commatis . De huiusmodi verò commatis
distributione doctissimè scripsit Excellentissimus Zarlinus in secunda parte Institu-
tionum Harmonicarum .

Sed quia sensus auditus non potest exactè cognoscere debitam quantitatem ex-
cessus, vel defectus, intendendo vel remittendo chordas instrumentorum, ideo hanc
viam sequutus sum .

Sit exempli gratia, hic subscriptus ordo lignorum tangentium seu pinarum inci-
piens ab. G. desinens ad. g. ita quod inter ipsos terminos sit ea consonantia quæ vo-
catur vigesimasecunda, quæro primum. b. inter. D. E. quod est nigrum ipsius Ela-
mi grauissimum, quod grosso modo facio consonans cum. G. grauissimo per sex-
tam minoré, deinde cū ipso primo. b. ipsius elami concordo suum octauum & quin-
tumdecimum, quo perfectius possum, deinde accipio. b. molle secundum ipsius. b
fabmi quod concordo cum. b. primo ipsius Elami per quintam imperfectam, dein-
de cum hoc. b. secundo ipsius bfabmi concordo secundum. f. per quintam similiter
imperfectam, cum quo. f. postea concordo tertium. c. per similem quintam, quem
tertium. c. postea confero cū secundo. b. ipsius elami, ita quod inter se consonent per
sextam maiorem tolerabilem, & si sic inuenio, tunc nihil muto has treschordas hoc.

G A B b C ✳ D b E F ✳ g ✳ a b b c ✳ d b e f ✳ g ✳ a b b c ✳ d b e f ✳ g ✳
1 1　1　1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3　3 3 3 3 3 3 3 3 3 4 4.

N n　　est

eſt. b. ſecundum ipſius bfabmi, f. ſecundum, et. c. tertium, ſed ſi dictum tertium. c.
valde diſſonans eſſet cum. b. ſecundo ipſius elami, tunc ipſum. c. intendo, aut re-
mitto, quouſque aliquo modo ſit conſonans per ſextam maiorem aliquantulum ex
ceſſiuam cum. b. ſecundo ipſius elami, cum quo poſtea. c. conſonare aliquantulum fa
cio. f. ſecundum per quintam defectiuam, & cum hoc demum. b. ſecundum ipſius
bfabmi, quo facto concordo ſecundum. c. cum tertio per octauam, cum quo ſecun-
do. c. poſtea concordo tertium. g. per talem quintam, quod ipſum tertium. g. cum ſe-
cundo. b. ipſius bfabmi conſonet tolerabiliter per ſextam maiorem aliquãtulum ex-
ceſſiuam, deinde cum iſto tertio. g. concordo tertium. d. per talem quintam, ita quod
ipſum. 3. d. concordet tolerabiliter cum. 2. f. per ſextam maiorem exceſſiuam, poſtea
cum hoc. 3. d. concordo. 2. d. per octauam perfecte, cum quo. 2. d. poſtea concordo.
3. a. per quintam, vt in alijs factũ eſt, ita vt cũ. 2. c. conſonet talis ſexta maior, vt ſupra
dictum eſt, cum quo. 3. a. poſtea concordo. 3. e. per quintam, vt dictum eſt, ita quod
cum. 3. g. faciat ſextam maiorem vt ſupra, poſtea cum hoc. e. concordo. 2. e. per octa
uam, cum quo concordo. b. quadrum tertium per quintam, vt dictum eſt, ita quod cũ
2. d. faciat ſextam maiorem ſimilem alijs ſuperius dictis, cum quo. b. quadrato tertio
concordo tertium nigrum ipſius. f. per quintam, ita quod cum. 3. a. faciat ſextam ma-
iorem, vt ſupra, deinde cum hoc concordo. 2. f. nigrum per octauam, cum quo, per
quintam concordo 3. c. nigrum ita quod cum. 2. e. faciat ſextam dictam, demum cũ
hoc concordo. 4. g. nigrum per quintam, ita quod faciat cum. 3. b. quadrato ſextam
dictam, & ſic ad vltimam quintam peruenio, ſupra quod. g. nigrum nulla quinta am-
plius reperitur, poſtea cum iſtis chordis concordo per octauas omnes alias ab acutis
ad graues.

G A B b C ＊ D b E F ＊ g ＊ a b b c ＊ d b e f ＊ g ＊ a b b c ＊ d b e f ＊ g ＊
1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 4 4.

Valde etiam admiratione dignum eſt, quod perfectiores quæque conſonan
tiæ, ita in harmonica diuiſione ſibi inuicem conueniant, vt diapaſon cum diapente,
cum diapaſondiapente, cum ditono, cum hexachordo maiori cum bisdiapaſon, cũ
decimaſeptima maiori. Nam in ipſa diapaſon, harmonicè locãtur diapente in par
te grauiori, & diateſſaron in acutiori. In diapente verò harmonicè locantur ditonũs
in parte grauiori, & ſemiditonus in acutiori. In ditono harmonicè locantur tonus
maior in parte grauiori, & tonus minor in acutiori. In hexachordo maiori, harmo-
nicè locantur diateſſaron in parte grauiori, & ditonus in acutiori. In diapaſondia-
pente, harmonicè locantur diapaſon in parte grauiori, & diapente in acutiori.
In bisdiapaſon, harmonicè locantur decima maior in parte grauiori & hexachor-
dum minus in acutiori. In decimaſeptima maiori, harmonicè locantur diapaſondia-
pente in parte grauiori, & hexachordum maius in parte acutiori. Ita quod tonus
ſeſquioctauus in ditono, proportionalis eſt ipſi ditono in diapente. Tonus verò ſeſ-
quinonus in ipſo ditono, proportionalis eſt triemitonio, vel ſeſquitonio ſeu ſemidi-
tono (quod idem eſt) in diapente. Ditonus autem in diapente, proportionalis eſt
ipſi diapente in diapaſon. Seſquitonus verò in diapente, proportionalis eſt diateſ-
ſaron in diapaſon. Et ſic de ſingulis. Ita quod tonus ſeſquioctauus in ditono, dito-
nus in diapente, diateſſaron in hexachordo maiori, diapente in diapaſon, diapaſon
in diapaſondiapente, decimamaior in bisdiapaſon, diapaſondiapente in decima-
ſeptima

septima maiori, omnia sibi inuicem sunt proportionalia, idem etiam dico de reliquis partibus, cum relatæ fuerint ad sua tota.

Nec alienum mihi videtur à proposito instituto, speculari modum generationis ipsarum simplicium consonantiarū; qui quidem modus fit ex quadam æquatione per cussionum, seu æquali concursu vndarum aeris, vel conterminatione earum.

Nam, nulli dubium est, quin vnisonus sit prima principalis audituǫ; amicissima, nec non magis propria consonantia; & si intelligatur, vt punctus in linea, vel vnitas in numero, quam immediate sequitur diapason, ei simillima, post hanc verò diapente, cæteræǫ;. Videamus igitur ordinem concursus percussionum terminorum, seu vndarum aeris, vnde sonus generatur.

Concipiatur igitur mente monochordus, hoc est chorda distenta, quæ cum diuisa fuerit in duas æquales partes à ponticulo, tunc vnaquæq; pars eundem sonum proferet, & ambæ formabunt vnisonum, quia eodem tempore, tot percussiones in aere faciet vna partium illius chordæ, quot & altera: ita vt vndæ aeris simul eant, & æqua liter concurrant, absque ulla intersectione, vel fractione illarum inuicem.

Sed cum ponticulus ita diuiserit chordam, vt relicta sit eius tertia pars ab vno latere, ab alio vero, duę tertię, tunc maior pars, dupla erit minori, & sonabūt ipsam diapason consonantiam, percussiones verò terminorum ipsius, tali proportione se inuicem habebunt, ut in qualibet secunda percussione minoris portionis ipsius chordæ, maior percutiet, seu concurret cum minori, eodem temporis instanti, cum nemo sit qui nesciat, quod quo longior est chorda, etiam tardius moueatur, quare cum longior dupla sit breuiori, & eiusdem intensionis tam vna quam altera, tunc eo tempore, quo longior vnum interuallum tremoris perfecerit, breuior duo interualla conficiet.

Cum autem ponticulus ita diuiserit chordam, vt ab vno latere relinquantur duæ quintæ partes, ab alio verò tres quintæ, ex quibus partibus generatur consonantia diapente; tunc clarè patet, quod eadem proportione tardius erit vnum interuallum tremoris maioris portionis, vno interuallo tremoris minoris portionis, quam maior portio habet ad minorem; hoc est tempus maioris interualli ad tempus minoris erit sesquialterū; quare non côuenient simul, nisi perfectis tribus interuallis minoris portionis, & duobus maioris; ita quod eadem proportio erit numeri interuallorum minoris portionis ad interualla maioris, quæ longitudinis maioris portionis ad longitudinem minoris; vnde productum numeri portionis minoris ipsius chordæ in numerum interuallorum motus ipsius portionis, æquale erit producto numeri portionis maioris in numerum interuallorum ipsius maioris portionis; quæ quidem producta ita se habebunt, vt in diapason, sit binarius numerus; in diapente verò senarius; in diatessaron duodenarius, in hexachordo maiori quindenarius; in ditono vicenarius, in semiditono tricenarius, demum in hexachordo minori quadragenarius: qui quidem numeri non absque mirabili analogia conueniunt inuicem.

Voluptas autem, quam auditui afferunt consonantiæ sit, quia leniuntur sensus, quemadmodum côtra, dolor qui à dissonantijs oritur, ab asperitate nascitur, id quod facilè videre poteris cum conchordantur organorum fistulæ.

Nn 2 De

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## DE IVSTITIA COMMVTATIVA.

*Francisco Ferrario Ancisa Iurisconsulto senatoriáq̃ apud subalpinos grauissimo.*

Sᴀᴇᴘɪᴠs inter nos dum oportunitas vicinarum ædium, & amoris mutui vis, ad familiaria trahunt colloquia ego de meis mathematicis, tu de tuis legibus, in quibus tractandis magnum tibi nomen comparasti loquuti su mus. Cum vero nonnunquam de mirabili iustitiæ commutatiuæ institu to non ingratus incidisset sermo, dixi modum, quo formam suam à proportionali tate arithmetica disiuncta, & non a coniuncta desumat, à nemine literis proditum esse, libet autem nunc per otium latius explicare. dixi enim à disiuncta, & non coniuncta proportionalitate, quia in coniuncta, seu continūa nullo pacto fieri potest talis commutatio, cum semper quatuor terminos ad minus transeat, vt nunc videbimus.

Exempli gratia, Petrus ex suis bonis tribuat Ioanni aliquid valoris quinquagin ta aureorum.

Vnde priusquam Ioannes aliquid ex suis bonis retribuat Petro, bona ipsius Petri diminuta erunt per quinquaginta aureos, bona verò ipsius Ioannis, aucta totidem aureis.

Ecce nunc quo pacto constituti sunt. 4. termini in proportionalitate aritmetica, per quos fit talis permutatio, sed nondum æquata, nisi fiat æqualis retributio à Ioanne ad Petrum, vt videbimus.

Cogitentur itaque. 4. termini aritmeticè proportionales. C. A. B. D. Ita quod. A. mediante significentur bona Ioannis. B. vero Petri, prius quam Petrus aliquid ex bo nis suis tribuat Ioanni. Tunc Petrus secat partem vnam ex. B. eamq̃; dat ipsi Ioanni, vnde ipsi Petro remanet. D. Ioanni autem. C. quatuor igitur termini constituti sunt. B. D. C. A. quorum. B. primus. A. quartus. C. uero tertius. D. aūt secundus, sed B. et. A. sunt in sua naturali mediocritate absque defectu vel excessu sui ipsius. Non ita tamen se habet. C. et. D. quia. D. deficit. C. autem excedit à sua priori quantitate. Nihilominus isti. 4. termini constituti sunt in ipsa aritmetica proportionalitate, nam eadem quantitate qua. D. diminuta est à. B. eadem. C. aucta est supra. A.

Sed quia. B. et. A. tantummodo iusti sunt termini. C. uerò et. D. iniusti, vt ad suam priorem æqualitatem reuertantur, oportebit ex. C. secare aliquam partem æqualis valoris ei, qua. C. superat. A. vel qua. D. minor est. B. & ipsam partem addere ipsi. D. vt bona Petri reuertantur ad priorem suam quantitatem ipsius. B. & bona Ioannis remaneant æqualia. A. vt prius.

Quare necessarium non est, vt talis proportionalitas sit coniuncta (vt inquit Eu stratius seu Michael Ephesius, super quinto capite libr. quin ti Ethicorum) tribus terminis contenta, imò oportet ut ipsa disiuncta sit, ut diximus, vbi

```
        C                    B
  |----------|         |----------|
        A                    D
  |----------|         |----------|
```

non est necesse quod. A. æqualis sit. B. aliquo modo.

                                                                  DE

## DE MOTV MOLAE, ET TROCHI, DE AMPVL-
lis aquæ, de claritate aeris, & Lunæ noctu fulgentis, de æter-
nitate temporis, & infinito spacio extra
Cœlum, Cœliq́; figura.

*Illust. Ioanni Paulo Capræ Nouariensi Sabaudiæ Ducis hospicij*
*Magistro, viro ingenij præstantia, & morum cando-*
*re, non minus quam familiæ nobili-*
*tate conspicuo.*

I vera esset animorum illa transmigratio quam sibi Italicæ sapientiæ Pa-
ter Pythagoras effinxerat, tuam, meamq́; existimarem animam canis,
quandoque venatici fuisse.

Quæris à me literis tuis, an motus circularis alicuius molæ molendina
rie, si super aliquod punctum, quasi mathematicũ, quiesceret, posset esse perpetuus,
cum aliquando esset mota, supponendo etiam eandem esse perfectè rotundam, &
leuigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec
etiam multum duraturum, quia præterquam quod ab aere qui ei circumcirca aliquã
resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quæ
cum motæ sunt, natura, impetum habent efficiendi iter directum, vnde cum simul
iunctæ sint, & earum vna continuata cum alia. dum circulariter mouentur patiuntur
violentiam, & in huiusmodi motu per vim vnitæ manent, quia quanto magis mo-
uentur, tanto magis in ijs crescit naturalis inclinatio recta eundi, vnde tanto magis
contra suammet naturam voluuntur, ita vt secundum naturam quiescant, quia cum
eis proprium sit, quando sunt motæ, eundi recta, quanto violentius voluuntur, tan-
to magis vna resistit alteri, & quasi retrò reuocat eam, quam antea reperitur habere.

Ab eiusmodi inclinatione rectitudinis motus partium alicuius corporis rotundi
sit, vt per aliquod temporis spacium, trochus cum magna violentia seipsum circun-
agens, omninò rectus quiescat super illam cuspidem ferri quam habet, non incli-
nans se versus mundi centrum, magis ad vnam partè, quam ad aliam, cum quælibet
suarum partium in huiusmodi motu non inclinet omnino versus mũdi centrum, sed
multo magis per transuersum ad angulos rectos cum linea directionis, aut verticali,
aut orizontis axe, ita vt necessariò huiusmodi corpus rectum stare debeat. Et quod
dico ipsas partes non omninò inclinare versus mundi centrum, id ea ratione dico,
quia non absolutè sunt unquam priuatæ huiusmodi inclinatione, quę efficit vt ipsum
corpus eo puncto nitatur. Verum tamen est, quod quanto magis est velox, tan-
to minus premit ipsum punctum, imò ipsum corpus tãto magis leue remanet. Id qđ
apertè patet sumédo exéplũ pilę alicuius arcus, aut alicuiꝰ alterius instruméti, seu ma
chinæ missilis, quæ pila quanto est velocior, in motu violento, tanto maiorem pro-
pensionem habet rectius eundi, vnde versus mundi centrum tanto minus inclinat,
& hanc ob causam leuior redditur. Sed si clarius, hanc veritatem videre cupis,
cogita illud corpus, Trochum scilicet, dum velocissime circunducitur secari, seu
diuidi in multas partes, vnde uidebis illas omnes, non illico uersus mundi centrum
descen-

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defcendere, fed recta orizontaliter, vt ita dicam, moueri. Id quod à nemine ad-
huc (quod fciam) in trocho eft obferuatum. Ab huiufmodi motu trochi, aut hu-
ius generis corporis, clarè perfpicitur, quàm errent peripatetici circa motum uio-
lentum alicuius corporis, qui exiftimant aerem qui fubintrat ad occupandum locum
à corpore relictum, ipfum corpus impellere, cum ab hoc, magis effectus contrarius
nafcatur.

Quod deinde ampullæ iungantur in aqua, non fit ratione fimpathiæ, de qua lo-
quitur Fracaftorus, nam per accidens iunguntur, quia cum alia ad aliam accedit, quę
libet earum tentat afcendere ab ea parte, à qua inuicem hærent, quemadmodum
efficiunt iuxta labrum vafis, ea enim fuperficies aquæ vicina circunferentiæ vafis ali
quantulum afcendit in vafe, qui non eft omnino plenus.

Ad id deinde quod de claritate noctium fcribis, miror cur non videas, quod quã
to magis obfcura nox apparet, non dico ratione nubium, fed diftantiæ Solis fub
orizonte ab eodem orizonte, tanto magis claram, & luminofam fefe nobis often-
dit Luna in quintadecima, quia cum Sol eft in Sagittario, & Capricorno, Luna eft
in Geminis, & in Cancro, vnde in media nocte, eius radius per valde exiguam quã
titatem vaporum tranfit, quia tunc ipfa eft valde propinqua axi orizontis, & præ-
terea in huiufmodi tempore anni & noctis, aer eft magis purgatus, quàm in qualibet
alia temporis parte, quia hieme Sol non poteft excitare multos vapores, & ij, qui
at tolluntur, nocte à frigore ftatim congellati ratione grauitatis decidũt, unde rema-
net aer multo clarior, qua ratione apparent ftellæ minutæ, & Cœlum ijfdem ma-
gis ornatum, quàm in quolibet alio anni tempore.

Dicere deinde, quemadmodum hic mundus eft ætatis feptem, aut octomillium
annorum, ita nunc potuiffet effe (fi Deus voluiffet) ætatis quinquagintamillium; er
go erat tempus; ita fe habet, ac fi diceremus, quemadmodum hic mundus eft ran-
tæ magnitudinis, ita etiam quinquagies maior effe potuiffet, ergo eft fpatium, aut
interuallum corporeum, quod eum capere potuiffet.

Illud, nihil, Ariftotelis extra Cęlum, nullo modo nobis inferuit pro eiufdem Cœ
li fphęrica rotunditate, cum cuiufque alterius ex infinitis figuris Cęlum ipfum effe
poffit, fecundum fuam fuperficiem conuexam. Nam Cœlum ea ratione fphęricum
non eft, quod magis fit capax, quia ei innumerahiles alias figuras adeo magnas po
terat concedere caufa diuina: fed fphæricum eft effectum, ne partem aliquam habe
ret fui termini fuperfluam, quia nullum corpus à breuiori termino quam à fphærico
terminari poteft.

### De reuolutione rotæ putealis & alijs
### problematibus.

### AD EVNDEM.

FVnis cui appenfa eft fitula, longè facilius axi inuoluitur, fi ipfi axi affixa fit rota,
atque item commodius eò fiet, quo amplior rota erit, & axis exilior.

Commodiffimè autem, fi ipfa rotæ extrema circunferentia, ex materia minori,
& denfiori, ac proinde grauiori conftabit. Cuius rei ratio multiplex eft. Nem-
pe quia omne corpus graue, aut fui natura, aut vi motum, in fe recipit impreffio-
nem

nem & impetum motus, ita vt feparatum à virtute mouente per aliquod temporis
fpatium ex feipfo moueatur. nam fi fecundum naturam motu cieatur, fuam veloci-
tatem femper augebit, cum in eo, impetus & impreffio femper augeantur, quia
coniunctam habet perpetuò virtutem mouentem. Vnde manu mouendo rotam, ab
eaq; eam remouendo rota ftatim non quiefcet, fed per aliquod temporis fpatium
circunuertetur.

Secunda caufa eft, quia quoduis graue corpus, aut per naturam, aut per vim mo-
tum, rectitudinem itineris naturaliter appetat, quod clarè cognofcere poffumus,
proijciendo lapides funda, & circunducentes brachium, nam funes tanto maius
pondus acquirunt, & manum tanto magis onerant, quanto velocius voluitur funda,
& incitatur motus, quod ab appetitu naturali infito ei corpori per lineā rectam pro-
grediendi procedit. Vnde fit, vt pondus circunferentiæ ipfius rotæ, tanto facilius cir-
cunuoluatur, & ex feipfo tanto longiori tempore moueatur, quanto longius diftat à
centro, cum eius iter tanto minus fit curuum. Hanc igitur ob caufam, rota, quanto
maior erit, eiufq; pondus tanto magis vicinum circunferentiæ, tanto magis durabit
impetus motus affumptus.

Tertia caufa eft, quod funis dum circunuoluitur, vicinius axi mathematico reuo-
lutionis, quam corpus graue circunferentiæ rotæ, ratione vectis, cum rota eft in mo-
tu, eius impetus non obtinet refiftentiam æqualem à contrario pondere aquæ in fitu
la pofitæ.

## De machina, quæ aquam impellit & fubleuat.

### AD EVNDEM.

Vnde fit vt in fonte mandauerim,
vas feu mortarium in quod in-
greditur inftrumentum, quod aquam
impellit, diametrum fuæ concauitatis,
habere non oportere maiorem dia-
metro fiftulæ, per quam debet afcende
re aqua, ratio eft, quia fi maius effet,
neceffarium effet aliquod inftrumen-
tum quo aqua impelleretur multo gra
uius toto corpore aqueo, quod aptum
effet implere aliquam fiftulam adeo
altam, vt eft fons, quæ tamen effet
adeo lata vt eft mortarium.

Sit exempli gratia, tota fiftula, feu
hirundo, per quam afcendit aqua. f.
mortarium verò fit. a. u. quod tam altū
fit vt. f. fed. f. anguftior ipfo. a. u. Nunc
cum repleta fuerint hæc duo vafa, ma-
nifeftum erit, quod aqua ipfius. f. fuffi-
ciens erit ad refiftēdum toti aquæ ipfi⁹
a. u. & aqua. a. u. refiftet aquæ. f. quam-
uis aqua. a. u. maioris quantitatis fit, &
ponderis ipfa. f. hoc autem euenit ex
eo quod aqua. a. u. nō impellit aquam
f. toto
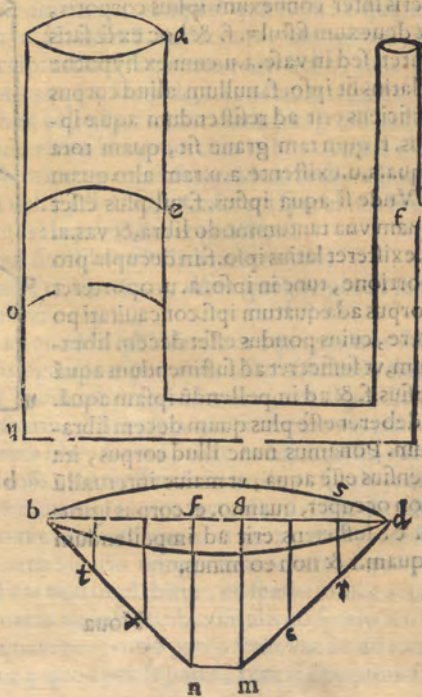
f.toto fuo pondere, propterea quod pondus diuiditur proportionaliter fupra ba-
fim vafis.

Sit exempli gra vas aliquod.b.d.n.m.conicæ figuræ, feu trũcus coni concaui aqua
plenus, cuius orificij diameter fit.b.d. & multiplex diametro. m. n. infimæ bafis. co-
gitemus etiam.b.d.diuifum in tot partes, quarum vnaquæq; æqualis fit, m. n. imagi-
nemurq; tot lineas perpendiculares defcendere verfus mundi centrum ad puncta.r.
c.m.et.t.x.m.vt in fubfcripta hic figura videre eft, per quas cogitemus tot fuperfi-
cies curuas conicasq;,inter quas, mente concipienda eft aqua, quę pondere fuo quie-
fcet fupra maiorem fuperficiem illa, quæ æque diftans effet mundi centro, feu quam
fupra bafim.m.n.vt exempli gratia confideretur aqua inter.g.m.et.s.r.cuius pondus
diftribuitur fecundum latitudinem.m.r.quæ maior eft.g. s.cogitemus igitur.m.c.æ-
qualem effe.g.s.manifeftum erit, quod.m.c.non fuftinebit totum pondus aquæ, quæ
inter.g.m.et.s.r.reperitur, eo quod omnis pars aquæ ad perpendiculum inclinat ver-
fus mundi centrum, quapropter fundus feu bafis.m.n.non fuftinet aliud pondus quã
aquæ.f.m. fed fi quis hoc in dubium reuocaret dicens, quod aqua circunfcribens fi-
tum corporis aquei.f.m.impellit lateraliter dictum corpus aqueum, refpondendum
eft, quod ex æquo huius corporis. f. m. aqua impellit etiam aquam circunftantem,
eo, quod funt corpora homogenea, cum in corporibus homogeneis æquales partes
habeant æquales vires.

Sed redeundo ad vafa.a.u.et.f.dico quod ficut aqua. f. fufficit ad refiftendũ aquæ
a.u.ita quodlibet aliud pondus equale.f.cuiufuis materiæ, in fiftula, f.pofitum, fuffi-
ciens erit, dummodo illud corpus ita fit adæquatum concauitati fiftulæ.f. quod non
permittat tranfitum aliquem aquæ vel
aeris inter conuexum ipfius corporis,
& deuexum fiftulæ. f. & hoc ex fe fatis
patet, fed in vafe.a.u.cum ex hypothe-
fi latius fit ipfo. f. nullum aliud corpus
fufficiens erit ad refiftendum aquæ ip-
fius. f. quin tam graue fit, quam tota
aqua.a.u.exiftente.a.u.tam alto quam
f. Vnde fi aqua ipfius. f. nil plus effet
quam vna tantummodo libra, & vas.a.
u.exifteret latius ipfo.f.in decupla pro-
portione, tunc in ipfo. a. u. oporteret
corpus adæquatum ipfi concauitati po-
nere, cuius pondus effet decem libra-
rum, vt fufficeret ad fuftinendum aquã
ipfius.f. & ad impellendũ ipfam aquã.
f.deberet effe plus quam decem libra-
rum. Ponamus nunc illud corpus, ita
denfius effe aqua, vt maius interuallũ
non occupet, quam.o. e. corpus igitur
o. e. fufficiens erit ad impellendum
aquam.f. & non eo minus.
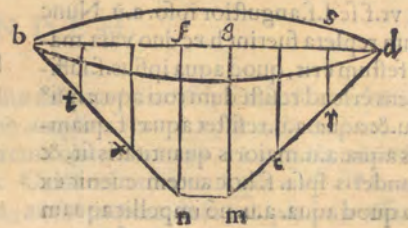
Noua

## NOVA SOLVTIO PROBLEMATIS DE VASE
pleno liquoris.

*Nicolao Caluxio Serenißimi Ducis Sabaudiæ à secretis.*

V o d à me postulas est problema ab alijs iam scriptum, sed illud tibi alio medio soluam.

Proponitur vas plenū liquore aliquo, puta aqua, ꝙ tres habeat fistulas ad basim, quarum vnaquæque possit euacuare ipsum vas, inæquales tamen, ita quod prima tam lata sit, vt spatio vnius horæ possit ipsum euacuare totum; secunda vero spatio duarum horarum, tertia autem spatio trium horarum. Tunc quæritur quanto tempore omnes tres fistulæ simul apertæ euacuabunt ipsum vas. Ad hoc volo vt quæratur primo quanta pars aquæ vnaquæquę fistula euacuabit in aliquo dato tempore, quod facilè est, vt puta, prima fistula, spatio dimidiæ horæ euacuabit dimidium vas, eo quod spatio integræ horæ potest totum euacuare, secunda fistula, eodem temporis spatio, euacuabit quartam partem ipsius vasis, tertia verò fistula, eodemmet spatio temporis dimidiæ horæ, euacuabit sextam partem ipsius vasis, quæ omnia fracta simul collecta faciunt vndecim duodecimas partes totius vasis, vnde manifestum erit, quod omnes fistulæ pariter apertæ, spatio dimidię horæ euacuabunt vndecim duodecimas partes totius aquæ, sed nos cupimus scire, quanto tempore, totum vas euacuabitur, apertis omnibus fistulis, quapropter dicemus ita; Si vndecim duodecimæ partes consumunt minuta. 30. temporis, quantum consument omnes partes aquæ? quæ sunt. 12. quare ex regula de tribus prouenient nobis minuta. 32. cum. 8. vndecimis vnius minuti, hoc est cum. 43. secundis horæ ferè, vel si accipiemus tres quartas vnius horæ, tunc prima fistula emittet tres quartas partes totius aquæ, secunda, tres octauas eiusdē aquę, tertia verò, quarta pars, tunc omnia, hæc collecta, faciunt vnum integrum cum tribus octauis. Si dixerimus igitur quando vnum integrum cum tribus octauis absumit. 45. minuta temporis, ergo illud solum integrum absumet idem vt supra hoc est min. 32. cum. 8. vndecimis vnius minuti vel. 43. secundis. Cuius rei speculatio tā coniuncta est operationi, quòd vna cognita, reliqua statim cognoscitur.

Idem eueniet de implendo vase tribus similibus fistulis mediantibus.

Secundum quæsitum ab alijs traditum, tuum etiam, aliter quoque potest solui, propterea non prętermittam tibi satisfacere.

Problema itaque tale est, vt sit vas aliquod in ꝗ̄ infunditur aqua per tres fistulas, sed dum infunditur aqua, eadem egreditur per duas alias fistulas in fundo vasis positas, sed tres superiores sint inuicem proportionatæ, vt supradictum est, primaque inferiorum talis sit, vt spatio. 4. horarum possit totum vas euacuare, secunda autem possit spatio. 6. horarum idem facere, vnde ex supradictis, vas implebitur à tribus fistulis superioribus, clausis existentibus inferioribus, spatio temporis minutorū. 32. cū. 8. vndecimis hoc est min. 32. cum. 43. secundis, deinde per duas fistulas inferiores posset euacuari spatio tēporis horarum. 2. et mi. 24. ex supradictis.

Supponamus igitur omnes fistulas operari spatio temporis minutorum. 32. cum secundis. 43. tunc manifestum est quod vas non implebitur, eo spatio min. 32. cum secundis. 43. sed tanta aqua deficiet, quanta ab inferioribus fistulis eo spatio temporis min. 32. secun. 43. potest euacuari, quare proportio partis vasis vacuæ, ad totum vas, erit vt min. 33. ferè ad horas. 2. min. 24. quod per se patet, tunc si demptum fue-

<div align="right">O o rit</div>

rit tempus.33.minutorum ex h oris.2.min.24. reliquum erit hora.1. min. 51. vnde
proportio aquæ, quæ in vafe reperitur,ad eam, quæ totum vas implet, erit vt. 111.
ad.144. Quare nunc poffumus rectè dicere ex regula de tribus fi.111.indigent mi-
nuta.33.temporis,ergo.144.indigent min.43.horæ, in quo tempore implebitur to-
tum vas omnibus fiftulis operantibus.

### *Alia circuli noua paßiones.*

### AD EVNDEM.

VT ad afcendendum ignis,& ad defcendendũ quicquid graue natum eft, ita ad
fpeculandum humanus intellectus. nec quiefcit, dum poteft, eft enim ver-
fatile, agitandoq́; fefe caufis rerum immifcere, & abditum aliquid rimari,
conatur,& eft in nobis, quafi Diogenes quidam in Dolio.

Tibi igitur mitto quod vltimò inueni, alias fcilicet nouas circuli paffiones,
quæ ita fe habét. Sit circulus.a.b.c.in quo fit.a.d.latus quadrati infcriptibilis in ipfo
circulo,et.b.c.fit diameter ad rectos cum.a.d.in puncto.e. quod medium erit inter
a.et.d.ex.3.tertij Eucli.fit fimiliter.a.f.contingens ipfum circulum in puncto.a. quæ
protractafit vfque ad punctum.f. interfectionis cum diametro protracto, quod ita
eueniet cum anguli.a.e.f.et.f.a.e.minores fint duobus rectis, eo quod angulus.f.a.e.
acutus fit, cum.a.d.tranfeat inter centrum et.f.

Dico nunc quod productum diametri.b.c.in parte.c.e.ipfius,æqualis erit produ-
cto ipfius.c.f.in.a.d. Protrahatur imaginatione.b.a.et.a.c.vnde ex.26.tertij Euclid.
habebimus angulum.d.a.c.æqualem angulo.a.b.c.fed ex. 31. eiufdem angulus. f. a.
c.æqualis eft angulo.b.quare æqualis erit angulo.d.a.c.& ita habebimus per.3.fexti
eandem proportionem.f.c.ad.c.e.quæ.f.a.ad.a.e.fed.a.f. eft æqualis femidiametro
circuli propofiti, propterea quod fi producta fuerit à puncto.a.ad centrum.o.femi
diameter.a.o.hæc cum.o.e.faciet dimidium anguli recti, cum ex fuppofito.a. d. la-
tus fit quadrati infcriptibilis in ipfo circulo.& cum.a.f.rectum ex.17.tertij, vnde an
gulus.f.erit fimiliter medietas recti ex.32.primi, quare ex.6. eiufdem. a. f. æqualis
erit.a.o. Ergo cum proportio.f.c.ad.c.e.fit.vt.f.a.ad. a.e.erit fimiliter vt.b.c.ad.a.d.
hoc eft ut dupli ad duplum, vnde ex.15. fexti
manifeftum erit propofitum, ex quo alia paf-
fio oritur, hoc eft, quod productum.f.c.in.a.
d.æquale fit quadrato ipfius.a.c.ratio eft,quia
quadratum.a.c.æquale eft producto.b. c. in.c.
e.eo quod.a.c.media proportionalis eft inter.
b.c.et.c.e.ex fimilitudine triangulorum.a.b.c.
et.e.a.c.nam anguli.b.a.c.et.a. e. c. recti funt
et.c.cõmunis,vnde.b.erit æqualis.e.a.c.ex.32
primi, fequitur etiam, quod.a.c.fit media pro
portionalis inter.a.d.et. f. c. & hæc etiam erit
alia circuli paffio, & quia.a.c.eft latus octago-
ni igitur tale latus mediũ proportionale erit
inter latus quadrati.et.f.c. eiufdé circuli, quę



quidem.f.c.eft una portio diametri quadrati circunfcriptibilis ipfum circulum inter
circulum & angulum ipfius quadrati.
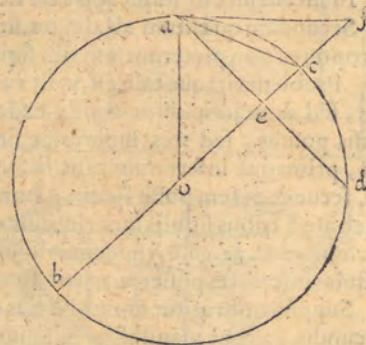
Quod

*Quod incendium, ex reflexione radiorum solarium , non fiat in cen*
*tro speculi sphærici , & aliquid contra Cardanum, &*
*de motu radiorum solarium.*

## AD EVNDEM.

Terum tibi dico, quod radij illi solares, qui à diuersis punctis ipsius solaris corpo-
ris veniunt, transeuntes per centrum speculi sphærici concâui, quamuis à super-
ficie speculi ad centrum ipsum reflectantur, vt aliàs tibi dixi, nihilominus nullo mo
do possunt aliquod obiectum incendere duabus ex causis, quarum vna est, quia cum
Sol valde remotus sit à nobis, valde etiam acutus generatur angulus coni radiorum
in centro speculi, vnde à parua superficie ipsius speculi reflectuntur, quare pauciffi-
mi radij sunt qui reflectantur in ipso centro , & propterea non sufficiunt ad combu
stionem alicuius obiecti. Alia verò causa est, quod quamuis multi , & sufficientes
radij fuissent ad côburendû velociter quoduis obiectum. impossibile tamen omnino
eset , vt aliquod obiectum comburerent, propterea quod cum radij incidentes de-
beant per centrum transire, obiectum combustibile, vt opacum, obstaret ipsis radijs,
ne vlterius transirent, vnde nulla fieret reflexio , sed etiam si dicti radij in centro re
flexi , sufficerent ad combustionem , incidentes hoc magis efficerent . & ita absque
vllo speculo, omnia & in quolibet loco comburerentur , quod manifestè falsum est.
Desine igitur mihi citare Lucillum Philalteum , qui in philosophia mathematica
fuit omnium imperitissimus. Verum speculum vstorium illud est quod ab Alhazem
Deinde à Vitellione describitur.

Quod deinde verum sit , vmbrâ vniuscuiusque corporis opaci à Sole productam
semper'esse centum nouemq; vicibus maiorem diametro eiusdem corporis, nego .

Imaginemur.s.l.diametrum esse illius circuli, quo vltimi radij solares veniunt tan
gentes corpus cuius diameter sit.c.e.et.a.i.sit diameter alterius circuli eiusdem cor-
poris solaris à quo vltimi radij veniunt tangentes corpus, cuius diameter sit. f. g. in
cadem distantia , & eodem situ prioris corporis . Tunc conus vmbræ ipsius.f.g.sit.f.
g.q. & ipsius.c.e.sit.c.n.e.centrum autem solare sit.o.conorum verò axes sint. t. n.q.
tunc ex supposito.q.f.a:n.c.s: n.e.l: et.q.g.i.erunt omnes contigui corpori solari, vn-
de ex.17. tertij Eucli.anguli.o.a.q.et.o.s.n.erunt recti. protracta deinde cum fu erit
a.s.habebimus angulos.u.a.s.et.u.s.a.minores duobus rectis. Quare. n. s. concurret
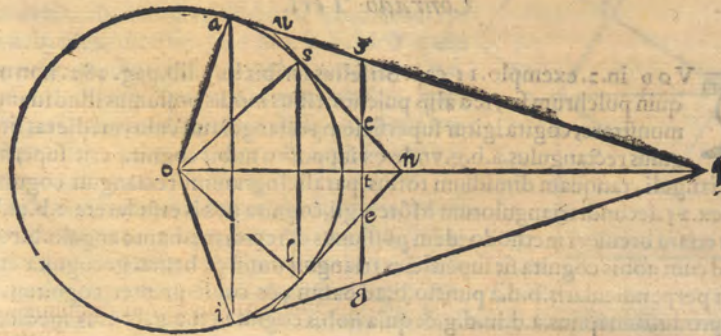
cum.a.q.in puncto.u.  Nunc verò si vmbra.t.q.tanto maior est.f.g. quanto. 109. est
vno et.t.n.etiam tãto maior.c.e.ergò eadem proportio erit.q.t.ad.t.f.quę.n.t. ad. t.
c.sed cum angulus.t.communis sit ambobus triangulis.q.t.f.et.n. t. c. sequitur ex. 6.
sexti dictos triangulos æquiangulos esse.  Vnde si anguli.t.n.c.et.t.q.f.æquales inui
cem sunt,ergo.q.f.æquidistans erit.n.c.quod est impossibile , quia nunc demonstra-
uimus ipsas concurrere in puncto.u. Quare non est eadem proportio.q.t.ad.t.f.quæ
n.t.ad.t.c.decipitur ergo Cardanus in.4.lib.de subtilitate.

   Circa illud deinde quod à me quæris,hoc est, quæ sit causa , quod nos videmus
radium solarem tardissimè moueri, cum alias tibi dixerim ipsum qualibet hora cir-
ca terram quindecim gradus perficer e,respondeo, quod radius ille quem videmus,
exempli gratia, in aliquo cubiculo,nunquam est idem numero, sed quia ipsi radij
nullo modo differunt inter se,nisi in numero,propterea putamus eundem semper esse,
cum semper alius,atque alius sit,quorum vnusquisque (de illis loquor , qui ad hunc
terræ globum perueniunt ) circa terram reuoluitur spatio.24.horarum,& cum quili
bet circulus diuidatur in.360.gradus , quorum vigesimaquarta pars est. 15 . verum
est igitur, quod tibi iam dixeram .
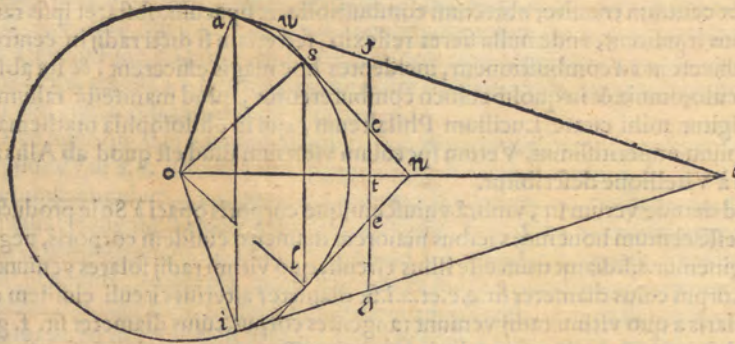


# OPERATIONES DIVERSAE AB ALIIS
## Michaelis Stifelij.

### Conrado Terl.

Vod in.2.exemplo.11.cap. Stifelius scribit in. 3.lib.pag. 282. non nego
quin pulchrum sit, sed alijs pulchrioribus modis possumus illud idem de-
monstrare; cogita igitur superficiem rectangulam, cuius medietas sit triã
gulus rectangulus.a.b.g.vnde ex supposito nobis cognita erit superficies
ipsius trianguli, tanquam dimidium totius parallelogrammi rectanguli cogniti .
Quare ex.25.secundi triangulorum Mõteregij,cognita nobis erũt latera.a.b.et.b.g.
   Alia etiam breuiori methodo idem possumus effi cere, mediante angulo.b.recto,
eo quod cum nobis cognita sit superficies trianguli simul cũ basi.a. g. cognita etiam
nobis sit perpendicularis.b.d.à puncto.b.ad basim , & consequenter cognitum no-
bis erit productum ipsius.a.d.in.d.g.& quia nobis cognita est.a.g. & eius medietas ,
                                                                              ideo

ideo vnaquæque eius pars.a.d.et.d.g. similiter nobis cognita erit ex quinta secundi Eucl. vnde ex penultima primi habebimus propositum.

Possumus item circulum mente concipere cuius.a.g. sit diameter, & ab eius centro. e. protracta cum fuerit.e.b.quæ nobis cognita erit, vt medietas ipsius.a.g. de cuius potentia, dempta cū fuerit potentia ipsi⁹ b.o.remanebit nobis potentia ipsius. d. e. & ita eius longitudo, quæ addita medietati.e.g. & detracta à dimidio.e. d. erunt nobis cognitæ.a.d.et. d. g. vnde.b.g.et.b.d.remanebunt nobis cognitæ ex dicta penultima primi Eucli.huiusmodi figuram videbis in dicto.25.problemate.2.li.Montisregij.

Aliter etiam possumus hoc idem efficere.

Sit rectangulus hic subscriptus.a.b.c.u. superficiei cognitę simul cum diametro.a. c.extendatur imaginatione.b.c.vsque ad,f. ita quod.c.f.æqualis sit.c. u. intelliganturq́; quadrata.g.f:g.u.et.u.f.vnde sūma quadratorū.g.u:u. f. cognita nobis erit ex penultima primi. nam.a.c.data nobis fuit, quare summā.g.u:u.b: et. u. f. cognoscemus, cui sūmæ addito suplemento.d.e. æquali. u.

b. dabit nobis cognitū quadratum.g.f.totale,quare cognoscetur eius radix.b. f.,cognita igitur. b. f. cum producto.b.u.illico ex.5. secundi cognoscetur.b.c. et. c. f.forte cognita.b.f. diuisa p æqualia in puncto.t.& per inæqualia in pūcto. c.Nam quadratū ipsius.t.f.cognitum, equatur rectāgulo.b.u. cū quadrato ipsius.t.c.depto igitur rectangulo,b. u.ex quadrato ipsius. t. f. relinquetur quadratum ipsi⁹.t.c.cognitum & eius radix.t.c.qua addita ipsi medietati.b.t.& depta ex medietate.f.t.relinquetur propositum.
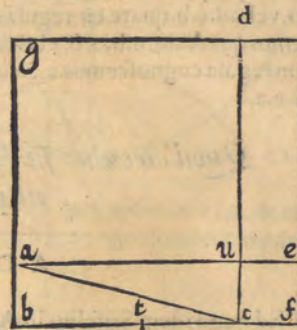
Similiter de tertio exemplo eiusdem Stifelij infero.

Sit rectangulus.a.b.c.u.cuius diametri.a.c.quantitas,simul cum proportione laterum.b.c.et.b.a.nobis data sit.cum autem scire voluerimus eius superficiem. b. u.clarum est,quod cum nobis data sit proportio.b.c.ad.b.a.illico cognoscemus etiā proportionem quadrati ipsius. b.c.ad quadratum ipsius.b.a.cum dupla sit ei quæ.b.c.ad.b. a. ita etiam & aggregati dictorum quadratorum ad quadratum ipsius. b. a. hoc est nota erit nobis proportio quadrati ipsius.a. c. diagonalis ad quadratum ipsius. a. b. idem dico de quadrato. b. e . idest quod proportio quadrati ipsius. a. c. ad quadratum.b.c. cognita nobis erit , sed . a. c. data nobis fuit, quare cognoscemus etiam omnia dicta quadrata eorumq́; radices.a.b.et.b.c. quare & superficiem rectanguli quæsitam.
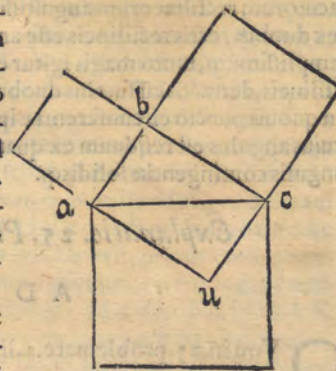
Quartum exemplum etiam faciliori via potest solui, propterea, quod cum nobis cognita sit basis trianguli cum summa reliquorum laterum , & cū angulo opposito basi ipsius reliqua cognita nobis emergunt ex.15.problemate secundi lib.de Triangulis ipsius Monteregii.

Vel

294     I O. B A P T. B E N E D.

Vel si tibi placet, accipe hanc aliam methodum à me excogitatum.

Duplicetur triangulū.a.b.c.orthogoniū, & fiat rectangulū.b.u. vt in mea figura secundi exempli hic vides. producaturq̃;.b.c.quousque.c.f.æqualis sit.c.u.vnde. b.f. cognita nobis erit ex hypothesi, quare cognoscemus etiam quadratum. g. f. à quo demptū cum fuerit aggregatū quadratorum.g.u.et.u.f.nobis cognitū ( nam quadrata.g.u.et.u.f.æqualia sunt quadrato ipsius.a.c.diagonalis datę ) remanebit aggregatum supplemétorū cognitum, quare eius medietas cognoscetur idest. b. u. vndæ ex. 5. secundi Eucli. vt superius diximus cognoscetur etiam.b.c.et.c.f.distinctæ.

Idem assero de exéplo Gemmæ Frisij à Stifelio citato in Appendice regulæ falsi.

Sit gratia exempli rectangulum hic subscriptum.a.b.datæ superficiei data etiam nobis sit proportio.a.e.ad.e.b.laterum producentium, cogitemusq̃;. a. e. producta vsque ad.o.ita vt.e.o.æqualis sit ipsi.e.b. imagine mus etiā perfectum esse quadratum.b.o. vnde ex prima sexti seu.18.vel.19. septimi vel. 15. quinti eadem proportio erit ipsius.a.b.ad.b.o.vt.a.e. ad e.o.vel ad.e.b.quare ex regula de tribus, cognoscemus quadratum.b.o.& eius radicé.e.o.& ex eadem regula cognoscemus.a.e.cum cognita nobis sit.e.o.simui cum proportione.e.o. ad. e.a.

## Quod circulus sit figura infinitorum angulorum hoc est ultima poligoniarum.

### AD EVNDEM.

S Ed quod idem Stifelius in Appendice secundi libri dicat circulum esse figuram poligoniam, non est ita mirandum, nam & alij multi doctissimi viri hanc veritatem cognouerunt, de Leone Baptista Alberto nihil dicam, cum ipse fateatur hoc accepisse à philosophis, vt etiam refert Arist.de sphæra tertio de cœlo. considera quæso in circulo, quod cum angulus contingentiæ sit angulus, quamuis omniū acutorum rectilineorum angustissimus, vnde ex communi ratione sequitur reliquum ex duobus rectis rectilineis esse angulum, & si omnium obtusorum rectilineorum sit amplissimum, tanto magis igitur erit angulus, id quod remanet ex duobus rectis rectilineis, detractis cū fuerint duobus angulis contingentiæ, qui quidem angulus erit in quouis puncto circunferentiæ ipsius circuli, idem intelligendum est de sphæra, cuius angulus est residuum ex quatuor rectis solidis, detractis cum fuerint quatuor angulis contingentiæ solidisq̃;.

## Explanatio. 25. Problematis lib.2. Monteregij.

### AD EVNDEM.

Q Vod in.25.problemate.2.lib.de triangulis Monteregium non intelligas, mirum non est, eo quod quandoque bonus dormitat Homerus. Puto enim illud problema ab ipso Monteregio non fuisse visitatum. Sed ne me aliquo modo culpes, accipe hanc aliā methodū à me aliter etiā excogitatā in eadem ipsius figura.

Propo-

Propositū sit nobis triangulum.a.b.g.cuius basis data sit cum area, seu perpendi-
culari.a.d.cum angulo etiam.a.ad cognoscendum autem.a.b.et.b.g.cogitemus circu
lum.a.b.q.g. circunscribere ipsum triangulum cuius diameter.p.q.ad rectos se-
cet basim.b.g.in puncto.m. cogitemus etiam.b.p.et.p.g.vnde ex.20.ter-
tij Euclid.angulus.b.p.g.æqualis erit
angulo.a.& angulus.m.p.b.erit eius di
midium,quod ex te ipso cognosces,&
angulᵒ.p.b.m.similiter cognoscetur,
quare ex.29.primi eiusdem Montere
gij cognoscemus.p.m.et.p.b. (nam.b.
m.datum fuit, vt dimidium totius ba-
sis.b.g.) ducta postea.b.q.ex eadē.29.
cognoscemus.p.q.cum.p.b.iam cogni
ta fuerit, à qua.p.q. (diametro) dēpta
p.m.remanebit.q.m.cognita, cū qua
iuncta cum fuerit.m.t.æquali.a.d. per
pendiculari,dabitur.q.t. et.t.p.inter
quas.a.t.media proportionalis loca-
tur, quare cognoscemus.a.t.quæ sinus
est arcus.a.p.vnde cognitus erit arcus
a.p.sed arcus.p.g.cognitus est median
te angulo.p.b.g.cognito, qui quidem
arcus.p.g.si coniunctus fuerit cum arcu.p.a.cognoscemus compositum.a.g.& eius
chorda similiter (hoc est secundū latus) qua cognita, illico cognoscemus chordam
a.b.hoc est tertium latus trianguli propositi.
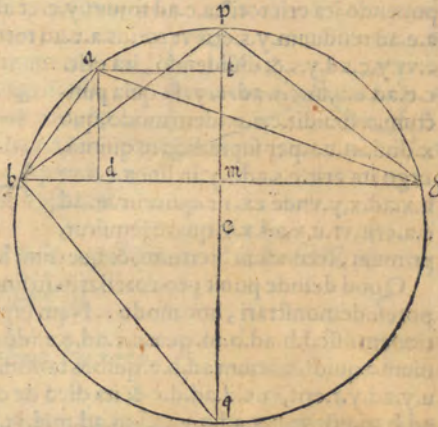
## Quædam notanda in Federicum Comandinum.

### AD EVNDEM.

PVtabas enim me ioco dixisse Federicum Comandinum non omnino irrepræ-
hensibilem esse, vide igitur, quod scribit in quinto lemmate in decimam
propositionem libr.2. de insidentibus aquæ Archimedis, volens demonstra-
re eandem esse proportionem.l.b.ad.b.m.quæ.c.e.ad.e.a.vbi est aliquo modo pro-
lixum, mediante linea.c.p.cum suis partibus, citans etiam antecedens lemma extra
propositum, eo quod nec in antecedēte lemmate, nec in alio, ipse vnquam proba
uerit proportionem.c.d.ad.d.q.esse,vt.l.b.ad.b.m.sed ne putes me falli, tibi demon
strabo non esse necessarium ducere lineam.c.m.p.vel.q.p.eo quod cū per quintam
lib. de quadratura parabolę Archimedis, ita sit.c.d.ad.d.e.vt.l.b.ad.b.m. existente
a.c.dupla ipsi.d.c.et.e.e.dupla ipsi.g.c.et.l.d.dupla ipsi.l.b: erit, primo componen-
do.c.e.ad.e.d.vt.l.d.ad.d.m.& per æqualitatem proportionum, ita erit.e.g.ad.e.d.
vt.b.d.ad.d.m.& per.19.quinti Eucli.ita erit.e.g.idest.g.c.ad.g.d.vt.b.d.idest.l.b.
ad.b.m.sed.c.g.ad.g.d.est vt.c.e.ad.e.a.ratio est, quia componendo ita est.c.d.ad.d.
g.vt.c.a.ad.a.e.& hoc est,quia permutando, ita est.a.c.ad.d.c.vt.a.e.ad.d.g.& hoc
verum est ex.19.quinti eo quod totius.a.c.ad totum.d.c.est vt abscissi.c.c.ad abscis
sum.g.c.vt supradixi.

Sed

Sed etiam alio vniuerſaliori modo potes probare, quod ita ſit.u.x.ad.x.y.vt.c.e.
ad.e.a.cogitando in linea.c.a .punctum quoddam quod vocabimus ſimiliter . y . in
tali ſitu locatum , quod diuidat.c.a.eadem proportione qua.y.diuidit.u.s.vnde cum
e.s.diuiſa eodem modo etiam ſit à puncto.s.ex ſupradicta quinta lib. de quadratura
parabolæ, erit igitur proportio.a.y. ad.y.c.vt.e.s.ad.s.c.per.11.quinti Eucli.& com
ponendo ita erit toti⁹.a.c.ad totum.y.c.vt abſciſſi.s.c.ad abſciſſum.s.c.quare reſidui
a.e.ad reſiduum.y.s.erit vt totius.a.c.ad totum.y.c.& permutando, ita erit.a.c.ad.a.
e.vt.y.c.ad.y.s.& diuidendo , ita erit.

c.e.ad.e.a.ut.c.s. ad. s. y. & quia pun-
ctum.s. diuidit.c.a.eodem modo,quo
x.diuidit.u.s.per ſupradictam quintã,
ergo ita erit.c.s.ad.s.y.in linea.c.a.vt
u.x.ad.x.y.vnde ex.11.quinti.c. e. ad
e.a.erit,vt.u.x.ad.x.y.quare ſequitur,
primum , ſecundum , tertium, & quartum lemma ſuperflua eſſe.
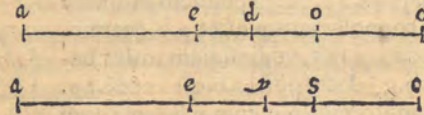
Quod deinde ponit pro corellario in fine.6.lemmatis, aliter quam per.6.lemma
poteſt demonſtrari , hoc modo . Nam ſuperius demonſtrauimus eandem propor-
tionem eſſe.l.b.ad.b.m.quæ.c.e.ad.e.a.idē dico de proportione.u.x.ad.x.y.& om-
nium æquidiſtantium ad.h.e.quibus rationibus mediantibus eodem modo ſcies, ꝗ
u.y.ad.y.r.erit,vt.c.d.ad.d.c & ita dico de omnibus æquidiſtãtibus.ad.h.e.vnde.l.b.
ad.b.m.erit vt.u.x.ad.x.y et.l.m.ad.m.d.vt.u.y.ad.y.r.per.11.quinti , ſed cum ſit. l.
b.ad.b.m.vt.u.x.ad.x.y.componendo erit. l. m. ad.b.m.vt.u.x. ad.x.y.& euerſim.b.
m.ad.m.b.erit,vt.x.y.ad.y.u.& per æquam proportionalitatem erit.b.m.ad.m.d.vt
x.y.ad.y.r.quod eſt propoſitum.

Non video etiam, quare ipſe ducat lineam.s.r.cum in ipſo contextu nihil faciat
de dicta.s.r.

Comentum poſtea contextus.P. pulchrius eſſet,ſi diceret,quod cum ita ſit totius,
l.a.ad totum.a.d.ſic ſe habebit abſciſſum.a.i.ad abſciſſum.a.z.eo quod ita eſt,vt ſcis,
hoc eſt in proportione dupla, ergo reſidui.i.l.ad reſiduum.d.z.erit vt totius. a . l. ad
totum.a.d.hoc eſt in proportione dupla.

## De Viſu.

## AD EVNDEM.

Atio vnde fiat, vt videamus diſtinctè omnes eolores,cum in qualibet aeris par
te, quo lumina reflexa poſſunt peruenire mixta ſint, & non diſtincta,oritur à
paruitate ipſius pupillæ oculorum , & à magna expanſione virtutis viſiuæ in ſuperfi-
cie concaua orbis continentis humores diaphanos oculorum per ramuſculos nerui
optici remotè ab ipſa pupilla. & quamuis radii luminoſi frangantur ab vnoquoque
humore diuerſimodè, hoc nihilominus maximè iuuat ad diſtinctionem radiorum ,
ſed & ſi directè procederent, idem ferè eueniret, non tamen ſuis locis,cogita exem-
pli gratia lineam.a.u.e. vt communis ſectio cuiuſdam plani ſecantis ſphæram oculi ,
per centrum ipſius, & pupillæ , et. o. punctum ſit proximum centro ipſius pupillæ ,
ſed interius aliquantulum,extra autē oculũ,ſint varij colores,vt.c.n.t.in dicto plano.

Iam nulli dubium eſt quod lumina quæ producuntur ab.c.n.t.ad.o. in ipſo.o.mi-
xta,

xta, & non diftinca, procedendo igitur vlterius ipfi radij citra.o.tunc difgregatur, & feparantur abinuicem, & cũ peruenium ad lineam. a.u.e. fentiuntur diftincti alij ab alijs. Cuius quidem rei , exemplum manifeftum accipere poffumus à quouis cubiculo ex omni parte claufo, quod tranfitum nullũ permittat radijs luminofis,ni fi per aliquod paruum foramen, in quo foramine , & extra ipfum cubiculum, omnes radij mixti erunt, fed in obiecto pariete ipfius cu-
biculi videb untur diftincti, vnde fequitur, quòd
quo remotius erit obiectum.c.n.t.ab.o.tanto acu-
tior erit angulus.c.o.t.& fuus contrapofitus fimili-
ter, & perconfequens linea. e.u.a.breuior erit, &
puncũ. o . propinquius etiam erit ipfi lineæ.a.u.e.
quæ omnia efficiunt, vt nobis obiectum.c.t.paruũ,
& minus diftinctum, feu magis confufum appareat .

## DE APPARENTI DISTANTIA PARTIVM hæmifphærij .

### Anfelmo Fucaro.

G Ratæ mihi tuæ literæ fuerũt, quibus oftẽdis non paruũ defideriũ fciẽdi vnde
fiat, quod cum dies illucefcit, & eft ferena pars Cœli, circa axem orizontis
demiffior appareat, quam aliæ partes,cũ ab alijs ( quod fciã)fatis expreffum nõ fue
rit, fed quia de eo à me aliquid fcire defideras dicam quod mihi vr . Scias non folũ
multitudinẽ obiectorũ oppofitorũ efficere, vt aliqua res alia longius diftare videat,
vt alij putarũt, fed etiam diuerfitates colorum, quamobrem cum decipiamur, cre-
dentes Cœlum effe præditum colore cęruleo , cum is color , aeri , non Cœlo
conueniat,& videntes huiufmodi colorem circa axem orizõtis magis denfum, quã
verfus ipfum orizontem , ratione exiguæ reflexionis, à pauca quantitate vaporum
inter noftrum fitum , & reflexionis locum,iudicamus Cœlum proximiorem effe cir-
ca dictum axem, quam fint aliæ partes ; præterquam,quod is color, qui videtur
terminare, aut impedire radium vifualem(aduertas tamen me hac in re platonicum
non effe ) eo femper propinquior effe videtur, qui ei locum dat, & hanc ob caufam
videntes nos dẽfitatẽ cęrulei circa axem orizontis, & cernentes amplitudinem gy
ri aliarum partium,adducimur,vt putemus eã parte
viciniorem effe. Neq; illud etiã omittã hoc etiã fie
ri ratione imaginationis,vnde etiã multis contrariũ
euenire poteft,ideft vt eis magis profundum videa
tur Cœlũ,circa axem orizontis, quam vicinum gy
ro eiufdẽ orizõtis,iudicantibus eã parté lõginquio
rẽ effe, quæ fefe magis obfcurã oculo demõftrat,&
eã propinquiorẽ quę fefe clariorẽ oftendit,vt ei ẽt
contingere poteft,qui fubfcriptã figurã cubicã non
quidẽ ductã fecundũ ordinẽ opticè,fed ita , vt om-
nia latera oppofita inuicẽ fint parallela,profpiciet,
ideft. a.i.ad.e.t.et.c.u.ad.o.n.et.a.i.ad.c.u.et.e.t.ad
o. n. vnde fequitur , vt aliquando quadratum. a.o.
videbitur citra,et.i.n.vltra dictũ cubum aliquando
verò èconuerfo.
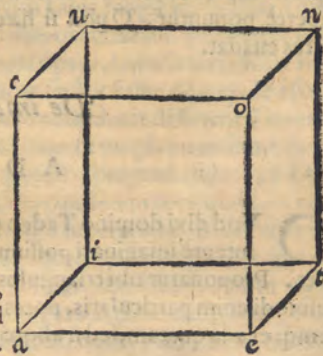
Pp De

# DE PHILOSOPHIA MATHEMATICA.

## *Dominico Pisano.*

SI omnia vno colore constarent, & corporum vmbræ à luminibus non di-
stinguerentur, neque diuersitas situs, lumina, quæ veniunt ad oculum non al-
teraret; perinde esset, ac si essemus cœci . Miror quod cum in Aristotele sis
versatus, in tuis tamen scriptis philosophum à Mathematico separes, quasi mathe-
maticus non sit adeò philosophus, vt est naturalis, & metaphysicus, cum multo ma
gis quam ij philosophus sit appellandus, si ad veritatem suarum conclusionum respi
ciamus . Verum quidé est, te in huiusmodi errore solû non versari; sed grauius est,
quod cum vos videatis etiam res morales sub philosophię appellationé cadere , non
animaduertatis diuinas scientias mathematicas etiam philosophiæ nomine ornan-
das esse . Quod si eiusdem nomen penitius considerare velimus, inueniemus aper-
tè, mathematico magis illud ipsum quàm cuilibet alio conuenire, cum nullus ex alijs
tam certo sciat id quod affirmat quam mathematicus, neque aliquis sit, qui in co-
gnitionis, & scientiæ cupiditatem magis ducatur, vt apertè patet, cum nec etiam ipsi
sensui det locum, neque aliquid præsupponat, quod non sit ita verum & intellectui
notum, vt nulla quæuis porentia , illud esse falsum ostendere queat. Sed quia
Græci, qui ad placitum nomina rebus imposuerunt, voluerunt etiam , non solum
mathematica, sed etiam naturalia, metaphysica, & moralia, sub communi philoso-
phiænomine contineri. Vt aût tibi satisfaciam authoritate Aristotelis, quem tanto-
pere colis, primum considera, nunquam eum de philosopho métionem facere quin
prius aperiat de quo philosopho loquatur, atque hoc semper præstat, exceptis qui-
busdam locis, vt cap. 2. lib. 4. Metaphysicorû, vbi de philosopho in genere loqués,
ait, proprium esse philosophi. vt res omnes speculetur atque hoc in principio quin
ti textus asserit, cum in quarto iam ostenderit mathematicum esse philosophum:
omitto quod in. 2. textu secundi physicorum idem affirmet , æquum esse appellare
philosophiam scientiam veritatis, & finem speculatiuæ existere veritatem. An non
idem in primo cap. 6. metaphisicæ philosophiam speculatiuam, mathematicis phy
ficis & supernaturalibus rebus contineri? An non idem paulo inferius scribit physi-
cam primam futuram, si aliæ substantiæ quam naturales non reperirentur? considera
deinde quid dicat in fine tertij cap. lib. 11. quo loco nil clarius esse potest, lege etiam
quæ. 6. cap. eiusdem libri ab eodem adducuntur, & quæ in. 8. cap. 12. libri textu. 44.
apertè ponuntur. Quod si hæc tibi non sufficiunt, vereor ne tuus morbus despe-
ratus euadat.

## *De imaginatione specierum.*

### AD EVNDEM.

QVod dixi domino Tadeo est, quod aliquas particularium species, perfectè &
integrè imaginari possumus, alias non item, id tibi melius exemplo innote-
scet. Proponatur tibi triangulus æquilaterus datæ magnitudinis, datiq; coloris, hu
iusmodi enim particularis, potes imaginatione tibi fingere integram speciem, tora
lemq; ei adæquatam, sed si aliquam speciem aliquando vniuersaliorem imaginatio

ne

Pp          De

ne concipere velles , quemadmodum vnius trianguli equilateri, tali magnitudine ,
sed non præfinito colore constantis, hoc minime præstare posses. quia nullam rem
visibilem priuatam colore imaginari possumus. nec etiam potes imaginari speciē ali
cuius trianguli æquilateri, indeterminatæ magnitudinis, & indefiniti coloris, quæ
cuilibet particulari cuiusuis magnitudinis,& coloris postea applicari queat. Species
deinde alicuius trianguli equicruri , aut vnius trianguli laterum inequalium, aut triā
guli in genere, aut tandem figuræ, considerato tu ipse, an possit sub imaginationem
cadere. Possumus quidem huiusmodi speciem ( ratione mediante ) intelligere, vn
de quamlibet speciem rei particularis visibilis, compositæ, ex figura,magnitudine,
& colore, perfectè imaginari possimus , & huiusmodi conceptus erit specialissima
species, quia in infinito suorum indiuiduorum,nunquam fiet, vt aliquod eorum, ali
quo modo ab alijs differre possit ; admonens te , nil reperiri , quod differat, aut in se
partem aliquam habeat, quod aliquid aliud non obtineat, quin dicta differentia sit
specifica, eius tamen solum partis quæ differt ab alia duorum indiuiduorum , vnius,
eiusdemq; speciei . quia si est in magnitudine nulla planè magnitudo reperitur, quę
sua specie non sit dotata , quod si non esset , inter res omnes nulla æqualitas
eluceret : & si in figura , & colore, idem affirmo, aliter nulla res similis esset
alteri , neque aliq̄ia similitudo reperiretur . Idem de quolibet alio obiecto
sensibili dico. Ratio autem eorum omnium quæ dixi est , quia imaginatiua nihil
aliud intellectui ostendere potest , quam id quod recipit à sensu, & cum sensus ,
alio modo moueri non possit quam supradicto, hanc ob cau sam verum est , qcquid
scripsi . Vnde triangulum equilaterum datę magnitudinis,erit genus triangulorum
equilaterū eiusdē datæ magnitudinis,sed diuersorū colorū,erit etiā species trianguli
æquilateri indeterminatæ magnitudinis, & hic deinde erit species trianguli , & hic
postea species figuræ. Idem de alijs omnibus rebus per gradus dico, quę sicut à sen
su,ita etiam ab imaginatione longè recedunt,adeo vt has species specialissimas tan-
tum,idest eas solùm , quas hic superius descripsi, integrè capere possit : at verò gene
ra, quanto vniuersaliora sunt, ab eadem imaginatione,tanto longius distant.

## *De maculis Lunæ, & eius lumine.*

## AD EVNDEM.

MAculæ Lunæ,nihil aliud sunt,quàm partes ipsius Lunę magis perspicuæ,à qui
bus, lumen non reflessum , sed penetrans, nobis occultatur; quemadmodū
via lactea,nihil aliud est, quam pars octaui orbis magis opaca,à qua lumen Solis re-
flessum, sese nobis ostendit . Quod autem Maurolicus scribit folio.64.cap.de astro
rum fulsionibus,circa Lunā,est falsum; primo,quia non considerat differentiā inten-
sionis luminum inter Venerem,& Lunam, cum lumē illius sit magis intensum,quam
Lunæ, quia quilibet qui sano sit oculo , facile potest compræhendere, si Lu-
na esset, vbi est Venus, aut Venus vbi reperitur Luna(quibus in locis eiusdem ma
gnitudinis nobis apparerent ) ipsa Luna à Venere longè superaretur , & excedere-
tur splendore, & lumine, ita vt si etiam verum esset , quod per tres gradus inter-
ualli sese nobis proderet sexagesima pars luminis ( quod in quadraturis nec in vllo
alio situ verum euadit, respectu ad Solem, idest vt tres gradus differentiæ situs, con
stituant sexagesimam partem differentiæ luminis respectu nostri) non ideo tamen

dictum lumen conspiceretur, quia non sufficit extensio luminis, cum eiusdem inten
sio sit etiam necessaria. Sed id quoque tibi dico, quod etiam si dicta sexagesima
pars totius luminis lunaris, eadem intensione splendoris, & luminis Veneris, in tali
distantia trium graduum à Sole praedita esset, non eam tamē videremus, ratione ob
liquitatis curuę, & sphæricę superficiei Lunæ, respectu nostri, in huiusmodi situ: id ꝗ
tibi ita demonstratum volo.

Pars superficialis lunaris globi, quæ nos respicit sit.a.p.u. quam accipere possu-
mus pro medietate ipsius superficiei totalis, eo quod respectu nostri visus, insensibi
liter, ab ipsa medietate differat, pars autem à Sole visa sit.u.q.a.cogitemus etiam cir
culum.a.p.u.q.vnum esse ex maioribus ipsius globi, cuius superficies trāseat per ocu
lum videntis, vnde pars eius.a.p.u.diuidet vmbram per æqualia, reliqua verò pars.
a.q.u.diuidet per æqualia lumen ipsius Lunæ à Sole receptum, ita quod pars illumi
nata, erit medietas.u.q.a.excessus verò, cum nostro visui incompræhensibilis sit, pro
nihilo reputetur, cuius causa est, maxima illa distantia, quæ inter Solem, & Lunam
reperitur, quamuis Sol maior sit Luna multis millibus vicium, eo quod tunc inter So
lem, & Lunam reperiantur plus quam.570.diametri terræ.

Supponamus nunc Lunam remotam esse à loco ipsius cōiunctionis cum Sole per
3. gradus. vnde quéadmodum prius
lumen erat in gyro.a.q.u. nunc re-
periatur in gyro.x.q.t. ita quod.t.u.
erit sexagesima pars ipsius.a.p.u. qđ
à vero sensibiliter non discedit.
Imaginentur nunc duæ rectæ lineæ
ductæ ab oculo.d.ad puncta.t.et.u.
verum tamen est quod linea.d.u. se-
cabit arcū.t.u. sed ita propinqua pū
cto.u. quod erit ei ferè contingens,
vnde absque sensibili errore possu-
mus arcum.t.u.intelligere inter duas
lineas.d.t.et.d.u.quapropter tale lu-
men compræhendetur, ferè, sub an-
gulo.t.d.u. quem quidem angulum
oportet nos videre, cuius magnitu-
dinis existat, respectu totalis anguli
a.d.u.protracta cum fuerit.d.a.

Producatur primo.d.t. vsque ad
diametrum in puncto.i. deinde per
puncta.a.et.u.ducatur arcus.a.e.u.cir
ca.d.cétrum, ad quem ducatur linea.
d.t.i.in puncto.e. sed quia, cum dia-
meter.a.u.tam breuis sit respectu di
stantiæ à terra, tempore interlunij,
vnde minor cétesima parte ipsius di-
stantiæ existit, sequit nos posse absꝗ
sensibili errore cogitare, à puncto.d.
ad quoduis punctum ipsius diametri
omnes lineas ad angulos rectos cum
ipso diametro, & insensibilis inæqua
litatis
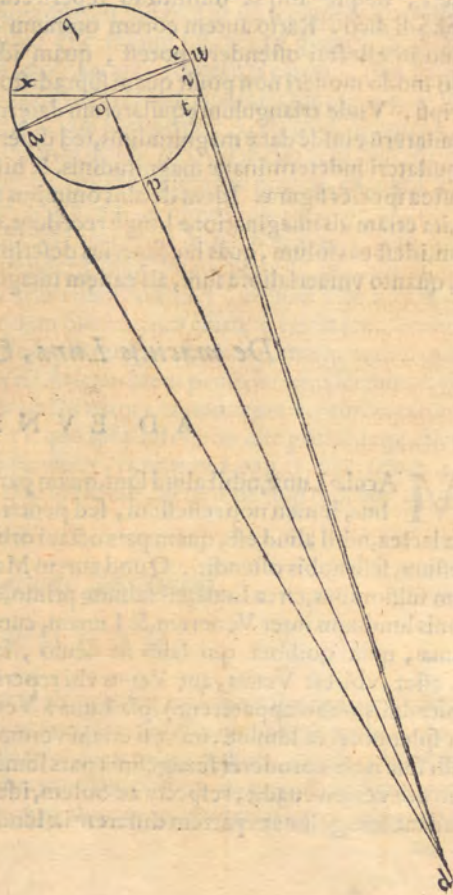
litatis à linea.d.o. Accipiemus igitur.t.i.pro finu arcus.t.u. qui eſt graduum. 3. hoc
eſt ſexageſima pars ſemicirculi graduum. 180.quapropter.t.i.erit partium.5233.ta-
lium qualium.o.u.eſt.100000.cuius. t.i.quadratum demptum cum fuerit à quadra-
to ſemidiametri.o.t.relinquet nobis quadratum ipſius.o.i.quæ quidem.o.i.vt radix
quadrata,erit partium. 99862.talium qualium ſemidiameter eſt. 100000. vnde.i.u.
reſiduum diametri, remanebit partium.138. Vel ſic, cum cognitus ſit nobis arcus.
t.u.illicò cognoſcemus ſinum arcus.p.t.complementū vnius quartæ,qui ſinus æqua-
lis erit ferè arcui.o.i.partium.99862.vnde.i.u.erit, vt dictum eſt, partium.138.quę
quidem.i.u.æqualis eſt ferè ſinui arcus.u.e.& ita etiam.u.e. quare ſi diuiſa fuerit to-
ta.a.u.partiū.200000.per.138.proueniet nobis.1449.& ſic angulus.t.d. u. erit vna
partium.1449.anguli.a.d.u. Conſideremus igitur quomodo fieri poteſt , vt oculo
compræhendatur hæc tam parua particula luminis lunaris.

## SOLVTIONES ALIQVAE.

### *Paulo Aemilio Raiſeſtaim.*

POſt eas literas quas proximè ad te dedi, Franciſcus Monardus mihi retulit tuas
nonnullas dubitationes circa noſtrum Theorema Arithmeticum.116.quarum
prima eſt,quod ſi numerus.a.cogitat⁹,eſſet æqualis.4.tunc ipſe non eſſet multiplex
ipſi.4.de quo tamen nullam mentionem feci. Idem etiam inquis,ſi.a.fuiſſet.5.6.7.
nec non.1.2.et.3. Cui reſpondi, quod quãuis nullam fecerim mentionem de æqua
litate ipſius.a.cum.4. nihil tamen refert, propterea quod quando ita fuiſſet, nihi-
lominus eaſdem conditiones ſubiret,quemadmodū ſi fuiſſet duplus,triplus,aut qua
druplus. eò quod à genere multiplici, æqualitas, formam diuerſam non induat.Qua
re idem eueniet ſi.a.fuerit.4.5.6.7.vt ſi eſſet.8.9.10.et.11.& ſic de cæteris, excepto
quod in proprijs multiplicibus, vel in ſuperantibus ipſis multiplicibus.a. menſurare
tur ab ipſo. 4. plus quam ſemel. Quod autem dicis.de. 1. 2. et. 3. nihil eſt, quia,
vt in ſecunda ſumma, hoc eſt in tertio termino maximo, reliquus tertius terminus,
ideſt.9.non compræhendetur, ita nobis indicabit primum numerum ſumptum mi
norem eſſe quaternario. Quæ omnia, ex ipſa noſtra theoria ibidem expreſſa ma-
nifeſtantur. Quid autem circa hoc Frater Lucas dicat, neſcio, quia ipſius opus
ad manus meas nunquam peruenit, ſatis enim mihi fuit , in Tartalea hanc praxim
vidiſſe, ratio vero nullibi à me reperta fuit. Tartalea enim multos citat authores,
quorum ſcripta ego nunquam vidi, vt Leonardi Piſani, Proſdocimi, Petri Borghi,
Fratris Lucæ,Ioannis Sfortunati, cæterorumq; ſimilium.

ELI-

## ELIPSIM PROPOSITAM QVALITER
### quadrare valeamus.

*Illustri Viro Francisco Mendoza.*

V o d antea tuo nomine fecerat Marcus Antonius amicus noster sufficie-
bat . Sed quia, quæ nunc à me petis, talia sunt, vt sine tripartita equa-
liter aliqua data proportione non possit aliquis exactè intentum perfice-
re, nihilominus, supposita di
cta diuisione, reliqua facilia erūt. Primū
enim est. Propositam Ellipsim qua-
drare.

Sit igiī Ellipsis proposita.a.b.d.c. cu-
ius axes sint.a.b et.d.c.dati, seu repti ex
47.secūdi Pergei, sintq; duo circuli.a.e.
b.f.et.g.d.h.c.circa easdem diametros,
tūc proportio.a.b.ad. d.c.dimidiū erit
proportionis circulorum ex.2.12. Eu-
clid. sed proportio.a.b.ad.d.c.æqualis
est proportioni maioris circuli ad Elli
psim.ex.5. Archimedis in lib.de cono
idalibus, quapropter proportio Elli-
psis ad minorem circulum altera me-
dietas erit totius proportionis circulo-
rum, hoc est maioris ad minorem,qua
re Ellipsis media proportionalis erit
inter eos circulos . Nunc verò cum
ex Archimede repertę fuerint duæ fi-
guræ rectilineæ æquales duobus circu
lis iam dictis, & inter has, reperta fue
rit alia media proportionalis proposi-
tum obtinebimus.
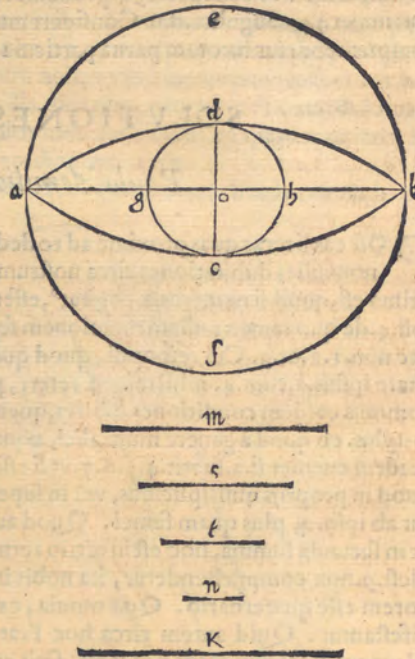


*Sphæroidem propositam cubare.*

### AD EVNDEM.

P Roposita sphæroides erit, aut prolata, aut oblonga, sit prius prolata, sitq;.a.b.
diameter circuli, qui eam per æqualia secat,circa quam.a. b. vt circa axem in-
telligatur sphæroides oblonga, cuius spissitudo sit.d.c.axis prolatæ, cogitemus nūc
duas sphæras.a.e.b.f.et.g.d.h.c.circa dictos axes . Vnde quatuor corpora habebi-
mus,hoc est duas sphæras, & duas sphæroides, quas probabo continuas proportio-
nales inuicem esse.

Consideremus igitur duos conos rectos,quorum.a.b. diameter sit eorum basium,
altitudo autem maioris, æqualis sit semidiametro maiori, hoc est medietati.a.b.al-
titudo

titudo verò minoris, æqualis fit femidiametro minori, hoc eft medietati.d. c. vnde
habebimus proportionem coni maioris ad conum minorem, eádem quæ eft diame
tri maioris ad diametrum minorem,quod ex.2.parte.11. duodecimi Eucli.nec non
ex.9.eiufdem manifeftum eft, fed conus minor, eft quarta pars fphæroidis prolatæ
ex.29. Archimedis in lib.de conoidalibus , & conus maior, eft etiam quarta pars
fphæræ,ex.32.primi lib.de fphæra, & cyllindro, quare ex communi fcientia, eadé
proportio erit fphæræ maioris ad fphæroidem prolatam, quæ.a.b.ad. d. c. fed pro-
portio.a.b.ad.d.c.eft tertia pars proportionis maioris fphæræ ad minoré.Confidere
mus nûc alios duos conos rectos,vnius & eiufdé bafis,cui⁹ diameter fit.d.c. fed altitu
do maioris, æqualis fit femidiametrofphęre maioris, altitudo verò minoris,fit æqua
lis femidiametro minoris fphæræ, vnde ex dictis rationibus habebimus proportio-
né maioris coni ad minoré,vt quæ eft.o.b.ad.o.d.hoc eft vt.a.b.ad.d.c.& ex dictis ꝓ
pofitionibus ita fe habebit fphæroides oblonga ad fphęram minorem vt.a.b. ad. d.
c. hoc eft tertia pars proportionis fphæræ maioris ad minorem. Quare proportio
fphæroidis prolatæ ad oblongam, erit reliqua tertia pars proportionis maioris fphę
ræ ad minorèm. Quapropter hæc quatuor corpora continua proportionalia inui-
cem erunt.

Nunc verò quærenda eft inter.a.b.& fuas duas tertias partes vna media propor-
tionalis, quæ fit. K.& ex Archimede,inuentum fit quadratum ęquale circulo, cuius
fit.K. diameter. Vnde proportio circuli ( cuius.a.b.eft diameter ) ad circulum cu-
ius.K.eft diameter,fefquialtera erit ex.2.12.Eucli.

Ducatur deinde quadratum lineæ. K. in lineam. a. b. & proueniet nobis cor-
pus quoddam,quod æquale erit fphærę maiori, ex corellario.32. primi de fphęra &
cyllindro, cuius corporis,latus cubus fit.m.

Idem facere oportebit mediante.d.c.minoris fphærę, cuius corporis cubica ra-
dix fit.n.

Nunc verò inter.m.et.n.inueniantur duę medię proportionales.s. t.& ex. s. pro-
ducatur cubus, qui ęqualis erit fphęroidi prolatæ propofiti, cubus vero . t. æqualis
erit fphęroidi oblongę , cuius axis effet.a.b.

Si autem fphęroides oblonga nobis propofita fuiffet, eodem methodo foluere-
tur problema.

## *Quadratum circulis mediantibus defignare.*

### AD EVNDEM.

**M**Odus autem conficiendi quadratum ex circulis fupra datam lineam, vt Do-
minum Gafparem docui,facillimus eft.

Sit enim linea.b.a.46.propofitionis primi Euclidis,pofitoq; pede immobli circi-
ni in puncto.a.fecundum quantitatem lineæ.a.b.propofitę fiat circulus, fimiliter cir-
ca punctum.b.alius circulus eiufdem magnitudinis, erecta deinde fola.a.c. perpendi
culari ipfi.a.b.ex puncto.a.ipfa fecabitur à circunferentia circuli. cuius centrum eft.
a.in puncto.c.vnde. a.c.æqualis erit.a.b. pofito demum pede immobili ipfius circi
ni in puncto.c.fecundum longitudinem ipfius.c.a.fiat alius circulus, qui æqúalis erit
reliquis duobus circulis cum eorum femidiametri æquales fint , & hic vltimo factus
fecabit circulum, cuius centrū eft.b.in pūcto.d.à quo cum ductæ fuerint.d.c.et.d.b.
rectè

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rectè habebimus quod volumus. nam omnia latera sunt inuicem equalia ex condi-
tionibus circuli, angulus autem.a.rectus effectus fuit, tunc si imaginatione cogita-
ta fuerit diameter.b.c.ex.8.primi, concludemus angulum.d.esse rectum deinde ex.5
et.32.eiusdem concludemus etiam reliquos angulos rectos esse.

Circa verò id quod mihi scripsisti de igne perpetuo putans nugas esse, quod Ro-
mæ inuentæ fuerint lucernæ ardentes in sepulchris antiquorum . Egò quidem mi-
nimè puto eas nugas esse, propterea quod tales lucernas non vnus tantum aut duo
viderint, sed multi homines fide dignissimi . Prætera cum ais id nulla ratione posse
fieri . Respondeo quod maxima ratione possibile esse puto, quam quidem ra-
tionem ita esse oportet, quod  primum lucerna sit perfectè circunclusa, vt
materia in ea constituta nullo modo exire possit, deinde quod materia inflamabilis
talis sit, vt excrementum fuliginosum ex flamma transmissum, tangendo superfi-
ciem deuexam ipsius lucernæ, aptum sit in pristinū humorē congelari, siue transfor-
mari, vnde materia prima per tres formas perpetuò transibit, hoc est per humorem,
siue oleum tale, vt diximus, per ignem, seu flammam,& per vaporem, seu exhala-
tionem fuliginosam aptam condensari, atque in priorem humorem illicò reuerti.

## DE DIVISIONE TRIANGVLI SECVNDVM
### propositam proportionem.

### Michaeli Angelo Muciasco.

V o d  mihi proponis, tale est, vt scilicet tibi modum scribam diuidendi
triangulum propositum secundum datam proportionem à linea transeun
te per punctum notatum extra triangulum.
    Triangulū igit à te mihi propositum sit.n.o.u. considero primū quod si
quis ipsum diuiserit in duas partes mediante.e.s. parallela ad. n. u. ea proportione,
quam mihi proponis. deinde inuenerit in dicta. e. s. punctum. r. per quod transiens
alia linea à puncto. p. proposito, ita quod efficiat duo triangula.m.r.e.et.r.s. x. inui-
cem æqualia, problema solutum erit.

co quod triangulum.m. o. x. æquale
esset triangulo.e. o. s. & quadrilate-
rum residuum.m.n.u.x.etiam equale
esset quadrilatero.e.n.u. s.

Sed dum punctum.r.uenarer, alia
via mihi in mentem venit, cognoui
igitur quod quum propositum expe-
ditum fuisset, hoc est,ꝙ si à puncto p.
protracta esset linea.p. m. quę trian-
gulum.n.o.u.in duas partes inuicem
ita proportionatas diuisisset, vt se ha
bet.A. et.B. ita se haberet productū
n.o.in.o.u.ad productum.m. o. in.o.
x. vt trianguli.n. o. u. ad triangulum
m . o. x. quod quidem non est diffi-
cile speculari, ex methodo. 24.sexti,
                eo quod
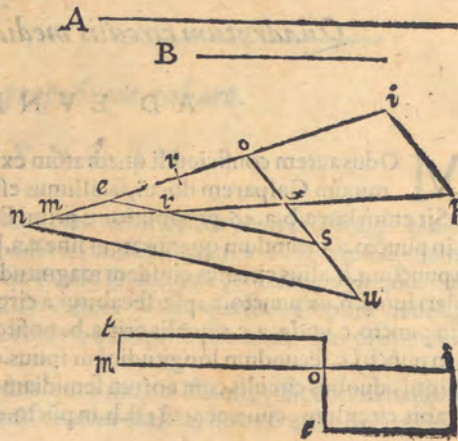
eo quod tam proportio producti.n.o.in.o.u.ad productum.m.o. in. o. x. quam pro-
portio trianguli.n.o.u.ad triangulum.m.o.x.componitur ex proportione.u.o.ad.o.
x. & ex proportione.n.o.ad.m.o. vnde proportio dictorum productorum nobis co-
gnita erit, eo quod cum nobis cognita sit proportio. A.ad.B.vt data, cognita etiam
nobis erit coniuncta, hoc est. A.B.ad.B.& propterea ea quæ trianguli.n.o.u. ad triá-
gulum.m.o.x.& similiter productorum . Quæsiui postea modum inueniendi duas
dictas lineas. m . o . et. o. x . & cognoui quod si producta fuerit.p.i.æquidistans li-
neæ.o.x.producendoq́;.o.n.quousque cum.p.i.se intersecarent in puncto.i. inuenien
do postea lineam quandam, quæ ducta cum.p.i.efficeret rectangulum æquale rectan
gulo cognito quod ex.m.o.in.o.x.potest fieri , quod cognitum dico, eo quod nobis
cognita est proportio data, & rectangulum etiam.n.o.in.o.u. deinde secando ab.o.
n.partem æqualem lineæ iam inuentæ, quæ sit.o.t. Inueniendo postea,ex.2 8.sexti
lineam.o.m.cuius productum in.m.t.æquale sit producto.t.o.in.o.i. vnde ex.15.eius
dem proportio.o.i.ad.m.o.eadem esset,quæ.m. t. ad. o.t.& componendo,ita se ha-
beret.m.i.ad.m.o.vt.m.o.ad. o. t. sed ex. 4. sexti,ita esset.p.i.ad.o.x.vt.m.i.ad.m.o.
quare ex.11.quinti,ita esset.p.i.ad.o.x.vt.m.o.ad.o.t.vnde ex.15.sexti productum.
o.x.in.m.o.æquale esset producto.p.i.in. o.t.& sic haberemus intentum.

Sed si punctum.m.caderet in punctum.n.idem esset , si verò punctum.m.transiret
n.oporteret nos facere hoc in latere.n.u.ipsum quærendo in linea.n.u. ducendo pri
mum lineam.p.i.æquidistanté.u.x.& producendo.u.n.ad partem.u.prosequendo, φ
superius iam dictum est.

### *Idem facere de parallelogrammo.*

### AD EVNDEM.

D Atum parallelogrammum in duas partes diuidere,secundum aliquam datam
proportionem à linea transeunte per punctum propositum.
Sit exempli gratia, datum parallelogrammum.b.u.datum verò punctum.o.extra
figuram, proportio autem ea sit, quæ.A.ad.B.vt supra.Nunc diuidatur primò re-
ctangulum datum per æqualia,mediante linea.r.c.parallela ambobus lateribus.b.x.
et.s.u.quæ quidem linea diuidatur in puncto.i. ita quod eadem proportio sit.r.i. ad.
i.c.vt.A. ad.B.protrahatur deinde à puncto.o.linea.o.i.q.quæ secabit ambo duo la-
tera.b.x.vel.s.u. intra terminos eorum , vel tantum.b.x.reliquum verò extra termi-
nos.s.u.

Nunc autem si intra dictos terminos transibit, vt in prima figura videre potes ,
problema solutum erit,eo quod
si à puncto.i. protracta fuerit. p.
d.parallela ad. u. x. habebimus
ex prima sexti eandem propor-
tionem.s .d.ad.p.x. ut.r. i.ad.i.c.
hoc est vt. A. ad.B. sed triágulus
i.e.d.æqualis est triangulo.i.q.p.
vt tibi facilè patebit , vnde qua-
drilaterum.e.q u. x. æquale erit
quadrilatero. d. u. ex communi
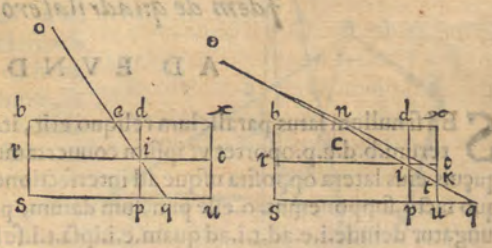
Q q          scien-

ſcientia. Quare ex.9.quinti,ita erit.s.d.ad dictum.d.u. vt ad quadrilaterum.e.q.u.
x.hoc eſt vt.A.ad.B.ex.11.eiuſdem.

Sed ſi punctum.q.fuerit extra ut in.2.figura videre eſt.tunc manifeſtum erit, ꝗ
triangulus.e.x.t.maior erit pa-
rallelogrammo.d.u.per triangu
lum.q.t.u.cum triangulus.q.i.p.
æqualis triangulo.d.i.e. excedat
quadrilaterum.i.t.u.p. per trian
gulum dictũ.q.t.u. quapropter
cum diuiſus fuerit triangulus.e.
x.t.mediante linea.o.n.K.ita ꝗ
quadrilaterũ.e.n.K.t. ſit æquale
triangulo.q.t.u. ex doctrina præ
cedenti, habebimus propoſitum.
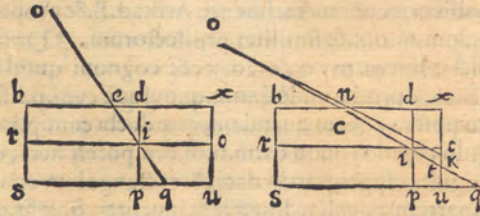
*Idem de fruſto trianguli.*

### AD EVNDEM.

Ed ſi quadrilaterum dictum eſſet fruſtum alicuius triãguli ut in figura.A.hic ſub
ſcripta videre eſt,ſuppoſita,b.d.parallela ad.u.p.ita faciendum eſſet, ducendo
ſcilicet parallelam.u.x.ad.b.p.quæ producatur vſque ad concurſum cum.b.d.
in puncto.x.ſitꝗ; proportio data inter.t.a.et.a.e.quas duas lineas cogitemus inuicem
directè coniunctas, tunc diuidatur tota.t.e.
in puncto.i. ita vt.t.i.ad.i.e.ſit vt quadrilate
ri.p.d.ad trigonum.u.d.x. deinde diuidatur
t.i.in puncto.r.tali modo vt.t.r.ad.r.i.ſe ha-
beat vt.t.a.ad.a.e. quo facto ex doctrina prę
cedenti diuidatur totum parallelogram-
mum.p.x. mediante linea.o.q.ſecundum
quod ſe habet.t.r.ad.r.e. Atque ita ſolu-
tum erit problema, vt ex te ipſo ratiotina-
rifacile p otes.
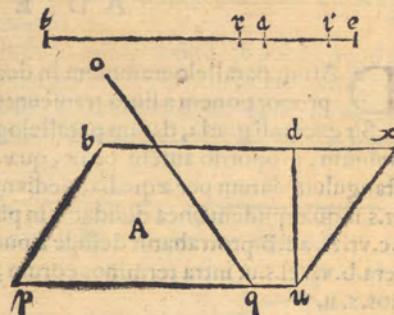
*Idem de quadrilatero in genere.*

### AD EVNDEM.

Ed ſi nullum latus parallelum reliquo erit, ita faciendum erit.ſi ſit tale quadrila
terum.b.d.u.p.oportet vt ipſum conuertamus in triangulum, producendo duo
quęuis eius latera oppoſita vſque ad interſectionem ut pote.u.p.et.d.b.in puncto.x.
quo tacto,ſupponemus. o.eſſe punctum datum, proportio verò data ſit.t.r.ad.r.i.ad
iungatur deinde.i.e.ad.t.i.ad quam.e.i.ipſa.t.i.ſe habeat vt quadrilaterum.b.d.u.p.

ſe

se habet ad triangulum.b.p.x.ducatur postea.o.q. quæ diuidat totale triangulum.d.
u.x.in duas partes inuicem ita proportionatas, ut se habent.t.r.et. r. e. quæ quidem
partes sint.c.d.u.q.et.c.q.x.ut in primo problemate tibi monstraui, & habebis pro-
positum, dato quod punctum.c.sit inter
b. et. d.

Sed si forte linea.o.q.secabit.b.x.hoc
est si punctum.c. esset inter.b.et. x. mani-
festum est, quod.c.q.secaret.b.p.in pun-
cto . y.vnde in tali casu , alio modo ope-
randum esset, hoc est ducendo.b.u. quæ
diuideret quadrilaterum in duo triangu-
la, & ut se haberet triangulum.b.d. u. ad
triangulum.b.p.u.vellem vt ita secaretur
t.i.in puncto.n.vt ita se haberet.t.n.ad.n.
i.ut dictum est de istis duobus triangulis,
deinde prout se habet.n.r.ad.r.i.ita seca-
res triangulum.b.p.u. mediante linea. o.
K.ex doctrina primi problematis, & ita haberes propositum.
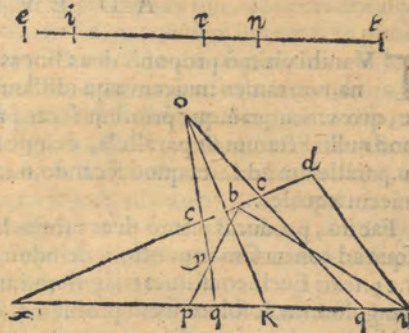
## Idem de Pentagono, Exagono, & de reliquis.

### AD EVNDEM.

PEntagonum,seu hexagonum, vel alias quasuis multilateras figuras propositas ita
diuidere, vt dictum est de trilateris, & quadrilateris.

Sit exempli gratia pentagonus.a.d.u.p.b.quem secare volumus mediáte linea. o.
q. in duas partes inuicem se habentes, vt se habent.t.r.et. r.i.oportet igitur ut ipsum
pentagonum reducas ad quadrilaterum.x.a.d.u. quod diuidatur secundum præce-
dentem doctrinam , vt se habet.t.r. ad. r. e.
vnde si punctum.q.incidit inter.p.et.u.tunc
habebis propositum, si verò incidet inter.
p.et.x. clarum erit quod linea.o. q. secabit
latus.p.b. trianguli.b.x.p.in puncto.y.qua-
propter duces lineam.a.p. vt claudat trian-
gulum.a.b.p.diuidaturq́;.t.i.in puncto.n.ita
vt.t.n.ad.n.i.se habeat,vt quadrilaterum.a.
d.u.p.ad triágulum.a.b.p. deinde húc trian
gulum.a.b.p.diuidas mediante linea. o. K.
vt.n.r.ad.r.i.ex doctrina primi problematis
& habebis propositum. Idem dico de hexa
gono,reducendo ipsum ad pentagonum, &
item de eptagono,ipsum reducendo ad exa
gono , & idem infero de infinito ipsarum
superficialium figurarum rectilinearum.

Qq   2           De
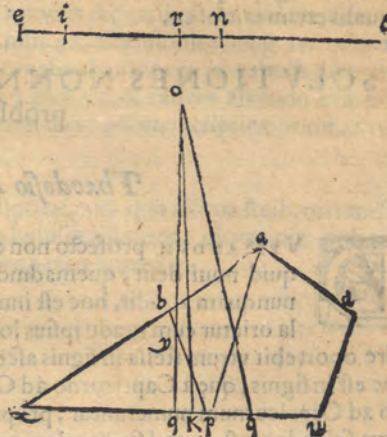
## De duobus triangulis æqualibus inter lineas inuicem inclinatas.

### AD EVNDEM.

TV mihi vltimò proponis dvas lineas rectas.b.f.et.q.s.in eadem superficie plana,non tamen inuicem æqu idistantes, proponis etiam.n.t.in eadem superficie, quæ vnamquamque priorum secat,proponis etiam lineam. h. tali conditione, quod nulli dictarum sit parallela, deinde scire cupis qua arte aliquis posset ducere. e.u.parallelam ad.h.ita quod secando.n.t.constituat duos triangulos.n.o.u.et.t.o.e. inuicem æquales.

Fac ita, producas primò dvas primas lineas à parte, in qua inuicem inclinantur, vsque ad concursum in puncto.i. deinde à puncto.n.duces.n.c.parallelà ad.h.postea ex.25.sexti Eucli.constitues triãgulum.i.u.e.simile triangulo. i . c . n. æquale tamen triangulo.i.t.n.& solutum erit problema.

Vel sic,inuenies.i.e. mediam proportionalem inter.i.c, & i.t. duces postea.e.u.parallelam lineç.h.vel.c.n.quòd idẽ erit ex.30.primi Eucli.& solutum erit problema .

Nam ex. 17 . sexti eadem proportio erit trianguli. i . c. n.ad triangulum.i.e.u. ut.i.c. ad.i. t. Quare ut trianguli. i. c.n.ad triangulũ.i.t.n.ex prima sexti, et.11.quinti. Vnde ex.9.eiusdem.i. e. u. æqualis erit.i.t.n. Quapropter.o.n.u. æqualis etiam erit.o. e.t.
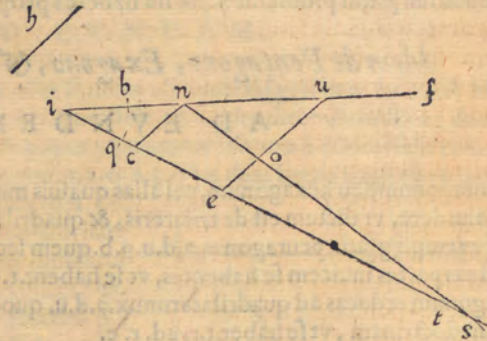


## SOLVTIONES NONNVLLAE QVORVNDAM problematum.

### Thæodosio à Raifestaim.

DVBITANDVM profecto non est, quin quotidie hominibus studiosis aliquid noui desit, quemadmodum, quod tibi nunc occurrit, mihi nonnunquam accidit, hoc est inuenire orizontem, cui aliqua proposita stella oriatur cum gradu ipsius longitudinis. pro cui? rei operatione te prius scire oportebit vtrum stella in signis ascendentibus, vel descendentibus reperiatur, hoc est in signis, quç à Capricorno ad Cancrum procedunt, vel in illis, quç à Cancro ad Capricornum numerantur, propterea quod si in signis ascendentibus inuenitur,sciendum est, quod supra talem orizontem polus mundi australis attollitur, sed si in signis descendentibus reperitur, tunc polus borealis eleuatur supra dictum

orizontem

orizontem, vt exempli gratia, canicula quæ à Græcis Prochyô vocatur, reperitur in 24. minuto vigesimi gradus Cancri, quapropter polus borealis eleuatur supra orizontem , cui ipsa oritur cum eodem gradu, & minuto eclipticę illius signi . sed quia volumus etiam scire veram quantitatem arcus eleuationis huiusmodi poli, pro pterea accipiemus in tabula generali Monteregij numerum qui vocatur radix ascen sionum, è regione numeri longitudinis ipsius stellæ, qui quidem numerus in præsen ti exemplo erit gra.107. cum minutis. 53. qui est cuiusdã arcus æquatoris, qui incipit in principio Arietis , & in circulo latitudinis desinit, hoc est ab orizonte quæsito, ita quod talis numerus erit ascensio obliqua huiusmodi puncti ecliptice illi orizonti, qua ascensione mediante, simul cum gradu, & minuto longitudinis in tabulis ascensionum obliquarum, inueniemus gradum , & minutum altitudinis pollaris, qđ quærebatur, eodem ordine ac methodo, quo vtimur ad inueniendum in tabulis positionum, polum circuli positionis alicuius astri, mediante declinatione & distantia à meridiano eiusdem astri, vt scis . Vnde in præsenti exemplo eleuatio poli borea lis supra talem orizontem erit gra.7. cum minutis.45.

Sed si stella fuerit in medietate ascendente, tunc certi erimus polum australem su per dictum orizontem attolli, nam idem est quærere altitudinem vnius polorũ mun di à tali orizonte, quod distantiam dicti poli à circulo secundum quem longitudo terminatur, qui etiam latitudinis dicitur , eo quod tunc temporis talis circulus vnus & idem est cum orizonte . Sumatur ergo exempli gratia stella, quæ in ore piscis au stralis est, quę, pro nunc, sit in gradu.20. cum minutis. 14. Aquarij longitudinis , & in gradu.23. cum nullo minuto meridianæ latitudinis. Tunc certi erimus orizontem, cui dicta stella oritur cum eiusmodi puncto ecliptice , depressum esse à parte australi sub illoq; polo , sed quia propositum est scire etiam quantitatem huiusmodi depræssionis, reperiemus in tabula generali gradum , & minutum æquatoris, correspondentem tali puncto longitudinis à circulo latitudinis terminato, qui quidem numerus in præsenti exemplo erit gra.317. cum minutis.46.& hic numerus, vt dixi mus est ascen.obli.ad dictum orizontem, vbi polus australis attollitur , & descensio obliqua, vbi polus borealis eleuatur . Quapropter si à.317. gradibus cum minutis 46. demptus fuerit dimidius circulus gra.180. remanebunt gra.137. cum minutis.46 & punctus oppositus gradibus.20. cum.14. minutis Aquarij est in eodem numero Leonis, & mediantibus istis gradibus.137. min.46. ascensionis, cum grad. 20. min. 14. Leonis inueniemus eleuationem poli borealis ab orizonte in tabulis ascensionum obliquarum Monteregij, hoc est gra.17. min.53.& eadem altitudo erit poli australis supra orizontem à quo Fomahant cum dicto puncto ecliptice oritur, in qua longitudine dicta stella reperitur.

Sed si propositus nobis fuerit punctus ecliptice, cum quo aliqua stella oritura sit, & oporteat inuenire vbi, hoc est orizontem huiusmodi ortus, eleuatione poli arti ci, seu antarctici supra talem orizontem, ita operandum esset.

**Sit** up

Sit exempli gratia stella.o. ecli
ptica verò.d. q. æquator autem.
p.q.punctus verò eclipticæ, cum
quo stella oritura sit.e.orizon de
mũ.o. e . vbi stella oriri possit cũ
puncto.e. Nam cum stella pro-
ponitur, datur etiam eius longi-
tudo,nec non latitudo , quare ar-
cus.a.q.& arcus.a.o. nobis cogni
tus erit, cum supponatur arcus.a.
o.esse circuli latitudinis, et. a . o.
latitudo ipsius stellæ , & angulus
a.rectus erit , & quia punctum.e.
datur,ergo arcus.a.o. & arcus. a.
e.simul cũ angulo.a.recto cogni-
ti sunt , vnde ex.11.primi lib. co
pernici,angulus.a.e.o. cognosce-
tur,& angulus.q.e.o. similiter, vt
residuum ex duobus rectis quo.e
mediante cum angulo.q.declina
tionis ab æquatore , medianteq́;
latere.q.e.cognito,cognitus quo
que nobis erit angulus.e. t. q. ex
12.eiusdem.qui quidem angulus
erit altitudinis æquatoris ab ori-
zonte quæsito , qui demptus à
90 . gradibus, dabit altitudinem
poli ab orizonte quæsito.

Inuenire postea gradum eclipticę,cum quo stella data oriatur ad orizontem pro
positum , nullius est difficulratis.

Ponamus exempli gratia,aliquem scire velle gradum eclipticæ, cum quo canicu-
la oritur ad orizontem, cui polus boreus eleuatur per gradus.19.quæ canicula sup-
ponatur habere gradus.19.cum min.24.Cancri longitudinis,& gra.16.min.10.lati-
tudinis meridianæ, quærere primum oportet eius declinationem ex doctrina.2.pro
blematis tabularum directionum Monteregij , quæ erit graduum.6.cum minutis.5.
septentrionalis, deinde inuenire eius ascensionem rectam ex doctrina.4.problema
tis eiusdem Monteregij, quæ erit gra.108.mi.42.deinde mediãte declinatione iam
inuenta in tabulis differentiarum ascensionalium sub polo.44.accipiemus differen-
tiam ascensionum, qua differt recta ab obliqua , quæ in præsenti exemplo erit gra.
5.min.55.quæ dempta ab ascensione recta stellæ, vt præsens exemplum exigit,relin
quet nobis ascensionem obliquam stellæ propositæ ad polum.gra.44. quæ erit gra.
102.minu.47.qua mediante,in tabulis ascensionum obliquarum poli. 44. habebi-
mus gradum & minutum eclipticę cum quo stella oritur. quod in casu nostro erit
gra.1.min.8.Leonis, sed si tecum non fuerint tabulæ dictæ, potes eleganter omnia
hæc perficere via triangulorum sphæricorum.

                                                                    Via
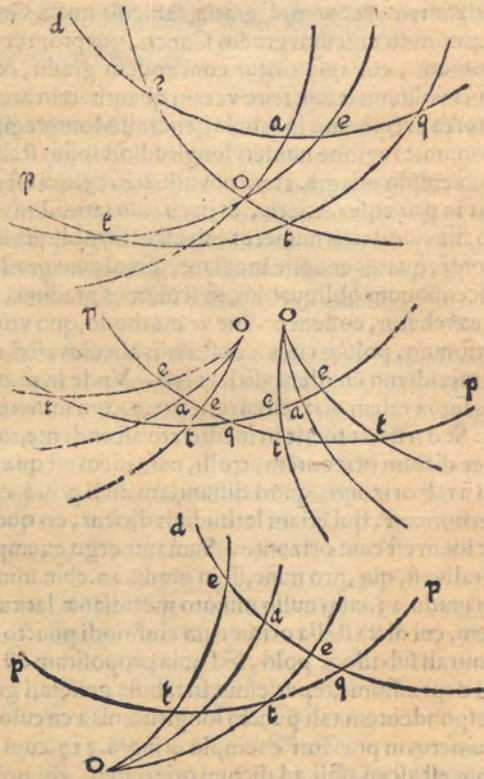
Via triangulorum idem facere.

Sit exépli gratia.q.b.æquator, ecliptica verò.q.a.propofitus aût orizon fit.o.c.d. & ftel!a data fit.o.in orientali parte orizontis, circulus verò.a.o.ille fit, qui tranfiés per polos eclipticæ & per centrum ftellæ terminat longitudinem ipfius ftellæ, & in ipfo fit eius latitudo . Nunc propofitum fit inuenire arcum.d.q. eo quod illicò fcie mus punctum.d.qua propter oportet nos prius cognofcere arcum.d.a.qui demptus, vel additus arcui.a.q. prius cognito ex fuppofito ( nam data nobis eft longitudo, & latitudo ftellæ ) dabit nobis.d.q.

Cum igitur voluerimus arcum.d.a.cognofcere, ita faciemus. nam. q. a. cognitus nobis eft ex fuppofito vt dictum eft.angulus quoque.a.q.b.qui declinationis eclipti cæ ab æquatore eft, angulus deinde.a. ( trianguli.a.b.q.) rectus eft, ergo ex.4.primi copernici cognitus nobis erit arcus. a.b.nec non angulus.a.b.q.vnde an- gulus.o.b.e. refiduus ex duobus re- ctis in duobus primis hic fubfcriptis figuris nobis itidem cognitus erit , etiam & arcus.b.o.refiduus fiue com pofitus ex arcu.a.o.cognito ex fup- pofito cû fit arcus latitudinis ab ecli- ptica. Tunc in triangulo.o.b.e.co- gnofcimus!atus.o.b. & angulum.o. b.e.nec non angulum.b.e.o.qui eft altitudinis æquatoris ab orizonte , quare ex.12.dicti lib.cognitus nobis erit angulus.b.o.e. Confideremus deinde triangulum.a.o.d.cuius angu lus.a.rectus eft , & angulus.a.o.d.cû latere.a.o.etiam cognitus, vnde ex fupradicta.4.nobis cognitus erit ar- cus.a.d.& confequenter cognofce- mus arcum.d.q.eius refiduum,feu compofitum, quem quærebamus.

Sed fi hac via inuenire defideras, cui orizonti propofita ftella oriatur cum eodem ecliptice puncto.a.lon- gitudinis, hoc aliud nihil effet,quam cognofcere amplitudinem anguli.a. b.q.eo quod talis orizon , idem cir- culus effet.a.b.o.vnde cum quis fci- ret vnum illorum angulorum quem æquator efficit cum orizonte , reli- qua illicò ei innotefcent, fed dictus angulus.b. iam diximus quomodo cognofcatur.
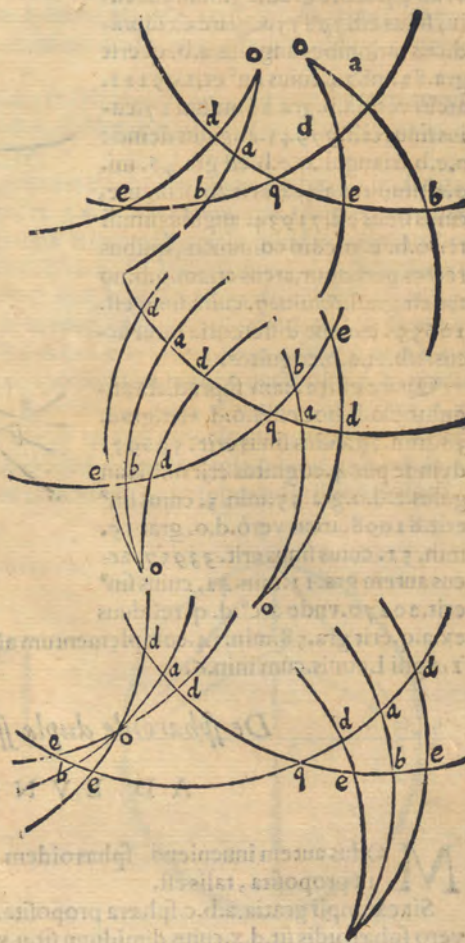
Pona-

Ponamus nos ſcire velle pūctum
ecliptiçe, cum quo Procyon oritur
polo.44.0.dato, quod ſtella in gra.
19.cum min. 24. Cancri, reperiatur
diſtans ab ecliptica per gra. 16.min.
10.meridiem verſus.vnde arcᵍ. a.q.
erit gra.70.min.36. eiusq̷ ſinus par-
tium.94321.talium qualium totalis
eſt.100000.arcus verò.a.o.gra. 16.
minut. 10. ſinus erit 27845. angu-
lus autem.a.q.e.declinationis zodia
ci ab çquatore grad.23.min.30. cu-
ius ſinus eſt.39875.Quare ex ſupra-
dictis rationibus angulus.a.b. q. erit
gra.82.mi.24.cuius ſinᵍ erit.99122.
arcus vero.a.b.gra.22.minu. 17.cu-
ius ſinus erit.37945.angulus deinde
o.e.b.trianguli.o.c.b.eſt gra.46. mi.
0.altitudinis æquatoris ab orizonte,
cuius ſinus eſt.71934. angulus ſimili
ter.o.b. e. medio coniuncti, quibus
rectus perficitur,arcus etiam.o.b.no
tus eſt grad.6.min. 7. cuius ſinus eſt.
10655. cum ſit differentia inter ar-
cus.a.b.et.a.o.cognitos.

Quare ex.12. iam ſupradicta an-
gulus.e.o.b.hoc eſt.a.o.d. erit.grad.
36.min.39.cuius ſinus erit.59693.
deinde per.4.cognitus erit nobis an
gulus.a.d.o.gra.55.min.5. cuius ſinᵍ
erit.81998.arcus verò.d.o. gra. 19.
min.51. cuius ſinus erit.33957. ar-
cus autem gra.11.min.42. cuius ſinᵍ
erit.20270.vnde arcᵍ.d. q. reſiduus
ex.a.q.erit gra.58.min.54.complementum aũt quartæ erit gra.31.mi.6.hoc eſt gra.
1.ſigni Leonis.cum min.6.

## De ſphæroide dupla ſphæra propoſita.

### AD EVNDEM.

M Odus autem inueniendi ſphæroidem ex dato axe, quod duplum ſit ſphæ-
ra propoſita, talis eſt.

Sit exempli gratia.a.b.c.ſphæra propoſita. cuius ſemidiameter ſit. o. c . ſemiaxis
vero ſphæroidis ſit.d.x.cuius dimidium ſit.u.x.tunc ex doctrina.9.ſexti Euclid. inuε
niatur.g.h.media proportionalis inter.u.x.et.c.o.deinde ſicut ſe habet.u.x.ad. g. h̷
facie-

faciemus, quod diameter.a. b. dictæ sphæræ ita se habeat ad.e.f.ex.10. sexti, quæ
e.f.erit reliqua axis quæsita. Vnde constituta cum fuerit ellipsis.d.f.t.e.ex dictis axi-
bus, deinde circumuertendo ellipsim circa maiorem axem, constituemus sphæroi-
dem oblongam, si autem circumuertemus ipsam circa minorem axim constituemus
sphæroidem prolatam.
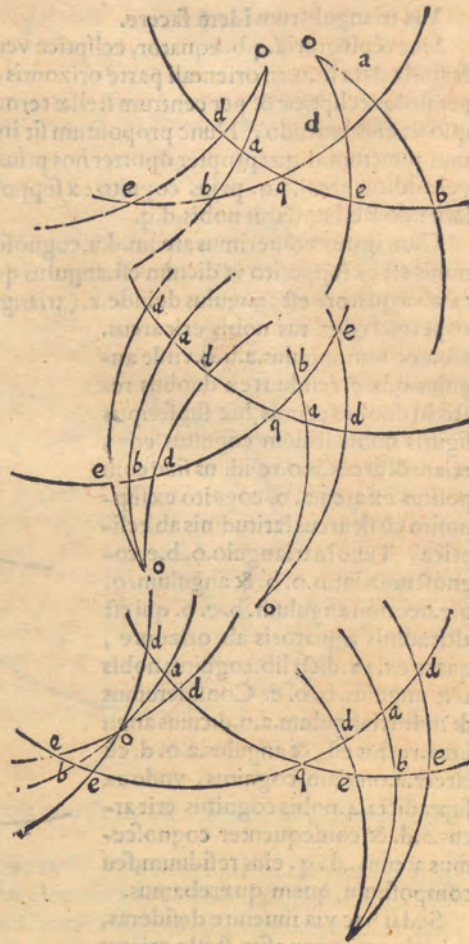
Quod autem talis operatio rationalis sit, nulli dubium erit, quotiescunque co-
gnoscet conum rectum.e.u.f.æqualem esse cono recto.a.c.b ex.2.parte.12.duodeci
mi Euclid.& quod cum conus.e.d. f. duplus sit cono. e. u. f. ex lemmate collecto ab
11.duodecimi, conus.e.d.f.duplus existit etiam cono.a.c.b.ex.7.quinti. Cum de-
inde ex. 32. primi lib. de sphæra, & cyllindro sphæra.a.c.b.q.quadrupla sit cono.a.
c.b.ipsa consequenter dupla erit cono.e.d.f.sed ex.29.primi de conoidalibus, dimi
dium sphæroidis.e.d.f.t.hoc est. e. d. f. dupla est cono.e.d. f. Quare talis medietas
æqualis est sphæræ propositæ, totaque sphæroides dupla erit sphæræ datæ. Quod
autem dico de proportione dupla,idem infero de qualibet alia,sumendo.u.x.ita pro
portionatam ad.d.x. vt proponitur.

Sphæram autem inuenire quæ dimidia sit sphæroidis propositæ nullius erit nego-
tij,quotiescunque inuentus fuerit modus diuidendi vnam datam proportionem in
tres æquales partes.

Sit proposita sphæroides.e.f.d.t.cuius axes ex consequentia dantur.e.f.et.d. t.quæ
quidem sphæroides sit primo oblonga, et. u.x. sit dimidium axis maioris. imagine-
tur etiam conus.e.u.f.vt supra. Imaginetur etiam factum esse,quod proponitur,hoc
est,vt sphæra.a.b.c.q.sit dimidium ipsius sphæroidis, vnde conus.a. c. b. æqualis erit
cono.e.u.x.vt supra demonstratum est, & sit.g.h.media proportionalis inter.u.x.e t
o.c.Iam visum superius fuit, quod eadem proportio erat ipsius.u.x. ad.g h.quæ.a.b.
ad. e.f.quare eadem quæ.o.b.ad.e.x.sed.u.x.et.e.x.dantur.inter quas.g.h.et.o.b.vel
o.c.(nam.o.c.æqualis est.o.b.) mediæ proportionales sunt,eo quod cum.g. h. media
proportionalis sit inter.u.x.et.o.c.& proportio.o.b.ad.e. x. æqualis sit ei, quæ. u. x.
ad.g.h.hoc est ei quæ.g.h.ad.o.c.vel.ad.o.b. quare quotiescunque inuentæ fuerint.
g.h.et.o.c.vel.o.b.mediæ proportionales inter.d.x.et. x.e.ipsa.o.c.vel. o.b.erit semi
diameter sphæræ quæsitæ.eodem modo faciendum erit si sphæroides fuerit prolata.
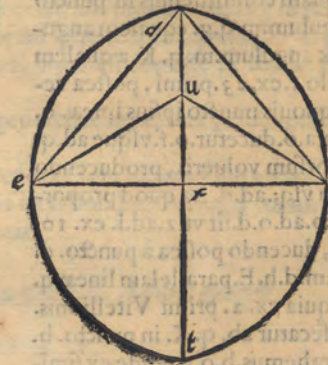
## Modus inueniendi duo triangula varijs conditionibus affecta.

## AD EVNDEM.

QVod etiam quæris ita se habet, duo scilicet triangula inuenire, æqualia duabus superficiebus rectilineis propositis, quę quidem triangula sint eiusdem altitudinis, & quod vnūquodque habeat angulum æqualém angulo proposito, & ꝗ alius angulus vnius, cum alio alterius, æquetur duobus rectis.

Sint exempli gratia duæ propositæ superficies.c.y.duo verò anguli dari sint. r. s. cum voluerimus inuenire duo triangula ( quæ sint. a. i. u. et.n.t.x. ) tali conditione prædita , quod angulus, a. æqualis sit angulo.s.& angulus. t, angulo. r. & quod angulus.x.simul cum angulo. u. æquetur duobus rectis, & quod triā gulū. a. i. u. æquale sit superficiei. c. reliquum verò superficiei. y. Ex duabus superficiebus.c. et. y. constituemus duo quadrata,per vltimam secundi Eucli.accipiemus, deinde duo latera tetragonica ipforum quadratórum , & inueniemus tertiam lineam in continua proportionalitate cum illis lateribus ex.10.sexti , seruabimus postea extremas illarum, quæ sint. z. et.l.quarum proportio, eadē erit, quæ inter duas propositas superficies reperitur ex.18.sexti , accipiemus, deinde lineam aliquam cuiusuis longitudinis, quæ sit.q.g.supra quam constituemus in puncto q.angulum.m q.g. æqualem angulo.s.& angulum.m. q. K. æqualem angulo.r.ex.23.primi , postea verò à quouis puncto ipsius lineæ. q. m.puta.o.ducetur.o.f.vsque ad.q. g.quorsum volueris, producendo ipsam vsq;.ad.d. ita quod proportio.f.o.ad.o.d.sit vt.z.ad.l. ex. 10. sexti, ducendo postea à puncto. d. lineam.d.h. E.parallelam lineæ.q. g.& quia ex. 2. primi Vitellionis. h.E.secatur ab. q. K. in puncto. b. protrahemus.b.o.p. vnde ex similitudine triangulorum habebimus proportionem.p.o.ad.o. b. vt.f.o. ad.o.d.
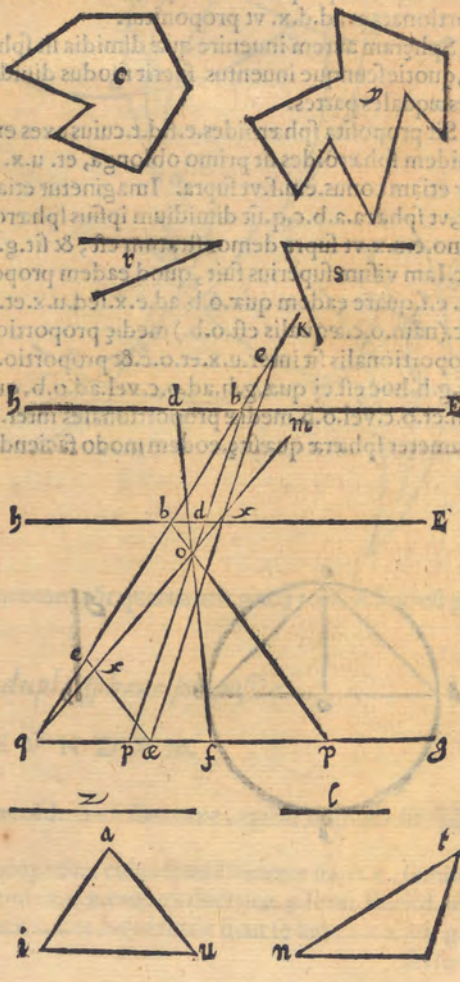
ad.o.d.hoc eſt vt.z.ad.l. hoc eſt vt.c.ad.y.quare triangulū. p.q. o.ita erit proportio
natū triangulo.o. q.b. vt.c.ad.y.conſtituo deinde ex. 25. ſexti duo triangula ſimi-
lia duobus.p.q.o.et.o.q b.æqualiaq̉;.c.et.y.quę ſint.a.i.u.et.n.t.x.ſecetur poſtea . q.
g.in puncto.æ.ita, quod.q.æ.æqualis ſit.i.a.duco poſtea.æ.e.æquidiſtantem.ad.p.b.
& ſic habebimus duo triangula. q. x. æ.et.q.x.e,vt quærebantur,quamuis duo trian
gula.a.i.u.et.t.n.x.eaſdem habeant conditiones.

# DE IMPERFECTA SOLVTIONE PROBLE-
### matis Nicolai Tartaleæ ad Cardanum. De animad-
### uerſione in Ptolomeum. De incendio carbo-
### num à vento.

## *Clariſſimo Dominico Moreſino.*

CIo propoſitam tibi quæſtionem te diu agitauiſſe, nec tamen ſolutio-
nem aſſequi potuiſſe, aduerte igitur ipſam falſam, ideſt impoſſibilem
eſſe, quemadmodum etiam decimumoctauum quæſitum propoſitum à
Cardano Tartaleæ, ab ipſo Tartalea ſolutum minimè fuit. Quiquidem
Tartalea vult circulum deſcribi circa triangulum per quintam libri quarti Euclidis,
vt in fine ferè quintæ partis ſuarum menſurarum affirmat, nèque videt in quinta
quarti Euclidem vti vndecima primi, & in vndecima primi,quarta aut octaua eiuſ-
dem, quas,ipſe Euclides oſtenſiuè non demonſtrauit. Quapropter oportebat Tar-
taleam demonſtraſſe omnes propoſitiones ad hoc neceſſarias oſtenſiuè vſq; ad pri-
mas indemonſtrabiles, quia ad demonſtrandam ſcientificè aliquā propoſitionem,
aut à propoſitione in propoſitionem vſque ad prima principia vniuerſalia ( vt ali-
quando ego feci) eſt retrogradandum, aut ab ipſis principijs incipiendum ſucceſſi-
uè eouſque progrediendo donec ad propoſitionem quam demonſtrare volumus
perueniamus.

Quod ad Ptolomeum in geographia attinet, dico eum mihi non ſatisfacere,cum
ſumit portionem arcus circuli maioris inter vnam ciuitatem, & aliam, ea ratione
quam deſcribit. Quod ſi vſus fuiſſet módo Menelai, ab ipſomet deinde in ſuū Al-
mageſtum vſurpato,aut Monteregij triangulorum ſphæricorum,quem Copernicus
adhibuit (qui tamen modus, tempore Ptolomei, nondum fortaſſe in lucem vene-
rat) bene egiſſet.

Quod deinde ad ſuum illud inſtrumentum geometricum attinet, eſt imperfectū,
vt oſtendi domino Pandulfo.

Motum autem aeris, aut mauis ventum,accendere ignem,non ſolum ratione an
tiperiſtaſis, quam affers euenit, ſed etiam quia à carbonibus accenſis totam excre
mentitiam materiam, quæ eos circundat, auferat.

## *Alia dilucidatio propoſitionis. 25.lib. 2. Monteregij.*

### AD EVNDEM.

SCribis te non intelligere. 25. propoſitionem lib. 2. Monteregij. cum nec ſcias
reperire diametrum circuli circunſcriptibilis circa propoſitum triangu-
lum,

lum, cuius data fit b afis tantummodo fimul cum angulo, qui ipfi bafi opponitur.

Imagineris igitur triangulum datum effe obtufiangulum. a. b. g. cuius bafi.b. g.fit nobis data fimul cum angulo.a. ei oppofito, obtufoque; Confidera etiam cir-culum.a.b.g.q.ipfum triangulum circunfcribentem, cuius diameter.q.e.p. tranfeat per.m.punctum medium ipfius.b.g. tũc protractis imaginatione.e.g.et.g.p.certi eri-mus angulos.circa.m.rectos effe ex.3.tertij Eucli.angulumq;.q.e. g. duplum effe an gulo.q.p.g.ex.19.eiufdem, vnde æqualem angulo.a.qui etiam duplus eft angulo.q. p.g.quapropter proportio arcus. q. g. ad arcum g.p.tibi cognita erit; & proportio etiam chor-de.p.g.ad finum.m.g.arcus.g.p.& quia. m. g. vt dimidium ipfius. bc g. tibi data eft, cognofces etiam.p.g.vt.m.g.& fic tertium latus.m.p.trian-guli orthogonij.p.m.g.& qa ex.34.tertij quod fit ex.p.m.in.m.q.eft æquale ei quod fit ex.b.m. in.m.g.ideo cum diuifum fuerit productum. b. m.in.m.g.per.p.m.proueniet. m. q. quapropter habebis totum.q.p.
Idem efficies,fi cũ angulus.a.acutus fuiffet.
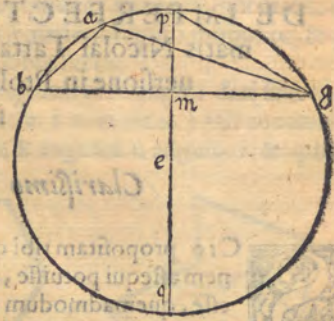
## Modus inueniendi puncta elliptica via Pergei.

### A D E V N D E M.

MOdus inueniendi puncta elliptica, via.21.primi lib. Pergei ex datis axibus, vt vbi alias fignificati,talis eft. Sit exempli gratia maior axis propo-fitus.a.c.minor autem.b. d. cum ergo volueris inuenire punctum circunfe-rentiæ correfpondentem puncto. e. maioris axis, inueniemus primo la-tus tetragonicum producti.a.g. in. g. c.quod fit.h.latusq; tetragonicũ pro-ducti.a.e.in.e.c.quod fit.i.deinde in-ueniemus lineam. K.tertiam in con-tinua proportionalitate cum. h. et. i. vnde.i.erit media proportionalis in-ter.h.et.K.& vt.h.proportionalis erit ad.K.inueniemus.e.f.cui.g.d.medie-tas fecundi axis ita fe habeat, quæ po

ftea iuncta axi maiori, ad angulos rectos in puncto.e.dabit fitum puncti.f.quæfiti ex dicta.21.primi lib. Pergei, fed talis modus prolixus eft.
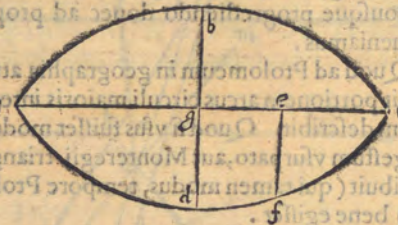
Accipe

Accipe igitur hunc alium.

Sit propositus maior axis. q. p. minor verò. e. c. ad angulos rectos se inuicem secantes in puncto. o. describatur circulus. q. n. p. a. cuius diameter sit axis maior, in quo accipiatur punctum, quod volueris, vt puta. u. à quo protrahatur. u. b. parallela ad. o. c. n. designetur postea separatim circulus. u. b. n. cuius diameter æqualis sit se midiametro prioris circuli, ita etiam fiat circulus. u. i. c. contingens circulum. u. b. n. in puncto. u. cuius diameter sit. u. c. æqualis dimidio axi minori. accipiatur deinde in circulo maximo longitudo. u. b. quæ collocetur in circulo mediocri à puncto. u. quæ secabitur à minimo circulo in puncto. i. cum itaque longitudo. u. i. mensurata fuerit in. u. b. maximi circuli à puncto. u. habebimus propositum.

Cuius rei ratio est, quia. u. b. mediocris circuli diuiditur à gyro minimi in puncto i. eadem proportione, qua diuisa est. u. n. in puncto. c. quòd manifestum est ex similitudine triangulorum. u. b. n. et. u. i. c. imaginatæ cum fuerint duæ. b. n. et. i. c. sed ita esse oportet parallelas maximi circuli, quotiescunque circunferentia ipsius ellipsis transitura sit per. c. vt in. 51. cap. meæ gnomonicæ ostensum fuit.



## Modus designandi angulum, certo modo conditionatum.

### AD EVNDEM.

NVllius reuera difficultatis mihi videtur esse, quotiescunque nobis proposita fuerint duo puncta. a. et. b. simul cum angulo. d. necnó linea. g. ducere duas lineas à dictis punctis terminatas, quæ constituât angulum æqualem dato, & ipsæ directè cóiunctæ constituant lineam æqualem datæ. Nam ducatur linea indefinita per puncta proposita, cuius lineæ, pars illa, quæ intercepta fuerit inter dicta puncta, diuidatur per æqualia in puncto. o. etiam & linea data, quarum medietates accipio in linea indefinitè protracta à puncto. o. medio
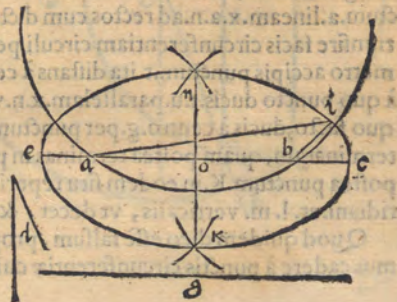
dio, vt vna earum fit.o. c.reliqua verò fit. o. e. deinde aperiatur circinus quantum.
o.c.positoq; vno pede in.b.designétur cum altero duo arcus.n.K.posito iterum vno
pede in.a.designentur alij duo arcus inter-
secantes primos in punctis. n. K. Deinde à
puncto.n.ad.K.ducetur linea.n.K.quæ per
punctum.o.transibit,quam.n.K.mente cō-
cipio, vt axis minor vni⁹ ellipsis,cuius.e.c.
sit axis maior, quibus axibus mediantibus
designetur ellipsis.n.c.K.e. considero dein
de.a.b.vt chordã vnius circuli , seu portio-
nis circularis, quæ capax fit vnius anguli
æqualis angulo.d. proposito,ex. 3 2 . tertij
Euclid. cuius circunferentia, circunferen-
tiam ipsius ellipsis secabit in duob⁹ punctis
quorum vnū sit.i.à quo protractæ cum fue

rint duæ lineæ.a.i.et.i.b.habebis propositum,cum.a.i. iuncta cum.i.b.æquetur.e. c.
ex.5 2.tertij Pergei.
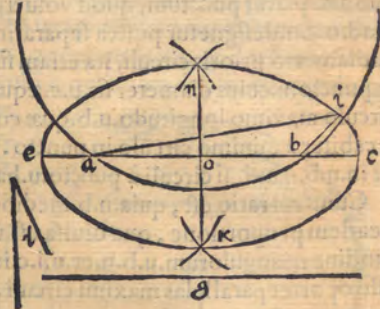
## EMENDATIO CVIVSDAM FALSI MODI
### delineandi horologia Italica orizontalia.

*Joanni Paulo Dardano.*

O D V s delineandi horologia Italica orizontalia, quem tibi monstrauit
nescio quis, ni fallor,talis est.Designato meridiano.l.b.m.q.ductisq; duo
bus diametris.l.m.et.b.q,inuicem ad rectos in centro.g. quorum. l. m. fit
verticalis.b.q.vero orizōtalis , ductoq; diametro. f.h. tropici Cancri se-
cundum altitudinem poli datam,descriptoq; dimidio circulo.f.z. h. ipsius paralleli,
inuentoq; puncto.z. horæ propositę, & ab eo ducta per pendiculari. z.r.ad.f. h. & à
puncto.r.ducta.r.o.y. parallela ad diametrum.q. b.orizontalem,ducis postea. f.ω.et.
r.t.vsque ad orizontalem.q.b. parallelas ad diametrum.l.m.verticalem. Determi-
nato postea gnomone.g.s. in orizontis axe, ductaq; vmbrarum linea. s. K. parallela
orizontali, ductaq;.y.g.K.ad terminandam.s.K.delineas deinde separatim circulum
q.x.b.n.magnitudinis prioris, qui quidem circulus significet orizontem ipsum , in
quo ductis diametris.q.g.b.et.l.g.m.accipis in diametro.q.g.b.puncta.a.et.ω. ita à cē
tro.g.distantia , vt sunt in diametro orizontali prioris circuli , ducis postea per pun-
ctum.a.lineam.x.a.n.ad rectos cum dicto diametro, deinde per tria puncta. n. ω. x.
transire facis circunferentiam circuli per quintã quarti Euclidis,postea in dicto dia-
metro accipis punctum.t.ita distans à centro, & ex eadem parte, vt in priori circulo,
à quo puncto ducis.t.u.parallelam.x.n.vsque ad circunferentiam.x.ω.n.in puncto. u.
quo facto,ducis à centro.g.per punctum.u.ipsius circularis circunferentiæ.g.u. inde-
terminatam, quam postea terminas in puncto.K.ita quod.g.K.æqualis fit.s.K. Dicis
postea punctum.K.in eodem situ reperiri, respectu duorum diametrorum. q. b.me-
ridiani.et. l. m. verticalis, vt decet , & oportet punctum horæ propositę existere.
   Quod quidem dico esse falsum , propterea quod perpendiculares quas cogita-
mus cadere à punctis circunferentiæ cuiusuis paralleli supra quemuis orizontem ob
                                                                            liquum

liquum ſceantem æquatorem, omnes cadunt in gyro elliptico, oxygonio, ſeu de-
fectionali, & non circulari. Vnde per ſupradicta tria puncta.n.ω.x. oporteret tranſi
re talem circunferentiam, & non circularẽ, quæ circunferentia eſſet vnius ellipſis,
cuius minor axis in diametro.b.q́.eſſet.ab.ω.vſque ad.i.terminum ſini h.i.arcus.h.b.
in analemate, maior verò axis eſſet magnitudinis.f.h.diametri paralleli, quæ trãſiſ-
ſet per punctum.c.medium inter.ω.et.i.quę quidem circunferẽntia tota eſſet intra cir
culum.q.n.b.x.cõtiguaq́; gyro.q.n.b.x. in punctis.n. x.

Si ergò circunferentia.n.ω.x.eſſet elliptica tunc punctum.u.in orizonte illud eſſet
vbi caderet ſinus altitudinis horę, et.t.u.æqualis eſſet.r.z. communi ſectioñi paralle
li cum almicantarat ex.34.primi Euclid.et.u.g.æqualis eſſet.o.y. communi ſectioñi
almicantarat cum meridiano, vel cum azimut illius horæ ex.4.primi, cum. g.t.æqua
lis ſit ipſi.o.r.et.t.u.ipſi.r.z.& angu
lus.t trianguli.g.t.u.rectus, quem-
admodúm.r. qui compræhenditur
ab.z.r.et.r.ó.vnde anguli. K. g. m.
et.K.g.b.rectè ſe haberent, diſtan-
tia verò inter.K. et. g.iã rectè ſum-
pta fuit.

Sed quia punctum. u.vt plurimũ
(in gyro circulari ſumptum) extra
puncta interſectionum ipſius circu
laris gyri cum elliptico repetitur,
propterea efficit angulos. K. g. m.
et.K.g.b.falſos, & non æquales il-
lis, qui fiunt ab azimut horæ cum
verticali, & cum meridiano, quæ
omnia ex cap.52.meæ gnomonicę
facilè videre potes.

Nec tacere volo quod punctum
u.verum, hoc eſt ellipticum, inue-
niri poſſet ea via quam ſcripſi in
eodem.52.cap.qua mediantè do-
cui demum inuenire punctum.π.
orizontis, quamuis in præſenti ca
ſu.σ.λ. perpendicularis eſſet ſupra
minorem axem ipſius ellipſis, quã-
uis ſupra maiorem axem, quod ta-
men minimè mutat ordinem, imò
rationes eędem ſunt, tam in vna,
quam in alia operatiõne, ſed vt il-
licò idipſũ habeas, fac vt.t.u. æqua
lis.ſit.r. z. & tunc punctum. K. erit
quæſitũ, quod ego in.52. cap. meę
gnomonicę, ijs verbis ſignificaui.

» Itaq; mediis binis triangulis ijs,
» medioq; azimut Solis pariter ho-
» rologia fabricari poterunt.

320　　　　　I O. B A P T. B E N E D.
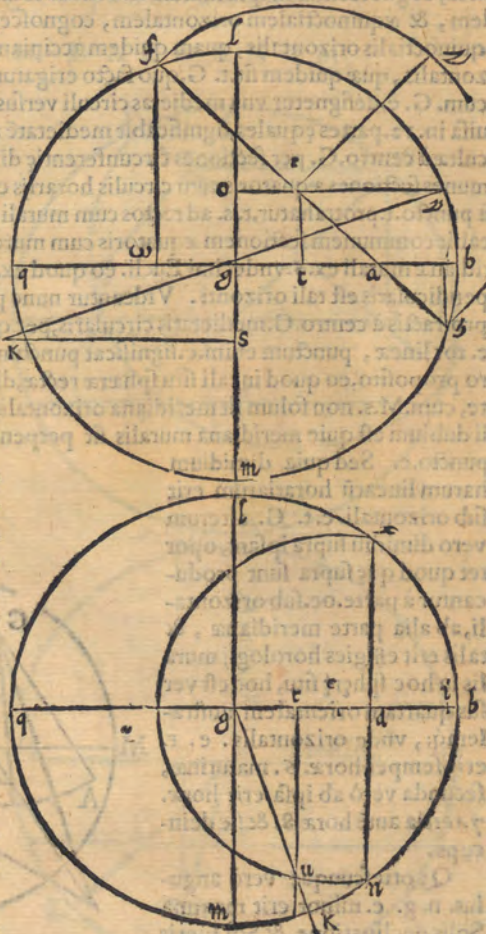
## De Horologio perpendiculari ad orizontem rectum.

### AD EVNDEM.

MOdus quem tibi fcribere promifi delineandi lineas horarias communes in pariete perpendiculariter ad orizontem rectum, declinantem à meridiano, fumendus eft ex.46.cap.meæ gnomonicæ, hoc fcilicet ordine.

Sit exempli gratia, orizon hic fubfcriptus. or . oc. M . S. diuifus à meridiana. M. S.et verticali feu æquinoctiali. or. oc . Sitque. e. t.communis fectio muri cum orizonte,et.g.n.fit gnomon perpendicularis ipfi muro, vnde ex dictis in mea gnomonica, cognofcemus in ipfa murali orizontali totam.e.t. inter meridianam orizontalem, & æquinoctialem orizontalem, cognofcemus etiam partem. g . t. ipfius æquinoctialis orizontalis, quam quidem accipiamus in rectitudine ipfius muralis orizontalis, quæ quidem fit.t. G.quo facto erigatur. G. A.ad rectos cum.G.t.e.& circum. G. e.defignetur vna medietas circuli verfus. e. cuiufuis magnitudinis, quæ diuifa in.12.partes æquales,fignificabit medietaté æquatoris,protrahanturq; lineæ oc cultæ à centro.G. per fectiones circunferentię dimidij circuli, quæ fignificabunt có munes fectiones æquatoris cum circulis horarijs communibus, quo facto oportet, vt à puncto.t.protrahatur.t. s. ad rectos cum murali orizontali,quæ quidem.t.s. fignificabit communem fectionem æquatoris cum muro propofito, & erit ęquędiftans me ridianæ murali ex.6.vndecimi Eucli. eo quod ex.19.eiufdem vnaquæq; illarum,per pendicularis eft tali orizonti . Videantur nunc puncta communia ifti.t.s.& occultis protractis à centro.G.medietatis circularis,per quæ puncta protrahantur à puncto. e. tot lineæ, punctum enim.e.fignificat punctum axis mundi , & meridianæ in muro propofito,eo quod in tali fitu fphæræ rectæ,dictum punctum reperitur in orizon te, cum.M.s. non folum fit meridiana orizontalis,fed etiam axis mundi , deinde nul li dubium eft quin meridiana muralis fit perpendicularis orizontali murali. e. t. à

puncto.e. Sed quia dimidium harum linearũ horariarum erit fub orizontali. e. t. G. alterum vero dimidiũ fupra ipfam, opor tet quod quę fupra funt produ cantur à parte.oe.fub orizonta li,ab alia parte meridianæ , & talis erit effigies horologij mura lis in hoc fphęrę fitu, hoc eft ver fus quartam orientalem auftra lemq́;, vnde orizontalis. e. t. erit femper horæ. 6. matutinæ, fecunda verò ab ipfa erit horæ. 7.tertia autę horæ.8. & fic dein ceps.

Quotiefcunque verò angu lus. n. g. e. minor erit maxima Solis declinatione, & Sol fuerit in parte auftrali ab æquatore cũ maiori



De
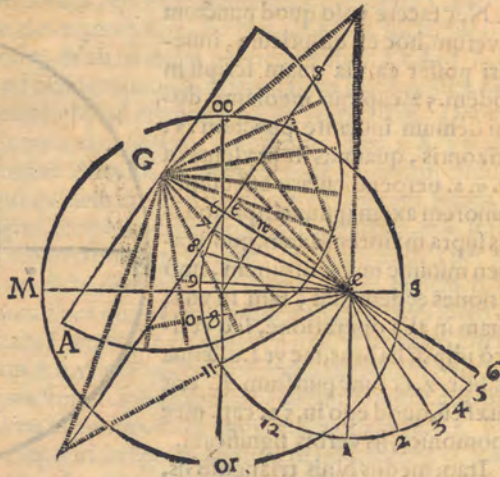
maiori numero declinationis quam fuerit angulus.n.g.e.tunc talis paries illumina-
bitur ab ipfo Sole à mane vfque ad vefperam.

Huius quidem rei fpeculatio, vnicuique manifefta erit, qui rationes.46.cap.no-
ftræ gnomonicæ prius intellexerit,vbi manifeftè apparet proportionem femidiame-
tri horologij ( fi ita eam appellare licet ) ad femidiametrum æquatoris horarij fem-
per effe,vt.c.t.ad.t.g.hoc eft proportio maioris inæqualitatis. nolo etiam prætermit
tere.quin te admoneam,vt nullo pacto confidas in longioribus vmbris,eo quod val
de nos decipiant, cum femper iufto breuiores fint.

### *Declaratio quorundam verborum noftræ Gnomonicæ.*
### *Defenfioq́; noftra contra Chriftophorum Clauium.*

## AD EVNDEM DARDANVM.

Vas demum accepi literas,qui
bus mihi fignificas te totum.
52.caput meæ gnomonicæ intelle-
xiffe , præter illa verba, quæ etiam
fuperioribus diebus ad te fcripfi ,
hoc eft.

Itaq; medijs binis triangulis ijs,
medioq́; azimut Solis, pariter ho-
rologia fabricari poterunt.

Quapropter ne aliquid tibi de-
fit, fcire debes , me nihil aliud, eo
in loco inferre voluiffe , quàm qd̃
punctum horæ propofitæ in plano
horologij orizontali reperiri po-
teft , ope longitudinis vmbræ gno-
monis, & eius declinationis à ver-
ticali linea,feu à meridiana orizon
tali , iam in ipfo horologij plano
ductis.

Exempli gratia , fit analemma.
l.q.m.b.in quo.l.m.fit verticalis. q.
b.verò orizontalis.f.n. h. autem fit
femicirculus,cuiufuis paralleli æqui
noctiali, cuius diameter fit.f.h. et.
n.fit Solis locus in ipfo parallelo: n.
r.autem fit rectus finus arcus.f.n:et.
r.o.z.fectio communis ipfius almi-
cantarat cum meridiano,et.s. a. có
munis fectio azimut Solis cum pla-
no horologij,et.s.g.gnomon, et. z.
g.a.radius Solis.z.u.verò finus alti-
tudinis ipfius Solis, vbi videre po-
tes duo triangula dicta effe. z. u.g.

Ss                    &
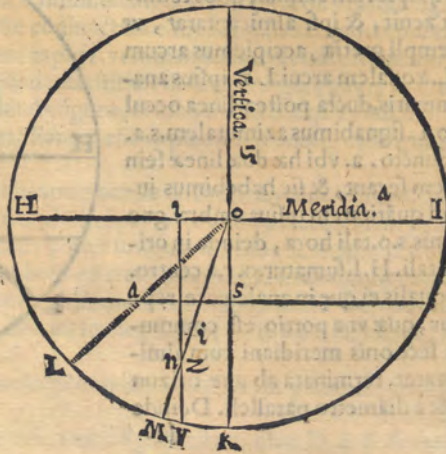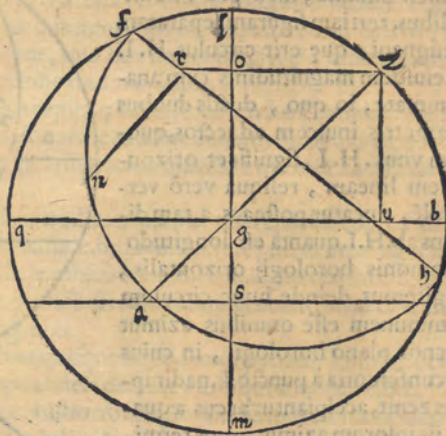
et. g. s. a. quibus mediantibus cognoscitur longitudo vmbræ gnomonis hoc est .s.a.

Cum autem dico, medioq; azimut Solis, nihil aliud significare volo , nisi angu-
lum , quem terminat linea azimutalis horologij , hoc est vmbra gnomonis cum li-
nea meridiana, seu cum verticali in ipso plano horologij. qui quidem anguli , æqua
les sunt ijs, qui in triangulo constituto ex.n.r.ex.r.o.& ex.o.z.reperiuntur, cuius qui
dem trianguli, angulus puncti.r.rectus est, angulus verò terminatus ab.n.r.et.o.z.il
le est quem constituit azimut cum verticali , vel ipsi æqualis, vt coalternus, reliquus
verò in pũcto.o. ille est quẽ azimut facit cũ meridiano, vel ipsi ẽqualis vt coalternus.

Vnde quotiescunque volueris in aliquo plano, orizonti parallelo , lineas hora-
rias ducere, iudico optimum fore si separatim designatæ fuerint hæ tres figuræ, hoc
est analemma meridianum , vel azimutale, vt ita dicam, deinde parallelus inseruiés
pro tropicis , vt ego feci cap. 5 1.meæ gnomonicæ , quæ duæ figuræ , sufficiẽtes erũt
pro omnibus horologijs , tam ori-
zontalibus quàm muralibus , non
tamen omninò , ideo pro orizon-
talibus, tertiam figuram separatam
designaui, quẽ erit circulus. H. I.
K.eiusdem magnitudinis cum ana-
lemmate, in quo , ductis duobus
diametris inuicem ad rectos,quo-
rum vnus . H. I .significet orizon-
talem lineam , reliqua verò ver-
ticalẽ , ducatur postea. s. a.tam di-
stans ab.H.I.quanta est longitudo
gnomonis horologij orizontalis ,
cogitemus, deinde hunc circulum
communem esse omnibus azimut
necnon plano horologij , in cuius
circunferentia à puncto.k.nadir ip-
sius zenit, accipiantur arcus æqua-
les ijs ipsorum azimut, quos termi-
nat zenit , & ipsi almicantarat , vt
exempli gratia , accipiemus arcum
k.L.æqualem arcui.L.z. ipsius ana-
lemmatis,ducta postea linea occul-
ta.o.L.signabimus azimutalem.s.a.
in puncto . a. vbi hæ duæ lineæ sein
uicem secant, & sic habebimus iu-
stam quãtitatem ipsius vmbræ gno
monis.s.o.tali hora, deinde in ori-
zontali. H.I.sumatur.o. r.à centro.
o.ẽqualis ei quẽ in analemate repe
ritur , quæ vna portio est commu-
nis sectionis meridiani cum almi-
cantarat, terminata ab axe orizon
tis,& à diametro paralleli. Deinde
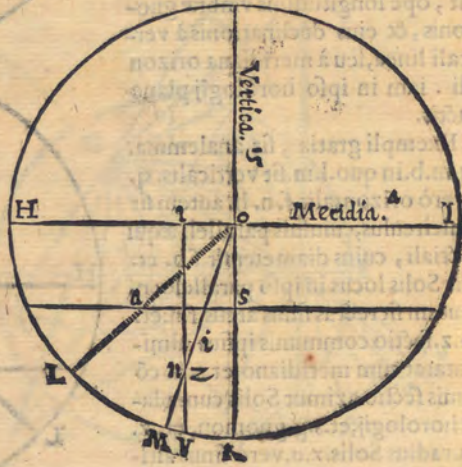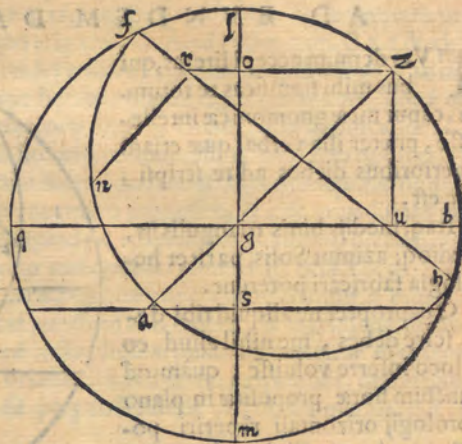
du-

ducaf.r.V.ad rectos cum.H.I.vsque ad circunferentiam, in qua accipiatur.r.n.æqua
lis ei quæ est in parallelo, ducatur postea.o.n.M. & habebimus triangulum.o.r.n.si-
milem æqualemq; triangulo iam supradicto. Vnde angulus.H.o.M.ei æqualis erit,
quem azimut facit cum meridiano, & angulus.M.o.k.ei equalis, quem azimut con-
stituit cum verticali, ita quod si talis circulus.H.k.I.esset planum horologij orizon-
talis, supposito.o.pro pede gnomonis, secando postea.o. M. in puncto.i. ita vt.o.i.
æqualis esset.s.a.dato quod.o.M.ducta sit ad partem sibi conuenientem, respectu.o.
k.ipsa pro verticali supposita, quod tibi relinquo, cum hoc facillimum sit, tunc pun
ctum.i.esset quod quærebamus. Quod verò de vno puncto dico, idem de omni-
bus infero.

Vbi verò mihi significas Christophorum Clauium, me duobus in locis meæ gno
monicæ redarguere, iam vidi. Circa primum locum igitur, qui est in pagin. 161.
ita inquit.

"     Non enim desunt, qui vel omninò negent, inter quos est Ioannes Baptista Be-
"  nedictus in sua gnomonica cap.70.et.71.vbi alia, & multo longiore ratione cona-
"  tur arcus signorum describere, vel certe dubitent, hoc modo rectè posse describi ar-
"  cus signorum, cum rationem non videant, qua hæc nostra descriptio quam quidem
"  omnes scriptores sine vlla demonstratione tradunt nitatur.

Absque dubio raptim trãscurrit illa capita.70.71.Reuerendus Clauius alioquin
non scripsisset, quòd ego alia & multo longiore ratione conatus sim arcus signorum
describere &c. præsertim cum eadem prorsus ratio, quæ ibi à me tradita est, illa sit,
quam ipse suis scriptis inseruit.

Meus igitur modus in dictis capitibus traditus, minime discrepat ab eo, sed ab il-
lorum modo, quorum opinio est interualla.e.h:h.u:u.n;n.m.et.m. d. meæ figuræ in
pagi.75.positæ, æqualia esse interuallis.e.h:h.u:u.n:n.m.et.m.d.præcedentis figuræ,
qui etiam supponunt.t.e.meæ figuræ.75.esse directè coniuncta cum linea. e. h. u.n.
m.d.& propterea versus finem.73.pag.dixi.

"     Aduertat autem quam diligentissime quisque ne se decipi patiatur à subscripta fi
"  gura semicirculi. Q. æ. m. cum reliquis lineis ductis, ex antiquorum more,&c.

Eo quod non defuerunt aliqui, ex vetustioribus (quorum scripta ad meas manus
peruenerunt) qui sumentes interualla.e.h:h.u.&c.figuræ.pag.75.æqualia illis figu-
ræ pag.74.putauerũt lineam.t.e.directè coniunctam esse cum.e.h.&c.quod quidem
maximi erroris causa erat, & propterea cap.71.verum modum ostendi, seruando il
lam eandem suppositionem, hoc est quod interstitia.e.h:h.u:&c.figurẹ pag.75.æqua
lia sint interstitijs.e.h: h.u.&c.præcedentis figuræ, & ideò in dicto cap.71.dixi.

"     Supposito deinde.f.e.b.lineam meridianam esse in plano orizontali, cẹterẹ lineẹ
"  horariẹ erunt prẹdictẹ.

Stantibus igitur his suppositis, vt habeantur omnia scientificè, volui, vt intellige-
retur pyramis quadrilatera, eo modo quo dixi, cap.71.vbi clarè patet eandem pyra
midem esse, quam Pater Clauius (tacitè) posuit in figura horologij, vt ipse docuit
propositione secunda, lib.secundi, cuius basis est triangulum.H.I.F.suæ figurẹ (exem
pli gratia pro quinta hora post meridiana) Alterum verò triangulum à me cogita-
tum, terminatum ab.t.e:e.d:et.ab.t.d.eleuata in mea figura, est in sua triãgulum. D.
I. F.& propterea dixi.

"     Nam.t.e.et.e.d.vtræq; in plano horologii non sunt, quamuis in plano æquatoris
"  tres sint,&c.

Angulus verò.e.quem dico rectum esse, in sua figura est angulus. D. I. F. & mea

t.d.imaginata, eſt ſua.D.F. Tertium deinde triangulum, quod in mea figura ter-
minatur ab.t.d.ab.f.d.& ab.f.t.in ſua eſt triangulum.D.F.H. vnde mea. f. t. reſpon-
det ſuę.H.D & mea.f.d.ſuæ.H.F.& mea.t.d.ſuę.D.F.  Quartum autem triangulum
f.t.e.in mea figura, reſpondet ſuo.H.D.I.& meum punctum.t.ſuo.D, Nunc triangu
lum rectangulum, quod dico ſeparatim conſtituere, eſt illud tertium dictum corre-
ſpondens ſuo.D.F.H.vt ipſe facit in ſequęti figura, quod ipſe vocat.D.C.H. & me⁹
ra dius.t.x.in ſua figura, ille eſt qui terminatur ab.D.& ab initio Tauri, & Virginis.

Et quamuis ego non ſcripſerim talem figuram, vt ipſe fecit, nihilominus ipſam
verbis deſcripſi eomet modo, & propterea dixi.

,,          Quam diuiſionê, ſi in triangulo ſeorſum deſcripto inuenire voluerimus, res erit
,,      inuentu facillima, cum rectum angulum.f.t.d. (reſpondentem ſuo.H.D.C.)prędicti
,,      trianguli tertij ea ratione diuiſerimus,&c.

Quapropter Reuerendus Clauius non animaduertit meam rationem aliam non
eſſe, nec puncto longiorem ſua, cum eademmet ipſa ſit.

Citaui etiam Munſterum cap.30.eo quod in ea impreſſione,quam tunc pre mani
bus habui, vidi in ea figura, quam ipſe vocat fundamentum horologiorum, literam
c.poſitam eſſe loco.f.et.f.loco.c.quod cauſæ fuit, vt omnia mendoſa viderentur,re
centiores autem impreſſiones correctæ ſunt.

Rurſus alio in loco mihi accidit vt repręhenderim Alexandrum Piccolomineum
in libris de ſphęra, qui quidem dicebat eas figuras ſuperficiales, quæ paucioribus an
gulis circunſcriberentur, capaciores eſſe alijs,dummodo earum periphæriæ eſſent
æquales.

Nunc autem correctę ſunt eo in loco impreſſiones, & qui non viderit primas, pu
tabit me immeritò ipſum repræhendere.

Idem etiam dico de eo capite ipſius Piccolominei, in ijſdem libris,vbi tractat de
modo, quo vſi ſunt antiqui ad diuidendum zodiacum in.12.ſigna, quod erat circa
finem quarti libri.

Nunc verò,in recentioribus impreſſionibus, illud caput poſitum non eſt. Impreſ
ſiones autem illæ, vbi talia dixit, duæ fuerunt, quarum prima erat anni.1540.ſecun
da verò. 1552.Venetijs apud Andream Puteum.

Alius verò locus ipſius Reuerendi Clauij, contra meas repręhenſiones, eſt circa
finem pag.298.& circa.299.vbi ita ſcribit.

,,          Ex his liquido conſtat,non rectè à Ioan.Baptiſta Benedicto in ſua gnomonica ca
,,      pit.49.repręhendi hanc rationem deſcribendi horologij declinantis,qua omnes fe-
,,      rè alij ſcriptores vtuntur, quoniam, vt ex demonſtratione à nobis allata conſtat,re-
,,      ctè per eam lineæ horarię in plano, quod à verticali declinat ducuntur. Modus au
,,      tem quem eo loco pręſcribit differentem ab eo, quem nos tradidimus certus etiam
,,      eſt, ſed nulla ratione noſtro contrarius, quia nos conſtituimus.D.E.F. angulum de-
,,      clinationis plani à verticali circulo propriè dicto,ipſe autem loco huius anguli aſſu-
,,      mit angulum declinationis eiuſdem plani à Meridiano circulo, vnde mirum non eſt
,,      modum ipſius à noſtro diſcrepare. Quod ſi côſtitueremus.D. E. F. angulum decli-
,,      nationis plani à Meridiano, ut ipſe ( quemadmodum forſitan ab alijs putauit fieri)
,,      & in reliqua deſcriptione progrederemur, vt tradidimus, proculdubio horologiû
,,      declinans perperam deſcriberetur,vt rectè docet.

Optimè ſcripſiſſet Reuerendus Clauius, ſi verum fuiſſet, quod antiqui ſumerent
declinationem ſuperius dictâ à verticali propriè dicto,& non à meridiano. Sed ego
dico, authores à me citatos. capit. 49. meę gnomonicę ſumere dictam declinatio-
nem

nem plani à meridiano , & non à dicto verticali .

Con sidera primum in Munstero cap. 16.suæ horologiographiæ , vbi clarè docet accipere angulum compræhensum inter meridianum , & planum propositum , vbi etiam ponit quandam figuram ædificij cum pariete super quo designatum est quod dam horologium, & vbi se manifestè declarat , ita dicens.

     Nam ipsarum partium complementum . propositum indicabit angulum , quan-
" tus videlicet fuerit arcus eiusdem circuli.d.e.f.g.à puncto.g. vsque ad productam li-
" neam meridianam interceptus,qui vnà cum ipso.f.g.quadrantem integrare videtur,
" vt in sequenti figura:quoniam arcus.f.g.est sexaginta partium,qualium.e.f. quadrās
" nonaginta, vnde concluditur reliquam partem hoc est, datum inclinationis angulū,
" fore partium triginta similium .
"      Orontius verò cap.13.ijsdē vtitur verbis,cum figura simili ad reliqua autem ipsius
℞. Clauij, videnda nondum mihi otium fuit. quod si dabitur , tibi libenter dicam quid sentiam.

# DE MODO DVCENDI LINEAS HORARIAS
## super cyllindro immobili.

### *Hieronymo Ferrerio artium & Medicina Doctori peritissimo.*

D ESIGNARE horarias lineas super cyllindro immobili, ad orizontemque perpendiculariter erecto difficile tibi non erit, (quod à me postulasti ) si modum. 53.cap. meæ gnomonicæ obseruaueris , accipiendo tamen pro linea orizontali in tabula non aliquam rectam lineam , sed circularem, similemque circunferentiæ ipsius cyllindri, dico autem similem , eo quod si gno-mon.o.x.supra tabulam signatus , & perpendicularis ipsi orizontali circulari. b. i. x. esset dimidia, vel tertia vel quarta pars gnomonis cyllindro infixi, oporteret, vt semidiameter circuli.b.i. x. etiam
esset medietas,vel tertia, aut quar
ta pars semidiametri cyllindri , vt
omnes arcus huiusmodi circuli in
ter ipsos azimut intercepti similes
sint arcubus cyllindri , quod à
te ipso facilè videre scientificè po
teris.reliqua nihil mutanda erunt
ab eo, quod scripsi circa figuram.
53. cap. vt dixi . Vnde inuenta
cum fuerit distantia orizontalis
puncti.b. à pede gnomonis.x,nec
non quantitas azimutalis muralis
b.t.quæ semper ab orizontali per
pendiculariter descendit , illicò
punctum.t. horæ propositæ in cy-
lindro inuenietur .
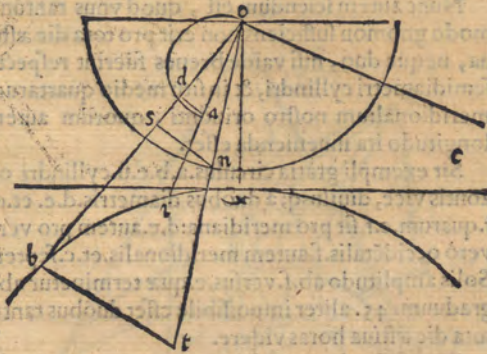
Nunc

Nunc verò cum duo puncta alicuius horariæ lineæ inuenta fuerint, quæ à Solis fi-
tu in diuerfis parallelis efficiuntur, fi voluerimus ipfam lineam horariā ducere,fcien
dum primò eft ipfam lineam horariam effe communem fectionem circuli horarij,
illius horæ cum fuperficie cyllindrica ,& propterea ellipticam, vt oftendit Serenus
in. 19.primi lib.quod etiam ellicere poffumus ab eo,quod Archimedes in. 10. pro-
pofitione libr. de conoidalibus , fcribit . Quapropter oportet nos inftrumen-
tum prius componere , modo circini , fed trium crurum , quæ omnia in eadem
plana fuperficie fint , ea tamen arte factum, vt quodlibet illorum poffimus pro-
longare , necnon contrahere, ut cum duo extrema firmata fuerint , media pof-
fit circunduci circa centrum,feu punctum commune illarum interfectionum fimulq;
poffit produci,necnon abbreuiari vel augeri , & diminui, vt mediante fua extremi-
tate inferiori poffimus delineare gyrum ellipticum horarium , dum cētrum ipforum
crurum adhæreat extremitati gnomonis, reliquæ vero extremitates ipforum crurū
fint fupra puncta inuenta ipfius horæ . oportet etiam vt hoc inftrumentum à tergo
ipforum crurum habeat in fuperiori parte fuperficiem quandam femicircularē,quę
fit vice vnius partis illius fuperficiei, in qua fupponuntur omnia crura inftrumenti ,
& hoc quantum fieri poteft, quod quidem fieri debet, ne crus medium,hoc eft mo-
bile,exeat à tali fuperficie, feu declinet ab ea, quæ femper fupponitur in fitu circuli
horarij talis horæ . oportet etiam, vt iuxta circunferentiam dimidij circuli fint duo
gyri eiufdem materiæ inter fe parum diftantes,ita ut crura poffint moueri, intra hos
gyros , & dimidium circulum , & quod inter hos gyros locatæ fint duæ cochleæ,feu
duo helices, vt quando voluerimus , poffimus fir-
mare ipfa crura extrema, dum eorum extremitates
fuerint fupra puncta inuenta illius horæ, deinde in
dorfo iftius inftrumenti , circa centrum coniunctio-
nis,rectè factum erit fi aliqua concauitas fuerit , in
qua,extremitas gnomonis poffit locari,dum duce-
re voluerimus aliquam horariam lineam.

Tale inftrumentum excogitaui ad fugiendum
tædium inueniendi dictam ellipticam ex punctis

Nunc autem fciendum eft , quod vnus tantum-
modo gnomon fufficiens non erit pro tota die æfti-
ua, neque duo, nifi valde breues fuerint refpectu
femidiametri cyllindri, & in fitu medio quartarum
meridionalium noftro orizonti , quorum autem
longitudo ita inuenienda effet.

Sit exempli gratia circulus.a.b.e.u.cyllindri ori
zontis vice, diuifusq; à duobus diametris.d.e. et.c.
f.quarum.c.f.fit pro meridiana:d.e.autem pro verticali,fitq;.e.punctus orientalis:d.
verò occidētalis.f.autem meridionalis.et.c.feptentriōalis , computeturq; maxima
Solis amplitudo ab.f.verfus.e.quæ terminetur ab.q.ita quod arc⁹.f.q. minor fit quā
graduum. 45. aliter impoffibile effet duobus tantūmodo gnomonibus mediantibus
tota die æftiua horas videre.

Quo facto ducatur ab.q:q.p.contingens circulum & à centro circuli. o. per pun-
ctum.u.medium quartæ ducatur.o.u.i.vfque ad contingentem.q.p.vnde.u.i.longitu
do erit vniufcuiufque gnomonis, qui gnomones infixi erunt in medio dictarum
quartarum.
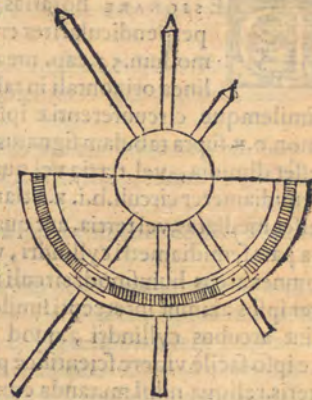
Huiufmodi

Huiuſmodi rei ratio per ſe nota erit quotieſcunque cogitauerimus verum arcum e.b.amplitudinis æſtiuæ, protractaq;.o.b.quę parallela erit.q.p. vnde cum Sol tempore æſtiuo orietur, tunc radios ſuos emittet via iſtarum æquidiſtantium linearum.

Sed ſi longiores gnomones cuperes, oportebit eos tres eſſe, quorum vnus erit orientalis in puncto.e. alter occidentalis in puncto.d. reliquus verò meridionalis in puncto.f.quorum vnuſquiſq; poteſt eſſe maior tertia parte ſemidiametri cyllindri, ſed ſi voluerimus ſcire quantũ ad plus poſſit eſſe longus vnuſquiſque illorum, ita faciendum erit.
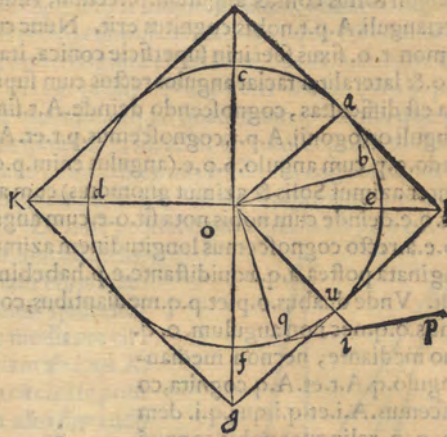
Faciemus quadratũ.o.a.h.u. ex ſemidiametro dicti circuli, à diametro poſtea.o.h.huiuſmodi quadrati ſubtrahatur ſemidiameter. o.e.circuli,reſiduum verò.e.h.ipſius diametri.o.h.quadrati,erit lógitudo gnomonis, vbi ſimul appareret huiuſmodi rei ratio, eo quod cum gnomon.e.h.orientalis deſinet operari, illico meridionalis

f.g.ſubintrabit, poſt hunc verò occidentalis.d.K.monſtrabit reliquum diei.

## Earundem linearum deſcriptio ſuper conum rectum.

## AD EVNDEM.

CVm ſuper datum conum rectum idem facere volueris eſto conus. A.&.R̃.qui diuiſus imaginatione ſit à quodam plano per axem, & communis ſectio ſit triangulus. A.&.R̃.in quo plano cogitemus gnomonem infixum ad rectos vbi volueris,qui ſit.p.t.o cogitem? etiam.l.t.m.aliud eſſe planum ( in quo ſit gnomon) quod conum ſecet, quæ quidem ſectio, circularis erit, ex. 4. primi Pergei.imaginemur etiam ſuperficiem.p.s.eſſe azimut in quo gnomon reperirur, ſuperficiemq́;.e. s. azimut propoſitæ horę, angulumq́;.e.o.a. contrapoſitũ angulo altitudinis Solis ab orizonte;
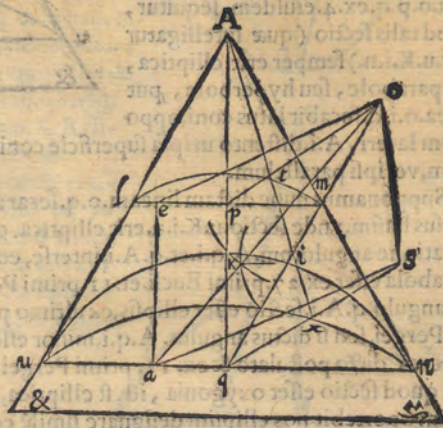
zonte; cogitemus etiam lineam.A.t.i.x.illud coni latus effe, quod à fummitate ver
fus bafim tranfit per medium latitudinis ipfius gnomonis, concipiamus etiam mente
e.a.communem fectionem effe trianguli fupradicti cum azimut horæ, necnon pun-
ctum.K.effe commune radio Solis.o.a.& fuperficiei conicæ, quod quidem eft illud
quod quæritur, hoc fcilicet modo. Primum cognofcimus angulum.p.A.t.vt medie
tas anguli totius coni, & angulum.p.rectum, vnde.t.tam intrinfecus, quam extrinfe-
cus trianguli.A.p.t.nobis cognitus erit. Nunc cum angulus.A.t.o. cognofcatur, fi
gnomon.t.o. fixus fuerit in fuperficie conica, ita qd̃ cum latere.A.t. efficiat angulũ
A.t.o.& lateraliter faciat angulos rectos cum fuperficie conica, ad quod efficiendum
nulla eft difficultas, cognofcendo deinde.A.t.fimul cum angulis.A. et. t. intrinfecis
trianguli ortogonij.A.p.t.cognofcemus.p.t.et.A.p.vnde etiam totã.o. p. fed cogno
fcendo.o.p.cum angulo.p.o.e.(angulus enim.p.o.e.cognofcitur ex hypothefi cum
fit inter azimut Solis & azimut gnomonis) cum angulo.o.p.e.recto cognofcemus.p.
e. et.o.e.deinde cum nobis nota fit.o.e.cum angulo altitudinis Solis.e.o. a. & angu-
lo.o.e.a.recto cognofcemus longitudinem azimutalis.e.a.necnon quantitatem. a.o.
Imaginata poftea.a.q.æquidiftante.e.p.habebimus.p.q.æqualem.a.e. ex. 34. primi
Eucli. Vnde duabus.o.p.et.p.q.mediantibus, cognitifq̃ cum angulo recto.p. cogno

fcemus.o.q.nec non angulum. o. q.
p.quo mediante, necnon median-
te angulo.q.A.t.et.A.q.cognita, co
gnofcemus.A.i.et.q.i.quę.q.i. dem
pta à. q. o. relinquet nobis cognitã
i.o. Et quia.o.i.q.et.o. K. a. femper
funt in eadem fuperficie fecante co
num, quæ etiam fecat fuperficiem
trianguli.A.q.x.ad rectos ex.18.vn
decimi, cum linea. u. n.perpendicu
laris fit fuperficiei trianguli. A. q.i.
ex.8.dicti,quia parallela eft.l.p.quę
perpendicularis eft fuperficiei triã-
guli.o.p q.ex.4.eiufdem, fequitur,
quod talis fectio ( quæ intelligatur
per.u.K.i.n.)femper erit elliptica,
vel parabole, feu hyperbole, ꝓput
linea.o.i.q.fecabit latus coni,oppo



fitum lateri. A.i.diftento in ipfa fuperficie conica,feu ad fuperiorem partem produ
ctum,vel ipfi parallelum.
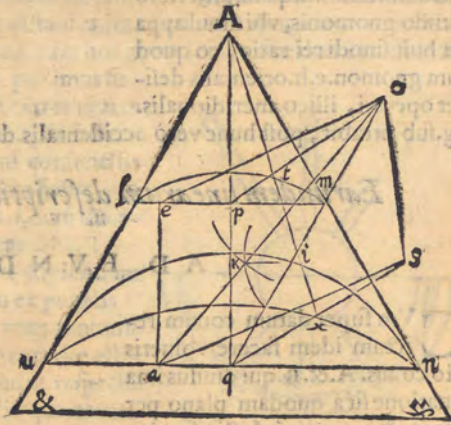
Supponamus nunc dictam lineam.o.q.fecare dictum oppofitum latus lateri. A.i.
verfus bafim,vnde fectio.u.K.i.n.erit elliptica. quod facile cognitu eft mediãte com
paratione angulorum.A.q.i.et.q.A.i.interfe, eo quod fi effent equales, dicta fectio
barabola effet ex.27.primi Eucli.et.11.primi Pergei, fed fi angulus.A.q.i.maior ef-
fet angulo.q.A.i.fectio effet ellipfis,ex ultimo poftulato primi Euclid. & ex.13.pri
mi Pergei,fed fi dictus angulus. A.q.i.minor effet angulo.A.tunc fectio effet hyper-
bole ex dicto poftulato & ex.12. primi Pergei. Sit ergo primum vt dictũ eft, hoc
eft, quod fectio effet oxygonia, ideft elliptica,feu defectio(quod idem eft,) fepa-
ratim oportebit nos ellipfim defignare fimilé equaléq̃ ei, quæ eft.u.K.i.n.qd̃ quidé
difficile non erit, quotiefcunque fuos axes inuenerimus, maiorem fcilicet, & mino-
                                                                                              rem,

rem, quæ ita reperientur, efficiemus primo angulum coni, qui fit. i. A. b. quem diui-
demus per æqualia mediante. A. q. conftituendo. A. i. huius anguli æqualem. A. 'i. fu-
perficiei conicæ et. A. q. diuidentem, æqualem parti. A. q. axis coni, ducendo poftea
ab. i. per. q. lineam vnam quoufque concurrat. A. b. in puncto. b. habebimus. i. b. pro
maiori axi ipfi ellipfis, quod per fe clarum eft, cuius medietas fit. i. c. fed. i. q. ipfius. i.
b. æqualis eft ipfi. q. i. ipfius coni, ex quarta primi Eucli. et. q. b. ipfius. i. b. æqualis alte-
ri parti inuifibili. Reliquum eft, vt reperiamus minorem axem, quem vocabimus.
f. r. ducatur ergo primum. q. a. u. n. ad rectos cum. i. b. æqualisq; ei quæ eft coni, & diui-
fa fimiliter in. a. quæ. u. n. ipfius coni nobis cognita eft ex lateribus. A. u. et. A. n. & ex
angulo coni, et. a. q. æqualis eft. e. p. ex. 34. primi. Nunc certi erimus ex. 21. primi
Pergei, quod eadem proportio erit quadrati. u. q. ad quadratum ipfius. f. c. quæ pro-
ducti ipfius. i. q. in. q b. ad productum ipfius. i. c. in. c. b. & cum cognita nobis fint
hæc tria producta hoc eft. i. q. in. q. b. et. i. c. in. c. b. et. u. q. in feipfa, cognofcemus etiã
quartum ipfius. f. c. & fic. f. c. eiufq; duplum. f. r. cogniti nobis itaque cum fint hi duo
axes. i. b. et. f. r. formabimus ellipfim. Deinde producemus axim. b. i. à part e. i. quo-
ufque. i. o. æqualis fit ei quæ extra conum eft, dein-
de ducemus. o. a. quæ circunferentiam ellipticam
fecabit in puncto. K. vnde habebimus quantita-
tem ipfius. o. K. et. K. i. rectam. inde mediante cir-
cino fi acceperimus rectam diftantiam ab. i. ad. K.
in ellipfi, deinde firmando pedem circini in pun-
cto. i. in fuperficie conica, & cum alio fignando
lineam vnam curuam ad partem. K. in fuperficie
conica, fumendo poftea interuallum. o. K. extra el-
lipfim, deinde firmando vnum pedem circini in
extremitate gnomonis, cum alio poftea fignan-
do aliam lineam curuam in fuperficie ipfius coni,
quæ primam fecet in puncto. K. hoc erit punctum
quæfitum horę propofitæ in fuperficie conica
propofita.

Sed fi talis fectio fuerit parabole, vel hyperbo
le, tunc mediante fuo diametro. i. q. cum bafi. u.
q. n. cognita, defignabimus ipfam fectionem. u. i. n
ope mei inftruméti in calce meę gnomonicæ de
fcripti, deinde diuifa. u. q. in. a. pductaq; q. i. vfq;
ad. o. ductaq; . o. a. habebimus punctum. K. Reli-
qua facienda funt, vt dictum eft de ellipfi.

Inuenta modo cum fuerint duo puncta eiuf-
dem horæ propofitę, ducemus ab vno ad a-
liud, lineam horariam, mediante circino trium
crurum, quem tibi fcripfi nudius tertius pro cyl
lindro, quæ quidé linea erit portio gyri ellipfis,
feu hyperbole, vel parabolę, vt à te ipfo cogi-
tare potes.

Tt Quę-

**IO. BAPT. BENED.**
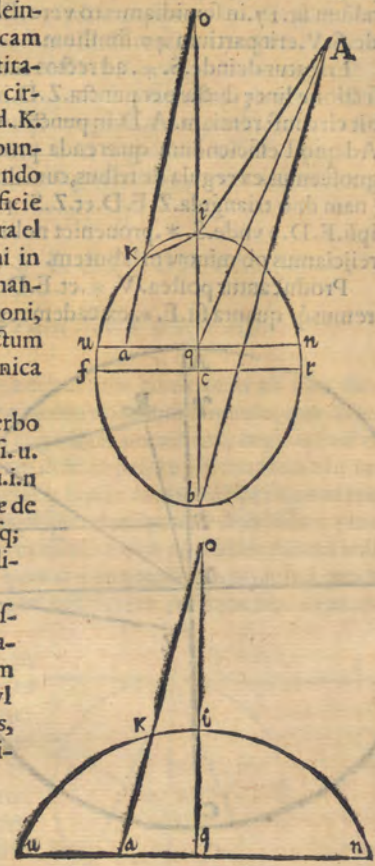
# QVAEDAM NOTATV DIGNA IN
Ptolomeum.

*Bartolomeo Christino Serenißimi Sabaudiæ Ducis apparitore.*

EX tuis literis cognoui quo erga me animo esses, qualiq; voluntate, sed ne tua pulcherrima studia aliquo modo imperfecta relinquant, vel ego tibi deesse videar, dum Problemata geographica Magni Ptolomei consideras, aduerte, quod si putares in figura.6.cap.libr. 7. geographię eiusdem (vt multi credunt) lineam. V. ✱. secare circunferentiam. A. D. in puncto. G. ita vt punctus. G. sit tropici æstiui, idest arcum. D. G. esse graduum. 24. cum illis incideres in maximum errorem. Quapropter considera quæ nunc tibi scribo.

Sit circulus. A. B. C. D. huius centrum. E. supponaturq; semidiameter. E. D. esse partium. 120. quarum. E. Z. in alio semidiametro. C. E. ei orthogonaliter coniuncto, talium sit.17. in semidiametro vero. E. A. accipiatur. E. S. talium. 24. et. E. V. 64. vnde. S: V. erit partium 40. similium.

Erigatur deinde. S. ✱. ad rectos cum. E. A. in puncto. S. quæ terminetur ab intersectione lineę ductæ per puncta. Z. D. in puncto. ✱. ducatur demum. V. ✱. quæ secabit circunferentiam. A. D. in puncto. G. Quæratur nunc quantitas ipsius. G. D. Ad quod efficiendum quærenda primum est quantitas ipsius. S. ✱. quam illico cognoscemus ex regula de tribus, cum dixerimus, si.17. dat nobis. 120. quid dabit. 41. (nam duo triangula. Z. E. D. et. Z. S. ✱. sunt inuicem similia, cum. S. ✱. parallela sit ipsi. E. D.) vnde. S. ✱. proueniet nobis ex similibus partibus. 289. cum fracto, quod reijciamus ob minorem laborem.

Producantur postea. V. ✱. et. E. D. vsque ad eorum concursum in puncto. ω. quæremusq; quanta sit. E. ω. ex eadem regula, cum dixerimus, si.40. dat nobis. 289. quid
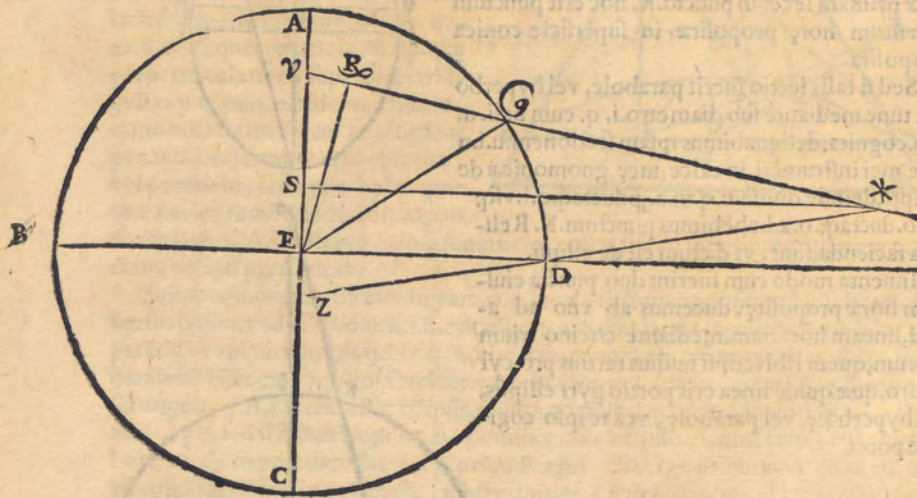


dabit

dabit.64. (nam duo triangula. V.S. ✳.et.V.E.ω.funt inuicem similia eadem ratio-
ne) vnde. E. ω. veniet nobis ex talibus partibus. 462.

Coniungatur nunc quadratum ipsius. E. V. quod est. 4096. cum quadrato ipsius.
E.ω. quod est. 213444. & habebimus quadratum ipsius. V.ω. talium partiũ. 217540.

Dicemus postea si. 217540. dat nobis. 4096. quid dabit quadratum ipsius. V. ω. vt
sinus totus quod est. 10000000000. vnde veniet pro quadrato ipsius. V. E. talium
partium, superficialium scilicet. 18827211. cuius radix erit. 13721. & erit sinus an-
guli. V.ω. E. qui erit grad. 7. min. 53. vnde angulus. ω. V. E. erit grad. 82. min. 7. eius
vero sinus erit partium. 99054.

Nunc autem quia angulus. E V.ω. est acutus, imaginemur. E. ℞. ductam esse ad re
ctos ipsi. V. ω. sitq; etiam ducta ipsa. E. G. Vnde habebimus angulum. ℞. E. V. gra-
duum. 7. min. 53. eius vero sinus. ℞. V. partium. 13721. (propter similitudinem trian
gulorum. E. ℞. V. et.ω. E. V.) talium scilicet, qualium. E. V. fuerit. 100000. Sed qua-
lium. E. V. est. 64. talium erit. 8. cum tribus quartis, cuius. ℞. V. quadratum erit par
tium. 76. cum dimidio similium sed superficialium, quo quidem quadrato dempto
ex quadrato ipsius. 64. quod est. 4096. remanebit quadratum ipsius. E. ℞. partium.
2871. quo etiam quadrato. E. ℞. dempto ex quadrato. E. G. partium. 14400. remane
bit quadratum ipsius. ℞. G partium. 11529. cuius radix. ℞. G. erit partium. 107. taliũ
qualium. E. G. est. 120. sed qualium. E. G. erit. 100000. talium. ℞. G. erit partium.
89166. quæ vt sinus anguli. ℞. E. G. habebit pro ipso angulo, gra. 63. min. 5. qui colle
cti cum gra. 7. min. 53. anguli. V. E. ℞. dabunt totum angulum. A. E. G. grad. 70.
min. 58. cuius complementum ex grad. 90. erit. G. D. graduum. 19. min. 2. & non. 24.
vt omnes ferè putant.

# DE REFLEXIONIBVS RADIORVM.

## *Excellentissimo Philosopho Francisco Vimercato.*

VONIAM non videbatur quiescere animus tuus, cum paucis ab hinc die-
bus tibi siscitanti respondissem, nec tamen rationem omnium, quæ dixe-
ram exactè explicare per temporis angustiam potuissem, cogitaui ad te
per hanc occasionem scribens, & iam dicta repetere, & omnium tibi ra-
tionem subiungere, & vt mihi plenius satisfaciam, & tibi commodè perlegenti faci
lius sit veritatem intueri. Scripsisti enim in tuis disputationibus, vir doctissime, quod
omnis res visa per speculũ quodcũque, sub breuissimis lineis cõpræhendatur à visu.

Propositio hæc non est vniuersaliter vera (quamuis etiam ab alijs omnibus pro ta
li posita sit) cum in speculis concauis non semper verificetur, vt nunc tibi demon-
strabo.

Esto quod linea recta. b. d. tangat circulum
b. o. q. n. qui sit communis sectionis superficiei re
flexionis, & sphæricę alicuius speculi sphærici
concaui, & punctum contingentiæ sit. b. à quo
exeant duæ lineæ. b. q. et. b. n. efficientes duos an
gulos inuicem æquales circa perpendicularem.
b. c. res autem visa primò sit in ipsa circunferen-
tia huiusmodi circuli in puncto. n. oculus vero in
puncto. q. ipsius circunferentię. Dico nunc duas
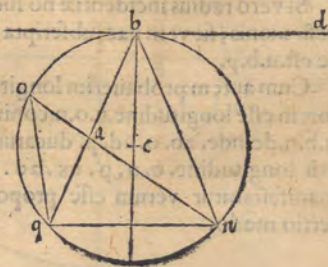
Tt 2 lineas

lineas.b.q.et.b.n. simul sumptas longiores esse omnibus alijs lineis exeuntibus ab ip
sis punctis.q.n.quæ in aliquo puncto dictæ circunferentiæ simul concurrant.

Sint igitur aliæ duæ.q.o.et.n.o.quas probare volo simul sumptas, esse minores dua
bus simul sumptis.q.b.et.n.b. Nam ex.20.tertij Eucli.cognoscimus angulos.q.b.n.
et.q.o.n.inuicem æquales esse, & similiter angulos.b.n.o.et.b.q.o.deinde ex.15.pri
mi eiusdem habemus angulos contra se positos,
circa. a.esse etiam inuicem æquales. Vnde ex.4·
sexti, habebimus proportionem. a . b.ad.a ·
o . eandem esse, quæ.a . n.ad.a.q. & sic.b·
n.ad.o. q. Quare ita erit. a.b.n.ad.a.o. q.vt.a.n·
ad.a.q.sed cum.a.n.maior sit.q.a.ex. 18. primi,
eo quod angulus.b.q.n.(qui æqualis est angulo.
b.n.q.ex.5.eiusdem ) maior est angulo. a.n. q.
qui pars est ipsius.b. n. q. ergo latera simul sum
pta.a.b.n.maiora erunt lateribus. a. o q. sed ex.
20.primi.a.b.n.etiã maior erit.a.n.vnde ex. 25.
quinti.q.a.b.n.maior erit.n.a. o. q. quare sequi
tur verum esse propositum.
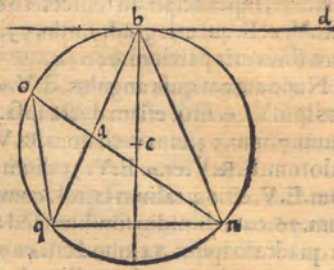


Sed si oculus esset in.u.quemadmodum in subscripta hic secũda figura videre est,
res autem visibilis in.n.ambo extra dictum circulum, esto etiam primum.b.u.æqua
lis.b.n.probabo similiter.u.b.n.maiores esse.u.o.n. Nam angulus.o. maior est angu
lo.b,eo quód si circulum.u.b.n.cogitemus circunscribere triangulum.u.b. n. ducen
do vsque ad suam circunferentiam.o.n.in puncto.s.deinde ducendo.u.s.habebimus
ex.20.tertij angulum.u.s.n.æqualẽ angulo.u.b.n.sed cũ angulus.u.o.n.exterior trian
guli.u.o.s.existat, ipse maior erit angulo.s.ex.16.primi.duco postea.o.q.parallelam
ad.u.s.quæ secabit.a.u.in puncto.q.& habebimus angulum.a. o. q. çqualem angulo.
n.s.u.ex.29. eiusdem,hoc est angulo. n. b. u. sed ex su
pradictis rationibus, lineæ.q.b. n. simul sumptæ maio
rem efficient longitudinem, quam.q.o. n. Nunc cum
ipsi.q.b.addita fuerit.u.q.& vice.q.o.sumpta fuerit ali
qua linea minor ipsa.u.q.o.eo amplius.u.q. b. n. maior
erit, quod quidem hoc modo faciendum . Acci
piatur.o.u. vt comes.o. n. quæ minor est ambabus. o.
q. et.q.u.ex.20. primi, ita enim habebimus propositũ.
sed breuiori modo hoc ipsum videbis ex præcedenti,
& ex.21.primi Euclid. Nam ex præcedenti.u.b.n.lon
gior est ipsa.u.s.n.ex.21.autem primi.u.s.n.longior est
ipsa.u.o.n.ergo verum est propositum.
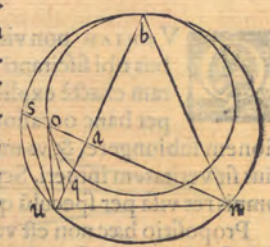


Si verò radius incidentiæ nõ fuerit æqualis radio
reflexionis, sit vt in hac subscripta tertia figura vide
re est.u.b.p.

Cum autem probauerim longitudinem.u.b.n.ma
iorem esse longitudine.u.o.n.coniungatur.n. p. cum
u.b.n.deinde. ab. o. ad. p. ducatur. o. p. quæ minor
erit longitudine. o. n. p. ex. 20 . primi, & illicò
manifestabitur verum esse propositum, etiam hoc
tertio modo.



Si

Si auté res viſibilis oculuſq; ambo fuerint intra circulum, túc poſſibile eſſet quod lógitudo.u.b.n.modo maior,modo minor,modo verò æqualis eſſet ipſa.u.o.n. núc. Quod etiam affirmo de.u. b.p.ſimiliter etiam eueniet ſi vnus terminorum.u.vel. n. fuerit intra circunferentiam, reliquus verò extra ipſam.
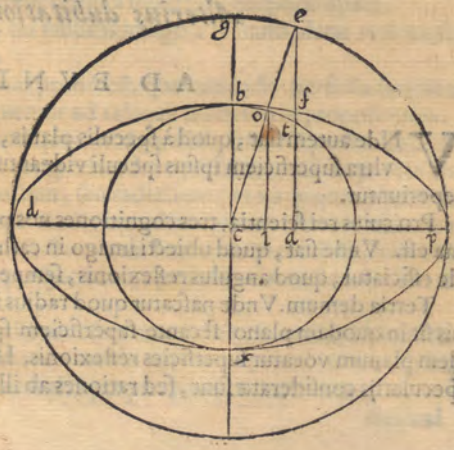
Conſideremus nunc hic infraſcriptam.4.figuram vbi.d.b.p.ſit circunferentia oxygonia ſeu elliptica (quod idem eſt) cuius maior axis ſit.d.p.in quo,duo termini. u. n. ſint centra eius generationis:b.x.verò ſit minor axis. Imaginemur etiam circulum. b.o.x.cuius ſemidiameter ſit.c.b.non maior mediatate minoris axis, ne circunferen- tia huiuſmodi circuli ſecet circunferentiam oxygoniam. Cogitemus etiam circu- lum. b.e. cuius ſemidiameter, minor non ſit minori axe.b.x. ipſius oxygoniæ, ne ſe inuicem ſecent huiuſmodi circunferentiæ,ſint etiam ambo eorum centra in linea.b. x.minoris axis, & punctum.b.ſit commune vnicuique earum periphæriarum, vnde minor circulus, totus intra,maior autem, totus extra ipſam figurâ oxygoniam erit. Nunc ad partem.o.r.e.vbi non communicant inuicem ipſæ circunferentiæ ducan- tur.n.o.r. e:u.o:u.r:et.u.e.& per. b.et.r.cogitetur tranſire alium circulum, cuius cen- trum in axe.b.x. ſit.t. omneſq;iſti circuli imaginentur trium diuerſorum ſphærico- rum ſpeculorum, vnde pro genera tione ipſi⁹ oxygoniẹ,ſeu ex.5 2. ter tij Pergei, habebis longitudinem. u.r.n.ẹqualem eſſe longitudini.u.b. n.& ei, quæ eſt.u.o.n. ( vt minor ip ſa.u.r.n.ex.2 1.primi Euclidis) mi- nor ipſa.u.b. n. & longitudinem. u. e.n. ( vt maior ipſa.u.r.n.ex eadem. 2 1.primi Eucli.) maior ipſa. u.b.n. Sed ſi quis vellet hoc demonſtrare ope circuli, vni⁹ tãtũmodo ſpeculi, multiplicãdo ipſas oxygonias qué- admodum de ipſis circulis fecimus, obtineret ſimiliter propoſitum.
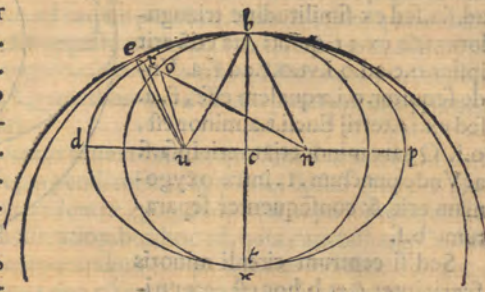


## Solutio dubitationis.
### AD EVNDEM.

Rationalis eſt dubitatio tua, vtrum ( cũ circulus minor hóc eſt.b.o.habeat ſuum centrum in mi nori axe inter centrum oxygóniæ, et.b: exiſtente. b. extremo axis mi- noris,communeq; ambobus circun- ferentijs circuli ſcilicet & oxigoniẹ) dictus circulus minor, plura puncta communia habeat cum ipſis circun- ferentijs.

Cui dubitationi reſpódeo quod quotieſcunque centrum alicuius cir culi fuerit idem cum.c. centro oxy- goniæ, vel inter.c.et.b. in interual- lo ſcilicet minoris axis, exiſtente.b. ſua extremitate communi ambabus cir-

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circunferentijs, ipsas circunferentias inuicem contiguas esse oportebit in puncto.b. tantummodo.

Esto primum quod centrum.c.commune existat, vt dictum est. sit etiam centrum vnius circuli, cuius diameter sit idē cū maiori axe. d. p.& in gyro oxygoniæ accipiatur punctum.f.proximum.b.quantum fieri poterit, tunc protrahatur.f.a.e.parallela ipsi. g. c. vsque ad gyrum maioris circuli in puncto.e.quæ cum. d. p. rectos efficiet angulos.ex.29.primi Eucli.secabitq; gyrum circuli.b.o.minoris in puncto.t.quod di co esse intra oxygoniam,separatumq; ab.f. Quapropter duco.c. e. quæ secabit circumferentiam circuli minoris in pūcto.o.à quo puncto duco etiam.o.i.parallelam ad e.a. Deinde considero, quod ex rationibus ab Archimede adductis in quinta propositione libri de conoidalibus , & sphæroidibus, eadem proportio erit ipsi⁹.g.c.ad.b. c. quę ipsius.e.a.ad.f.a. vnde permutando ita erit ipsius.g.c.ad.e.a.vel.b. c. ad f.a.hoc est ipsius.e.c.ad.e.a. vt. o.c. ad.f.a.sed ex similitudine triangulorum,& ex.11.quinti, ita etiā erit ipsius.o.c.ad.o.i.vt.o.c.ad.f. a. Vnde sequitur.o.i.æqualem esse . f..a. sed ex.14.tertij Eucli.t.a.minor est. o.i. Quare minor etiam erit ipsa.f. a. Vnde punctum . t . intra oxygoniam erit, & consequenter separatum.ab.f.

Sed si centrum circuli minoris fuerit inter.c.et.b.hoc est eccentricum ipsius oxygoniæ, ipse tanget concentricum in puncto.b.tantummodò , vt in.3. Euclidis libro probatur. Vnde tanto magis distans erit punctum.t.à puncto.f. quod erit propositum .

### *Alterius dubitationis solutio.*

## A D   E V N D E M.

V Nde autem fiat , quod à speculis planis, obiectorum imagines, ita distantes vltra superficiem ipsius speculi videantur, vt obiecta citra ipsam superficiem reperiuntur.

Pro cuius rei scientia, tres cognitiones nos primum habere oportet, quarum prima est. Vnde fiat, quod obiecti imago in catheto incidentiæ videatur. Secūda.vnde efficiatur, quod angulus reflexionis, semper æqualis sit angulo incidentiæ.

Terria demum. Vnde nascatur quod radius incidentiæ simul cum radio reflexionis sit in quodam plano secante superficiem speculi semper ad rectos, quod quidem planum vocatur superficies reflexionis. Huiusmodi tres passiones,ab omnibus specularijs consideratæ sunt , sed rationes ab illis traditæ,mihi non satisfaciunt.

Nam

Nam circa æqualitatem angulorum reflexionis & incidentiæ, iam tibi probaui
illud non vniuerfaliter euenire à breuitate aggregati radiorum incidentiæ reflexio-
nisq; . Sed hoc nafcitur potius ab eo, quod cum radius incidentiæ non poffit fuper
ficiem corporis opaci penetrare, reflectit, vt citra ipfam cũ angulo æquali ei, quem
faceret cum eadem fuperficie vltra ipfam fi tranfiuiffet.

Exempli gratia fit.a.obiectum.b.autẽ oculus in figura. A. et.c.e. fuperficies ipfius
fpeculi.d.verò fit punctum ipfius fuperficiei, à quo ad oculum reflectitur imago ip-
fius.a. Nunc fi radius.a.d.incidentiæ, recta
incederet fub.c.e.efficeret angulum.e. d..h.
æqualem angulo.c. d. a. eius contrapofito,
fed quia impeditur ipfæ radius ab opacitate
ipfius fpeculi.c.e.ne vlterius incedat, propte
rea reflectitur ab ipfa fuperficie fpeculi, con-
ftituens cum ipfa angulum.e. d. b. æqualem
angulo.e.d.h.fed quia angulus.c.d.a.eft etiã



ęqualis ipfi angulo.e.d.h.propterea angulus.e.d.b.ęqualis exiftit angulo . c. d. a; per
accidens igitur fequitur.a.d.et.d.b. fimul fumptas, breuiorem facere longiludinem
omni alia, quæ ab ipfa fuperficie.c.e.ad eadem puncta.a.b.ducta effet, quare natu-
ræ intentio eft efficere angulum.e.d.b.æqualem angulo.e.d.h.vnde ex accidenti po
ftea fequitur, ipfum æqualem effe angulo.c.d.a.& deinde qđ lineæ.a.d.et.d.b. con-
ftituant longitudinem breuiorem. Quare illud quod omnes putabant effe primum
& per fe, vltimum eft, & ex accidenti.

Quare verò fuperficies, quæ vocatur reflexionis, in qua funt duæ lineę, hoc eft
incidentię, reflexionisq;, femper fit perpendicularis fuperficiei ipfius fpeculi : Hæc
eft ratio, quia cum quilibet radius incidentiæ, perpendicularis ipfi fuperficiei fpe-
culi, in feipfo reflectit, ex ijfdem dictis rationibus, hoc eft, quia cum tali angulo vult
reflecti, cum quali tranfiret, ita etiam putandum eft, quod radius incidens obliquus,
cum in feipfum non poffit redire, quia non eft perpendicularis fuperficiei fpeculi,
reflectitur tamen per planum erectum ipfi fuperficiei fpeculi, vt in eo, cui magis re-
fiftit fuperficies corporis opaci, quàm alicui alij plano ipfius infiniti inclinatorum
planorum, ab vtraque parte ipfius plani perpendicularis, quod vnum etiam tan-
tummodo eft, & in quo, radius maiorem vim obtinet reflectendi, feu in eo, in quo
radius ipfe cum maiori refiftentia repercutitur à fuperficie corporis opaci.

Poftremo fciẽdũ vnde oriatur, ꝗ rei vifibilis imago, à fpeculo plano reflexa, fem
per in catheto incidentiæ videatur.

Pro cuius rei ratione cognofcendum primò eft, quo modo fit perfecta fimplexq;
vifio, & non reflexa, deinde profequemur ad reliqua huius tertiæ propofitionis.

Animaduertendum igitur eft, quod quotiefcunq; obiectum aliquod vifibile afpi
cimus, nos nunquam perfectè illud comprehendere poffumus, nifi in puncto con-
curfus, feu interfectionis axium vifualium, feu radialium ( vt ita loquar) quã inter-
fectionẽ, nos efficimus ope reuolutionis oculorum adinuicẽ, hoc eft voluendo vnum
verfus alium, ita vt in fitu ipfius obiecti, feinuicem fecent axes iam dicti, tunc enim
vtroque oculo mediante, exacte rem perfpicimus, cęteris. 8 . circunftantijs non ob-
ftantibus.

Vnde ftantibus oculis in tali fitu, altero refpectu alterius, fi eorum alter tectus,
feu velatus fuerit, tunc alio tantummodo oculo mediante, videbimus obiectum,
in ea diftantia, exactius, quam in quauis alia propinquiori, & remotiori.

Animal

Animal igitur, fecundum diftantiam obiecti, oculum accommodat ad recipiendum quam exactiffimè fpeciem ipfius obiecti, & hoc voluendo ambos oculos, vnum verfus alium, ita quod interfectio axium fit in fitu feu loco dicti obiecti, nam tunc vi deant ambo vel aliquis eorum folus, in tali diftantia exactè obiectum videbit.

Vnde fequitur obiectum vifibile, compræhenfibile non effe ab vno tantummodo oculo in quolibet fitu axis ipfius oculi, fed in eo, vbi alius axis interfecatur à dicto. Quæ quidem interfectio poteft fieri propinqua, vel remota à vifu, ad certos tamen terminos vfque.

De huiufmodi axium vifualium interfectione fcribit Alhazem in. 2. et. 15. propofitione tertij lib. Vitellio verò in. 32. et. 45. eiufdem.

Quod igitur dico, verum eft, ideft, quod fi vno tantummodo oculo afpiciemus obiectum aliquod, ipfum nunquam perfectè profpicietur, nifi cum oculus ita fitus fuerit, vt eius axis cum axe alterius in loco obiecti fe inuicem fecent, quamuis alter oculus nihil videat, cũ aũt duobus oculis in tali fitu cõftitutis obiectũ videmus, vnum tantummodo nobis cernere videbimur, & fi extra talem punctum interfectionis ipfum obiectum pofitum fuerit, tunc duo talia obiecta nobis apparebunt, fed huiuf modi rei caufam alias tibi manifeftabo.

His igitur cognitis, ponamus aliquam fpeculi fuperficiem effe. g. h. in figura. B. obiectum autem vifibile. b. oculos vero. a. et. u. punctum autem. n. in fuperficie fpecu li, à quo imago ipfius. b. reflectit ad. a. & punctum. t. à quo reflectitur ad. u. et. c. e. fit cõmunis fectio fuperficiei reflexionis radiorum. b. n. a. et. c. f. fit communis fectio fuperficiei reflexionis radiorum. b. t. u. qua rum vnaquæq; fuperficies reflexionis, erecta eft ad fuperficiem fpeculi. g. h. vt fupra diximus. Nunc ex. 19. vndecimi Eucl. fequitur communem fectionem harum duarum fuperficierum. (b. c. d. fcilicet) ad rectos etiam effe fupra fuperficiem fpeculi. g. h. cũm qua. b. c. quælibet linearum. a. n. vel. u. t. reflexarum ( productę cum fuerint ) feinuicem interfecabunt eo quod duo anguli. d. c. n. et. d. n. c. fimul collecti minores funt duobus rectis, & ita. d. c. t. cum. d. t. c. cum anguli. a. n. e. et. u. t. f. reflexi, ipfis contrapofiti, æquales fint angulis. b. n. c. et. b. t. c. incidentiæ, quorum vnufquifq; ex. 32. primi, minor eft recto.

Dico etiam quod in eodem puncto huiufmodi catheti. b. c. d. in quo interfecabitur à linea. a. n. in eodem fecabitur à linea. u. t. & quod punctum dicti concurfus, tantum depreffum erit fub fuperficie fpeculi. g. h. quantum. b. fupra ipfam reperietur. Nam anguli. b. n. c. et. d. n. c. funt inuicem æquales, anguliq; b. c. n. et. d. c. n. recti. c. n. verò communis ambobus triangulis. b. c. n. et. d. c. n. vnde ex. 26. primi Eucli. latus. d. c. commune, vt trianguli. d. c. n. æquale erit lateri communi. b. c. vt trianguli. b. c. n. Idem etiam dico de latere. d. c. vt ipfius trianguli. d. c. t. quod æquatur lateri. b. c. vt trianguli. b. c. t. Vnde cum. b. c. vnum, & idem fit: d. c. igitur etiam erit, & ipfum vnũ & idem, quod erit propofitum.

Nunc autem cum hi duo radij feinuicem fecent in puncto. d. ergo in ipfo puncto. d. videbimur nobis videre imaginé obiecti. b: cũ ope duorũ iftorũ radiorũ. n. a. et. t. u. ita inuicem fitorũ, videamur nobis imaginé profpicere. Vnde fi in tali cafu, vnus
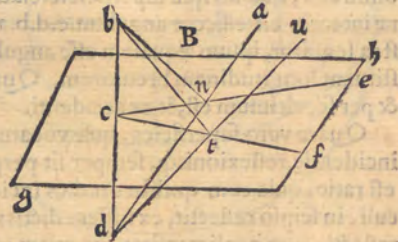
ocu-

oculorum clauderetur, nihilominus cum reliquo obiectum vidissemus in eodē ipso loco.d. & non in alio ex superius dictis rationibus.

Et si stantibus ijs terminis volueremus pupillam oculi.u. versus aliam.a. ad aspiciendum punctum.n.in superficie.g.h. ipsius speculi, hoc est si fecerimus quod axes visuales seinuicem secarent in ipso puncto. n. tunc videremur nobis videre duas imagines ipsius obiecti. b . intra speculum , eo quod obiectum, propter hoc non cessaret reflectere ad oculos ab ipsis punctis .n. et. t. quapropter recipiendo radium.t.u.in situ axis oculi.u. & radium.n.a.in situ axis oculi.a. hi axes ex necessitate (vt probauimus) seinuicem secant in puncto.d.vnde vnam tantummodo imaginem ipsius obiecti nobis apparebit.

Ex his igitur omnibus potes facilè videre omnem imaginem, cuiusuis obiecti, reflexam à speculo, reperiri in ipso catheto incidentiæ, cum ipse semper sit communis sectio duarum superficierum reflexionis , in quo catheto concurrunt ipsæ axes visuales.

Ex ijsdem etiam dictis rationibus facile compræhendere poteris, vnde fiat, vt videamus imaginem reflexam à speculis sphæricis concauis citra ipsorū superficiem, & non vltra. Quod nunquā euenit, nisi quando punctū.d. intersectionis ipsorū radiorū visualium (quod alio in loco non fit, nisi in catheto incidentiæ hoc est in communi sectione duarum superficierum reflexionis. Dato quod obiectum non sit in vna eademque superficie, in qua reperti fuerint axes visuales, hoc est dato, q̃ ambo axes visuales non sint in vna eademq́; superficie reflexionis) reperitur citra & non vltra superficiem ipsius speculi.

Ad cuius rei euidentiam non prætermittā dicere, quod cum debeant semper superficies reflexionum perpendiculares esse, vel ad rectos secare superficiem ipsius speculi, ipsarum communes sectiones cum superficie speculi sphærici, semper erunt circunferentiæ magnorum circulorum illius sphæræ , cuius portio est speculum propositum, vt etiam Vitellio affirmat in prima sexti libri. Vnde vnusquisque cathetus incidentiæ transibit per centrum speculi, cum ipse sit communis sectio duarum superficierum reflexionis, quare in ipso catheto erit punctum intersectionis ipsorum axium visualium ex necessitate, vt videbimus, si vnam tantummodo imaginē obiecti nobis videremur videre.

Exempli gratia , sint duæ superficies reflexionis speculi sphærici concaui.b.n.c.a. et.b.t.c.u.obiectumq́; sit.b.oculi autem sint.a.u.punctum verò superficiei speculi , à quo obiectum emittit reflexionem suæ imaginis ad oculum.a. sit.n.punctum autem à quo eandem reflectit oculo.u. sit t.communis autem sectio harum duarum superficierum sit.b.c.sed.x. centrū sit speculi,radius verò incidentiæ super ficiei.b.n.e.erit.b.n.cuius reflexus sit.n. a.radij autem alterius superficiei erunt b.t.et.t.u.Imaginemur nunc duos semidiametros.x.n.et.x.t.quæ angulos.b.n. a.et.b.t.u.per æqualia diuidant ex supposito.
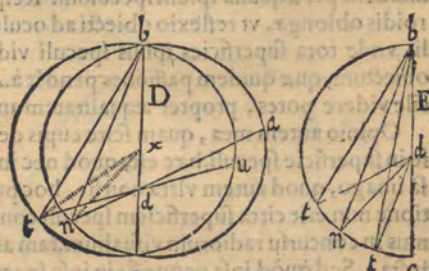
Nunc ijs suppositis, si vnam tantummodo obiecti imaginem videbimus ,

Vu clarum

clarum erit ex rationibus supradictis nos ipsam videre in cõmuni concursu ipsorum
axium visualium, qui axes cum reperiantur vnà cum ipsis radijs reflexis.n.a. et. t. u.
ex necessitate seinuicem secabũt in catheto.b.c.cum extendantur in ipsis superficie-
bus reflexionum, quæ superficies nihil aliud commune inuicem habent,quam cathe
tum dictum.b.c.sit igitur in puncto.d.

Ex his dictis alia oritur necessitas, hoc est, quòd quotiescunque vnam tantummo
do imaginem obiecti.b.videmus, dato quod duæ superficies reflexionis sint, & non
vna tantum, tunc angulos.n.et.t.semper inuicem æquales esse oportebit. Vnde ar-
cus.n.c.et.t.c.ex necessitate inuicem æquales erunt.

Scimus enim ex. 3. sexti Euclid. quod eadem proportio erit ipsius. b. n . ad.n.
d. quę ipsius. b. x. ad. x. d. & ipsius.b. t. ad. t. d. similiter, quare ipsiusb. n. ad. n.
d. erit vt ipsius. b. t. ad. t. d. Vnde sequitur. b. n. æqualem esse ipsi. b. t. et. n. d.
ipsi.t.d.vt à medio circulo.E. potes videre , quamuis etiam. b. non esset extremum
diametri, sed vbicunque volueris in ipso diametro,vel etiã protracta, eo quod pun-
ctum.n.& punctum.t.in eodem semicirculo,vel in æqualibus semicirculis,non possét
aliter in ipsa circunferentia locari,eãdem seruando proportionem.b.n.ad.n.d.vt. b.
t.ad.t.d.propterea quod in omni alio situ existente puncto.t.ipsa.b.t.esset aut maior
aut minor ipsa.b.n.et.t.d.aut minor, aut maior ipsa.t.d.ex.7.& 14.tertij Eucli.vnde
aut maior,aut minor proportio esset ipsius.b.t.ad.t.d.quam ipsius.b.n.ad.n.d.& non
eadem.

Nunc è conuerso si.b.n.et.b.t.sunt sibi inuicem æquales, & sic.n.d.cum.t.d.sequi-
tur ex.8.primi Eucli.angulos.n.et.t. inuicem æquales esse.

Ab ijsdem speculationibus potes etiam videre vnde accidat quod partes superio
res alicuius obiecti reflexæ à tali speculo concauo videntur nobis inferiores esse, &
inferiores appareant superiores, & dextræ sinistræ, & sinistræ dextræ. quod autem
hucusque demonstraui de speculis planis, & sphæricis concauis,ratiocinare tu ijsdem
medijs circa sphærica conuexa,vbi clarè videbis puncta huiusmodi speculi conuexi,
à quibus reflectitur imago obiecti ad ambos oculos, semper oportere æquidistantia
esse à pũcto communi ipsius superficiei speculi,& catheto incidentiæ, dum unam tan
tummodo imaginem ipsius obiecti videmus,& à diuersis superficiebus reflexionum.

Nolo etiam prætermittere, quod nunc mihi succurrit , hoc est quod posset ali-
quis duos situs inuenire, vnum pro oculo,alterum verò pro obiecto,respectu alicu-
ius speculi concaui, sphęroidis prolatæ, vt reflexio ipsius obiecti videretur , vt linea
diuidens per æqualia ipsum speculum. Respectu verò alicuius speculi concaui sphæ-
roidis oblongæ, vt reflexio obiecti ad oculum veniret à tota superficie ipsius specu-
li, vnde tota superficies ipsius speculi videretur colorata illo colore cuius esset
obiectum, quæ quidem passiones pendēt à.48.tertij lib.ipsius Pergei,vt ex te ipso fa
cile videre potes, propter æqualitatem angulorum reflexionis, & incidentiæ.

Opinio autem mea, quam scire cupis de imagine obiecti reflexa,quam putas es-
se in superficie speculi,hæc est, quod nec in superficie,nec ultra, nec citra eam est ip
sa imago, quod autem vltra non sit, hoc puto nulli dubium esse. eadem etiam ra-
tione non erit citra superficiem speculi concaui, quamuis ipsam nos compræhenda-
mus in concursu radiorum visualium,tam ab vno speculo quam ab alio reflexione
facta. Sed quòd ipsa neque sit in ipsa speculi superficie, manifestum erit ex hoc, ꝗ
duo spectantes in eodem speculo,duas diuersas imagines vident,tres, aũt tres, qua-
tuor , quatuor , & sic deinceps, vnde tot essent imagines supra superficiem speculi,
quot obiecta, ꝗ tamen ita non est, nec plus est in vno loco ipsa imago,quam in alio,

nisi

nisi in obiecto ipso, lumen enim ab ipso obiecto reflexum, seipsum diffundit vndique, & radij ipsius luminis reflexi, vt plurimum seinuicem secant. Vnde in ipso aere sunt omnes misti. Quapropter natura sagacissima pupillam oculi animalibus tam paruam construxit ad superficiem tam amplae sphaerae ipsius oculi, vt distinctae viderentur omnia obiecta.

Nolo etiam tibi tacere, quod quotiescunq; oculorum pupillae positae fuerint inter cathetum incidentiae, & superficiem speculi sphaerici concaui. vt puta in lineis. d.t. et.t.n. in figura. D. tunc nullo pacto possemus videre vnam imaginem obiecti, sed duas nec non confusè, propterea q nullo pacto radij.t.d. et.n.t. reflexi poterint ambo vniri cu ambobus axibus visualibus, eo quod axes visuales nunquam possunt inuicem intersecari post visum, sed semper ante ipsum, vnde nec inuicem paralleli possunt esse.
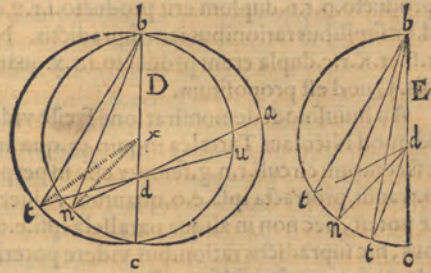
Dico etiam, quod si obiectum inciderit in eadem superficie, in qua duo axes visuales, vel radij reflexi reperiutur, hoc est in vna eademq; superficie reflexionis, tunc locus imaginis non erit in catheto incidentiae, eo quod intersectio axium uisualium non erit in ipso catheto sed extra, in qua intersectione fit visio vnius tantummodo imaginis, quod antiqui non animaduerterunt. Hoc autem dico de speculo sphaerico concauo.

## Speculatio cuiusdam propositionis arithmeticae.
### AD EVNDEM.

SPeculatio vltimae propositionis quam numerorum via inueni, haec est. Imaginemur triangulum.r.e.o.abscisum à circulo, in cuius circumferentia sit punctum r. superioris anguli ipsius trianguli, vel etiam non sit abscisum dummodo protrahantur lineae vsq; ad circumferentiam,à quo ad oppositum latus descédant duae.r.K.et.r. f.ita q.K. o. aequalis sit.f.e. vnde haec.4. lineae secabuntur à circulo dicto in punctis.n. c.b.u. Dico nunc q producta.o.r.n.et.e.r.u.aequalia erunt productis. K. r. c. et.f.r.b.

340     IO. BAPT. BENED.
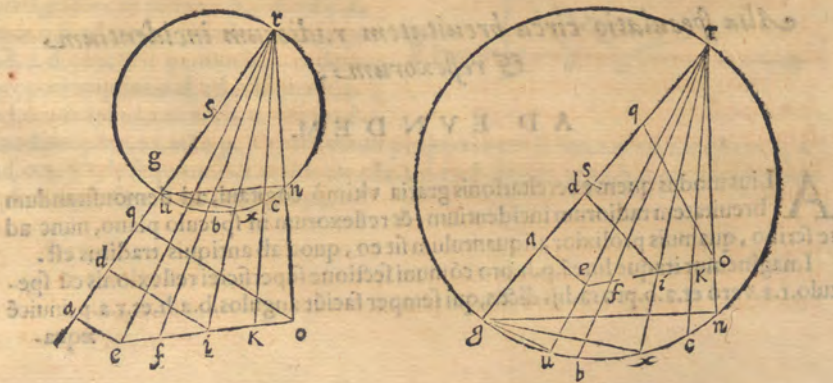
quapropter cogitemus.r.a.indeterminatam transire per centrum.s. ipsius circuli , si-
militer etiam.r.i.ad punctum medium lateris.e.o.deinde à tribus punctis,e.i.o. ima-
ginemur tres perpendiculares ad.r.a.hoc est.e.a:i.d.et.o. q. & ybi circulus secat. r. a.
sit punctum.g.protractis deinde.g.n:g.x:et.g.u.habebimus triangulum. a. e. r. simi-
lem triangulo.g.u.r.vnde clarum erit productum.g.r.a.æquale esse producto. e. r. u.
productumq;.g.r.q.æquale esse producto.o.r.n.nam trianguli.g.r.n.et.o.r.q.sunt in-
uicem similes,sed productum.g.r.a.simul cum producto.g.r.q. duplum est producto.
g.r.d.ex prima sexti , eo quod.a.r.q. dupla est.d.r.& ideo productum.e.r.u. simul cū
producto.o.r.n.duplum erit producto.i.r.x.quod quidem æquale est producto. g. r.
d.ex similibus rationibus iam supradictis. Nunc ex similibus rationibus producta.f.
r.b.et.K.r.c.dupla erunt producto.i.r.x.quare prima producta æqualia erunt secun-
dis. Quod est propositum.

    Ab huiusmodi demonstratione facilè videre poteris non esse generaliter verum,
id quod Nicolaus Tartalea inquit.43.quæsito vltimæ partis suorum tractatuum,hoc
est centrum circuli.r.n.g.semper esse in perpendiculari,quæ à puncto.r.ad lineam. e.
o.transit,protracta ipsa.e.o.quantum volueris,imò in quacunque alia linea ipsum es
se potest , nec non in aliqua parallela ipsi.e.o.quemadmodum ex te ipso, medianti-
bus , hic supradictis rationibus videre poteris,vnde ex necessitate sequitur illud pro
blema semper ferè falsum esse.
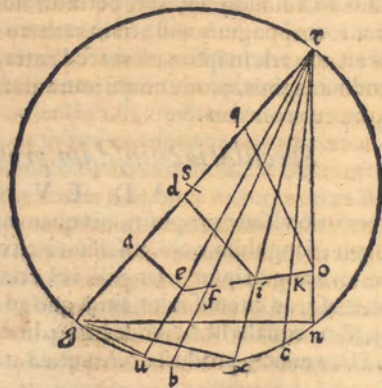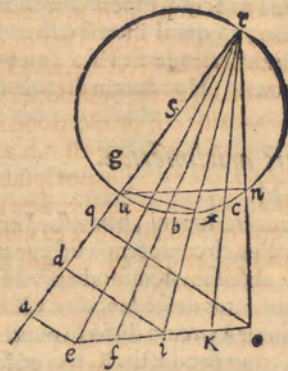


### Alia speculatio circa breuitatem radiorum incidentium & reflexorum.

### AD EVNDEM.

Alius modus quem exercitationis gratia vltimò cogitaui, ad demonstrandum
breuitatem radiorum incidentium , & reflexorum in speculo plano, nunc ad
te scribo , quamuis prolixior aliquantulum sit eo , quod ab antiquis traditus est.
    Imaginemur itaque lineā.p.h.pro cōmuni sectione superficiei reflexionis cū spe-
culo.r.a.verò et.a.b.pro radijs dictis,qui semper faciūt angulos.b.a.h.et.r.a.p.inuicē
æqua-

æquales. Nunc protrahantur duæ.r.o.et.b.o.ab iisdem punctis.b.r.ad aliud punctum,
quod volueris ipsius lineæ.p.h.quas probabo longiores¹ (simul sumptas) esse priori-
bus. Imaginemur igitur duas perpendiculares, seu cathetos.b.i.et.q.r.à punctis.b.
r.ad.p.h.abscissaq; sit linea.o.b.in puncto.x. ita quod . b.x.æqualis sit ipsi.b.a. quod
nulli dubium erit posse effici, cum.o.b.lógiot sit.b.a.eo quod opponatur angulo ob-
tuso ipsius trianguli.b.a.o.quę.o.b. similiter protrahatur vsque ad.d. ita quod. b. d.
æqualis sit.x.b.deinde protrahatur.o.i.quousque.i.h.æqualis sit.a.i.In alia parte po-
stea idem faciendum est secando.a.r.in puncto.u.ita quod.u.r.æqualis sit.r.o.efficien
do.r.s.æqualem.r.u.et.q.p.æquale.q.o.vnde habebimus productũ.o.d.in.o.x. æqua
le producto.o.h.in.o.a.& productum.a.s.in.a.u.æquale producto.a.p.in.a. o. ex istis
rationíbus . Nam cum quadratum ipsius.o. b.æquale sit duobus quadratis.o.i.et. i.
b.ex penultima primi Eucli.ipsa quadrata.o.i.et.i.b.æqualia erunt producto.o. d.in
o.x.simul sumpto cum quadrato.b.x.ex.6. secundi,hoc est ipsi producto simul sum-
pto cum quadrato.b.a.hoc est ipsi producto simul sumpto cum duobus quadratis.a.
i.et.i.b.sed quia productum.o.h.in.o.a.simul sumpto cum quadrato.a.i.ęquatur qua
drato.o i.ideo productum.o.h.in.o.a.simul sumptum cum quadrato.a.i.& cum qua-
drato.i.b.æquale erit producto.o.d.in.o. x. simul sumpto cũ duobus quadratis dictis
hoc est ipsius.a.i.et.i.b.quę quadrata dempta cum fuerint ab vtraque parte,tunc cer
ti erimus producta esse inuicem æqualia. Idem dico de alijs ex altera parte. Nunc
imaginemur protractam esse.a.e.parallelam ipsi. o. b. & habebimus proportionem
ipsius.a.b.ad.a.i.maiorem esse ea quæ est ipsius.a.e.ad eandem.a.i. cum. a. b. maior
sit ipsa.a.e.vt opposita angulo obtuso, quapropter proportio.x.b.ad. a. i. maior erit
ea quæ est.o.b.ad.o.i. Iam enim scis proportionem.o.b.ad.o.i.esse,vt.a.e.ad.a.i. ex
similitudine triangulorum. quare proportio.b.d.ad.i.h.maior erit proportione.o.b.
ad.o.i.tũę ex.27.quinti pmutãdo ͛pportio. b.d.ad.b.o.maior erit proportione.i.h.
ad.i.o.& ex.26.eiusdē cóponédo maior ͛pportio erit.o.d.ad.o.b.ea quę est.o.h.ad.o
i. & permutãdo maior ipsius.o.d.ad.o.h.ea quæ.o.b.ad.o. i.& ex.33.maior ipsius.b.
d.ad.i.h.ea quæ.o.d.ad.o.h. Sed vt.b.a.ad.a.i.ita est.a.r.ad.a.q.ex similitudine triã
gulorum . Erit igitur.a.r.ad.a.q.maior proportio,ea quæ est.o.b.ad.o.i. & ex ijsdem
supradictis rationibus maior erit proportio ipsius.s.a.ad.p.a.ea quæ est. a. r. ad. a. q.

sed cum iam probatum fuit proportio
nem.b.d.ad.i.h.hoc est.a.b.ad.a.i. ma
iorem esse. o.d.ad. o. h. ergo eo ma-
gis maior erit proportio ipsius.a. s.ad
a.p.ea quæ.o.d.ad.o.h.sed cum ex.15
sexti,eadem sit proportio.o.d. ad.o.a.
quæ.o.h.ad.o.x.et.s.a.ad.o.a. quę a.p.
ad. a. u. tunc erit permutãdo eadem
proportio ipsius.o.d. ad.o.h.quæ.o.a.
ad.o.x.& ipsius.a.o. ad. a. u. quemad-

modum ipsius.a.s.ad.a.p. Quare maior proportio erit ipsius.a.o.ad.a.u.quam. a. o.
ad.o.x. Vnde sequitur.o.x.maiorem esse.a.u.ex.8.quinti, ergo.b.x.o.r. longior erit
ipsa.b.a.u.r.Quod est propositum.

**Alia**

Alia etiam via poſſumus idem concludere. Imaginemur maiorem axem alicu-
ius ellipſis tranſire per duo puncta.r.et.b.ſupponendo ipſa puncta, ea eſſe, quæ ita
axem diuidunt, vt ſingula produ-
cta ſectionum ſint, vt inquit Per-
geus.imaginemur, etiam.p.h.con
tiguam eſſe ipſi ellipſi in pũcto. a.
vnde ſi protractæ fuerint duæ.r.a.
et.b.a.habebimus ex.48. tertij ip-
ſius Pergei angulos.b.a.h. et. r. a.
p. inuicem æquales. Ducendo
poſtea ad quoduis punctum ipſius
p.h.duas.b.o.et.r.o. certi erimus,
quod ſecabuntur à gyro oxygo-
nio, quarum vna ſecta ſit in pun-



cto.i.ducta poſtea.i.r.clarum erit ex.5 2.dicti, quod longitudo.b.i.r.æqualis erit lon
gitudini.b.a.r.& minor ipſa.b.o.r.ex.2 1.primi Euclid.
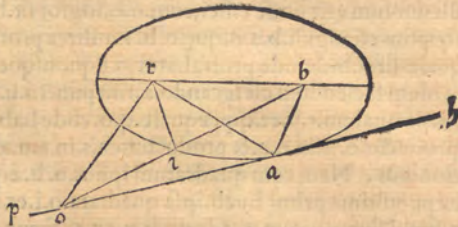
## De errore Euclidis circa ſpeculum vſtorium.

### A D  E V N D E M.

V Erum ſpeculum vſtorium, illud non eſt, quod ab Euclide traditum fuit, & ꝗ
tu etiam putas, Nam Euclides errat, cum credat radios reflexos à ſuperficie
ſphærica concaua ſeinuicem in centro ſpeculi interſecare. Nam cum omnes lineę
recte à centro, & circunferentia alicuius ſphæræ terminatæ, ſint eidem circunferen-
tiæ perpendiculares, ſequeretur ex neceſſitate radios incidentiæ etiam perpendicu
lares eidem ſuperficiei eſſe, cum anguli incidentiæ ſemper æquales ſint angulis re-
flexionis, vnde etiam ex neceſſitate ſequeretur punctum corporis lucidi, à quo radij
luminoſi exeunt, in centro ſpeculi reperiri. quod quidem falſiſſimum eſt.
Alia etiam via poſſum hanc oſtendere impoſſibilitatem, & tibi probabo, quod
in nullo aliquo puncto poſſunt inuicem conuenire ipſi radij reflexi omnes.
Sit igitur.l.a.c.cõis ſectio ſuperficiei reflexionis cum ſpeculo, cuius centrum ſit.o.
punctum verò lucidum ſit.g.protrahaturꝗ;. g. o. a.Nunc autem primum dico, quod
radij reflexi à punctis diuerſarum diſtantiarũ ab.a.non coincidĕt inuicem in aliquo
puncto lineę.g.o a:ſint ergo duo puncta.u.et.r.diuerſarum diſtantiarũ ab.a. à quibus
veniant duo radij incidentiæ.g.r.et.g.u.radius verò reflexus ab.r. ſit.r.e.protrahatur
u.e.quam dico eſſe non poſſe radium reflexum ab. u. quotieſcunque eius incidens
deſcendat ab.g. Protrahantur ergo duæ lineæ.o.r.et.o.u. vnde cum dixerit aliquis
u.e.reflexũ eſſe ipſius. g.u.igitur anguli.g.u.o.ct.o.u.e. erunt inuicem æquales, & ſic
etiam erunt duo. g.r.o.et.o.r.e. vnde ex tertia ſexti &.1 1.quinti Eucli.proportio. g.
u.ad.u.e.æqualis eſſet ei, quæ.g.r.ad.r.e.quod quidem impoſſibile eſſe demonſtra-
bo, eo quod cum.g.u.maior ſit.g.r.ex.8.tertij,erit ex.8.quinti proportio ipſius.g.u.
ad.r.e.maior proportione ipſius.g.r.ad.r.e.ſed ex.7.tertij.u.e.minor eſt.r.e. erit igi-
tur ex dicta.8.quinti maior proportio ipſiˀ.g.u.ad.u.e. quam.g.u.ad.r.e. vnde eo ma
gis

gis erit maior proportio ipfius.g.u.ad.u. e. quam ipfius. g. r. ad.r.e. ergo non æqualis, quapropter impoffibile eft.u.e. effe radium reflexum incidentis radij.g.u. Vnde fequi tur concurfum radiorum reflexorum à fpe culo fphærico concauo non effe in vno, & eodem puncto ipfius catheti incidentiæ, quando à fitu non æquidiftanti ab ipfo ca theto reflectûtur, ex hac fpeculatione etiã videre licet, verum effe id quod in. 3. Epifto la tibi fcripfi nempe, quod quotiefcunque axes vifuales, vel radij reflexi, in vna ea demq́; fuperficie reflexionis fuerint, tunc imago obiecti nullo modo videbitur in ca theto incidentiæ, in fpeculo fphærico con cauo.

## Alterius dubitationis folutio.

### AD EVNDEM.

NOn abfque ratione dubitas, vtrum etiam in fphæricis fpeculis conuexis idem accidat, hoc eft, an radij reflexi à punctis inæqualis diftantiæ à catheto inciden tiæ conueniant inuicem in eodem catheto.

Ad quod refpondeo, non concurrere in dicto catheto, fed extra ipfum, & fimi liter extra ipfum videbitur imago.

Pro cuius rei ratione, imaginemur fuperficiem reflexionis alicuius fpeculi fphæ rici conuexi. b. d. h. g. cuius communis fectio cum fuperficie fphærica fit linea circularis.d.e.h.et.o. eius cêtrum, à quo protrahatur.g.b.indeterminata, et.o.g.fit fe midiameter circuli.d.g.h.et.o.c. fit plus medietate ipfius.o.g. accipiaturq́; linea.e.c. minor ipfa.o.c. fed maior ipfa.c. g. quod difficile non erit, locando im mobilem pedem circini in puncto. c. aperiendo ipfum aliquantulum plus quam. c. g. fed minus quam. c. o. fignando circunferentiam. d.e.h. in puncto.e.quod ex. 7.tertij poffi bile eft, protrahatur poftea.o. e. f. indeterminatè. Faciemus deinde angulum.f.e.b. æqualem angulo. o. e.c.protracta poftea cum fuerit . c. e.K.indeterminatè, habebim°duos angulos.b.e.f.et.f.e. K. æquales in uicem mediante. r 5. primi, ita qd̄ fi radius incidens veniet à puncto. b. ad.e.reflexus erit.e. K. qui quidem reflexus
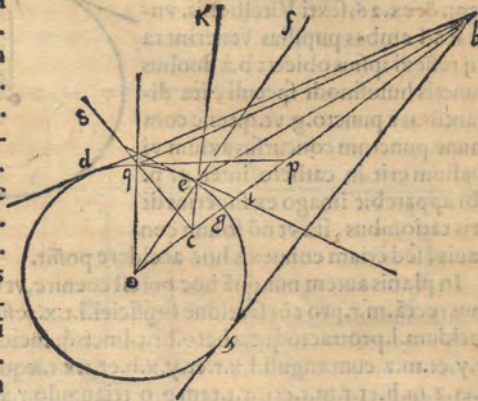
reflexus fecabit cathetum.b.o.in puncto.c.intra fpeculum,nec dubitandum eft quin
linea.e.b.fectura fir.b.o.eo quod cum angulus.o.e.c.fit maior angulo.e.o.c.ex.19.
primi, & fimiliter angulus.b.e.f.fequitur ex.13.dicti,angulos.b.e.o.et.e.o.b. effe mi
nores duobus rectis, vnde ex penultima petitione primi,duæ lineæ. b.e.et.o.b.inuicè
concurrent. Quare poffumus ex hoc , quoddam corollarium extrahere , hoc eft
neceffariũ feper exiftat,vt linea.c.e.minor effe linea.c.o. Sed vnde eueniat quod ip
fa neceffario debeat femper maior effe ipfa.c.g.clarum eft ex.7.tertij Eucli. Nunc
imaginemur ductas effe duas tägentes.b.d.et.b.h.& ab.e.ipfa,e.i.vnde certi erimus,
quod ab interuallo inter.h.et.d.punctum.b. poffibile fit vt reflectatur. Accipiamus
nunc.p.c.minorem medietate ipfius.b.c.& à puncto.p.imaginemur tangentem.p.q.
in puncto.q.prorractaque fit. b. q.vt radius incidentiæ, tunc dico, radium reflexum
ipfius.b.q.nó concurrere in eodem puncto.c.ipfius catheti, fi vero dixeris ợ fic.Efto
igit radius dictus.c.q.s.Imaginemur tägentè.e.i.in puncto.e.vnde ex.18.quinti Alha
zem, vel. 12.fexti Vitellionis proportio.b.i.ad.i.c.erit,vt.b.o.ad.o.c.& fimiliter erit
ipfius.b.p.ad.p.c.vt.b.o.ad.o.c. ex eadem. Quare ex.11.quinti Eucli.proportio ip
fius. b. p. ad. p. c.erit vt ipfius.b.i.ad.i.c.fed quia.p.b.vt pars ipfius.b. i. minor eft ip-
fa,ergo ex.14.dicti.p.c.minor erit ipfa. c. i. hoc eft totum minus fua parte , quod eft
impoffibile, quare non in ipfo cathero videbitur imago ipfius obiecti.

    Aliud notandum etiam cernere potes ex ipfis fpeculis fphæricis conuexis,hoc eft
quod poffibile fit aliquoties , radium reflexum concurrere cum catheto incidentiæ
extra fpeculum inter puncta. g. et. p. vt exempli gratia. fi punctus. p. effet exactè
in medió inter.b.et.g.tunc punctum.c.ipfius concurfus cum catheto incidentiæ effet
inter.g.et.p,eo quod cũ linea.p.q.debeat diuidere angulũ.b.q.c.p æqualia,oportebit
c.pofitum effe inter.g.et.p.quia angulus.g.q.p.maior eft angulo.p.q.b.vt per te faci

le potes ratiotinari,imaginando cir
culum circa triãgulum.g.q.b. & dia
metrum perpendicularem. ad. g. b.
in puncto.p.producendo poftea. q.
p.vfq; ad alterã parte circunferen-
tiæ ipfius circuli. argumétado dein-
de mediante vltima fexti,illud idé
potes etiam fcire ex.22.quinti Alha
zeni. & ex.26.fexti Vitellionis. vn-
de fi ad ambas pupillas venerint ra
dij reflexi ipfius obiecti.b.à duobus
punctis huiufmodi fpeculi, ita di-
ftantibus à puncto.g.vt.q.tunc com
mune punctum concurfus axium vi
fualium erit in catheto inter. g. p.
vbi apparebit imago ex fuperius di
ctis rationibus, ita vt nó folum con
cauis, fed etiam conuexis hoc accidere poffit.

    In planis autem nunquã hoc poteft euenire,vt tibi alias dixi, eo quod fi accéperi-
mus rectã.m.r.pro cói fectione fupficiei.l.t.x.reflexionis &fupficiei fpeculi,pũctúq;
lucidum.l.protractoq; catheto.l.r.t.lineisq; incidentiæ.l.x.et.l.m. reflexionis etiam
x.y.et.m.z.cum anguli.l.x.r.et.y.x.h.et.r.x.t.æquales inuicem fint,& fic anguli.l. m.
r.et.z.m.h.et.r.m.t.erit.r.t.tam pro triangulo.r.x.t.quam pro triangulo.r.m.t. æqua
lis.r.l.ex.26.primi, ita quod femper in puncto.t.conueniét omnes radij reflexi ipfius
                                                                        puncti
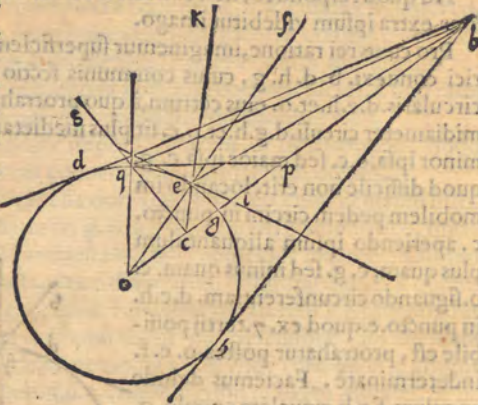
puncti.l.clarum igitur nunc habes , quod in fphærico concauo , feu conuexo,non omnes radij reflexi conueniunt in vno, eodemq́; puncto catheti incidentiæ, quemad modum in planis accidit, in quibus femper vnum, & idem punctum eft ipfis commu ne in ipfo incidentiæ catheto.

Non prætermittam etiam hunc alium breuiorem modum fpeculandi æqualita té depreffionis imaginis fub fpeculo plano, ei quæ fupra reperitur ipfius obiecti,in ca theto incidentiæ, quemadmodum nunc vltimò diximus,hoc eft quod cum imago obiecti. l. reflexa à puncto. x.reperiatur in linea.y.x.t. & ima go eiufdem obiecti reflexa à pun- cto.m.reperiatur in linea.z.m.t. & iftæ duæ lineę feinuicem fecent in puncto.t. ipfius catheti, exiftente. r.t.æquali.r.l.vt nunc vidimus , er- go femper imago reflexa à fpecu- lo plano, nobis apparebit í ipfo ca theto,tam vltra fpeculum,quam ci tra ipfum,reptú fuerit ipfú obiectú quod nec Alhazem , nec Vitellio , nec alius aliquis ( quod fciam) adhuc fcientificè demonftrauit. exempla enim vel ex perientia non faciunt fcire. Credo etiam te non dubitare quin duæ lineæ.y.x. et. z. m.inuicem concurrant,cum anguli. t.x. m. et. t. m. x. minores fint duobus rectis cum æquales fint angulis.l.x.m.et.l.m.x.
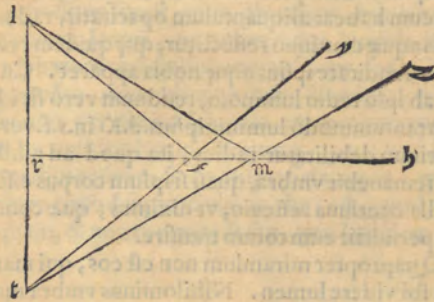
## De rotunditate vmbræ terræ in ecclipfibus Lunaribus.

### AD EVNDEM.

ROtunditas vmbræ in ecclipfi- bus lunaribus oritur tã à rotun ditate maris, quá terræ,& fi terra ef- fet etiã cuiufuis alterius figurę , quá fphæricę,dummodo aqua impleret locú fphęriceitatis à terra derelictú, nihilominus vmbra effet rotunda, quę quidem ab aqua produceretur, quáuis Alexander Piccolhomineus aliter fentiat in libro de magnitudi- ne terrę,& aquæ.

Sciẽdũ enim eft, quod omne cor pus in fe habens aliquantulũ opaci- tatis,femper debilitat radiũ lumino fum,& táto magis,quáto magis in ip fo corpore radius penetrat, etiã & fi ad rectos incideret ipfe radius fupra fuperficié ipfi⁰ corporis. Exẽpli gra tia,efto.q.p.corpus aqueũ,cuius pro funditas diuidatur in partibus.d. K: K.s: et. s. f. à puncto verò lucido. b.

Xx defcen-

346   IO. BABPT. BENED.
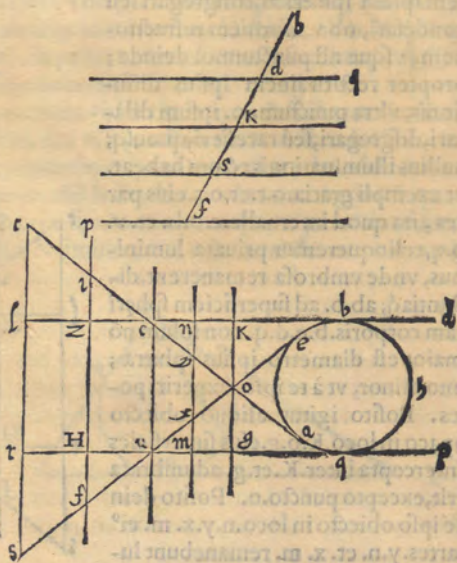
defcendat radius.b.d.K.s.f. ad libitū hoc eft rectè vel obliquè, cuius pars.b.d. in ipfo
aere exiftat. Nunc manifeftum erit partem.b.d.ipfius radij clariorem feu minus im
peditā effe quam.d.K.quod ex eo etiam cognofcere poffumus quia.b.d.reflectitur à
puncto.d.fuperficiei corporis aquei, quapropter minus luminofa remanebit pars. d.
K.cum non tota claritas.b.d.defcendat in corpore aqueo, fed vna eius pars reflecta-
tur,reliqua verò tantummodò defcendat, deinde pars. K. s. ex neceffitate debilior
erit ipfa.d.K.eo quod fuccedit poft ipfam. d. K. propter hoc etiam,quia cum corpus
aqueum habeat aliquantulum opacitatis, radius.d.K.ab omni puncto ipfius fpiffitu-
dinis aquæ continuo reflectitur, quę quidem reflexio eft illud lumen cęruleum,quod
in profunditate ipfius aquę nobis apparet. Cum igitur reflexio ipfa femper detra-
hat ab ipfo radio luminofo, refiduum verò fit id quod penetrat, ideo. K. s. erit vna
pars tantummodò luminis ipfius.d.K: in.s.f.verò aliqua pars luminis ipfius.K.s.& fic
continuò debilitatur radius, ita quod ad nihilum vfque deuenit, & vltra tale cor-
pus remanebit vmbra, quafi fi ipfum corpus effet perfectè opacum, cuius rei caufa,
eft illa continua reflexio, vt diximus, quæ continuò adimit aliquid ex ipfo radio,
nec permittit eum totum tranfire.

Quapropter mirandum non eft eos, qui margaritas quærunt in fundo maris nul-
lum ibi videre lumen. Nihilominus vmbra maris, quam dico nos poffe videre in
fuperficie corporis lunaris,ab alia etiam ratione prouenire poffet.Imaginemur enim
aggregatum terrę, marisq; effe tantummodò aqueum, quod quidem effet perfectè
fphæricum ratione centri grauitatis,fupponamusq; ipfū effe valde diaphanum, ita
quod radij folares ipfum penetraffent. Tunc dico quod in fuperficie corporis luna-
ris produceret vmbram. Pro cuius intelligentia cogitemus fubfcriptam hic figuram
b.h.q.a.e.effe fphęram aliquam cryftallinam,& ad partem.b.h.q. fit radius lumino-
fus folaris qui ipfam illuminet,cuius radij extremitates fint.d.b.l.et.p.q.r.fupponen-
do.d.l.et.p.r.terminos effe vnius plani fecantis ipfum radium per axem, tunc vide-

bis ipfum radium.b.p.q. d. tranfeū-
tem ipfam fphæram,congregari feu
condéfari, ob vniformem refractio-
nem, vfque ad punctum.o. deinde;
propter rectitudinem ipfius diffu-
fionis, vltra punctum. o. ipfum dila-
tari,difgregari,feu rarefieri,quoufq;
nullius illuminationis actum habeat.
vt exempli gratia.o.t.et.o.s.eius par
tes, ita quod interualla.c.o.b. et. u.
o.q.relinquerentur priuata lumini-
bus, vnde vmbrofa remanerent.di-
ftantiaq; ab.o. ad fuperficiem fphęri
cam corporis.b.e.d.q. non folum nó
maior eft diametro ipfius fphæræ;
imo minor, vt à te ipfo experiri po-
tes. Pofito igitur aliquo obiecto
opaco in loco.K.o.g.eius fuperficies
intercepta inter.K.et.g. adumbrata
erit,excepto puncto.o. Pofito dein
de ipfo obiecto in loco.n.y.x. m. eiº
partes.y.n. et. x. m. remanebunt lu-
mine
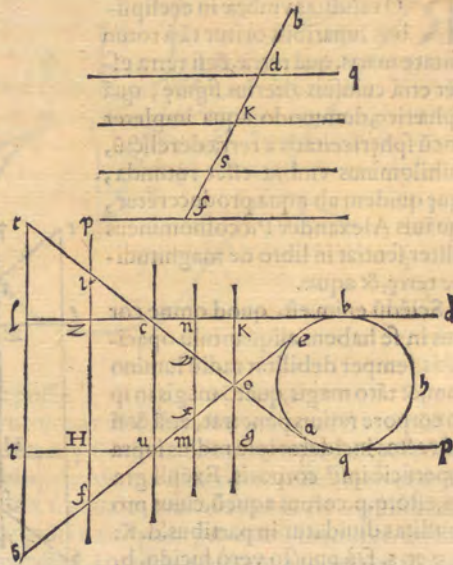
mine deftitutæ interuallumq; tantummodò inter. y. x. illuminatum erit , fed fi in
loco. c. u. pofitum fuerit, tunc totum. c. u. illuminatum erit, fed debili modo propter
detractionem factam à reflexione in fuperficie corporis fphærici, vt fupra diximus.
    Pofito deinde obiecto in loco. i. z. H. f. tunc partes. z. i. et. H. f. rectos Solis radios
habebunt cum aliquibus refractis, fed . z . H. pauciffimum habebit lumen , pro-
pter difgregationem radiorum . Pofito poftea ipfo obiecto in loco. t. l. r. s. tanto
minus lumen habebit pars. l. r. propter dictam difgregationé, feu diffipationé radio
rum, & fic fuccefliuè quanto remotius pofitum fuerit ipfum obiectum, tanto minus
illuminabitur . vnde ita remotum poterit locari , ut nullus actus luminis in eo
videatur, de radijs fcilicet , qui per fphæram chryftallinam tranfibunt, fed videbi-
tur vmbra ipfius fphęrę in obiecto propofito , cum nullum actum illuminationis in
eo loco obiecti habeant radij tranfeuntes per dictam fphęram. quapropter partes.
t. l. et. r. s. illuminatæ erunt à Sole, et. l. r. omnino lumine deftituta.
    Quòd vero tolerabilior fit oculis radius reflexus Solis à fuperficie aquæ , quàm
à fuperficie alicuius fpeculi, oritur ab eo, quod fupra diximus, hoc eft, quod ma-
gna pars ipfius luminis penetrat in aquam, & non totum reflectit, quod quidem non
accidit fpeculis opacis.

# DE LONGITVDINE DVORVM LATERVM
## cuiufuis trianguli fupra tertium.

### *Hieronymo Fenarolo.*

V o' d quælibet duo latera continentia rectum angulum cuiufuis triangu-
li orthogonij , longiora fint tertio latere, per diametrum circuli in eo in-
fcripti, ab alijs iam demonftratum fuit . Sed quòd quælibet duo latera
cuiufuis trianguli longiora fint tertio per latus tetragonicum , quadrupli
producti cuiufuis lineæ defcendentis ab angulo contento à dictis duobus lateribus
ad oppofitam partem circuli infcripti, in partem extrinfecam ipfius lineæ , nullus
( quod fciam ) vnquam fcripfit, vel animaduertit.
    Sit exempli gratia triangulus. a. b. c. quem volueris, in quo defcribatur circulus.
u. s. n. & puncta contingentiæ fint eadem. u. s. n. à puncto vero. a. defcendat linea. a.
i. e. quæ terminetur à circunferentia in puncto. e. ipfius circunferentiæ, vbi volue-
ris. Dico nunc latera. a. b. et. a. c. longiora effe latere. b. c. per latus tetragonicũ qua-
drupli producti ipfius. a. e. in. a. i. Nam certi fumus ex vltima parte penultimæ ter-
tij Eucli. n. c. et. s. c. æquales inuicem effe, & fimiliter. b. s. et. b. u. vnde ex communi
conceptu dicta latera maiora erunt
ipfo. b. c. per. a. u. et. a. n. quæ duæ
partes funt inuicem æquales di-
cta ratione , & quadratum lineæ
æqualis aggregato earum, effet qua
druplum quadrato cuiufuis earum
ex. 4. fecundi , fed ex penultima ter
tij, productum. a. e. in. a. i. æquale eft
quadrato ipfius. a. u. vel ipfius. a. n.
    X x    2     Ve-
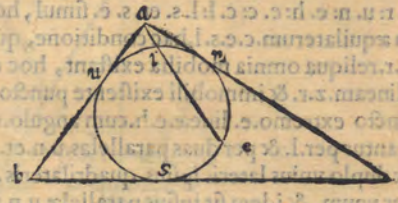
Verum eſt igitur quod. a.b. cum. a. c. longiores ſint ipſa. b. c. per latus tetragonicum quadrupli eius quod ſit. ex. a. e. in. a. i. quod fuit propoſitum.

Illud etiam non eſt ſpernendum, quod quotieſcunque data fuerint omnia latera alicuius trianguli, illicò poſſumus cognoſcere puncta. u. n. s. contingentiæ circuli in ſcripti, ope vltimæ partis penultimæ tertij, eo quod ex illa iam ſcimus, quod detrahendo. b. c. ex aggregato aliorum duorum laterum, remanebit. u. a. et. a. n. quarum vnaquæque nota erit, cum illarum quælibet, medietas ſit reſidui cogniti, detrahendo poſtea vnam illarũ ab altero duorum laterum. a. b. vel. a. c. remanebit. u. b. vel. c. n. æqualis. b. s. vel. c. s. vnde ſimiliter nobis innoteſcet punctum. s. cum duobus punctis. u. et. n. à quibus duobus punctis, ſi duæ perpendiculares ad talia latera ductæ fuerint, vbi hæe perpendiculares ſeinuicem ſecabunt, ibi centrũ circuli inſcriptibilis erit in triangulo propoſito.
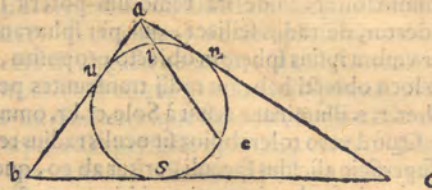


Inter alia, quæ tibi dixi de Iride, quod memoria non tenes, nihil aliud eſt niſi quod cum Iris videtur, non eodem loco ab omnibus videtur, quia reflexio eſt, & vt reflexio luminis à ſpeculo non omnibus ab eodem puncto ſit, ita etiam tibi dixi de Iride.

## De Inſtrumento oxygonio, ſeu elliptico.

### AD EVNDEM.

Qvod aliquando à me audiuiſti falſum non eſt, ſcilicet poſſibile eſſe ( vt ſpeculatus ſum ) particulare inſtrumentum fabricari ad deſignandum oxygoniam, ſeu ellipticam ſectionem, quæ à Pergeo defectio appellatur, quod quidem inſtrumentum valde diuerſum eſt ab alijs, quæ aliàs inueni, pro ipſis conicis ſectionibus delineandis. Occaſionem aũt huiuſmodi inſtrumenti inueniendi mihi præ buit ſecũda dubij ſolutio quã feci ann. 1568. grauiſſ. philoſopho Franciſco Vimer cato, nã cũ viderim in ea figura. f. a. ſemper æqualē eſſe. o. i. ſuæ parallelæ ſcilicet, vnde cum recta linea fuerit protracta per. o. et. f. ipſa foret ſemper æquidiſtãs. d. p. ex 33. primi Eucli. Venit mihi in mentem modus conſtruendi hoc ſubſcriptum inſtrumentum, tali ordine, videlicet, coniungẽdo ſeptem hic ſubnotatas lineas materiales. z. r: u. n: e. h: e. c: c. l: l. s. et. s. e. ſimul, hoc modo, ſcilicet fabricãdo quadrilaterum æquilaterum. c. e. s. l. hac conditione, quod immobili exiſtente puncto. c. in li nea. z. r. reliqua omnia mobilia exiſtant, hoc eſt quod punctũ. s. moueatur per dictam lineam. z. r. & immobili exiſtente puncto. e. vt extremum lineæ. e. h. hoc eſt coniuncto extremo. e. lineæ. e. h. cum angulo. c. e. s. reliqua puncta lineæ ipſius. e. h. moueantur per. l. & per duas parallelas. u. n. et. z. r. longitudo vero. e. h. ſit compoſita ex duplo vnius lateris ipſius quadrilateris. Oportet deinde quod punctum. f. ſemper vnum, & idem ſit ipſius parallelæ. u. n. moueatur tamen per. e. h. quod quidem punctum illud erit, quod vnam portionē circunferentiæ oxygoniæ ſectonis
deſi-

designabit, puncta verò . o . et . K . vt puncta laterum. c. e. et. s. e. æquædistantia à
punctis. c. et. s. eadé semper sint, ita tamen vt puncta lineæ. u. n. semper diuersi exi
stát, & quodlibet ipsius quadrilateri latus, æquale sit medietati maioris axis ipsius
oxygoniæ sectionis delineandæ, et. c. o. seu. s. K. (quod idem est) sit æqualis medie
tati axis minoris dictæ sectionis, et. z. r. æqualis duplo. e. h. vnde, quando puncta. c. et.
l. coniuncta simul erunt, similiter coniunctæ simul erunt. c. e. et. e. s. cum. c. l. et. l. s.
Quapropter



Instrumentum
oxigonium

350 **IO. BAPT. BENED.**
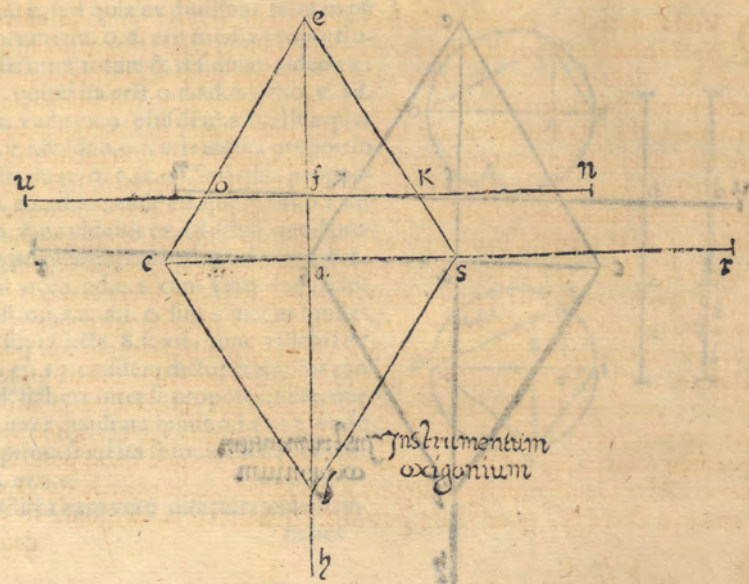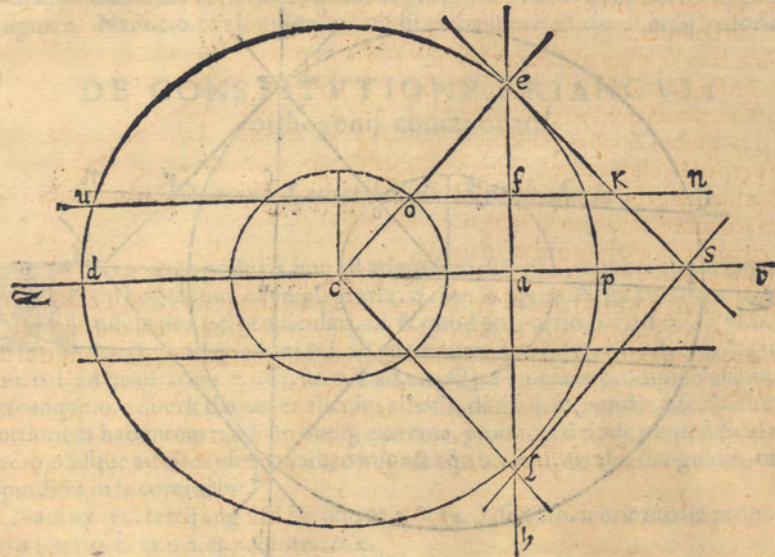
Quapropter puncta.e.l.f. et.p.extremium axis maioris, in eodem met loco erunt, hoc eft in aliquo extremorum maioris axis, & cum punctus.s.coniunctus fuerit cum centro.c.punctus.f. parallele.u. p. in extremo axis minoris erit, & in eodem loco erit cum.o. & cum. K.In extremitatibus verò lineæ. z.r.necefse eft, vt fint duo pucta fer ra,ad firmandum ipfam.z.r.fuper fubiectam lineam fignificantem maiorem axem propofitæ fectionis.
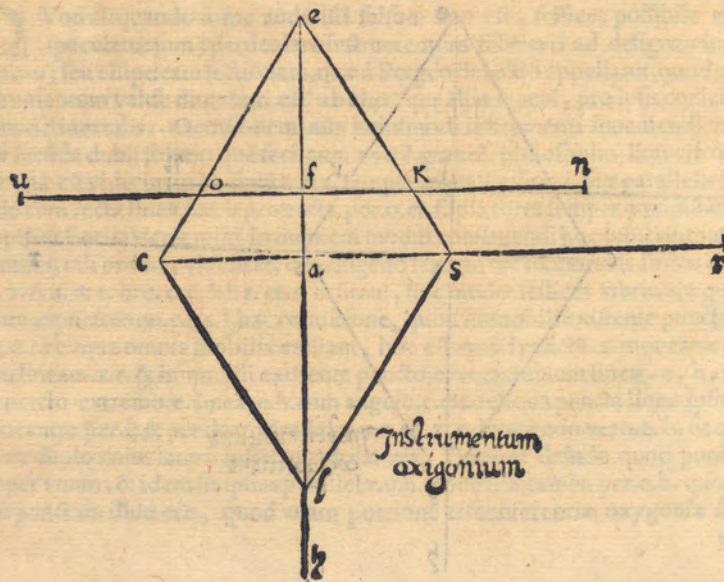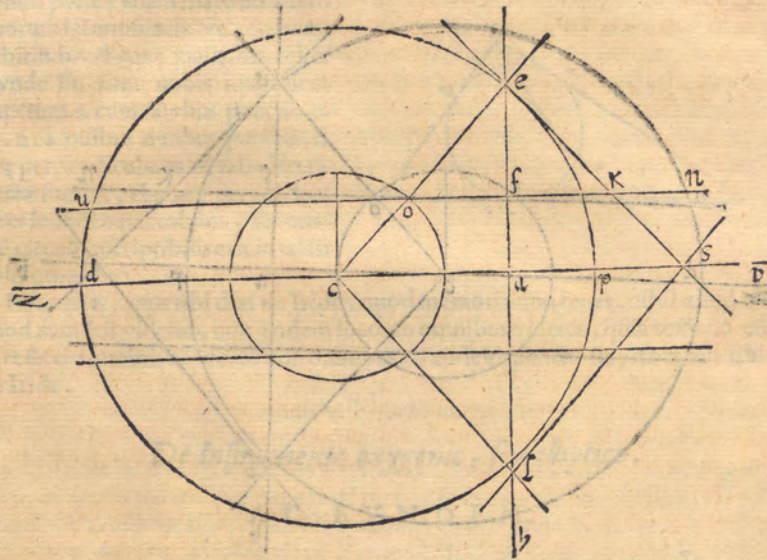
Volo



*Instrumentum oxigonium*

Volo etiam quod ad partem.c. l. s. quadrilateri conftituta fit alia parallela ad. z. r. & in æquali diftantia ab ipfa quemadmodum.u.n.diftat ad eademmet. z. r.ad eandem operationem faciendam. Vnde in vno tantummodo itinere puncti.s.ab. r. vfq; ad.c. defignabimus quartam partem fectionis, conuerfo poftea inftrumento, hoc eft pofito puncto.r. vbi prius erat. z. et. z.vbi erat.r. aliam delineabimus quartam, & fic ad oppofitam partem ipfius.z. r. faciendum erit. Hoc inftrumentum poffumus etiam ita conftruere, vt puncta.o. et.K. poffint collocari in laterihus. c. e.et.e.s.vbi nobis magis libuerit, ita vt licebit in qualibet proportione axiū propofita, oxygoniam defignare. Nam.c.o. erit longitudo dimidij axis minoris, er.c. e.dimidij maioris.

# DE CONSTITVTIONE TRIANGVLI
## orthogonij conditionati.

*Domino Ludouico de Rocchaforte.*

Vod à me poftulas, non eft admodum difficile , cupis enim triangulum orthogonium, exempli gratia . o. i. e. in figura. A. ita conftituere, vt diuifum fit à perpendiculari.a.i. & quod proportio.o.e.ad.o.i.fit vt.o. i. ad i. e.& quod quadrati.o.i. ad quadratum.o.a. fit vt. e.i. ad. e. a. & quadratum. o. i. ad quadratum. e. i. fit. ut. o. a.ad.e.a. Quæ omnia in promptu veniunt, quotiefcunque.o. e.fuerit diameter alicuius circuli, diuifaq; in puncto.a.fecundum proportionem habentem medium duoq; extrema, protracta deinde perpendiculari. a. i.ad.o.e.ufque ad circunferentiam, coniunctæq;. o. i.et.i. e: tale triangulum, omnia fupradicta in fe continebit.

Nam ex. 30. tertij angulus.i.rectus erit, & ex. 8.fexti.o. i. erit media proportionalis inter.o. e. et.o.a.et.e.i. inter. o. e. et. a. e. fed quia ex diuifione facta in pū cto.a.etiam. o. a. erit media proportionalis inter totum & refiduum , ideo ex. 11. quinti ita erit.o.e.ad.e.i.vt.o. e. ad o.a. vnde ex.9. eiufdem.a.o. erit æqualis. e. i.& ideo.o.i. erit media proportionalis inter. o. e.et.e.i.Sed quia proportio. e.i.ad.a.e.eadē eft, quę ipfius.o.e.ad o. a. tunc videbis ex.18.fexti, quod proportio quadrati.o.i.ad quadratum.o. a. erit vt.e.i. ad.e.a. cum vero duo trianguli.o.i.a.et.a.i. e. fint inuicem fimiles ex fupradicta. 8.fexti, tunc videbis ex 18.et. 17.eiufdem dictos triāgulos eandem habere inter fe proportionem, quę eft inrer quadrata ipfius.o.i.et.i.e. vnde ex prima fexti ita fe inuicem habebunt. a.o. et.a.e.

Circa eam verò difficultatem quam
habes
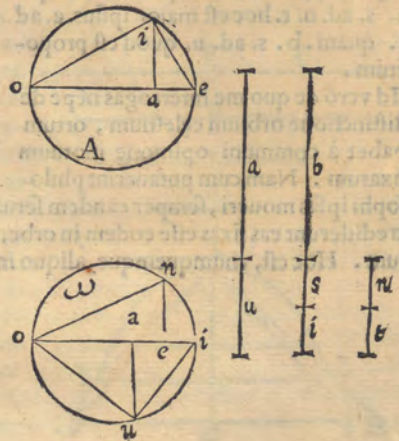
habes in circulo. ω. vbi fateris te non videre qua ratione eadem proportio fit qua-
drati.u.o. ad quadratum.o.n. vt lineæ.o.a. ad lineam.o.e. partes diametri. o. i. ipfius
circuli, terminatæ à perpendicularibus.u.a.et.n.e.

Hoc neceffario contingit, propterea quod cum fuerint protractæ.u.i. et.n.i. tũc
habebimus ad partem.o.u.i. triangulum.o.u.i. diuifum in duo triangula fimilia ipfi
totali triangulo. Idem etiam dico ad partem.o.n.i. vnde ex tali fimilitudine habe-
bimus.o.u. mediam proportionalem inter.o.i. et.o.a. et fic.o.n. erit media proportio
nalis inter.o.i. et.o.e. quare ex. 16. fexti, quadratum.o.u. æquale erit producto ipfius
o.i. in.o a. & quadratum.o.n. æquale producto.o.i. in.o.e. fed ex prima eiufdem, ea
dem proportio eft ipfius.o.a. ad.o.e. quæ producti ipfius.o.i. in.o.a. ad productum.o.
i. in.o.e. quare, ex cõmuni conceptu, ita erit quadrati.o.u. ad quadratum.o. n. Et hęc
eft alia circuli paffio.

Reliqua verò difficultas quam te habere fcribis, eft, quare cum duæ lineę
a.u. et. b.s.i. fint inuicem ęquales, diuifæ verò non æquali modo, fed tali, quod. a.
maior fit quam.u. et. b.s. maior quam.i. quomodo poteft fieri, quod fi.u. maior fue-
rit.i. proportio.a. ad.i. maior fit quam ipfius.b.s. ad.u.

Hoc etiam ex neceffitate cuenit, eo
quod fi accepta fuerit.t.n. æqualis. u. ab
ipfaq; abfcifa fuerit. t. æqualis. i. & ab.
b.s. abfcifa. s. æqualis.n. habebimus.a. et
b. inuicem æquales, vnde habebis ma-
iorem proportionem ipfius.b. ad.t. quã
s. ad.n. quod cum clarum per fe fit, tibi
relinquo. fed ex. 27. quinti, proportio
b. ad. s, maior erit quam. t. ad. n. & ex
28. eiufdẽ pportio.b.s. ad.s. maior erit,
quam. t.n. ad. n. & ex. 27. maior propor
tio erit ipfius. b.s. ad.n.t. quam. s. ad. n.
ergo ex. 33. maior erit ipfius. b. ad.t. quã
b. s. ad. n. t. hoc eft maior ipfius. a. ad
i. quam. b. s. ad. u. quod eft propo-
fitum.



Id verò de quo me interrogas nẽpe de
diftinctione orbium cęleftium, ortum
habet à communi opinione motuum
fixarum. Nam cum putauerint philo-
fophi ipfas moueri, femper eandem feruãdo inuicem diftantiam, non fine ratione
crediderunt eas fixas effe eodem in orbe, idem etiam poftea de planetis opinaue-
runt. Hoc eft, vnumquemque, aliquo in orbe, fixo exiftere.
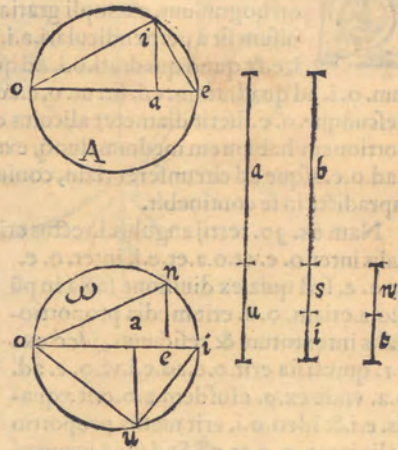
DE

## DE MODO DIVIDENDI PARABOLAM
propositam secundum datam proportionem.

### Pamphilo Gothfrid.

Vod à me quæris, est quidem possibile, non tamen adhuc inuentum, quo niam nemo ad hûc vsque diem diuisit vnam datam proportionem in tres æquales partes, sed si hoc pro facto concesseris, nunc tibi morem geram. Nam proponis mihi parabolem.x.b.e. cum proportione.p.ad.q. cupisq; scire modum diuidendi ipsam parabolem vna mediante linea parallela ipsi basi, ita vt eandem habeat proportionem tota parabola ad partem abscissam, quæ est inter. p.et. q. Ad quod faciendum, supponendum primò datam proportionem inter. p.et.q.diuisam esse in tres partes æquales, duabus lineis mediantibus.n.et.u. quæ me diæ proportionales vocabuntur inter. p. et.q.deinde à quouis puncto circunferentię ipsius figuræ ducatur parallela basi. x. e. postea verò per puncta media harum dua rum æquidistantiũ protrahatur.g.b. quæ diameter erit sectionis, ex 28. secundi Per gei, diuidatur deinde hæc diameter in puncto.a.ita quod eadem proportio sit ipsius b. g. ad.b.a. quæ ipsius.p.ad.u.quod tibi facile erit, secando à linea.p.partem.i.æqua lem ipsi.u. tali modo postea diuidendo.b.g.ex.12.sexti, ducatur a puncto. a. ipsa.d. h.parallelam ipsi.x.e.& habebitur propositum.

Pro cuius rei ratione, scies primum quod.h.d.diuisa erit à diametro.b.g. per æqua lia ex. 7.primi Pergei, vel si cogitabimus aliquam lineam tangentem ipsam parabo lam in puncto.b.tunc ex quinta secundi ipsius Pergei habebimus ipsam esse paralle lam.e.x.& ex.30.primi Eucli.erit similiter æquidistans.d.h.vnde ex.46. primi eius dem Pergei.h.a.æqualis erit.d.a.Protrahatur deinde.e.b:d b:x.b.et.h.b.vnde ex.17 lib. de quadratura parabolæ Archimedis, habebimus eandem proportionem super ficiei totalis parabolæ.x.b. e. ad trigonum.x.b.e.quæ portionis.h.b.d. ad suum tri gonũ, eo quod tã vna quàm alia erit sesquitertia, ei² etiã medietates sic se habebũt.

Vnde permutando, proportio medietatis totalis parabolę ad medietatem partia lem ipsius, æqualis erit proportioni trianguli g.b.e.ad triangulum. a. b. d. sed ex. 20. primi Pergei, eadem est proportio quadrati ipsius. g.e.ad quadratum ipsius.a.d.quæ.b.g. ad. b.a. hoc est, vt.g.e.ad.a.o.ex similitudine triangu lorum,& quia.b.g.ad.b.a.est sicut.p. ad. u. ita igitur erit quadrati ipsius.g.e.ad quadratum ipsius.a. d. quare. g. e. ad.a, d. erit ut p. ad. n. ex. 18. sexti Euclid. sed cum ex.24. eiusdem proportio trianguli.b. g. e. ad triangulum. b. a.d.composita sit ex proportione, g. e. ad. a. d. et. ex. g. b ad. b. a. hoc est. g. e. ad. a. o. & quia ,ppo.tio.g.e.ad. a.o.æqualis est ei quæ.p. ad.u ex. 11. quinti Euclid. & proportio.g.e. ad.a.d.æqualis est ei quæ.p.ad.n.hoc est vt. u. ad.q.ergo proportio trianguli.b.g.e.ad trian gulum.b.a.d.composita erit ex ea quę.p.ad.u. & ex ea quæ.u.ad.q.æqualis ergo erit ei, quæ p. ad.q. & ita medietates parabolarum, & e orum dupla.
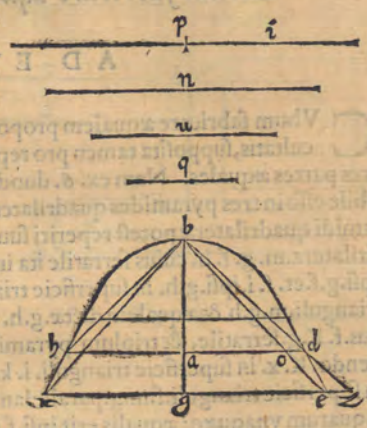
Yy Cor-

### COROLLARIVM.

Proportio maioris portionis ad minorem semper erit sesquialtera proportioni ipsius.b.g.ad.a.b. eo quod cum sit proportio totalis portionis ad partialem vt trianguli.b.g.e.ad.b.a.d. & hæc sesquialtera proportioni ipsius.g.e.ad. a. o. hoc est vt ipsius. b.g.ad. b.a. ideo proportio ipsarum portionum erit similiter sesquialtera proportioni diametrorum.

Deinde si protractæ fuerint.b.d.et.g.e. quousque conueniant in puncto. z. habebis inter.g.z.et.a.o.duas.g.e.et.a.d.medias proportionales in proportionalitate continua, eo quod cum ( ex ijs quæ supra diximus. ).a.d. media proportionalis sit inter. g.e.et.a.o.& proportio. g.z.ad.g.e. vt ipsius.a.d.ad.a.o. eo quod ipsius.g.z. ad. a. d. & ipsius.g.e.ad.a.o.est vt ipsius. b. g. ad. b.a.ex similitudine triangulorum, ideo dictæ pportiones erunt inuicé æquales. Vnde permutatim ita erit ipsius.g. z.ad.g. e. vt ipsius.a.d.ad.a.o.& ut ipsius.g.e.ad.a.d.

Amplius etiam dico, quod proportio parabolæ totalis ad partialem, eadem est, quę cubi ipsius.g.e.ad cubum ipsius.a.d.& ex cõsequenti, vt cuborum earundem basium,eo quod cum sit, ex. 36.vndecimi Euclid.proportio cubi ipsius. g.e.ad cubum ipsius. a.d. tripla ei quæ ipsius.g.e.ad.a.d. ideo æqualis erit ei quę trianguli.b.g.e.ad triangulum.b. a.d.cum proportio horum duorum triangulorum composita sit ( vt supra vidimus ) ex ea quæ.g.e.ad.a.o.& ex ea quæ.g.e.ad. a. d. & hæc medietas illius, sed trianguli ita se in uicem habent, vt parabolę,quare ipsæ parabolæ seinuicem habebunt, vt cubi ipsarum basium.
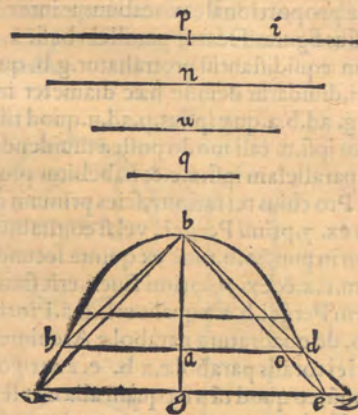
*Cubum fabricare æqualem pyramidi propositæ.*

### AD EVNDEM.

CVbum fabricare æqualem propositæ pyramidi quadrilateræ,nullius erit difficultatis,supposita tamen pro reperta diuisione cuiusuis datæ proportionis in tres partes æquales. Nam ex. 6. duodecimi Eucli.patet omne corpus serratile diuisibile esse in tres pyramides quadrilateras æquales,scimus etiam quod cuilibet pyramidi quadrilateræ potest reperiri suum serratile. Sit igitur proposita pyramis quadrilatera.m. g. f. h. cuius serratile ita inueniemus, ducendo primum.h.i.parallelam ipsi.g.f.et. f. i.ipsi.g.h. in superficie trianguli. f. g. h. et. m. K.ipsi. g. h. in superficie trianguli.m.g.h.& æqualem dictæ.g.h. ducetur postea.K.h.et. K.i.& habebimus corpus.f. K.g.serratile, & triplum pyramidi propositæ. Nunc duplicemus ipsum,ducendo. K. x. in superficie trianguli. i. k. h. parallelam,æqualemq̃; ipsi. i. h. et. m. y. in superficie trianguli.f.m.g.parallelam,ęqualemq̃; ipsi.f.g.ducatur postea.g.y.et.h. x.quarum vnaquæq; æqualis erit ipsi.f.m. vnde habebimus corpus.f.x. parallelepipidum, & sexcuplum ipsi pyramidi propositæ.

Inue-

Inueniatur nunc quadratum.u.n.æquale fextæ parti fuperficiei.f.i. g. h. quod per
fe facile erit, deinde accipiatur altitudo corporis.f.x. ducendo vnam perpendicula
rem à puncto. m.ad bafim.f.g.h.quę fit. n.e.qua mediante, cum quadrato. u.n. fabri
cetur folidum parallelepepidum.u.e.quod erit æquale dictæ pyramidi ex.33.vnde-
cimi Euclid.

Repertæ nunc fint duæ mediæ proportionales.r.s. inter. n.e.et.n.p. quarum.s. fit
proximior ipfi.u.p. ex qua. s. fi conftitutus fuerit cubus, habebimus propofitum.

Pro cuius rei ratione, cogitemus corpus.u.e.productum effe vfque ad.a.o. per lon-
gitudem.s. latus dicti cubi, qui quidem cubus fit.d.b.vnde proportio corporis. u.e.
ad corpus.e.o.erit, vt fuperficiei.p.e.ad fuperficiem.t.e.ex. 33. undecimi, ipfæ verò
fuperficies fibi inuicem erunt vt.n. e.ad.e.a. ex prima fexti, quare proportio corpo
ris.u.e.ad corpus.e.o.dupla erit proportioni ipfius.s.ad.n.p. fed cum ex. 33. vndeci-
mi, proportio cubi.d.b.ad corpus.e.o.fit vt quadratū.q.b.ad quadratum.o.a. & cum
proportio.q.b.ad.o.a.dupla fit ei quæ.q.o.ad.o.t.ex. 18.fexti, erit igitur proportio
cubi.d.b.ad corpus.e.o.dupla ei quæ.q.o.ad.o.t.hoc eft ei quæ.s.ad. n. p. fed ita erat
corporis.u.e.ad corpus.e.o.quare ex.9.quinti, cubus.d.b.æqualis erit corpori. u.e.
hoc eft pyramidi propofitæ.

Sed fi oportebit cubum maiorem vel minorem ipfa pyramide reperire, in qua
proportione tibi placuerit, tunc opus erit aliud quadratum inuenire, quod in ea
proportione fe habeat ad quadratum.u. n.quam volueris, quo mediante fimul cum
altitudine pyramidis confequemur propofitum.

Aduertendum tamen quod fabri-
care ipfum corpus ferratile.k.f.h.& fo
lidum.f.x.neceffarium non eft, nifi pro
demonftratione. idemq; dico de alijs
folidis, nam pro fimplici operatione
huiufmodi problematis, abfque ali-
qua re neceffaria ad fpeculandum, ita
faciendum erit.

Data pyramide.m.f.g.h.accipe ei⁹
altitudinem à pūcto.m. vfque ad fuper
ficiem bafis.f.g.h. quæ fit. n. e. accipe
deinde latus tetragonicum quadrati.
u.n.æqualis tertiæ partis ipfius bafis.f.
g.h. quod latus fit. n. p. inter quod,et.
n. e. inuentæ cum fuerint duæ lineæ
mediæ proportionales. s. et.r. quarū.
s.proximior fit. n. p. quæ quidé. s. erit
latus cubi quæfiti.
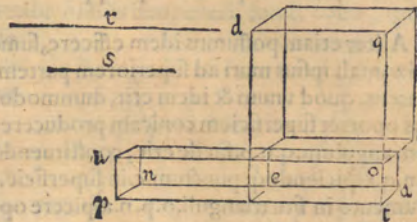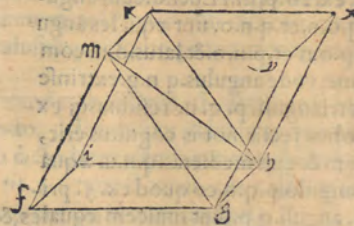


Yy 2 Duplex

## Duplex modus parallelam orizontalem alicui muro proposito una tantummodo statione ducendi.

### AD EVNDEM.

DVcere parallelam orizontalem alicui muro recto proposito vna tantummodò statione, non solum possibile est sed etiam facile.

Sit exempli gratia murus rectus. a. d. situs verò. o. n. Si cupimus ducere. n. u. parallelam dicto muro, accipiatur quadratum geometricum, seu scala altimetra vel aliquod simile instrumentum, quo mediante a situ. o. videbimus punctum. q. quod volueris ipsius muri, dexterâ versus, inferius tamen. ipso. o. vnde formatum habebimus triangulum. n. o. q. Quo facto ad partem sinistrã cum eodem angulo. n. o. q. oportebit nos inuenire punctum aliquod. p. in dicta superficie muri, & tunc habebimus angulum. n. o. p. æqualem angulo. n. o. q. vnde angulus. q. n. p. nobis cognitus erit, duoq́; latera. n. q. et. n. p. erunt inuicem æqualia, ex 26. primi Euclid. cum anguli. q. o. n. et. q. n. o. sint æquales angulis. p. o. n et. p. n. o. & latus. o. n. commune, vnde angulus. q n. g. extrinsecus trianguli. p. q. n. residuusq́; ex duobus rectis nobis cognitus erit, etiam & eius medietas. q. n. u. æqualis angulo. p. q. n. eo quod ex. 5. primi, anguli. q. p. sunt inuicem æquales, & ex. 32. eiusdem, æquales sunt extrinseco. q. n. g. & ex 27. n. u. erit parallela ipsi. q. p.
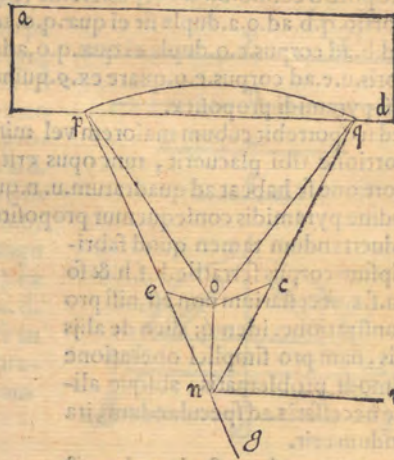


Aliter etiam possumus idem efficere, sumendo duo illa puncta in suprema linea orizontali ipsius muri ad superiorem partem aspiciendo, quemadmodum ad inferiorem, quod vnum & idem erit, dummodò non aspiciamus orizontaliter, eo quod nos oportet superficiem conicam producere, linea visuali mediante. cognoscere autem angulum. q. n. p. facile erit, constituendo primò instrumentum in situ trianguli. o. n. q. aspiciendoq́; punctum. c. in superficie. n. q. o. & sic in alia parte, existente instrumento in situ trianguli. o. p. n. aspicere oportet punctum. e. proximum puncto. n. vbi possit metiri angulum. c. n. e.

Sed si situs puncti. n. talis esset, vt ab eo non posset aliquis murum videre ad rectos angulos, aspiceremus punctum. q. sub orizontali ab oculis nostris, in orizontali tamen puncti. n. ita quod angulus. o. n. q. rectus existat, quo facto obseruando angulum. n. o. q. eo mediante, medianteque. n. o. cum angulo. o. n. q. cognoscemus quantitatem distantiæ. n. q. idem etiam faciendum est cum alio puncto. p. quod volueris, & mediantibus duobus punctis inuicem proximis. c. e. cognoscatur angulus
gulus

gulus. p. n. q. vnde ex methodo. 56.
primi triangulorum Monteregij,
cognoscemus reliqua trianguli.
q. p. n. Constituendo postea angu-
lum. q. n. u. æqualem angulo. n. q. p.
propositum habebimus.

Si etiam puncta. q. p. lineæ. q. p.
orizontali in eodem plano non exi
sterent cum puncto. n. nihil refer-
ret, dummodo in pauimento notē
tur pūcta. c. e. proxima. n. in ijsdem
superficiebus triangulorum. n. o. p.
et. n. o. q. vnde. n. c. et. n. e. erunt cō-
munes sectiones dictarum superficierum cum superficie pauimenti supra quam sit
statio.
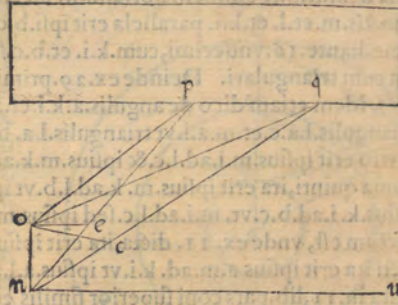
## CONI RECTI DIVISIO A PLANO
### parallelo basi secundum datam proportionem.

#### *Raphaeli de Auria.*

VOTIESCVNQVE volueris conum rectum diuidere à plano parallelo ba-
si secundum vnam datam proportionem, nullius tibi erit difficultatis, con
cessa tamē pro inuenta diuisione cuiusuis propositę proportionis per tres
æquales partes.

Sit exempli gratia conus rectus. a. b. c. secandus vt dictum est, accipiatur latus
ipsius, quod sit. a. c. ipsumq́; diuidatur in puncto. d. secundum illam proportionem
quam desideras, hoc est ipsius. a. c. ad. a. d. quo facto, inter totum. a. c. et. a. d. inueniā
tur duæ lineæ proportionales, quarum maior sit. a. i. tunc si conus. a. b. c. sectus fue-
rit à plano per punctum. i. parallelo basi, habebimus quod quærebamus.

Cuius rei ratio, primò est, quia quotiescunque conus aliquis sectus fuerit ab ali-
quo plano parallelo basi ipsius, pars superior similis semper erit totali cono, quod
ita probo, cogitemus conum sectum esse
à plano per axem. a. l. vnde ex. 3. primi
Pergei, talis sectio triangularis erit, quæ
sit. a. b. c. et. b. c. diameter erit basis.

Imaginemur deinde. K. i. communem
esse sectionem huiusmodi trianguli cum
plano parallelo ipsi basi, tunc tale planū,
circulare erit ex. 4. primi ipsius Pergei. K.
i. verò, eius diameter erit, et. a. m. su⁹ axis.

Cum verò. a. l. sit perpendicularis ipsi
basi coni totalis, eo quod rectus supponi-
tur, ideo eadem. a. m. l. erit perpendicula-
ris etiam ipsi secundo plano circulari, ex
conuersa. 14. vndecimi Euclid. vnde ex
secunda
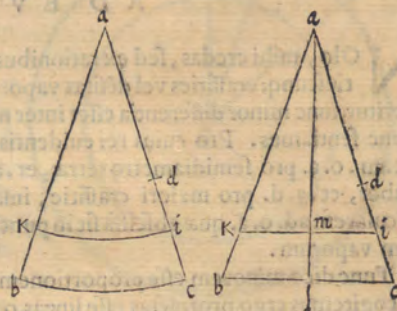
secunda definitione eiusdem libr.a.m.l.efficiet angulos rectos cum duabus.b.c.et.K.
i.in punctis.m.et.l.et.k.i. parallela erit ipsi.b.c.ex.28.primi, quod etiam potest con
cludi mediante.16.vndecimi,cum.k.i.et.b.c.sint communes sectiones duorum pla
norum cum triangulari. Deinde ex.29.primi anguli.a.i.m.et.a.c.l.erunt inuicem
æquales, idem etiam dico de angulis.a.k.i.et.a.b.c. anguli postea ad.a. communes
sunt triangulis.l.a.c.et.m.a.i.vt triangulis.l.a.b.et.m.a.k. Vnde ex.4.sexti, eadem
proportio erit ipsius.m.i.ad.l.c.& ipsius.m.k.ad.l.b.vt ipsius.a.m.ad.a.l. Quare ex
vndecima quinti,ita erit ipsius.m.k.ad.l.b.vt ipsius.m.i.ad.l.c.& ex.13.eiusdem,ita
erit ipsius.k.i.ad.b.c.vt.m.i.ad.l.c.sed ipsius.m.i.ad.l.c.est vt ipsius.a.m.ad.a.l.quod
iam dictum est, vnde ex.11.dicta,ita erit ipsius.k.i.ad.b.c.vt ipsius.a.m.ad.a.l.& ex
16.dicti ita erit ipsius.a.m.ad.k.i.vt ipsius.a.l.ad.b.c. Quare ex definitione ab Eu-
cli.posita in.11,lib.pars coni superior similis erit cono totali.

Deinde sciendum est illud quod Euclid.scribit in.10.duodecimi lib.hoc est , q̃
proportio duarum pyramidum inuicem
similium , triplicata est ei diametrorum
suarum basium , hoc est, quod proportio.
b.c.ad.k.i. tertia pars erit proportionis to
tius pyramidis.a.b.c.partiali pyramidi.a.
k.i.sed ita est ipsius.a.c.ad.a.i. vt ipsius.b.
c.ad.k.i. ex.4.sexti cum trianguli.a.b.c.
et.a.k.i.sint æquianguli, quod ex ijs, quę
superius diximus facile comprehenditur.
Quare proportio.a.c.ad.a.i.tertia pars erit
proportionis totius coni.a.b.c.ad eius par
tem abscissam.a.k.i.sed eadem proportio
ipsius.a.c.ad.a.i.erat etiam tertia pars pro
portionis ipsius.a.c.ad.a.d.Quare ex com
muni conceptu,proportio totius pyramidis, ad partem abscissam, æqualis erit pro-
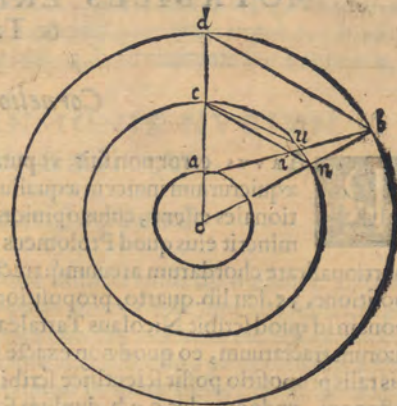portioni ipsius.a.c.ad.a.d.
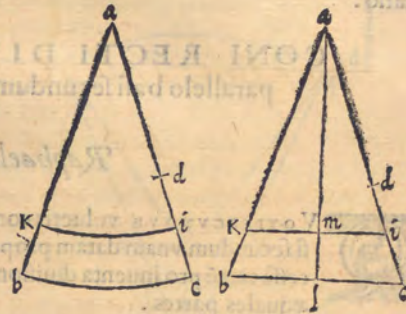
## De differentia caloris Solis propter vaporum altitudinem.

### AD EVNDEM.

NOlo, mihi credas, sed ex rationibus, quas tibi scribo considera , quod quo
tiescunq; crassities vel dẽsitas vaporũ,seu altitudo,maior esset ea,quę nunc re-
peritur,tunc minor differentia esset inter maiorem minoremq́; calorem Solis,quam
nunc sentiamus. Pro cuius rei euidentia,imaginemur in hac subscripta figura, li-
neam.o.a. pro semidiametro terræ, et.a.c. pro crassitie vaporum , vt nunc se
habet, et.a.d. pro maiori crassitie, imaginemurque lineam.a.b. quasi perpen-
dicularem ad.o.a. quæ abscissa sit in puncto u.à circunferentia.c.u. inferiori prio-
rum vaporum.

Tunc dico minorem esse proportionem ipsius.a.b. ad.a.d. quam ipsius.a.u.ad.a.
c. cogitemus ergo protractas esse lineas.o.b: d.b: c.u. et.c.n.quæ.c.n.secabit.a.u. in
puncto

puncto. i. ex communi conceptu, &
parallela erit ipfi.d.b. ex. fecunda par-
te fecundæ fexti, vnde ex prima parte
eiufdem, ita erit ipfius.b.i.ad.i. a. vt. d.
c.ad.c. a.& coniunctim ita erit ipfius.b.
a.ad. a. i. vt ipfius. d. a.ad.a.c.& permu
tatim ipfius . a. b. ad. a . d erit , vt. a. i.
ad. a. c . fed cum.a.u. maior fit ipfa.a.i.
vt omne totum maius eft fua parte .
maior proportio erit ipfius. a. u. ad. a.
c. quam ipfius. a. i.ad.a.c. hoc eft quam
ipfius . a . b. ad. a.d. Verum igitur eft
propofitum.

## De differentia caloris Solis refpectu altitudinis ipfius.

### AD EVNDEM.

Vod à me poftulas deinde, ita fe habet . Inquis enim, quod cum differentia
inter maiorem, minoremq; calorem, oriatur etiam ex differentia maioris
quantitatis vaporum ad minorem, per quam quantitatem vaporum rranfit lumen
Solis ( vt alias etiam tibi dixi) velles nunc fcire quantitatem ipfius differentiç, quæ
inter duas Solis datas altitudines fupra orizontem reperitur.

Quapropter imaginemur circulum. a. c.pro magno terræ, et. z.b. d. pro magno
vaporum, fupponatur etiam quod angulus. z. o. d. vel. z. a. b. qui funt inuicem fe-
rè æquales, fit angulus diftantiæ Solis à zenit, z. a. verò fit fpiffitudo vaporum,et.a.
b.radius tranfiens per vapores dictos. nunc
quæratur proportio,quæ eft inter. a. b. et. a.
z, qua inuenta, angulo. z . a . b. mediante ,
quæremus eandem mediante angulo.z.a.b.
maiore priori, vel ipfo minore, vnde cogno
fcemus differentiam duarum. a. b. quæ qui-
dem inæquales inuicem erunt , eo quod fup
ponatur.a. z. immutabilis, & hoc ita facie-
mus. Imaginabimur.o. b. quæ claudat trian
gulum.a.b.o.& quia. a. z.cognita eft quam
Alhazem docet inuenire, cognofcimus etiã
o.a.vt femidiametrum terræ, vnde. o. b. et.
o. a. duo latera trianguli.a.o.b.cognita erũt
fimul cum angulo.o.a.b. refiduo duorum re
ctorum,eo quod reliquus. z. a. b. datus eft.
Quare. a.b. cognita erit refpectu. o. a. et.o.
b. et.a. z.quæ eft eorum differentia . Nunc
fi idem faciemus cum alia.a. b. fub diuerfo
angulo , habebimus propofitum.

## IO. BAPT. BENED.
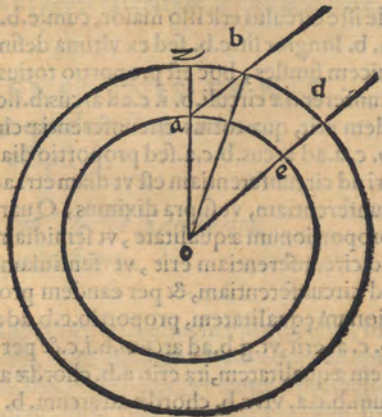
# NOTABILES ERRORES ORONTII
## & Tartaleæ.

### Cornelio Bitonto.

**P**ARVVS error non fuit, vt putabat Orontius, quod anguli triangulorum æquicrurium inuicem æqualium, baſibus oppoſiti, ijſdem baſibus propor tionales eſſent, cuius opinionis cauſa fuit quod nunquam viderit vel me minerit eius quod Ptolomeus ſcripſit lib. primo Almageſti, vbi de diſpro portionalitate chordarum arcuumǵ tractat, vel quod ſcribit Vitellio lib. primo pro poſitione. 35. ſeu lib. quarto, propoſitione. 21. quod idem eſt. Sed nec ego tibi pro ponam id quod ſcribit Nicolaus Tartalea diuiſioni. 28. quinti capitis quartæ partis ſuorum tractatuum, eo quod non exactè ſcientificè ſcripſerit, nec vniuerſaliter, quá uis talis propoſitio poſſit ſcientificè ſcribi, accipiendo. b. c. in eius figura, pro latere octagoni, vnde angulus. a.e.b. duplum foret angulo. b.e.c. collocato poſtea. b. c. in arcu. a.b. punctum. c. medium fuiſſet dicti arcus, et. e.c. diuideret. a. b. per æqualia, ex quinta primi, nec non ad rectos ex. 3. tertij, vnde ex. 18. primi, clare vidiſſemus non eſſe proportionem. a.b. ad. b. c. vt anguli ad angulum. Sed vniuerſaliori modo poſſumus hoc ſpeculari. Nam manifeſtè ſcimus, eandem eſſe proportionem circun ferentiæ ad diametrum in omnibus circulis tam maioribus, quam minoribus. Sint igitur duo anguli. a.e.b. et. c.e.b. cuiuſuis amplitudinis, quorum latera. e. a: e. b: et. e. c. ſint inuicem æqualia, protrahatur. b.a. et. b. c. Tunc dico maiorem proportio nem eſſe anguli. a.e.b. ad angulum. b. e. c. quam. a. b. ad. c.b. ducatur enim. b.g. ita ǫ faciat angulum. g. b. c. æqualem angulo. e.b.a. protracta poſtea. c.g. quæ idem faciat in puncto. c. vnde. g.b. et. g.c. æquales inuicem erunt ex. 6. primi, & quia angulus. a. æqualis eſt angulo. e.b.a. ex quinta eiuſdem, ideo ex. 32. dicti, et. 4. ſexti, horum duorum triangulorum latera, erunt inuicem proportionalia. Conſtituto deinde. g. centro, & ſecundum ſemidiametrum. g.b. vel. g.c. quod idem eſt, deſcripto circu- lo. b. i. c. necnon circulo. b. c. a. circa centrum. e. ope ſemidiametri. e. b. et. e. a. vn de iſte circulus erit illo maior, cum. e.b. maior ſit. g. b. ex. 14. quinti. cum ex. 14. tertij a. b. longior ſit. c.b. ſed ex vltima definitione tertij, arcus. b. i. c. et. b.c. a. erunt in- uicem ſimiles, hoc eſt proportio totius cir- cunferentiæ circuli. b. i. c. ad arcus. b.i.c. ea- dem erit, quæ totius circunferentiæ circuli b. c. a. ad arcus. b.c.a. ſed proportio diame- tri ad circunferentiam eſt vt diametri ad cir cunferentiam, vt ſupra diximus ; Quare ex proportionum æqualitate, vt ſemidiametri ad circunferentiam erit, vt ſemidiametri ad circunferentiam, & per eandem propor tionum equalitatem, proportio. e.b. ad arcũ b. c. a. erit, vt. g.b. ad arcum. b.i.c. & per ean dem æqualitatem, ita erit. a.b. chordæ ad ar cum. b.c.a. vt. c.b. chordæ ad arcum. b. i. c. & permutando, ita erit chordæ. a.b. ad chor dam. c.b. vt arcus. b.c.a. ad arcum. b.i. c. ſed arcus. b.i.c. maior eſt arcu. b.d. c. ex commu

ni
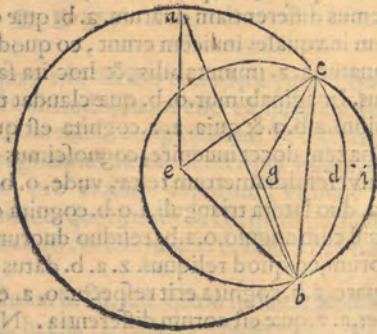
ni ſcientia. Quare maior proportio erit acus.b.c.a.ad arcum.b.d.c.quam ad arcum
b.i.c. ex. 8. quinti. Vnde ex vltima ſexti et.12.quinti,proportio anguli.a. e.b. ad an-
gulum.c. e.b.maior erit quam chordæ, ſiue baſis.a. b.ad chordam ſiue baſim.c. b.

## DE CAVSA SVSPENSIONIS NVBIVM
### in aere contra Antonium Bergam.

### Clariſſimo Franciſco Venerio.

E G o enim non tantum miror ea quæ mihi ſcripſiſti de opinione Ortenſij
quantum quod Antonius Berga putat nubes à Sole ſupenſas teneri, id pla
nè falſum eſt, vera cauſa huiuſinodi effectus, alia nulla eſt, niſi earundem
raritas hoc eſt, cum rariores ſint ipſo aere ſubiecto, propterea ſupra ipſū
natant & ſtant ſub eo qui rarior ipſis eſt, eo quod corpora rariora poſita in medio
non tam raro, aſcendunt, & denſiora in medio minus denſo deſcendunt. Nam ſi
Sol ipſas nubes ſuſpenſas in aere teneret, hoc interdiu tantummodo fieret, ſed no
ctu, cur non deſcendunt vſque ad terram, & in eodem loco ſemper manent ? Scien-
dum igitur eſt nubes aſcendere in altum quouſque inueniant aerem eiuſdem ra-
ritatis cuius ipſæ ſunt. Raritas enim & denſitas non ſunt res viſibiles niſi per acci-
dens, quemadmodum etiam leuitas, & grauitas, opacitas verò & diaphaneitas ma
gis compræhendūtur, opacitas enim ex reflexione radiorum luminoſorum, diapha
neitas verò compræhenditur ex penetratione ipſorum radiorum,opacitas autem nu
bis non eſt denſitas, cum valde diuerſa ſit denſitas ab opacitate, ſicut raritas ab dia-
phaneitate, vt aliàs dixi. Et quando dicit, quod Sol calefaciendo aerem ipſam nu
bem ambientem, rarefaciat eum magis quam ipſam nubem reſpondeo, hoc verum
non eſſe, propterea quod radius Solis non multum calefacit ea corpora,quæ ipſi per
mittunt liberum tranſitum. vnde corpora quanto magis diaphana ſunt tanto minus
ab ipſo radio luminoſo calefiunt,ſed ea quæ magis opaca ſunt,magis etiam calefiunt
& per conſequens magis rarefiunt, cum calidi ſit per ſe rarefacere,& non attrahere,
vt ipſe & ferè omnes alij putant.

## DE RATIONE EXTENSIONIS FVNIS
### cuiuſdam libramenti, & de quadam ſimboleita-
### te circuli cum ellipſi.

### Angelo Ferrario Sereniſſimi Ducis Sabaudiæ
### Agrimenſori expertiſſimo.

I B I in mentem veniet, quod cum ſuperioribus diebus in villa lucenti, in
qua degebat Sereniſſimus Dux noſter, dum viridarium ad æquilibrium
reducebas,eſſemus, à te quæſiui an ſcires vnde fieret, vt ſtante libramen-
to ad angulos rectos ſupra ſuum pedem, funis quæ extrema eiuſdem li-
bramenti cum pede in formam trianguli æquicruris coniungit, magis diſtentus exi-
ſteret,quam cum dictum libramentum cum pede obliquum remanet, ita vt huiuſ-

                                                    Z z            modi

modi funis cum libramento triangulum scalenum constitueret.

Exempli gratia, ponamus lineam.d.b.c.esse libramentum.et.b. e. u. eius pedem, funem autem, qui aliquando cum libramento facit triangulum isocellum,& aliquan do scalenum,esse.d.e.c.esto etiam quod in figura.A. dictus triangulus.d. e. c. sit iso cellus, & in figura. B. scalenus. Tunc quæsiui à te an scires rationem, quare funis.d.e.c.in figura. A. esset distensus, & in figura. B. laxus quemadmodum vide bamus. cum mihi responderis, nescio quid, quod nunc memoria nõ teneo, sed quia pollicitus sum me tibi eam afferre, propterea nunc ad te mitto. Scias ergo huius modi rationem nihil aliud esse nisi quod in figura. A. duæ lineæ. c. e. et. d. e. simul è directo iunctæ longiores sint illis, quę reperiuntur in figura. B. sed quia funis tam in figura. B. quam in figura. A. vnus,& idem est, ideo in figura. B.laxatus est,& non in tensus, ut in figura. A. Sed vt huiusmodi veritatis certam notitiam habeas, infrascri ptum circulum mente concipe. f. e.i. cuius semidiameter,æqualis sit. b. e. & diame ter sit. f. i. in quo imaginare esse tuum

libramentum.d.b.c.& figuras.A.et.B. & probabo lineas.d.e. c. figurę. A.lon giores esse lineis.d.e.c. figuræ. B.

Imaginemur igitur lineam. b. e. esse dimidium minoris axis alicui⁹ ellipsis cuius quidem figuræ ponamus. d.et.c. centra ipsius circunscriptionis esse, cu ius circunferétia,nullidubium est,quin extra propositum circulum transitura, & in vno tantummodo puncto ipsum circulum tactura sit, qui existat. è. figuræ. A. separatum tamen à puncto e. figuræ. B. Tunc si protracta fue rit linea.d. e. figuræ. B. vsque ad gi rum ellipticum in puncto. g. à quo ad punctum.c. ducta etiam sit linea g. c.tunc manifestũ erit duas lineas d.e.et. e.c.figuræ. A. simul iunctas, æquales esse duabus.d.g.et. g. c. si mul positis, vt etiam ex. 52.tertij Pergei facilè videre est,sed ex. 21.



primi Euclid.iam certò scimus.d.g.c. longiores esse.d.e.c.figuræ. B.ergo.d.e.c.figu ræ. A.longiores sunt.d.e.c.figuræ.B. quod est propositum.

Quod etiam mihi nunc circa hoc succurrit,tibi libenter significo, hoc est, quod sicut in ellipsi duæ lineæ.d.e.e.c.figuræ. A. simul iunctæ,sunt semper æquales duabus lineis.d.g.g.c.in longitudine, ita in circulo duæ. d.e.e. c.figuræ. A. æquales sunt in potentia duabus.d.e.e.c.figurę. B.

Manifestum enim primum est ex penultima primi in figura. A. quadratum. e. c. æquale esse duobus quadratis scilicet.e.b.et.b.c.& quadratum. e. d.æquale duobus. e.b. et.b.d.Quare quadrata.e. c.et.e.d.æqualia sunt quadratis.e. b. figuræ. A.et. e. b. figurę. B.et.b.c.et.b. d. hoc est duplo quadrati.e.a.(ducta cum fuerit.e.a.perpen dicularis ad.c.b.d.a. ) duplo quadrati.a.b. ex penultima primi, & duplo quadrati.b. c. Sed quadrata.d.e.et.e.c.figurę.B.æqualia sunt duplo quadrati.a.e.& quadrato a.d.

&
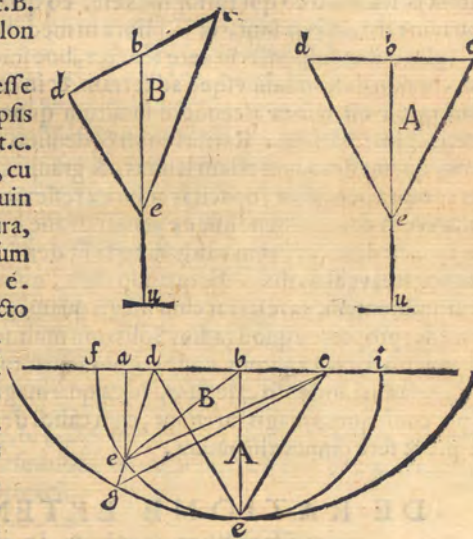
& quadrato.a.c. ex eadé. Nunc videndum eſt vtrũ duplũ quadrati.a.e.cũ duplo qua
drati.b.a. cũ duplo quadrati.b.c. ſit æquale duplo quadrati.a. e. cũ quadrato.a. d. &
cum quadrato. a. c. Sed quia tam ex vna parte quàm ex alia habemus duplum qua-
drati.a.e. Videndum igitur erit vtrum duplum quadrati.a.b.ſimul cum duplo quæ-
drati.b. c. æquale ſit quadrato.a.c.cum quadrato.a. d.ſed hoc manifeſtum eſt.ex.10.
ſecundi Euclidis, dato quod punctũ. a. ſit inter.f.et.d. ſed ſi fuerit inter.d. et. b. hoc
manifeſtum erit ex. 9.ſecundi dicti, nihilominus accipe hunc alium modum.

Sit hic ſubſcriptum quadratum. D. ex. a. c.in ſeipſa producta, cuius diameter ſit
a.n. protrahanturq; parallelę.d.h: b. K:l.m.o.et.r.q.s. eiq; addatur.c.p.ad. a.c.æqua-
lis tamen.d.a. ſitq; protracta.p.u. vſque ad.m.o.u. vndé habebimus. a. n. pro totali
quadrato,et.p.s.pro partiali,& æquali quadrato lineæ.a. d. Videndum nunc eſt, vtrũ
hęc duo quadrata æqualia ſint duobus quadratis lineæ.a. b.& duobus lineæ.b. c. Nã
duo quadrata lineæ. b. c.ſint. K. o.et. h.l.videndum nunc eſt utrum reſiduum ęquale
ſit duobus quadratis lineę. a.b. quorum vnum ſit.m.b.alterum verò.l.p. quod ſupe-
rat.l. c.et.s.p.figuræ.D. per ſupplementum.o.t.cui æquale eſt parallelogrammum.h.
m.figuræ. D. ſed ſi punctus.a. poſitus fuerit inter.d. et.b.conſtituto quadrato.d.u.cũ
omnibus parallelis, vt in figura. C. videre licet, in qua figura videbimus quadrata. r.
n.et.d.r.ęquari duplo quadratorum.l.n.et.r.l.nam in quadrato.r.n. ipſa duo quadra-
ta.l.n.et.r.l.capiuntur, reliquum eſt igitur vt videamus an duo ſupplementa.l.t.et.l.
s.cum quadrato.d.r.ſint æqualia dictis quadratis.l.n.et.r.l.ſed quadratum.d.l. æqua-

tur quadrato.l.n. videndum igitur eſt,
an duo ſupplementa.l.t.et.l.s.cum qua
drato.d.r.ſint æqualia duobus quadra
tis.d.l.et.r.l.ſed quadratum.d.l.æqua-
tur quadrato.d. r.& ſupplemento. l. t.
mediante.q.l.& ſupplemento.r.b.ſup-
plementum verò.l.s.ſuperat ſupplemẽ
tum.r.b.per quantitatem æqualé qua-
drato.r.l.quare duo ſupplementa. l. t.
et.l.s.cum quadrato.d.r.æquantur qua



drato.d.l. cũ quadrato.l. r. verum igitur eſt duas.d.e. e. c. figuræ. A. æquales eſſe in
potentia duabus d.e.e.c.figurę. D.quæ quidem affectio circuli,à nemine fuit adhuc
(quod ſciam) detecta.

Zz 2 DE

364          IO. BAPT. BENED.
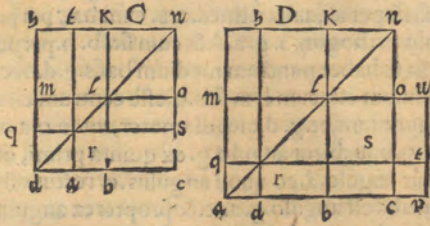
## DE AVGMENTO PONDERIS CORPORIS
ad stateram appensi, & quadam alia demonstratione,
& quibusdam erroribus Tartaleæ.

### Mutio Groto.

I ea quæ à me audiuisti non credis, considera quæso libram seu stateram o.a.cuius centrum non longitudinis sed ponderum sit.i.quę statera, vt ori zontaliter consistat, oportebit pondus extremitatis. o. ita se habere ad pondus extremitatis.a.ut.a.i.se habet ad. o. i. quod te scire puto, ima ginemur nunc duas lineas.a.e.et. o.n.parallelas infinitasq; & à puncto.n. immobili, & fixo extra stateram, transeat per.i. linea.n.i.e. Cogitemus etiam punctum.e. inter sectionis ipsius.n. i.e.cum.a.e.progredi vniformiter continuòq; ab.a.per lineam.a.e. vnde punctum.i.intersectionis ipsius.n.i.e.cum.a.i.o. semper vicinius fiet puncto. o. nec unquam cum illo vnum erit, quamuis moueatur tempore infinito. Nunc autem dico, quod cum stateram.o.i.a.oporteat semper orizontalem esse virtute ponderis, o.oportebit pundus.o.in infinitum etiam augeri, quotiescunq; pondus.a. nunquam diminui voluerimus vel econtra hoc in infinitum diminui, si illud nunquam augeri voluerimus.

Sed re vera non putabam te indigere aliqua demonstratione, quod linea.b.h.di uisa sit per æqualia à linea.c. a. cum hæc perpendicularis sit ab.a.ad basim.g.d.in triã gulo orthogonio. g. a.d.& cum sit. b. h.perpendicularis ad.a.o.ex supposito quæ. a. o. in se habet punctum medium basis.g.d.nec nõ illud anguli recti.a. quod per se cla rissimum est, cum iam scis.o.esse centrum circuli circundantis triangulum.g.a. d.or-thogonium, et.g. d.eius diameter, vnde.o.a.æquabitur ipsi.o.g.quapropter angulus o.a.g.æquabitur angulo.g. ex quinta primi, deinde ex.32.eiusdem,angulus. h.æqua bitur angulo.d.eo quod angulus.e. rectus est, quemadmodum et. a. sed angulus. d. æqualis est angulo.g.a. c.& propterea angulus.h.erit etiam æqualis angulo. h . a. u.

vnde. h. u. æqualis erit ipsi . u. a.ex.6.primi,cum postea angulus. o.a.d.æqualis sit angulo.d.ex quin ta primi erit angulus. a.b.e. æqua- lis angulo.g.ex.32.dicta, eo quod e. rectus est , & ex eadem æqualis erit angulo. d. a. c . vnde.u.b.'erit æqualis ipsi.u.a.ex.6. dicti,& ideo æqualis erit ipsi. u. h. Reliqua ve- rò illius propositionis credo ex te omnia posse ĩtelligere,excepto,q, vt tibi significaui si à pũcto.i.com- muni ipsi.a.e.u. & circunferentiæ, ducta fuerit.i.x.ad pũctum.x. com mune vni parallelæ à pũcto.g. ipsi h. b . & circunferentiæ , quod di- cta.i.x. ad rectos erit ipsi.a.b.d.eo quod cum angulus.a.g. x. æqualis sit
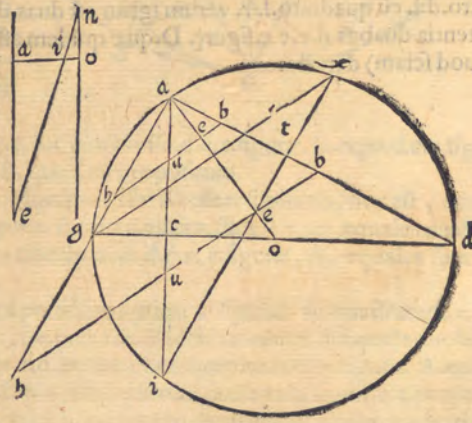
fit angulo.a.h.b. propter æquidiftantiam dictam, æqualis etiam erit angulo.d.& ar-
cus.a.x.æqualis arcui.a.g.vnde angulus.a.i. x. æqualis erit.d.fed angulus.i.a.d.com-
munis eft triangulis.c.a.d.et.i.a.t.quare angulus.a.t.i.rectus erit,vt.c.hoc eft.i.x.per
pendicularis erit ipfi.a.d.

Sed vbi tibi fcripfi circa finem illius epiftolæ, Tartaleam erraffe in quinta propo-
fitione primi lib.fuæ nouæ fcientiæ,non fine ratione illud fcripfi . Nam, inquit ipfe,
nullum corpus æquè graue poteft in aliquo temporis fpatio moueri motu naturali,
violentoq; fimul miftis. Vbi decipitur,eo quod non animaduertit incrementum ve
locitatis vnius motus, fimul effe cum decremento velocitatis alterius , eodemq; tem
pore, vt manifeftè patet in itinere corporis, ab ipfo pro exemplo affumpto , hoc eft
quod velocitas motus in fpatio.c.d. crefcit vt naturalis , & decrefcit vt violenta. nã
crefcit orizontem verfus & decrefcit in remotione à linea. a. b. fed fi à puncto. c. ad
punctum.d.motus effet purè violentus, vt putat Tartalea , corpus illud minimè de-
fcenderet, eo quod uirtus mouens,in.a. pofita,nullo pacto poteft talem effectum ef-
ficere, vnde ab ipfa natura prouenit defcenfio illius corporis propter grauitatê, quã
dictum corpus habet in tali medio, aeris fcilicet ,& non ex violentia aliqua . Sed fi
dixiffet ipfe , illum motum effe purum naturalem, hoc effet falfum , eo quod purus
naturalis motus alicuius corporis non impediti , extra locum fuum,fit per lineam re
ctam , & non per curuam, vt videre eft inter.c.et.d.

In vltima propofitione deinde eiufdem lib.quæ.6.eft decipitur fimiliter, & hæc
deceptio oritur ab ignoratione quintæ,& à putando motum naturalem non effe cau
fam ipfius defcenfus per fpatium.c.d. Sed quia tibi fignificaui expeditiorem viam
reperiri ad cognofcendã proportionem inter.a.h.et.a. e. in vltima propofitione fe-
cundi lib.ipfius Tartaleæ, ipfam nunc tibi fcribo. Nã iam fcis angulum.h. l. i. diui-
fum effe per æqualia ab.P. l.& quod.a.h.et.h.p.equales inuicem funt ex.6.primi Eu-
cli.vnde.p.i.et.a.h.æquales erunt inuicem fimiliter, fed ex.3.fexti ita eft ipfius . a. l.
ad.l.i.vt ipfius.a.p.ad.p.i.& coniunctim ita erit.a.l.i.ad.l.i.vt.a.i.ad.p.i.fed.a.l.cogni
ta eft ex eius quadrato,et.l.i.etiam, cum æqualis fit ipfi. a.i.vnde ex regula de tribus
notam habebimus.p.i.refpectu.a.i.& ita refpectu.a. e. fi hypothefes ipfius Tartaleæ
veræ funt.

## *Alia demonftratio impoffibilitatis diuidendi per æqualia proportionem fuperparticularem in difcretis .*

## AD EVNDEM.

Vod à me poftulas, hoc eft fcientiam impoffibilitatis diuidendi per æqualia
proportionem fuperparticularem in numeris fatis à Campano in. 8 . octaui
potes habere, Iacobus Faber Stapulenfis etiam idem tractat in libello fuç muficæ
demonftratæ. Sed fi etiam alia via idem defideras , quamuis longiori, nihilomi-
nus vniuerfaliori, confidera duos numeros.g. et.h.inuicem relatos fecundum pro-
portionem fuperparticularem, quam volueris . Tunc dico impoffibile effe, vt per
æqualia diuidatur, quod fi dixeris poffibile effe, fit per te. K. medius numerus
propor-

proportionalis inter.g.et.h.quare.g.et.h.non erunt minimi in ea proportione, quia
vnitas diuisibilis esset si.g.h.minimi fuissent, quod non conceditur, sint igitur mini-
mi in dicta proportione.a.et.b.quorum differentia erit vnitas, vt scis, sitq;.c.quadra
tum ipsius.g.et.d.quadratum ipsius.K.tunc clarum erit ex.11.octaui, quod propor-
tio ipsius.c.ad.d.eadem erit quæ.g.ad.h.hoc est vt ipsius.a.ad.b. vnde si vnus termi-
norum.a.vel.b.esset quadratus, reliquus etiam quadratus esset ex.22.octaui, & ex
16.eiusdem,inter.a.et.b.reperiretur aliquis medius numerus proportionalis, quod
fieri non potest ex hypothesi, cum inter.a.et.b. nullus sit numerus, quia differunt in
ter se per vnitatem tantummodo. Nunc autem cum nullus numerorum.a.vel.b.qua
dratus sit, ponatur quod.f. quadratus sit ipsius.b.et.e.sit productum ipsius. a.in.b.vn
de ex.18.septimi,proportio ipsius.e. ad.f.erit vt.ipsius.a.ad.b.hoc est vt ipsius. c.ad
d. quapropter.e. erit quadratus ex.22.octaui, cuius latus tetragonicum esset mediũ
proportionale inter.a.et.b.ex.20.septimi, quod est impossibile, vt iam dixi, cum.a.
et.b.sint inui cem consequentes, vnus post alium immediatè.

Superius enim dixi hunc modum esse vniuersalem,
hoc est quod hac methodo possumus in cognitionem
venire, quod non solum in duas æquales partes diui-
di non possit, sed nec in tres, nec quatuor nec quot vo
lueris. Primum enim quod non in tres diuidatur à te
ipso cognosces ope cuborũ vice quadratorũ,opevero
censuũ cẽsuũ, vel qui cognouerit eam proportionẽ
esse indiuisibilem per æqualia,illicò etiam cognoscet
indiuisibilem esse per quatuor partes, ope verò pri-
morum relatorum,cognoscetnon esse diuisibilem per
quinq; partes, & sic de cęteris, sed mediantibus ijs
quas scripsi de istis dignitatibus in libro Theorematũ
arithmeticorum.

Id autem quod Illustrissimus Daniel Barbarus scri
bit in quinta parte suæ perspectiuæ, si supra aliquo im
mobili, atque magno pariete facere volueris, te opor
tebit hoc ex reflexione radij solaris à speculo plano
perficere.
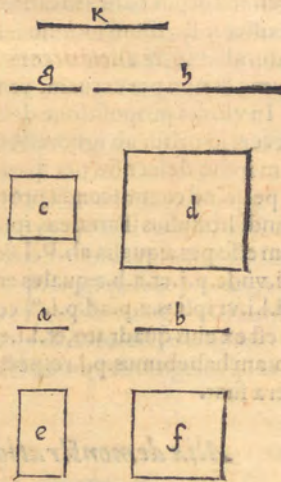
## DE INVENTIONE DIAMETRI
### circuli circunscribentis triangulum.

### *Franchino Triuultio.*

V o d mihi nunc proponis est triangulum, cuius basis cum angulo sibi op
posito dantur. Vellesq; diametrum circuli apti eum triangulum circun-
scribere inuenire in discreto.

Sit igitur triangulum.a.b.g. cuius basis.b.g.simul cum angulo. a. ei op-
posito data sit in numeris. Imaginetur ergo circulus circunscribens ipsum triangu-
lum.b.p.g.q.cuius diameter sit.q.p.perpendicularis eius basi. b. g. vnde. b. g. diuisæ
erit per æqualia ab ipso diametro in puncto.m. per tertiam tertij, protrahatur etiam
                                                                                    e.g.

e.g. vnde angulus.g.e.q. æqualis erit angulo.b.a.g. portionis , cum duplus fit angulo
q.p.g. medietati anguli ipfius portionis ex.19. tertij, ita quod angulus.q.e.g. nobis
cognitus erit,& fimiliter arcus.g.q. & confequenter ar-
cus.p.g.refiduum medij circuli, & fic.m.g. eius finus re
ctus, & etiam chorda.p.g. vt dupla finus dimidij arcus.
p.g.& fic.p.m.eius finus verfus, vel vt tertium latus trian
guli orthogonij.p.g.m.vnde nobis cognita erit propor
tio ipfius.b.g.( quæ dupla eft ipfi.m.g.)ad.m.p. & quia
productum.p.m.in.m.q.æquale eft ei, quod fit ex.b.m.
in.m.g. ex.34.tertij,quapropter nobis cognita erit pars
q.m.quæ cum.p.m.complet totum diametrum.q.p. vn
de nobis cognita erit proportio ipfius.b.g.ad.q.p. qua
mediante cognofcemus diametrum fecundum partes il
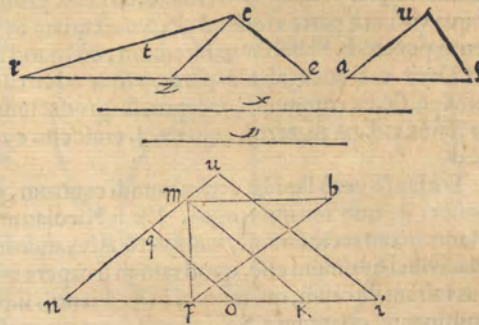las quibus propofita fuerit.b.g.
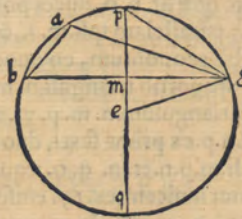
Hoc autem problema non in numeris fed in continuo ab Euclid. ponitur in.32.
tertij.

## De inuentione alterius trianguli conditionati.

## AD EVNDEM.

Q Votiefcunque etiam inuenire voluerimus triangulum aliquem, puta.n.q.o.
æqualem triangulo.t. (exempli gratia ) propofito,qui habeat angulum.n. æ-
qualem angulo.a.dato , latera vero continentia ipfum angulum.n.fint inuicem pro-
portionata vt.x.et.y.ita faciemus, accipiemus lineam.n.m. cuius volueris magnitu-
dinis, fupra quam conftituemus triangulum.m.n.p.æqualem triangulo.t.hac metho-
do, hoc eft prolungando latus.r.z.trianguli.t.quod fit.r.e. ita vt duplum fit ipfi.r.z.
ducendo poftea.c.e. habebimus ex.38. primi triangulum.t. effe dimidium totius
trianguli.r.c.e. defignabimus deinde ex.44.dicti fuperficiem.p.n.m.b.parallelo

grammam æqualemq; triangu
lo.r.c.e. habentem angulum.
n. æqualem angulo.a. ducatur
poftea.p.m.& habebimus triã
gulum.m.n.p.æqualem.t.cum
angulo.n.æquali angulo.a.pro
ducatur poftea.n.p. ita vt.n.K.
fe habeat.ad.n.m. quemadmo
dum.x.ad.y. quod erit facilli-
mum producendo.n.m.et.n.
K.indeterminatè fi oportuerit,
deinde eas ad æqualitatem fe-
cardo ipfis.x.et.y. efficiendo
exempli gratia quod.n.i.fit
æqualis ipfi.x.et.n.u. ipfi.y.du

cendo poftea.u.i. deinde à puncto.m. ducendo.m.K. æquidiftanter.u.i. ex.31.
primi.& fic habebimus ex.4. fexti proportionem.x.ad.y. effe inter.n.K.et.n.
m.inue-

n.K.
m.in

m. inuenies postea ex. 9. eiuſ-
dem lineam aliquam mediam
proportionalem inter.n. K. et.
n.p. quæ ſit. n. o. duces poſtea
o.q.parallelam ipſi.m. K.& ha
bebis propoſitum, eo quod cū
ſit proportio trianguli.n. m.K.
ad triangulum.n. m. p. vt.n.K.
ad.n.p.ex prima ſexti, duo triā
guli.m.p.n.et. n. q. o. æquales
erunt inuicem,ex.17. eiuſdem
& ex. 9. quinti , & proportio.
o.n.ad.n.q. erit, vt.x.ad.y. ex.
11.dicti, cum ex. 4. ſexti ſit vt.
n. k. ad.n. m.
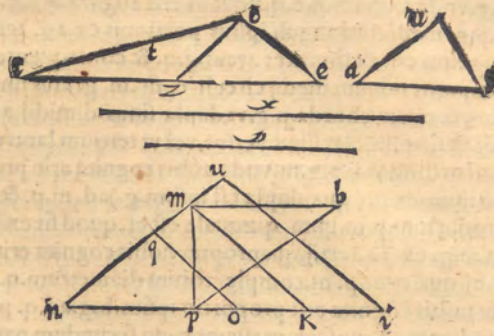
## De producto conditionato.

## AD EVNDEM.

PRoponis deinde mihi duas rectas lineas, vni quarum, vis vt aliam quandam di-
recte coniungam, ita quod productum huius aggregati in lineam adiunctam
æquale ſit quadrato alterius.

Vt exempli gratia ſi fuerint duæ lineæ.e. d. et. e.f.opor-
teretq; nos ad lineam.e. f.aliam lineam puta.f.c.vel.e. b.iū
gere, ita longam, vt productum totius compoſiti.e. c. vel.
f.b. in.f.c. vel. e. b.eſſet æquale quadrato ipſius. e.d.

Hoc enim nullius eſſet difficultatis, eo quod quotieſcū-
que.e.d.coniuncta erit cum. e.f. ad rectos, diuiſaq; per me
dium à puncto. a. à quo ducta.a.d. deinde ſecundum ſemi-
diametrum.a. d. deſignato circulo.b.d.c. & protracta. e. f.
à qua volueris parte vſque ad circunferentiam in pūcto . c.
ſeu in puncto.b. habebimus intentum, eò quod ſi produ-
cta fuerit. e. f. etiam ab alia parte, vſque ad circunferentiam,habebimus.b.e.æqua-
lem ipſi.f.c.ex communi conceptu, & productum. e. c . in . e . b. æqualem quadra-
to ipſius.e.d.ex.34.tertij, cum ex. 3. eiuſdem. e. d.medietas ſit chordæ arcus dupli
b. d.
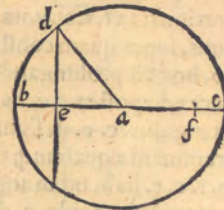
De lapſu verò lapidis verſus mundi centrum, dum ipſum attingere, ac præterire
poſſet, de quo me interrogas. Dico Nicolaum Tartaleam, nec non Franciſcum
Maurolicum recte ſenſiſſe, malè verò Alexandrum Piccolhomineum, & exemplum
Maurolici optimum eſſe, quod tamen ſi capere non potes, crede ſaltem authoritati
bus talium virorum, qui tantum in ijs ſcientijs ſuperant ipſum Alexandrum Piccol-
homineum, quantum à Sole cætera ſuperantur aſtra.

Lapis igitur ille tranſiret centrum, reddiretq;,cum diminutione tamen motus im
preſſi, eo fermè modo vt ſcribunt iudicioſiſſimi illi viri , donec poſt multas reddi-
tiones ſurſum,deorſumq; quieſceret circa centrum mundi. Lucidioris tamen intelli
gentiæ

gentiæ gratia cogita filum illum (exempli adducti ab illis doctissimis viris) cui pondus appensum est, æqualem esse axi orizontis, hoc est eius extremitatem immobilem esse in primo mobili, & in ipso zenit tui orizontis, tunc arcus motionis ipsius lapidis per tantum interuallum, quantum est diameter terræ, insensibiliter differret à linea recta, & cum lapis distans à centro mundi per semidiametrum terræ, iret rediretq́;, vt scis, ergo idem faceret si filum longius esset per dictum terræ semidiametrum, ita vt posset ipsum centrum attingere, nam differentia illa semidiametri terræ, ferè nulla est respectu semidiametri ipsius primi mobilis.

## AN PENTAGONVS AB ALBERTO DVRERO
descriptus æquiangulus sit.

### Conrado Neubart.

SI non credis Pentagonum ab Alberto Durero super datam lineam designatum, æquiangulum non esse. Fingamus hic subiectam figuram similem ei quæ à Durero ponitur, in qua primò, ducta sit linea. o. a. & habebimus angulum.a.o.b.graduum.60.talium qualium duo recti fuerint gra. 360. vel.30.talium qualium duo recti fuerint. 180.nam ex supposito, arcus. a. b. est sexta pars totius circunferentiæ, angulus vero.b.o.d. rectus est, eo quod.b.o.q.rectus etiam sit, quare angulus.d.o.a. residuus ex recto erit graduum.60.talium, ut rectus est.90.angulus verò.o.a.c. erit gra.15.eorundem.

Ducatur deinde perpendicularis.a.e.ad.o.d. quæ vt sinus anguli.a. o. e. erit partium.86602.talium qualium.a.o. erit.100000. quæ quidem.o.a. vt chorda arcus. a. o. est partium.51762.talium qualium.a.d. vel. a.c. semidiameter est.100000.

Nam sinus dimidij arcus.a.o. (existente.a.o.graduum. 30. ) est partiū. 25881. ex quo. a. e. erit partium. 44827.talium qualium. a. d. erit 100000.vnde angulus.a.d.o.cuius sinus est.a.e.erit graduum.26.min.38 qui quidem angulus, sumptus cum angulo.a.o.d.erit gra.86.min.38.Dempta denique hac summa ex duobus rectis gra.180.reliquum erit gra.93. min.22.idest angulus.o.a.d. cui additus cum fuerit angulus.o.a.c.gra. 15. talium, habebimus angulum.c.a.d. graduum.108.min.22.exuperantem verum angulum pentagoni per min. 22.vel sic, cum inuentus fuerit angulus.a.d.o.gra.26.min.38. si ex vno recto demptus fuerit, relinquetur angulus.d.a.e.gra.63.min. 22. qui quidem collectus cum fuerit cum angu-



lo.e.a.o.residuo ex recto dempto angulo.a.o.e.grad.60.qui.e. a. o. est grad. 30. & Aaa etiam
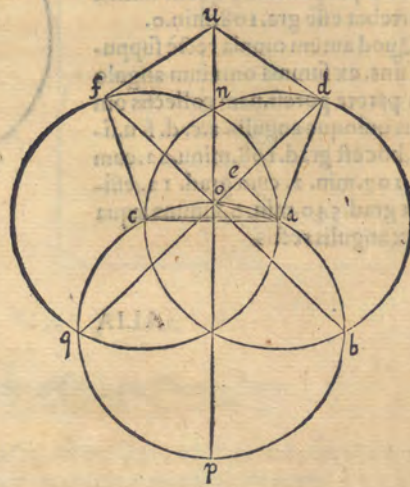
etiam collectus cum angulo.o.a.c.grad.15.hi tres anguli efficient angulum. d. a. c.
dictum grad.108.min.22.

## Examinatio anguli. u.

Ducatur.d.n. quam quidem.d.n.cognoscemus vt sinus anguli.d.o.n.gra.45.nam
angulus ei contrapositus.q. o.p. est dimidium recti, quare.d.n.erit partium. 70710.
talium qualium.d.o. fuerit.100000.sed.d.o.est partium.115270.qualium. a. d. est.
100000.nam.e.d.vt sinus anguli.e.a.d.gra.63.min. 22.est partium.89389. o. e. ve-
ro est partium.50000.talium qualium.a.o.est.100000.vt sinus anguli.e.a.o.gra.30.
sed vt.a.o. est partiū. 51762.hoc est vt.a.d.est.100000.ipsa.o.e.erit partiū.25881
quæ iuncta cum fuerit cum.e.d.efficiet.d.o.partium.115270.vt dictum est, quapro-
pter cum. d.n. sit partium.70710.talium qualium.d.o.fuerit.100000. ipsa. d. n. erit
partium.81507.talium qualiumi.d.o.erit.115270. idest qualium.d. a. vel. d. u. erit
100000.quæ quidem.d.n.est sinus anguli.d.u.n. graduum scilicet.54. 36. cuius du-
plum erit gra.109.mi.12.debebat tamen esse.108.m.0.

## Examinatio anguli. d.

Accipe angulum. a. d. o. gra.26.
min. 38. vt supra, cui applica angu-
lum.o.d.n.gra.45.min. 0. simul cum
angulo. u. d. n. residuo ex recto gra
duum.35.minu. 24. & conficies an-
gulum.a.d.u.grad. 107. minu. 2. &
habebis propositum, quem tamen
oportebat esse gra.108.min.0.

Quod autem omnia recte suppu-
tata sint, ex summa omnium angulo
rum patere potest.nam collectis om
nibus quinque angulis. a.c, d. f. u.si-
mul,hoc est grad.108.minu.22.cum
gra. 107.min. 2. cum grad. 12. effi-
cient grad.540.min. 0.summa æqua
lis sex angulis rectis.

**ALIA**
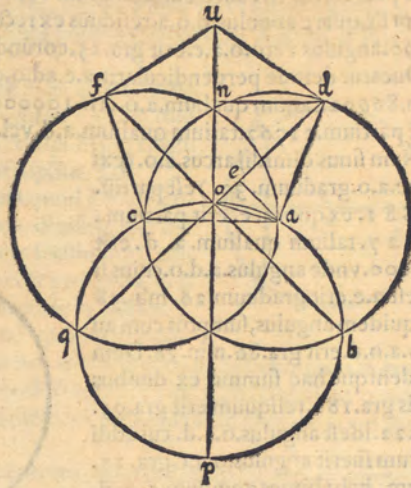
## ALIA DEMONSTRATIO NONÆ, ET DECIMÆ
secundi Euclidis.

### *Petro Catenæ.*

Qᴜᴀᴍᴠɪs nona ac decima secundi Euclid. aliter à Comandino & Maurolico demonstratæ fuerint, nihilominus mihi etiam visum est non nihil meo moræ in eas tibi scribere, vt sensibiliter quoque cognoscas illas veras esse.

Esto linea.a.b. pro nona propositione, diuisa per æqualia in. c. per inæqualia verò in.d. quadratum autem. a. d. sit.d.e. quadratum verò. d.b. sit.d. i. quadratum. a. c. sit. c. f. & quadratum.c. d. sit.c. K.clarum enim erit. K. h. æqualem existere ipsi.a.c. seccetur igitur. e. h. in. g. ita vt. h. g. equalis existat ipsi. K. h.vnde.g. e.æqualis erit c. d. perficiatur etiam quadratum.h. n. vnde in totali quadrato.a. h. habebis duplũ quadrati partis. c. d. nempe. c. K. et. f.g. & quadratum.a. u. cum gnomone.u.g.h.k. cui deficit quadratum æquale. d.i.quadrato, vt sint etiam duo quadrata partis.a.c.

In decima aũt propositione, quadratũ totalis lineæ.a.d.sit.d.e.& lineæ.b.d.sit.b.i.et c.d.sit. d.n. et.a.c.sit.c.f.et.f.e.sit.e.u.vnde.n.u.æquale erit quadrato.b.i.vnde in quadrato totali.a.h. videbis duo quadrata æqualia.f.c.et.g.k.partis.a.c.& quadratum.c. K.cum gnomone.n. f.e.g.cui addito quadrato.b.i.habebis duplum quadrati partis. c. d.
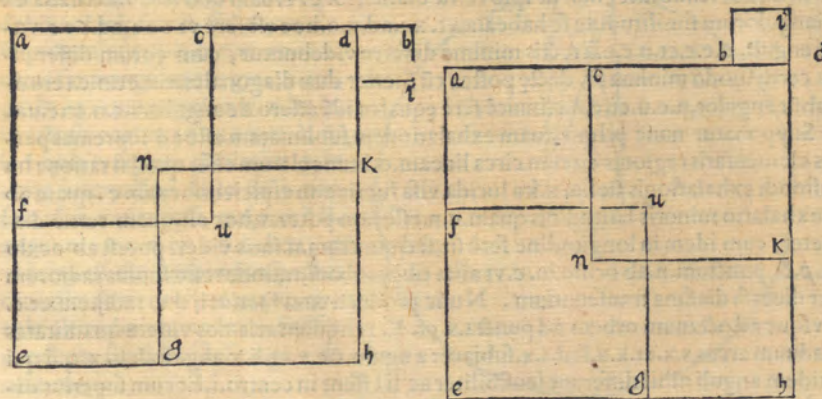


## DE STELLA CASSIOPEIÆ.

### *Annibali Raymundo Astrologo Peritißimo.*

Pᴏsᴛqᴠᴀᴍ tua doctissima scripta perlegi, consideraui, quod si à multitudine exhalationum in regione elementari acciderit anno. 1572. & 1573. vt totos sex menses ab omnibus per vniuersum terrarum orbem visa fuerit stella illa, quæ est in angulo septentrionali quadrilateri Cassio

Aaa 2 peiæ

I O.  B A P T.  B E N E D.

peiæ tam lucida, vt ipſo lucifero videretur rutilantior atq; cæterarum ( abſque vlla
aſpectus diuerſitate ) magis ſcintillans . Quî fieri poterat , vt ſtellæ quæ ab illa pa-
rum diſtant , alioqui multo maiores, non etiam illa clariores apparuerint ? ſed ſi ali-
quis diceret eam exhalationem non ita fortaſſe dilatari , vt inter nos , & aliam ali-
quam ſtellam interponeretur . Tunc ego reſponderem neceſſariò ſequi debere ta-
lem exhalationem , tantam latitudinem occupare , quod aliquibus populis aliam
aliquâ ſtellâ circunuicinâhac ipſa de qua mentionem fecimus redderet lucidiorem.
Sed cum hoc perſpectum fuerit nulli , ſequebatur lucem illam ab ipſis exhalatio-
nibus elementaribus haud poſſe oriri : quod nobis ſcintillatio illa maxima perma-
gno fuit inditio, ſi phas eſt credere , nâ quo magis aliquod cœleſte corpus ſcintillat,
eo longius à nobis diſtare .

    Verum quoniam efflagitaſti à me vt aliquid circa huiuſce rei ſpeculationem tibi
ſcribam , idcirco tibi morem gerere volens paucis ſubiungam .

    Conſidera primo hanc ſubſcriptam primam figuram, in qua. c. a. e . ſignatur pro
Globo terreſtri cuius. i. centrum ſit et. u. o. n. pro conuexo ignis, ſed. K. x. s. pro orbe
octauo. x. autem pro ſtella iam ſuperius dicta, quæ ſemper fuit, eſt, & erit, quamuis
cæteris tribus nunc obſcurior ſit . Accipiantur deinde duo loca in ſuperficie terræ,
quę ſint. c. et. e. diametraliter inuicem oppoſita, ita quod circa eorum orizontes poſ-
ſibile ſit ſtellam. x. videre, radijs ipſius ſtellæ mediantibus. x. n. e. et. x. u. c. quorũ par-
tes. n. c. et. u. e. ita breues ſint, reſpectu eorum totorũ, vt vix ſexcenteſima pars ſit vna
quæq; illarum , nec non. c. e. ita breuis reſpectu ſemidiametri octauæ ſphæræ , quod
vix ſit vna ex partibus decemmillibus, vt ſcis , ſequitur quod recta terminata ab. u.
et. n. minor ſenſibiliter non ſit ipſo terræ diametro. c . e. cum duo hæc interualla ex
triangulorum ſimilitudine ſe habeant vt. x. i. ad. x. o. hoc eſt ferè vt. 602. ad. 601. vn-
de anguli. n. e. c. et. u. c. e. à rectis minime differre videbuntur, cum eorum differen-
tia certo modo minima ſit. ductę poſtea cũ fuerint duæ diagonales. e. u. et. n. c. termi-
nabũt angulos. n. e. u. et. e. n. c. inuicē ferè equales, idē aſſero de angulis. u. c. n. et. e. u. c.

    Supponatur nunc primò tuam exhalationem ſublimatam eſſe ad ſupremas par-
tes elementaris regionis circum circa lineam. o. i: tunc clarum eſſet quod ſi ratione hu
iuſmodi exhalationis ſtella. x. ita lucida viſa fuerit tam aſpicientibus ab. e . quam ab
c. exhalatio minoris latitudinis quam. u. n. eſſe non poterat, hoc eſt, quam terræ dia-
meter, cum idem in longitudine ferè ſit, ſed punctum. u. ſatis videri poteſt ab oculo
in. e. & punctum. n. ab oculo in. c. vt alias tibi probaui, ratione refractionis radiorum
per diuerſa diafana tranſeuntium . Nunc producti cum fuerint ij duo radij. e. u. et. c.
n. vſque ad octauum orbem ad puncta. s. et. K. reliquum erit nos videre quantitates
graduum arcus. s. x. et. k. x. ſed. s. x. ſubiacet angulo. s. e. x. et. k. x. angulo. k. c. x. qui qui
quidem anguli nihil differunt ſenſibiliter ac ſi eſſent in centro. i. Et cum ſuperius di-
xerimus angulos. s. e. x. et. k. c. x. ſenſibiliter minime differre ab angulis. c. n. e. et. e.
u. c. ſi cognouerimus quantitatem iſtorum , cognita etiam nobis erit quantitas
ill orum .

    Cum igitur ſemidiameter elementaris regionis maior ſit ſemidiametro terræ, vt
33. ad vnum, & cogitata. c. n. vt dicta ſemidiameter, quia ſenſibiliter ab ea minime
differt, nunc ſi ſupponatur dicta. n. c. vt baſis triãguli orthogonij eſſe partiũ. 100000
& dixerimus ſi. c. n. vt partium. 33. præbet nobis. c. e. duarum partium , quid nobis
pręſtabit eadem. c. n. vt partium. 100000. vnde prouaniet nobis. c. e. vt partiũ. 6060.
cuius angulus. c. n. e. erit graduum. 3. & min. 29. ita etiam erit angulus. k. c. x. cuius ar-
                                                                                    cus

cus.s.x. eorundem graduum erit,
& minutorum. idem dico de arcu.
x.k. Sed circa dictam stellam om-
nes aliæ non distant huiusmodi in
teruallo.

Nihilominus nec tu nec alij pe
ripatetici qui hanc sequuti sunt
opinionem exhalationum, ad ser
uandam nullitatem diuersitatis
aspectus, affirmant posse tam lon
ge à terra ascedere exhalationes,
imo nec attingere supremas tertię
regionis aeris partes, ita ut non in-
grediatur suum igneũ orbem, qui
quidem orbis secundum illorum
opinionem incipit non valde lon-
ge à superficie terræ, vt iã in mea
consideratione contra Antonium
Bergam probaui, sed demus, φ
dictæ exhalationes ascenderint
per decem semidiametros terræ,
discurrendo postea sic, cum.c.n.ut
decem, nobis dat.c. e. vt duo, qd
dabit nobis ipsa.c.n. vt. 100000.
& proueniet nobis.c. e. vt 20000.
cuius sinus angulus erit gra. 11.mi.
32.& ita erunt anguli.s. e. x. et. k.
c. x.& sic eorum arcus.s.x. et. k. x.
sed quis vnquam dubitabit φ in
tanto interuallo à dicta stella non
sint aliæ multæ ipsa maiores? li-
neas vero.e.o.r. et. c.o.t. duxi, vt
videres effectum maioris aspectus
diuersitatis ab oculis.e.et.c.in cir-
culo altitudinis quando. o. fuisset
punctum illud lucidissimum, &
non. x.

374     I O. B A P T. B E N E D.
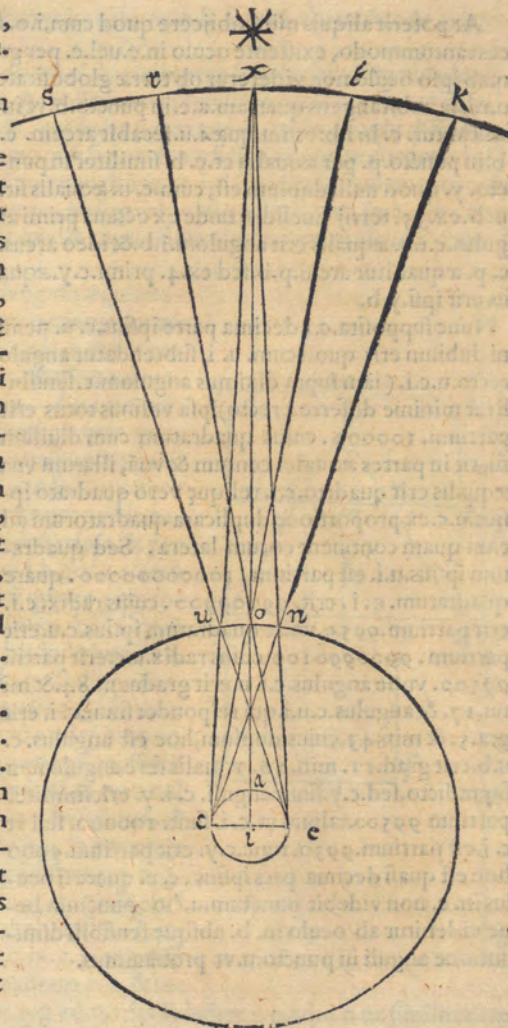
At poterit aliquis mihi obijcere quod cum.i.o. fuisset longior.i.e. per decem vi‑
ces tantummodo, existente oculo in.e.uel.c. per gradus.90.ab.a.tunc punctus.u.vel
n.ab ipso oculo non videretur ob terræ globositatem. Imaginemur igitur à puncto
u.recta. u.b.tangens quartam.a.e.in puncto.b.vt in secunda figura videre est, in qua
ducantur. c. b: i.b: et.i.u.quæ.i.u.secabit arcum. c.
b.in puncto.p. per æqualia et.c. b. similiter in pun‑
cto. y. quod nulli dubium est, cum.c. u. æqualis sit.
u. b.ex.35. tertij Euclidis, unde ex octaua primi an‑
gulus.c.i.u. æqualis erit angulo.u.i.b.& ideo arcus.
c. p. æquabitur arcui.p.b.sed ex.4. primi.c.y.æqua‑
lis erit ipsi.y.b.

Nunc supposita.c.i.decima parte ipsius.c. u. nemi‑
ni dubium erit quod cum. u. i. subtendatur angulo
recto.u.c.i. ( iam supra diximus angulum.c.sensibi‑
liter minime differre à recto)ipsa vt sinus totus erit
partium. 100000. cuius quadratum cum diuisum
fuerit in partes æquales centum & vnã, illarum vna
æqualis erit quadrto.c.i. reliquę vero quadrato ip‑
sius.u.c.ex proportione duplicata quadratorum ad
eam quam continent eorum latera. Sed quadra‑
tum ipsius.u.i. est partium . 10000000000. quare
quadratum. c. i. erit. 99009900. cuius radix. c.i.
erit partium.9950.vnde quadratum ipsius.c.u.erit
partium . 9990990100.cuius radix.u.c.erit partiũ
99500. vnde angulus.c.i.u.erit graduum.84.& mi‑
nu.17. & angulus.c.u.i.qui respondet sinui.c. i. erit
gra.5. & min.43.cuius duplum,hoc est angulus. c.
u.b.erit grad.11. min. 26. æqualis ferè angulo iam
supradicto.sed.c.y.sinus anguli. c. i. y. erit similiter
partium.99500.talium vt.c. i. sunt. 100000. sed vt
c. i.est partium.9950. tunc.c.y. erit partium.9900
hoc est quasi decima pars ipsius . c. u. quare si ocu‑
lus in. e. non videbit punctum.u. hoc punctum be‑
ne videbitur ab oculo in. b. absque sensibili dimi‑
nutione anguli in puncto.u.vt probauimus.
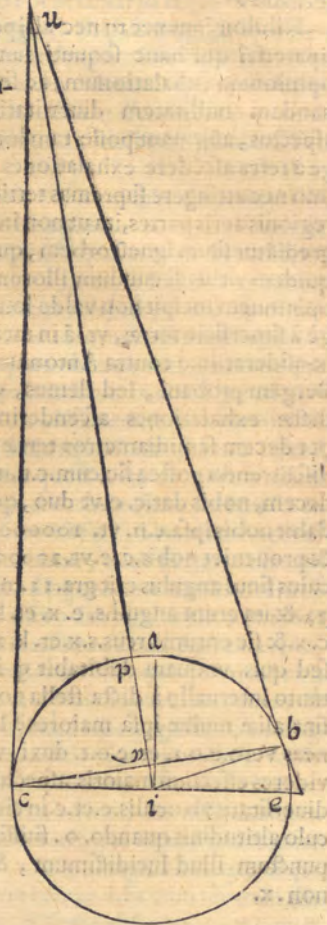
# DE MAGNITVDINIBVS FIGVRARVM
isoperimetrarum.

### Domino Ioanni Maria Agatio.

VAMVIS à Theone supra Ptolomei Almagestum sufficienter traditum
sit de magnitudinibus figurarum isoperimetrarum , nihilominus vt tibi
morem geram , ea nunc scribo , quæ mihi in mentem venerunt contra
Alexãdrum Piccolhomineum, antequã aliquid ipsius Theonis vidissem
Alexan‑

Alexander Piccolhomineus in libro primo de mundi sphæra vbi tractat de cęli rotunditate, ita inquit.

" Oltre di questo, douendo il decimo cielo contenere & in se chiudere tutte le co-
" se, è conueneuol cosa il pensare, che fosse fatto di quella più capace figura che es-
" ser possa, la qual è la figura rotunda, però che si può trar da molti luoghi d' Euclide
" che si come se noi ci immagineremo più figure superficiali talmente che tutte le li-
" nee de l'vna congionte insieme, sieno vguali à tutte le linee pur insiememente com
" poste di qual si voglia de l'altre figure, ne seguirà, che quella figura sarà più capa-
" ce la qual haurà manco angoli, & quella capacissima che sarà senza alcuno come è
" la figura circolare, &c.

Cogitemus igitur primò de triangulo æquilate-
ro & quadrato isoperimetris, sit enim triangulus æ-
quilaterus. o. b. g. quadratum verò. b. l. quorum pe-
riferiæ inuicem æquales sint. Dico quadratum ma-
ioris superficiei esse ipso triangulo. Accipio pri-
mum lineam. f. h. eiusdem longitudinis quæ vnius
periferiæ dictarum figurarum, quam punctis. r. K.
mediantibus diuido in tres equas partes, in quatuor
verò mediantibus punctis. l. x. i. vnde proportio to-
tius. f. h. ad. K. h. erit vt. l. h. ad. i. h. idest tripla, & per
16. quinti erit. f. h. ad. l. h. vt. k. h. ad. i. h. per. 19. verò
f. h. ad. f. l. vt. K. h. ad. K. i. sed. f. l. est quarta pars ip-
sius. f. h. ergo. k. i. erit quarta pars ipsius. k. h. Coniũ
gantur enim ambo istæ figuræ vt hic inferius vides,
vnde. a. g. erit quarta pars ipsius. b. g. diuisa postea.
b. g. per æqualia in. c. erit. a. c. æqualis. a. g. Ducatur
deinde. o. c. quę per. 8. primi, nec nõ ex definitione,
perpendicularis erit ipsi. b. g. ergo etiam quadratũ
b q. supra. b. g. producoq;. o. c. vsque ad. m. nam nul
li dubium est quin. o. c. breuior sit. o. g. ex. 18. vel. 48
primi cui æquatur. q. g. diuido etiam. c. m. per æqua
lia in puncto. e. ducoq;. t. e. p. æquidistantem. b. g.
vnde habebimus duo quadrata. e. g. et. e. b. sed
quadratum. b. l. æquatur quadrato ipsius. c. a.
cum duplo illius quod fit ex. b. c. in. c. g. vt patet
ex. 9. secundi, hoc est æquatur quadrato. c. a. & re-
ctangulo. t. g. Deinde vt se habet. p. g. ad. o. e. ita se habet. u. p. ad. u. e. ex similitudine
triangulorum. Sed. p. g. maior est ipsa. o. e. cum. p. g. æqualis sit. e. m. quare triangu-
lus. u. g. p. maior erit triangulo. o. e. u. ex. 17. sexti. Similiter dico maiorem esse trian
gulum. b. d. t. triangulo. e. o. d. vnde sequitur rectangulum. t. g. maiorem esse triangu-
lo. b. o. g. sed quadratum. b. l. est etiam maior ipso rectangulo. t. g. ex quadrato ipsius
c. a. vt diximus, tanto igitur maior erit triangulo. b. o. g.

**Possumus**

376        IO. BABPT. BENED.
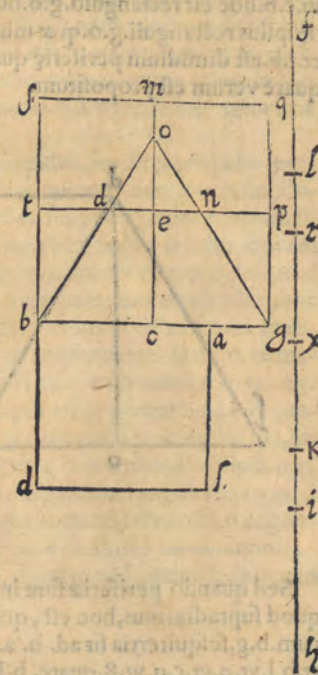
Poffumus etiam probare quod periferia quadrati æqualis triangulo æquilatero
minor fit periferia ipfius trianguli æquilateri . Cogita triangulum æquilaterum hîc
fubfcriptum.d.l.q. cuius bafis. l.q. diuifa fit per æqualia à perpendiculari. d. o.defcri
ptûq; fit rectangulum.o.g.quod æquale erit triangulo.d.l.q. fed periferia trianguli
maior eft periferia rectanguli,nam.l.q.æqualis eft.o.q.cum.d.g.fed.q.d.maior eft.o.
d. ex. 18. primi,vnde.l. d.maior etiam.q.g.cum ex.34.dicti latera oppofita ipfius re
ctanguli fint inuicem æqualia, accipiamus poftea. e.c.æqualem.o.d. et. c. h. indire-
ctum æqualem.o.q.circa quem diametrum.e.h.intelligatur circulus.e.i.h.k. et.à pun
cto.c. dirigatur perpendicularis.k.i.ad.e.h.vnde ex. 3.tertij.c.i.æqualis erit.c.k.& ex
34.quod fit ex.c.i.in.c.k.hoc eft quadratum ipfius.c.i.æquale erit ei quod fit.ex. e.c.
in.c.h.hoc eft rectangulo.g.o.hoc eft triangulo.d.l.q. fed.e. h. eft dimidium perife-
rię ipfius rectanguli.g.o.quæ minor eft di midio periferiæ trianguli.d.l.q. vt vidimus
et.i.k.eft dimidium periferię quadrati ipfius. i.c.&minor etiam ipfa.e.h.ex.14.tertij
quare verum eft propofitum.
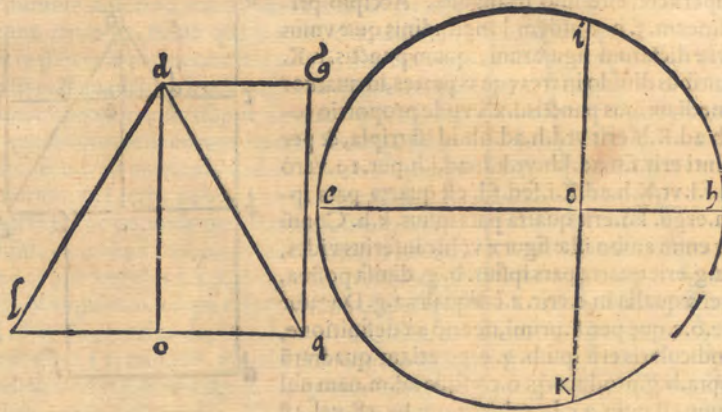


Sed quando periferiæ funt inuicem æquales,poffumus etiam breuiter videre id
quod fupradiximus,hoc eft , quod quadratum,maius fit triangulo æquilatero. Nam
cum.b.g.fefquitertia fit ad. b. a. ergo. b. g. erit vt.4.et.b.a.ut. 3.vnde.b.q.erit vt.16
et.b.l.vt.9.et.c.q.vt.8.quare. b.l.maius erit ipfo rectâgulo.c.q.fed. c.q.maius eft triâ
gulo.b.o.g.cum.q.g.quæ æqualis eft.o.g.maior fit.o.c.ex.18.vel penultima primi,
nam fi.q.g.æqualis effet.o.c.tunc.c.q.æqualis effet triangulo.b.o.g.ex.41. primi.
Alia etiam via maiores noftri vfi funt quæ generalis eft vt in Theone fupra Al-
mageftum videre eft, medijs perpendicularibus à centris ad latera figurarum, fed
quia differêtia longitudinum ipfarum perpendicularium alio medio inueniri poteft,
eo quo ipfi vfi funt, prætermittere nolo quin tibi fcribam .
Ego enim ita difcurro.
Sint duæ figuræ ifoperimetrę æquilaterę & æquiangulæ , puta primò trian-
gulum & quadratum quorum centra fint. e. et. o. à quibus centris ad latera fint per-
pendiculares.e.n.et.o.u.vnde.n.et.u. diuident latera per æqualia vt fcis , ducantur
poftea.e.t.et.o.a.ad angulos dictorum laterum,vnde habebimus angulum.o.a. u. di·
midiũ recti, et.e.e.t.n.tertia pars vnius recti, vt ex te ipfo videre potes, quare angulus
                                                                                   a.fefqui-

a.sesquialter erit angulo.t.quod vt clarius videas cogita lineam.b. d. cuius medietas sit.c.d.tertia verò pars illius sit.g.d.tunc dico.c.d.sesquialteram esse ipsi.g d.sit enim f.d.duplum ipsius.g.d.quare.f.d.erunt duæ tertiæ totius lineę.b.d.& quia eadem proportio est totius. b.d.ad.c.d.quæ.f.d.ad.g.d.ergo permutando eadem erit totius. b. d.ad.f.d.quæ.c.d.ad.g.d. Sed.b.d.ad.f.d.sesquialtera est,verum igitur erit quod angulus.a.sesquialter sit ipsi.t.deinde.t.n.est sesquitertia ipsi.a.u. vt superius vidimus. in eorum duplis. scimus etiam.n.e.esse dimidium ipsius.t.e. eo quod cum. e. t. n. sit tertia pars vnius recti,angulus.t.e.n.erit duo tertia vnius recti, vnde. e. n . erit latus. exagoni æquilateris inscriptibilis circulo cuius diameter sit.e.t.quare.e.t. dupla erit ipsi.e.n.in longitudine, sed quadrupla in potentia:t.n. vero tripla in potentia ipsi.n. e.ex penultima primi, quæ omnia etiam ex.8.tertijdecimi.Eucli. elicere potes, sed t.n.erat sexquitertia ipsi.a.u.in longitudine, hoc est ipsi.o.u.nam.o.u.æqualis est ipsi a.u.quare.n.t.erit minus quam dupla in potentia ipsi.o.u.hoc est, vt. 16. ad. 9. ergo maior proportio erit ipsius.t.n.in potentia ad.n.e.quam ad.o.u. quare etiam in lon gitudine , maior proportio erit ipsius.t.n.ad.n.e.quam ad.o.u.vnde.o.u. longior erit ipsa.n.e.quod est propositum .

Sed si.o.a.u.esset pentagonus æquilaterus & æquiangulus,similiter probabo per-pendicularem.o.u.longiorem esse.n.e.ipsius trianguli æquilateri,dummodo sint iso-perimetrę. Sit enim.a.u.dimidium lateris pentagoni ex supposito,cuius centrum sit o.tunc proportio.t.n.ad.a.u.erit superbipartienstertias,vt ex ordine iam hic supradi cto à te facillimè elicere potes, hoc est,vt.5.ad.3. et. a. u. minor erit. o. u. eo quod angulus.o.minor erit angulo.a.nam angulus.o.erit quinta pars duorũ rectorum,hoc est duæ quintæ vnius recti , vnde angulus. a. residuum vnius recti erit tres quin-tæ vnius recti, quare angulus.a.maior erit angulo.o. & consequenter latus.o. u. ma-ius latere.a.u. sed.t.n.minor est quam tripla in potentia ad.a.u.eo quod erit vt. 25. ad.9.cum in longitudine sit vt.5.ad.3.sed dicta.t.n.tripla est in potentia ad.e.n.qua-re.a.u.maior erit ipsa.e.n.sed.o.u.maior est ipsa.a.u.vt diximus, igitur multo magis. o.u.maior est ipsa.a.u.vt dixim⁹ & cõsequéter multo magis. o.u.maior erit ipsa.n.e.

Quotiescunque enim cognoscimus proportionem anguli.o.ad angulum. a. quod quidem facillimum est, nec non proportionem.t.n.ad.a.u.quod,etiam illico cogno-scitur , tunc ex scientia cordarum & arcuum omnia etiam facillimè innueniuntur . Verum circa triangulũ æquilaterum,& pentagonum,alium modũ inueni,sed aliquan tulum prolixiorem.
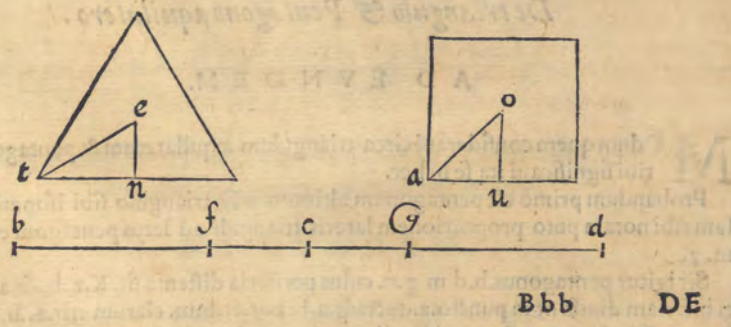
## De incommmensurabilitate, in longitudine perpendicularis trianguli æquilateri cum eiusdem latere.

### AD EVNDEM.

ID quod à me postulas est omnino impossibile, velles enim duos numeros inueni re inter se ita se habentes, vt se habent perpendicularis in triangulo æquilatero cum vno eius laterum, quod vero hoc fieri non possit, considera in figura præcedenti triangulum æquilaterum. d.l.q. cuius perpendicularis sit. d.o.quæ diuidit. l. q. per æqualia in.o.vnde ex.4.secundi Euclidis, quadratum.l.q. ( idest. d. q. ) quadruplum erit quadrato.o.q.& ex penultima primi equale quadratis.d.o.et.o.q.quare erit sesquitertium quadrato ipsius.d. o. & ita quadratum.d.o.erit triplum quadrato ipsius. o. q. hæc autem proportiones non sunt vt numeri quadrati ad numerum quadratum quod si ita fuissent,sequeretur ternarium numerum esse quadratum ex.22.octaui. Cum igitur non sint vt numeri quadrati ad numerum quadratum , sequitur ex septima decimi.d.o.esse incommensurabilem ipsi.l.q.seu. d.q.in longitudine.

Vel dicamus ita , proportio quadrati ipsius.l.q.ad quadratum ipsius.o.d.est in genere superparticulari , cum sit sesquitertia , vnde quadratum ipsius.d.o.numeris dari non potest, eo quod si dabilis fuisset , sequeretur , quod inter quadratum ipsius. l. q.& ipsius.d.o. esset aliquis numerus medius proportionalis ex.16. octaui , vnde ex octaua eiusdem vnitas diuisibilis esset , quod fieri non potest.
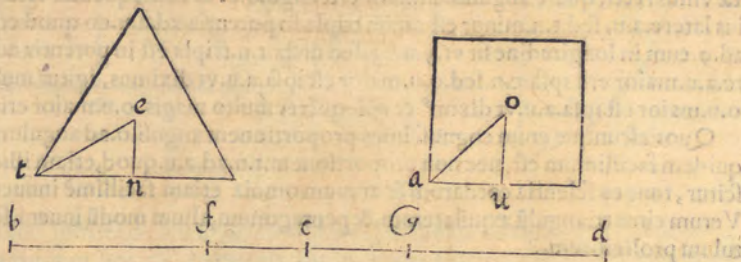


## De triangulo & Pentagono æquilatero.

### AD EVNDEM.

MOdum quem consideraui circa triangulum æquilaterum & pentagonum , vt tibi significaui ita se habet.

Probandum primò est pentagonum altiorem esse triangulo sibi isoperimetro. Iam tibi notam puto proportionem lateris trianguli ad latus pentagoni esse vt.5. ad.3.

Sit igitur pentagonus.b.d m.g.v. cuius periferia distenta sit. K.z. basis autem. m. g. bifariam diuisa sit in puncto.a.ductàq;.a.b: b.g.et.b.m. clarum erit.a. b. perdicularem esse ad.m.g.ex.8. primi Eucli.cum.b.m.et.b.g. (bases triangulorum. b.d. m.
et.b.u.

et. b.u.g.) fint inuicem æquales ex. 4. eiufdem .

Accipiatur deinde vel intelligatur. g.p. æqualis duabus tértijs ipfius.a.g.ducatur éque.b.p.quam probabo maiorem effe duplo ipfius.a.p. vnde maior erit latere ipfius trigoni æquilateris,cuius dimidium eft.a.p. fcimus enim ipfum latus fe habere ad.m. g. vt quinque ad.3. ita étiam.a.p.ad.a.g. vt diximus .

Cum auté angulus.a.b.g.fit quarta pars anguli,b.g.a.ex.10.quarti & quinta pars vnius recti ex.32.primi,dictus angulus erit graduum.18.et.a.g.erit partium.30902. et. a. b, partium.95015.et.a.p.51503.vnde ex penultima primi latus.b.p. erit partium.108075.duplum vero ipfius.a.p. erit.103006.latus igitur dicti trigoni, quod ab.p.erigitur,fecabit perpendicularem.a.b.fub.b. hoc eft inter.b.et.a. ex penultima primi. Finiatur enim triangulus æquicrurus.b.q.p.quem probaui maiorem effe æquilatero ifoperimetro pentagono propofito,ducaturq;.u.p.ducatur etiam. u. n. parallela ipfi.b.g.quæ concludet triangulum.g.u.n. fimilem triangulo.m.b.g.eo quod cum angulus.m.b.g. æqualis fit angulo.b.g.u.ex.16.tertij,per.27.primi.m.b.et. g.u. erunt inuicem æquidiftátes,vnde angulus.b.m.g.æqualis erit angulo.u.g.n.et.ex.29. angulus.g.u.n. æqualis erit angulo.u.g.b.quare etiam angulo.g.b.m.& angulus. u.n. g.angulo.b.g.m.ex.32.eiufdem,vnde ex. 4. fexti proportio.g.n.ad.g.m.erit.vt. g.u. ad.m.b.fed cum.g.u.maior fit dimidio ipfius.b.g.ex.20. primi, hoc eft maior dimidio ipfius.b.m.ergo.g.n.etiam maior erit ipfa.g. a. quapropter maior erit ipfa. g. p. cum.g.p.minor fit ipfa.g.a.ex hypothefi , ducta deinde cum fuerit. b. n. habebimus triangulum.b.n.g.æqualé triangulo.b.u.g.& maioré triāgulo. b.p.g. ex prima fexti vel quia totum maius eft fua parte . Triangulus igitur.b.u.g.maior eft triangulo.b.p.g.quare triangulus.b.u.o.maior erit triangulo.g.o.p.ex communi conceptu, idem infero ab alia parte dictarum figurarum. Quare pentagonus.b.d.m.g.u.maior erit triangulo.b.q.p.quem probauimus maiorem effe triangulo æquilatero fibi ifoperimetro.
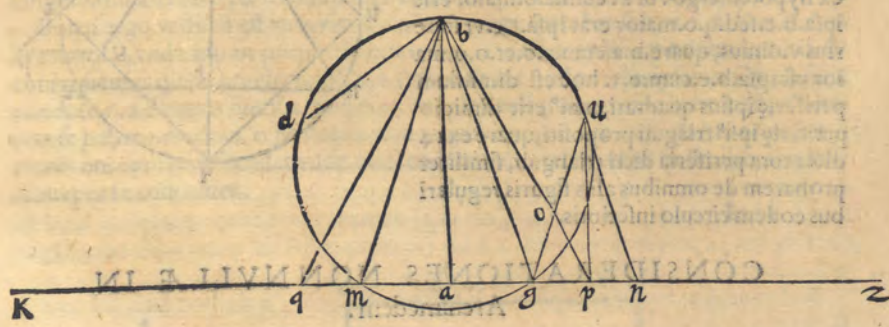


*Comparatio periferiarum quadrati & trianguli æquilateri circunfcriptorum ab eodem circulo.*

## AD EVNDEM.

Qvod autem periferia quadrati in eodem circulo infcripti, in quo fit triangulus æquilaterus, longior fit periferia ipfius trianguli æquilateri, abfque vllo

negotio cordarum & arcuum possumus geometricè demonstrare quod valde desideras.

Quapropter sit circulus.b.a.e.q.in quo sit triangulū æquilaterum.b.e.n.& quadratum.b.a.q.u. cuius periferiam probabo longiorem esse periferia trianguli. Sit enim diameter circuli.b.q.qui etiam erit diameter quadrati, vt à te scire potes. Sit etiam punctū.b.commune tam anguli quadrati quam trianguli. vnde sequitur quod dictus diameter secabit latus.n.e.trianguli ad rectos & per æqualia in.t. Nam cum arcus.b. e.æqualis sit arcui.b.n.ex.27.tertij, remanet vt arcus.q.e.equalis sit arcui. q.n.vnde angulus.q.b.e.æqualis erit angulo.q.b.n.ex. 26. eiusdem. quare ex. 4. primi anguli ad.t.erunt recti,et.h.t.æqualis erit ipsi.t.e. vt diximus.

Deinde.b.e.et.q.a.seinuicem secāt in puncto. o.vt ex se clarum patet, ducatur postea.q.e.vnde habebimus angulum.b.e.q.rectum ex.30.tertij, quare ex.18.primi.q. o. longior erit ipsa.q.e.et.q.e.longior erit ipsa.e.t. quare.q.o. longior erit ipsa.t.e.

Vt probemus postea.b.a.o.longiorem esse ipsa.b.e. producatur.b.a. ita quod. a. p.æqualis sit ipsi.a.o.ducaturq; o.p et.a.e.cum autem ex iam dicta.30.tertij angulus .b.a.o.rect⁹ sit, erit angulus.o.a.p. similiter rect⁹ ex.13. primi, vnde ex.5.et.32.eiusdē angulus.a.p.o.erit dimidium recti, & similiter, ex ijsdem, angulus.b.q.a. est dimidium recti quare angulus.a.p.o.æqualis erit angulo.a.q.b. sed angulus.a.e.b.æqualis est angulo.a.q.b.ex. 20.tertij, ergo angulus.b.p.o.æqualis erit angulo.b.e.a. angulus vero a.b. e.communis est ambobus triangulis.a.b.e.et.o.b.p.quare ex. 32. primi anguli. b.a.e.et.b.o.p. reliqui ex duobus rectis æquales inuicem erunt. Quare ex quarta sexti, et.18. quinti proportio.b.o.ad.b.p.erit, vt b.a.ad.b.e.sed ex.18. primi.b. o. maior est ipsa.b.a.quare ex.14.quinti.b.p.maior erit ipsa.b.e.sed.b.p.æquatur ipsis.b.a.cum.a.o ex hypotesi, ergo.b. a. cum.a.o.maior erit ipsa.b.e.sed.q.o.maior erat ipsa.t.e.vt superius vidimus, quare.b.a.cum.a.o.et.o.q.ma ior est ipsa.b.e.cum.e.t. hoc est dimidium periferię ipsius quadrati, mai⁹ erit dimidio periferię ipsi⁹ triaguli propositi,quare ex14 dicta tota periferia dicti trianguli, similiter probarem de omnibus alijs figuris regularibus eodem circulo inscriptis.



## CONSIDERATIONES NONNVLLÆ IN
### Archimedem.

*Doct̄ ßimo atque Reuerendo Domino Vincentio Mercato.*

V o D tibi aliàs dixi verum est. intellectum scilicet non omninò quiescere circa illas duas Archimedis propositiones, quæ in translatione Tartaleæ sunt sub numeris. 4. et. 5 . & in impressione Basileæ sub numeris. 6. et. 7. vbi tractat

tractat de centris libræ, seu stateræ: Aspice igitur in.4. supradicta, quod cum appen-
sæ fuerint omnes illæ partes ponderum, partibus longitudinis ipsius.l.K.in qua volo
vt à punctis.e.et.d.imagineris duas lineas.e.o.et.d.u.inuicem æquales, & ferè per-
pendiculares ipsi.l.K. hoc est respicientes mundi centrum; imagineris etiam.o.u.
quæ sit parallela ipsi.l.k.quæ diuisa sit in puncto.i.supra.g. Hinc nulli dubium erit,
cum.g.fuerit centrum totius ponderis appensi ipsi.l.K.quod.i.similiter erit centrum
cum directe locatum sit supra.g.hoc est in eadem directionis linea, quod quidem
non indiget aliqua demonstratione, cum per se satis pateat. Vnde ex communi
conceptu.o.erit centrum ponderis appensi ipsi.l.h.et.u. erit centrum ponderis ap-
pensi.ipsi h.K. Scimus igit. i.esse cetrum duorum, hoc est ipsius.l.h. & ipsius.h.k.con
tinuatorum per totam.l.k. Nunc ergo si consideremus.l.k.diuisam esse, hoc est di-
siunctam in puncto.h.inueniemus nihilominus.i.centrum esse dictorum ponderum,
& quod tantum est, ipsam esse continuá, quantum diuisam in dicto puncto.h. neque
ex hoc, punctum.i. erit magis vel minus centrum duorum ponderum.l.h.et.h.k.quo
rum vnum pendet totum ab.o.aliud verò totum ab.u. & hoc modo in longitudine.
o.u.diuisa vt dictum est, habebimus propositum.

Reliquam propositionem tibi relinquo.

Illa verò propositio, quam tibi dixi Archimedem tacuisse in huiusmodi materia
est, quod si duo pondera æquilibrant ab extremis alicuius stateræ, in certis præfixis
distantijs à centro. Tunc dico si eorum vno manente alterum moueatur remotius
ab ipso centro quod illud descendet, & si vicinius ipsi centro appensum fuerit ascen-
det. Hæc enim propositio quotidie omnibus in locis videtur, ipsam verò puto Ar
chimedem prætermisisse ob facilitatem, cum ab antedicta ferè dependeat.

Sit exempli gratia statera.a.u.cuius centrum sit.i.& pondera.u.a. appensa, se in-
uicem habeant vt.i.u.et.i.a.se inuicem habent. Nunc dico quod si pondus ipsius.u.
positum fuerit vicinius centro vt puta in.o. inmoto existente pondere.a. quod bra-
chium.i.o.u.ascendet, & è conuerso, si remotius positum fuerit, descendet.

Ponat ergo vt dictü est in.o.vicinius cetro, quapropter brachium. i. o. breuiꝰ erit
brachio.i.u.vnde minor proportio erit ipsius.i.o.ad.i.a.quàm.i.u.ad eundem.a.i. &
consequenter quam ponderis ipsius.a. (quod sit.n.e.) ad pondus ipsius.u. Quare si ex
pondere.n.e.dempta fuerit.e.pars eius, ita quod reliqua pars.n.se habeat ad pondus
o.vt se habet.i.o.ad.i.a. tunc statera non mouebitur, addita verò parte. e. ex com-
muni conceptu,a. descendet vnde.o.ascenderet conuersum verò ex similibus ratio-
nibus per te concludes.
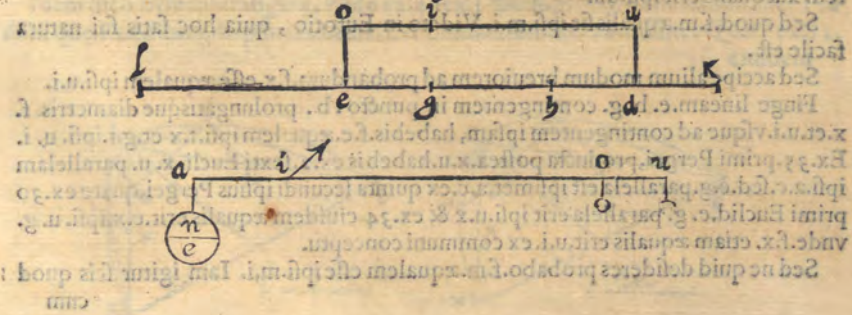
In eo quod à me petis, mittendo te ad Eutotium, tibi non satisfacerem, cum Eutotius citet sextum librum Pergei, quem nunquam vidimus, supponatq; ea, quæ nec ipse nec alius vnquam quod scimus probauit.

Desideras enim demonstrationem illius quod Archimedes dicit inter primam, & secundam propositionem secundi libri, vbi tractat de centris grauium, propterea quod illud supponit pro manifesto.

Sit enim figura hic subscripta, ferè similis parabolæ positæ in. 2. propositione dicti libri, vt in impressione Basileensi habetur, sintq; diuisæ duæ a.b. et.b.c. per æqualia à punctis.x.et.u. protractisq; f.x.et.u.i. ad. b.d. quæ inuicem etiam erunt parallelæ ex. 30. primi Eucli. vnde ipsæ etiam, diametri erunt ipsarum portionum: vt ex eo colligere est, quod in. 49. primi lib. Pergei probatur. Imaginando postea ad puncta. b. f. et. i. tres contingentes, manifestum erit punctum. b. illud esse quod terminat altitudinem huiusmodi portionis, et. f. et. i. terminantia altitudines partialium, ex. 5. secundi ipsius Pergei, eo quod dictæ contingentes paralellæ erunt ipsis basibus, vnde trianguli inscripti, easdem habebunt altitudines, quas portiones ipsæ, quod erit ex mente Archimedis. Et sic deinceps poteris multiplicare angulos figuræ rectilineæ in parabola, quæ designata erit vt desiderat Archimedes, qui quidem dicit, quod protractæ cum fuerint aliæ deinceps post. f. i. ipsæ inuicem æquidistantes erūt, diuiseque peræqualia ab.d.b. quod quāuis verū sit, tn ab Eutotio non satis demōstratū est, cum supponat. a. f. b. æqualem esse ipsi. b. i. c. probare volens eius diametros æquales esse absque aliqua citata ratione, quæ quidem ratio esset conuersum. 4. propositionis libri de conoidalibus. Sed oporteret nos etiā videre. 6. librum ipsius Pergei, & propterea tibi non satisfacerem.

Esto igitur, ut inuenta sit linea. K. cuius productum in.u.l. æquale sit quadrato ipsius.u.c. inuenta etiam sit linea.h. cuius productum cum.f. x. æquale sit quadrato ipsius.a.x. vnde ex conuerso. 49. primi ipsius Pergei, proportio ipsius K.ad.b.c. erit ut ipsius.b.c. ad.b.d. & ipsius.h.ad.a.b. vt ipsius.a.b. ad.b.d. Erit igitur ex. 1. sexti Eucli. quadratum. b.c. æquale producto ipsius.K.in.b.d.& quadratum.a.b. æquale producto ipsius.h.in.b.d.& ex prima sexti, ita erit ipsius. K.ad.h. vt producti quod fit ex.Kl in.b.d.ad productum ipsius.h.in.b.d. hoc est vt quadrati ipsius.b. c. ad quadratum ipsius.b.a. ex. 16. et. 11. quinti, hoc est vt quadrati ipsius.u.c. ad quadratum ipsius.a.x. hoc est ut productum ipsius.k. in.u.i. ad productum ipsius.h.in.x. f. Nunc si ipsius. k. ad.h. est vt producti ipsius.K.in.u.i. ad productum ipsius.h.in.f. x. ergo ex. 24. sexti, & communi conceptu, proportio ipsius. k.ad.h. composita erit ex ea quæ ipsius. u. i. ad.f.x. & ex ea quæ ipsius.k.ad.h. Cum ergo dempta fuerit proportio ipsius. k.ad.h. ( vt simplex ) à proportione ipsius.k.ad.h. ( vt composita) reliquum nihil erit. Quare.f. x. æqualis erit ipsi.u.i.

Sed quod.f.m. æqualis sit ipsi.m.i. Videto in Eutotio, quia hoc satis sui natura facile est.

Sed accipe alium modum breuiorem ad probandum.f.x. esse æqualem ipsi.u.i.

Finge lineam.e. b. g. contingentem in puncto. b. prolungatisque diametris. f. x.et.u.i. vsque ad contingentem ipsam, habebis.f.e. æqualem ipsi.f.x et.g.i.ipsi. u. i. Ex.35. primi Pergei, producta postea.x.u. habebis ex. 2. sexti Eucli.x. u. parallelam ipsi.a.c. sed.e.g. parallela est ipsimet.a.c. ex quinta secundi ipsius Pergei, quare ex.30 primi Euclid.e. g. parallela erit ipsi.u.x & ex.34. eiusdem æqualis erit.e.x. ipsi. u. g. vnde.f.x. etiam æqualis erit.u.i. ex communi conceptu.

Sed ne quid desideres probabo.f.m. æqualem esse ipsi.m.i. Iam igitur scis quod

cum

cum sit. f. x. æqualis ipsi. u. i. vt tibi probaui , & inuicem parallelæ ideo. f. i. parallela
erit ipsi. x. u. ex. 33. primi Euclidis. Vnde ex. 30. eiusdem, parallela erit etiam ipsi. a.
c. sed cum. x. u. diuisa sit ab. d. b. per æqualia, eo quod diuidit. a. c. eodem modo, quę
ipsi parallela est ex. 2. sexti. Reliqua tibi consideranda relinquo . cum verò ambæ. f.
x. et. u. i. parallelæ sint ipsi. b. d. sequitur quod cum ex. 34. primi vnaquæq;. f. m. et. m.
i. æqualis sit medietati ipsius. x. u. erunt inuicem æquales.
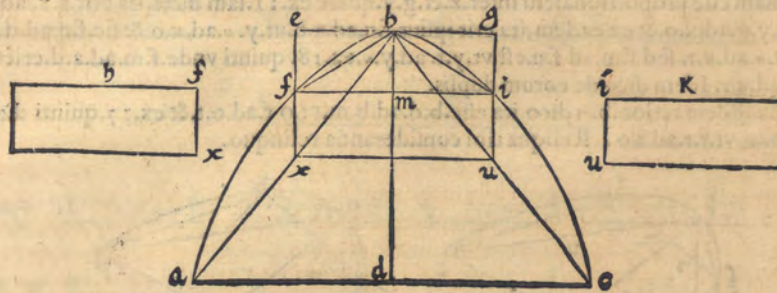


Minime dubitabam tibi non satisfacere Eutocium in. 3. propositione secundi
lib. de centris Grauium Archimedis , cum citet. 6. librum de elementis conicis, ad-
de quod si aliud in ipso. 6. libro ab eo citato non esset magis ad propositum , quàm
ea quæ ab ipso citata sunt, nihilominus adhuc irresolutus maneres.

Considera igitur eandem ipsam figuram præcedentem ; pro alia verò parabola si
mili dicta, accipe secundam figuram ipsius tertiæ dictæ propositionis. Deinde ima
ginabis duo latera. o. x. et. o. p. diuisa esse per æqualia in punctis. g. et. K. protractisq;
diametris. g. y. et. K. u. quæ , vt in præcedenti probaui , sunt inuicem æquales , scire
debes quod similes parabolæ inuicem aliæ non possunt esse, nisi eæ quæ diametros
proportionales suis basibus habeant, similiterq; positæ, hoc est, ut proportio ipsius
b. d. ad. a. c. sit eadem quæ ipsius. o. r. ad. x. p. & quod anguli ad. r. sint æquales angulis
circa. d. Notentur ergo primum puncta communia ipsius. o. g. cum. y. t. & ipsius. b. x
cum. f. m. characteribus. �. et. n. Nunc igitur scimus. f. m. æqualem esse. m. i. totamq;. f.
i. parallelam esse ipsi. a. c. Idem dico de. y. t. u. trianguliq;. x. f. n. et. g. y. �. esse similes
triangulis. n. m. b. et. �. t. o. quod ita probatur, nam ex. 15. primi Euclid. anguli ad. n.
sunt inuicem æquales, ex. 29. verò eiusdem anguli. f. x. n. et. n. b. m. similiter æquales
ita etiam. n. f. x. et. n. m. b.

Idem dico in secunda figura, vnde ex. 4. sexti Eucli. proportio. n. f. ad. m. n. erit ea
dem quę. f. x. ad. b. m. & ipsius. n. f. ad. x. f. vt. n. m. ad. m. b. ex. 16. quinti . Quare ex. 11.
                                                                                    eiusdem

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eiufdem erit vt. a.d.ad.d.b. Idem etiam dico in fecunda parabola , fed ipfius.x.o.ad
o . r . eft vt.a.b. ad. b. d. ex. 6.fexti Eucli. vnde ex.11.quinti.n.f.ad.f.x. erit vt. ω. y.
ad.y.g.  Sed in precedenti iam tibi dixi.a.b.mediam proportionalem effe inter. h.
et.b.d. Sit nunc.z. pro fecunda parabola, ita ut.h. eft pro prima, vnde.o.x.crit media
proportionalis inter.z.et.o.r.& ex.11.quinti ita erit.h.ad.a.b.vt.z.ad.x.o.& ex. 22.
h.ad.a.x.ut. z.ad.x.g.& quia ex.16. fexti. a.x.media proportionalis eft inter.h.et.f.
x. cum fupponatur productum.h.in.f.x.æquale effe quadrato . a. x. Idem dico . x . g.
mediam effe proportionalem inter.z.et.g.y.quare ex.11.iam dicta,ita erit.a.x. ad.f.
x.vt.y.g.ad.x.o.& ex eadem,ita erit ipfius.f.n.ad.a.b.ut.y. ω.ad.x.o.& fic.f.n. ad.d.a.
vt. y.ω.ad.x.r. fed.f.m.ad f.n.eft vt.y.t. ad.y.ω.ex.18. quinti vnde.f.m.ad.a.d.erit vt
y.t.ad.x.r. Idem dico de eorum duplis.

   Ex ijfdem rationibus dico ita effe.b.d.ad.b.m.vt.o r.ad.o.t.& ex.17.quinti.d.m.
ad.b.m.vt.r.t.ad.t.o . Reliqua tibi confideranda relinquo.



   In reliquis verò propofitionibus illius lib. nullo pacto poteris dubitare:Verum ne
in. 4. aliquid tibi noui exurgat, te fcire volo corollarium. 20. in libr. de quadratu-
ra parabolę docere poffibile effe infcriptionem rectilineæ,ea tamen conditione quâ
dicit Archimedes.

   In quinta poftea animaduertendum eft,quod prima pars,probat tantummodo de
centro trianguli,et.2.pars probat de centro pentagoni , à te ipfo deinde potes pro-
bare de centro nonanguli:& fic de cæteris: eo quod cum probatum fuerit de centro
figuræ in medio locatæ fi conftitutæ poftea fuerint fimiles figuræ in portionibus la-
teralibus habebitur propofitum in infinitum.

   Idem intelligendum eft in.3. propofitione quamuis exemplum vlterius non ex-
tendatur quam ad pentagonos.

   Sexta verò ,ppofitio tibi facilis erit,quæ nihilominus pôt demóftrari hoc mô fcili
cet. Sint.4.quâtitates.a.b.c.d. ipfius Archimedis fupponédo.a.pro figura rectilinea
infcripta in parabola,et.b.pro refiduo ipfius parabolę et.c.pro triangulo.a.b.c.in me
dio ipfius parabolę et.d.pro triangulo. r. Nunc cum. a. maior fit.c.prout totum ma-
ius eft fua parte, ideo ex.8.quinti maior proportio habebit.a.ad.b. quam. c. ad .b.
Cum autem.b.minor fit.d.ex fuppofito,ideo ex eadem dicta,maior proportio habe
bit.a.ad.b.quam.c.ad.d.cum verò centrum cuiufuis figuræ plenæ neceffariò fit intra
ipfam figuram, idcirco centrum refidui ipfius parabolę intra ipfam reperietur. quo d
ita clarū p fe eft,quéadmodū quoduis aliud axioma,& quia dictū centrū,ex.8.primi
de centris, neceffariò eft in linea.b.h.inter.b.et.h. Sit igitur.g.vnde ex eadem.8.ita
erit.g.h.ad.h.e.vt.a.ad.b.ergo.g.h.ad.h.e.maior proportio erit quâ.c. ad. d. hoc eft
quam.b.h.ad.f.ex.12.quinti. Sed cū.h.b.maior fit ipfa.h.g.prout omne totum ma-
ius eft fua parte, ideo maior proportio habebit.h.b.ad.h.e.quam.h.g. ad.h. e. vnde
multo maioré quâ.h.b.ad.f.ex côi côceptu,quare. h.e.erit minor ipfa.f.ex.10.quiti.
   Septima verò et. 8.propofitio nullius tibi erit difficultatis .

                                                                    Quam-

Quamuis Eutotius ſcribat ſuper duas vltimas lib. ſecundi de centris grauiũ, nihil miror ipſum tibi non ſatisfacere. Accipe igitur quod ego nunc tibi mitto.

Archimedes eo in loco primũ ſupponit in penultima dicti libri quatuor lineas proportionales. a. b: c. b: d. b: et. e. b: ſupponit etiam quod proportio quæ eſt ipſius. e. b. ad. e. a. eadẽ ſit quæ ipſius. f. g. ad tres quintas ipſius. a. d. & quod proportio com poſiti dupli ipſius. a. b. cum quadruplo ipſius. b. c. cum ſexcuplo ipſius. b. d. cum triplo



Ccc ipſius

ipsius.b.e.ad compositum quintupli ipsius. a.b.cum decuplo ipsius.c.b.cum decuplo ipsius.b.d.cum quintuplo ipsius. b.e.eadem sit quæ ipsius.g.h.ad.a.d. & vult proba-re.f.h.esse duas quintas ipsius.a.b.

Cum autem dicit proportionem ipsius.a.c.ad.c.d.& ipsius.c.d.ad.d.e.esse vt ipsius a.b.ad.b.c.& cętera, verum dicit ex.19.quinti Eucli.eo quod cum ex hypothesi sit ipsius,a.b.totalis ad.c.b.totalem vt ipsius.c.b. partialis (sumptæ vt pars abscisa ab.a. b.pro nunc) ad.d.b. partialem (abscisam ab.c.b.) erit ex.19.dicta ipsius.a.c.(residui ex.a.b.) ad.c.d.(residuum ex.c.b.)vt ipsius.a.b.ad.c.b.& ita probabitur de pro-portione ipsius.c.d.ad.d.e. eadem ratione.

Cum verò ex.18.quinti sit ipsius.a.b.cum.c.b.ad.c.b.vt ipsius.a.d.ad.d.e.ergo ex 22.eiusdem,ita erit ipsius.a.b.cum.c.b.ad.d.b.vt.a.d.ad.d.e.& ex ijsdem rationibus eadem proportio erit ipsius.c.b.cum.d b.ad.b.e.vt.a.d.ad.d.e.quod inquit Archi. Verum etiam erit (ex.13.quinti) cùm dicit eandem proportionem esse ipsius. a. d. ad. d.e.quę dupli primi antecedentis cum simplo secundi antecedentis ad duplum primi consequentis cum simplo secundi consequentis, hoc est dupli ipsius.a.b.c. cū simplo.c.b.d.ad duplum ipsius.d.b.cum simplo.e.b.hoc est dupli.a.b. cum triplo ip-sius.b.c.cum simplo.d.b.ad duplum ipsius.d.b.cum simplo.e.b. Nunc duplum.a.b. cum triplo.b.c.cum simplo.b.d.signatum sit charactere. D. suum verò consequens, **M** hoc est duplum.d.b.cū simplo.e.b.significetur à charactere.B.hinc proportio ipsius a.d.ad.d.e.erit vt.D. ad.B.

Inquit nunc Archimedes, si quis sumeret aliquod màius antecedens æquale scilicet duplo ipsius.a.b.cum quadruplo ipsius.b.e.cum quadruplo ipsius.b.d. cum du-plo ipsius.b.e. compararetq; illud cum côsequente.B.clarum esset ex.8.quinti quod tale antecedens maiorem proportionem haberet ad.B.quam ad.D.hoc est maiorem quàm ipsius.a.d.ad.d.e.ex.12.quinti.

Nunc si sumpta fuerit aliqua linea, puta.d o.cui.a.d.dictā habeat proportionem maiorem, clarum erit ex secunda parte decimę quinti quod.d.o.minor erit ipsa.d.e. Corrige igitur impressionem Basileę locando characterem.o.inter.d.et.e. eo quod ibi positum non fuit.

Volo nunc quod dictum maius antecedens æquale scilicet duplo ipsius.a. b. cum quadruplo ipsius.b.c.cum quadruplo ipsius.b.d.cum duplo ipsius. b. c. significetur à **ß** charactere.A.Hinc habebimus proportionem ipsius a.d.ad.d.o.ut.A.ad.B.

Ex.18.quinti postea habebimus.A.B.ad.B. vt.a.o. ad. d. o. & proportionalitate **T** euersa in.19.dicti ita erit.A.B.ad.A.vt.a.o.ad.a.d. Sed hoc vltimum antecedens in se continet id quod Archimedes scribit,hoc est duplum ipsius.a.b. quadruplū ipsius b.c. sexcuplum ipsius. b.d.& triplum ipsius.b.e. Consequens verò.A. continet du plum ipsius.a.b.quadruplum ipsius.b.c.quadruplum ipsius.b.d.& duplum ipsius.b.e.

Ex supposito deinde ipsius Archimedis & ex conuersa proportionalitate in. 19. dicta, verum est id quod dicit Archimedes, videlicet quod eadem proportio est ipsius.a.d.ad.g.h. quod quintupli ipsius.a.b.cum quintuplo ipsius. b.c.cum decuplo ipsius.b.c.cum decuplo ipsius.b.d.(quod quidem antecedens significetur per.V.) ad duplum ipsius.a b.cum quadruplo ipsius.b.c.cum sexcuplo ipsius.b.d. cum triplo ipsius.b.e.hoc est ad.A.B.
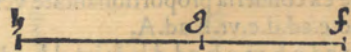
Erit igitur.V.ad.A.B.vt ipsius. a. d. ad. g. h. sed superius vbi signatum est.T.iam probatum fuit ita esse. A.B ad.A.vt ipsius.a.o.ad.a.d. Ergò ex.23.quinti Archime des verum scribit, hoc est quod ita erit ipsius. V. ad. A. vt ipsius.a.o.ad.g.h.

Clarum per se etiam est, id quod Archimed.dicit hoc est quod. V. ad A. est vt quinque

quinque ad duo, cum quodlibet ingredientium in compofito. V. ad quodlibet in-
gredientium in compofito. A. fit vt quinque ad duo. Quare ex.13. quinti verum
dicit. Vnde.a.o.ad.g.h.erit vt quinq; ad duo ex.11. eiufdē vt inquit Archimedes.

Corrige impreffionem vbi fcriptum eft, rurfus quoniam.o.a.quia oportet dicere
Rurfus quoniam.o.d.

Archimedes igitur verum dicit, quod ipfius.o.d.ad.d.a.eft vt ipfius.B. ad. A . ex



Ccc 2          con-

conuerfa proportionalitate in. 19. quinti,cum.a.d.ad. d, o, iam probatum fuit ( vbi B, ) ita effe ut.A.ad.B.

Sed in principio huius fpeculationis probatum iam fuit ita effe ipfius. d.a.ad.d.e. vt ipfius.D.ad.B.vbi notatum eft. M. quare ex.23.quinti,Archimedes verum dicit, quod.d.o.ad.d.e.erit vt.D.ad.A.

Sed cum.d.o.ad.d.e.fe habeat ut.D.ad.A.erit ex conuerfa proportionalitate iam dicta.d.e.ad.d.o. vt.A.ad.D.per euerfam vero erit. d.e.ad.a.o.vt. A. ad fuum refiduum.quod refiduum componitur ex fimplo.b.c.cum triplo.b.cum duplo.b.o.quod à te ipfo videre poteris detrahendo numeros ipfarum quantitatum quæ in.D. reperiuntur, ex numeris earundem,quæ in. A. quod quidem refiduum fignificetur à charactere.E. Vnde ex conuerfa proportionalitate verum dicit Archime. hoc eft quod ita fe habebit.o.e.ad.d.e.vt.E.ad.A.

Cum autem fit.a.b.ad.c.b.vt.c.b.ad.d.b.& ita.d.b.ad.e.b. ex fuppofito, ideo ex 17. quinti verum dicit Archim.hoc eft quod ita erit ipfius.d.e.ad.e.b.vt.a.c.ad.c.b. & vt.c.d.ad.d.b.& ex.13. eiufdem eadem proportio erit tripli ipfius.c.d. ad triplum ipfius.d.b. quæ dupli ipfius.d.e.ad duplum ipfius.e.b.vt inquit Archi.

Ex qua.13. compofitum ex.a.c.cum triplo ipfius.c.d.cum duplo ipfius. d. e. eandem proportionem habebit ad compofitū ipfius.c.b.cum triplo ipfi°.d.b.cum duplo ipfius.e.b.quam ipfius.d.e.ad.e. b. Sed horum compofitorum primum fignificetur per.H.fecundum verò fignificatum fuit per. E. vnde.H. ad.E. fe habebit vt.d. e. ad e.b.fed.E.ad.A.iam dictum eft effe vt.o.e.ad.d.e.vbi fignatum eft. ʌ. quare ex. 23 . quinti eadem proportio erit ipfius.o.e.ad.e.b.quæ.H.ad.A. vt ipfe inquit.

Ex.18. poftea eiufdem ita erit.o.b.ad.e.b.vt.H.A.ad.A.

Notandum etiam eft quod fi collectæ fuerint omnes partes compofiti. H. A. hoc eft duplum.a.b.cum duplo.b.e.cum quadruplo.b.c.cum quadruplo.b.d.cum fimplo a.c.cum triplo.c.d.cum duplo.d.e.habebitur triplum.a.b.triplum.b.d.& fexcuplum b.c.vt ipfe dixit . Quod autem hoc verum fit , cum diftinctæ fuerint omnes partes , vt in fubfcriptis his lineis videre eft, videbis quod fi ex.H.detracta fuerit fimplex.a. c.quæ quidem poftea iuncta vni ex partibus quadrupli.b.c.ipfius.A.refultabit nobis vna integra. a.b. Vnde habebimus triplum ipfius. a.b.& in.A. remanebit triplum ip fius.c.b. Deinde fi ex.H.auferatur triplum ipfius.c.d.& ipfum addatur tribus parti- bus quadrupli.b.d. ipfius.A. habebimus tres vices.b.c.quæ fi iungantur tribus, quæ remanebant in. A. vt dixi, habebimus fexcuplum ipfius.b.c. & in.A.remanebit fim plum.b.d. cum duplo ipfius. b. e. Vnde fi ex. H. demptum fuerit duplum ipfius. d. e. quod quidem iungatur cum duplo ipfius. b.e.habebimus duplum ipfius.b.d.quod coniunctum cum fimplo.b.d.quod in.A. relictum fuerat , habebimus triplum ipfius d.b. Verum igitur eft quod inquit Archimedes , hoc eft,quod. H. A. eft triplum ip- fius.a. b.fexcuplum ipfius.b.c.& triplum ipfius.b. d.

Verum etiam dicit ex eo(vt fupra probatum eft )quod.a.c: c.d: et.d.e. fe habebāt in continua proportionalitate, quare ex conuerfa proportionalitate erunt fibi inui- cem continuæ proportionales.

Nunc autem cum.a.c: c.d.et. d.e.fint continuæ proportionales in ea proportione in qua funt.a.b: c.b: d.b: et.c.b.vt in principio diximus , erit ex.22.quinti.a.c. ad. d. e.vt.a.b.ad.d.b.& fic etiam.c.b.ad.e.b. Vnde ex.24. eiufdem.a. d.ad.d.e.erit vt. a. b.cum.b.e. ad.d.b.& vt. c.b.cum.b.d.ad.e.b.& ex. 13. dicti vt . a . b. cum. b . e . bis fumpto, & cum.b.d.ad.e.b.Quare ex conuerfa proportionalitate, vt fe habet. e . d . ad.d.a.ita fe habebit.e.b.cū.d.b.ad d.b.cū.b.c.duplicato & cū. b. a . vt inquit Archi medes . Nunc antecedens vocetur.M.hoc eft. b.e. cūm.d.b.confequens verò , hoc eft

est.d.b.cum duplo.b.c.cum simplo.b.a.vocetur. N.

Animaduertendum tamen est quod impressio mendosa est ubi dicit.
vnaquæque.c.b: b.d. & cætera,
propterea quod dicendum est ita
vnaquæq;.e.b:b.d.

Nunc ex.18.quinti, quemadmodum se habet.a.e.ad.d.a.ita se habebit.M.N.ad.N.

a——————c————————d———o——e————————b

b————g————f
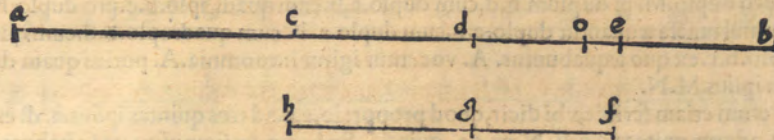
V a .5. b   b .10. o   b .10. d   b .5. e

AB a .2. b   b .4. c   b .6. d   b .3. e

A a .2. b   b .4. c   b .4. d   b .2. e

B b .2. d   b .1. e

D a .2. b   o .3. c   o .1. d

E b .1. c   b .3. d   b .2. e

H a .1. c   c .3. d   d .2. e

HA a .3. b   b .6. c   b .3. d

M b .1. d   b .1. e

N a .1. b   b .2. c   b .1. d

Q $\frac{3}{5}$   b .2. d   b .4. c   b .2. a

A b .2. e   b .4. d   b .4. o   b .2. a

HA o .3. d   b .6. c   b .3. a

Vbi

Vbi autem ſcriptum eſt
„ ad vtrunque ſimul.b.d; d.a.cum dupla.b.c.
dicendum eſt ita,
ad vtranque ſimul.b.d.b.a.cum dupla.b.c.

Inquit deinde Archi.quod ſicut ſe habet.e.a.ad.d.a.ita ſe habebit duplum.M.N. ad duplum.N. Quod quidem verum eſt ex.13.quinti, huiuſmodi verò antecedens & conſequens, Archi.manifeſtat ex ſuis partibus, ſumendo duplum.e.b.cum duplo b.d.pro duplo.M. & duplum.b.d.cum duplo.a.b.cum quadruplo.b.c.pro duplo.N. quę ſimul iuncta æquantur duplo.e.b.cum duplo.a. b. cum quadruplo.b.d.cum qua-druplo.b.c.ex quo æquabuntur. A. vocentur igitur hæc omnia. A. potius quàm du-plum ipſius.M.N.

Verum etiam ſcribit,vbi dicit,quod proportio.e.a.ad tres quintas ipſius.a. d. erit vt. A.ad tres quintas dupli. N.ex.22. quinti. Sed cum ex ſuppoſito ita ſe habeat.f. g.ad tres quintas ipſius.a.d.quemadmodum.b.e.ad.e.a. erit ex. 16 . quinti verum ꝙ dicit Archimed. hoc eſt , ita ſe habere. b. e.ad.f.g.vt.e.a.ad tres quintas ipſius.a.d.

Et per. 11. eiuſdem verum etiam erit quod ſicut ſe habet.e.b.ad.f.g. ita ſe habe-bit.A.ad tres quintas dupli. N. quod quidem duplum.N.ſignificetur per . Q .

Sed ſuperius iam demonſtratum fuit ( vbi.X.) quod.o.b.ad.b.e.ita ſe habebat vt H.A. ad. A.& nũc demum probatum fuit ita eſſe. A.ad tres quintas ipſius.Q.vt.e.b. Y ad.f.g.Quare ex.22.quinti ita erit. H.A.ad tres quintas ipſius.Q.vt.o. b. ad. f . g . vt idem inquit.

Sed. H.A.ad.Q. ( vt ex ſuis partibus videre eſt) ita ſe habet vt tres ad duo ex.13. quinti, vt inquit Archimedes.

Ipſe etiam dicit proportionem. H.A.ad tres quintas ipſius. Q. eſſe vt quinque ad duo . Pro cuius rei euidentia imaginemur tam.H.A.quam.Q. diuiſa per quinq; partes æquales, vnde ex.16.quinti habebimus quamlibet quintam partè ipſius. Q. æqualè eſſe duabus tertijs vniuſcuiuſque quintæ partis.H.A. vnde tres quintæ ipſius Q. erunt, ex communi conceptu, ſex tertiæ vnius quintæ ipſius. H. A. hoc eſt duæ quintæ.ipſius. H. A. Quare.o.b.ita ſe habebit ad.f.g. vt quinque ad duo ex commu ni cóceptu,cum.o.b.ad.f.g.probatum fuerit ſe habere vt.H.A. ad tres quintas ipſius Q. ( vbi. Y.) ſed iam probatum fuit ( vbi.ω. ) quod. o.a.ad.h.g. erat etiam vt quinque ad duo, hoc eſt quod.f.h.erit duæ quintę ipſius.a. b. Quod eſt propoſitum.

In vltima

γ   a .5. b   b .10. o   b .10. d   b .5. e

AB   a .2. b   b .4. o   b .6. d   b .3. e

A   a .2. b   b .4. c   b .4. d   b .2. e

B   b .2. d   b .1. e
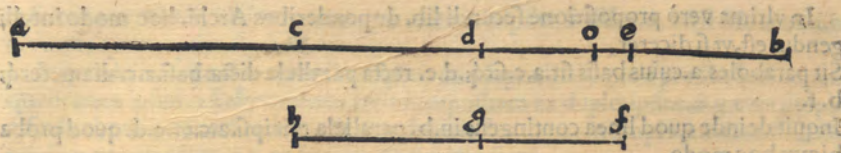
D   a .2. b   o .3. c   b .1. d

E   b .1. c   b .3. d   b .2. e

H   a .11. c   c .3. d   d .2. e

HA   a .3. b   b .6. c   b .3. d

M   b .11. d   h .1. e

N   a .11. h   b .2. c   b .11. d

Q $\frac{3}{5}$   b .2. d   b .4. c   b .2. a

A   b .2. e   b .4. d   b .4. o   b .2. a

HA   c .3. d   b .6. o   b .3. a

In

In vltima verò propositione secundi lib. de ponderibus Archi. hoc modo intelligendus est, vt si diceret,

Sit paraboles.a.cuius basis sit.a.c.sitq;.d.e. recta parallela dictæ basi.a.c. diameterq; b.f.

Inquit deinde quod linea contingens in.b. parallela erit ipsi.a.c.et.e.d. quod probabimus hoc modo.

Cum.b.f. diameter sit et.a.c.basis, clarum erit ex definitione quod.b.f.diuidet. a. c. per æqualia in.g. Vnde ex.7.vel etiam ex.46. primi Pergei. d.e.diuisa erit per æqualia à diametro.b.f. Quare verum dicit ex quinta secundi ipsius Pergei hoc est quod dicta contingens in puncto.b. parallela erit ambobus.a.c.et.e.d.

Inquit postea quod diuisa cum fuerit pars diametri quę inter.d.e.et.a.c.posita est (hoc est.g.f.) per quinque partes æquales, quarū partium media sit.h.k.diuisa etiam imaginatione sit in puncto.i. ita quod proportio ipsius.h.i.ad.i.K. eadem sit quæ inter duo solida quorum vnum(illud scilicet à quo relatio incipit, hoc est antecedens)

R  pro sua basi teneat quadratum ipsius.a.f.cuius etiam solidi altitudo composita sit ex duplo ipsius. d. g.cum simplo. a. f. Aliud verò solidum habeat pro sua basi quadratum ipsius.d.g.eius verò altitudo composita sit ex duplo ipsius.a.f.cum simplo.d.g.

Inquit nunc Archi.quod cum ita factum fuerit,ostendet punctum. i. centrum esse portionis abscissæ à tota sectione,quod frustū nominat signatū characteribus.a.d.e.c.

Sit igitur nunc.m.n.inquit, æqualis diametro.b.f.et.n.o.æqualis.b.g.sitq;.x.n.media proportionalis inter.n.m.et.n.o.et,t.n.in continua proportionalitate post.o.n. hoc est quod ea proportio quæ est ipsius.o.n.ad.n.t.eadem sit ipsius.x.n.ad.n,o.Hinc habebimus. 4. lineas in continua proportionalitate sibi inuicem coniunctas.m. n: x. n: o.n.et.t.n.

A    Vult etiam quod à linea.i.b.incipiens ab.i.versus.g.alia linea abscissa sit , cui lineæ,ita proportionata sit.f.h.vt.t.m.est ad.t.n.quæ quidem linea signata sit.i.r.

Dicit postea quod diameter.b. f. erit fortasse axis vel aliqua reliquarum diametrorum , quod quidem in. 46. primi Pergei videre est, cum omnes diametri sint inuicem paralleli ipsi axi.

Cum postea dicit , quod.a.f.et. d.g. sunt intentæ ductæque, ibi vult id em inferre,quod Pergeus vocat ordinatè, vt ex. 11. et.49.primi ipsius Pergei videre licet , vnde ex.20.eiusdem proportio.b.f.ad.b.g.erit vt quadrati. a . f. ad quadratum ipsius.d.g.vt ipse dicit .

α    Sed ita erit quadrati.m.n.ad quadratū.x.n.ex. 18.sexti Eucli.Quare ex.11.quinti quadratum ipsius.m.n.ad quadratum ipsius.n.x. eandem habebit proportionem, quam quadratum ipsius.a.f.ad quadratum ipsius.d.g. Vnde ex.18.& ex communi sciētia,eadem proportio erit ipsius.m.n.ad.n,x.quę ipsius.a.f.ad.d.g.vt inquit Arch.

Quapropter proportio cubi ipsius.m.n.ad cubum ipsius.n.x.erit vt cubi ipsius. a. f.ad cubum ipsius.d.g.vt etiam dicit ex communi scientia,nec non ex.36.vndecimi.

Inquit postea quod proportio totius sectionis.a.b.c.ad portionem.d.b. e. eadem est quæ cubi ipsius.a.f.ad cubum ipsius.d.g.quod verum est,vt aliàs tibi monstraui in diuisione parabolæ secundum aliquam propositam proportionem.

β    Quando autem dicit quod proportio cubi ipsius.m.n.ad cubum ipsius.n.x.eadem est quæ ipsius.m.n.ad.n.t. verum dicit ex. 36. vndecimi. Vnde ex. 11. quinti ita se habebit totalis sectio.a.b.c.ad portionem.d.b.c.vt.m.n.ad.n.t.& ex.17. eiusdem ita erit ipsius.m.t.ad.t.n.vt frusti.a.d.e.c.ad sectionem.d.b. e. quemadmodum ipse dicit . Sed quia superius,vbi.A.ipsa.f.h.(quæ est tres quintæ ipsius.f.g.)ad.i.r.ita relata fuit

ta fuit vt.m.t.ad.t.n. idcirco ex.11. quinti ita erit ipsius frusti.a.e. ad sectionem.d.b.
c. vt tres quintę ipsius.f.g.ad.i.r.

Inquit deinde quod proportio corporis iam supradicti, quod pro sua basi habeat
quadratum ipsius.a.f.altitudinem verò compositam ex duplo ipsius.d.g.cum simplo
a.f.ad cubum ipsius.a.f.eadem erit quæ dupli ipsius.d.g.cum simplo.a.f.ad.a.f.Quod
quidem verum est ex.33. vndecimi & ex prima sexti.

Sed superius( vbi.α.) iam probauimus eandem proportio nem esse inter.m.n.&
n.x. quæ inter.a.f.et.d.g.ideo ex conuersa proportionalitate ita erit ipsius.x.n.ad.n.
m. vt ipsius.d.g. ad. a.f.sed dupli.x.n.ad simplum.x.n.est vt dupli.d.g.ad.d.g.Qua
re ex.22. quinti dupli.x.n.ad.m.n.erit vt dupli.d.g.ad.a.f.& ex.18.eiusdem ita erit
dupli.x.n.cum simplo.m.n.ad.m.n.vt dupli. d.g. cum simplo.a.f.ad.a.f.Quare solidi



Ddd        iam

♪ iam dicti ad cubum ipsius.a.f.ex. 11. quinti erit vt dupli.x.n.cũ simplo.m.n.ad.m.n.

Superius autem vbi. β. demonstratum fuit ita esse ipsius.m.n.ad.n.t.vt cubi.m.n. ad cubum. x. n. & inter. α. et. β. probatum fuit ita esse cubi.a.f. ad cubum. d. g. vt cubi.m.n.ad cubum. x. n. Vnde ex. 11. quinti.m.n.ad.n.t.erit vt cubi.a. f. ad cubum d. g.

Dicit postea quod eadem proportio erit inter cubum. d. g. & corpus illud quod pro basi habeat quadratum ipsius.d.g. altitudinem verò vt dictum est, quæ est inter d.g. & compositum ex duplo.a.f.cum simplo. d. g. quod compositum est altitudo dicta,& verũ dicit ex ratione superius allegata pro reliquo corpore & cubo ipsius.a.f. Quare etiam quemadmodum. t. n. se habet ad duplum ipsius.o. n.cum simplo.t.n. ex ijsdem rationibus supradictis, vbi loquuti sumus de.x.n.cum.m.n.

Disponantur nũc omnia tali ordine, ita vt. u. primum sit corpus quod pro sua basi habeat quadratum ipsius.a.f.&c.

Et.y. sit cubus ipsius.a. f.et.s. sit cubus ipsius.d.g.et. z.sit corpus quod basim habet quadratum ipsius.d.g.altitudinem verò vt supradictum est,et. p. sit compositum dupli.n.x.cum simplo.m.n.et. l. sit compositum dupli ipsius. n. o. cum simplo. t. n. Sed.u.locata sit è regione.p.et.y.è regione.m. n.et.s.è regione.n.t.et.z. è regione.l. & habebimus proportionem ipsius.u.ad.y. vt.y.ad.m.n.& ipsius.y.ad.s. vt.m. n. ad. n. t. quod superius iam demonstratum fuit, vbi,δ. et.s.ad.z. ita se habebit vt.n.t.ad. l. vt vltimò probatum fuit . Quare ex. 22. quinti ita se habebit.u. ad. z. vt. p. ad. l. quemadmodum dicit Archi.

Et quia vt se habet.u.ad.z.ita facta fuit.h.i.ad.i. K.vbi.R. ideo ex.11. quinti vt se habet. h.i.ad. i.K. ita se habebit.p.ad. l. vt ipse dicit : Et ex.18. quinti ita erit. h. K. ad. K.i. vt.p.l.ad.l. & ex communi conceptu.g.f.se habebit ad. h. K. vt quintuplum ipsius.p. l.ad.p.l. & ex.22.eiusdem ita se habebit.f. g.ad.i.k.vt quintuplum ipsius.p. l.ad.l. quintuplum autem ipsius.p. l.compositum est ex quintuplo ipsius. n. m. cum decuplo ipsius.n.x.cum quintuplo ipsius.n.t. cum decuplo ipsius. n. o. vt à te facilè computare potes.

Verum etiam erit ex communi scientia quod.g.f. ad. f.k.est ut quintuplum ipsius p.l.ad duplum ipsius.p.l.eo quod superius suppositum fuit.h.K.esse quintã mediam, vnde. k. f. relinquebatur prò duabus quintis inferioribus, duplum autem.p. l. compositum est ex duplo ipsius.m.n.cum duplo ipsius.n.t.cum quadruplo ipsius . n. x. & cum quadruplo ipsius. x.o.

Ex conuersa proportionalitate deinde ita se habet,i.K.ad.i.k.ad.f.g.vt.l.ad quintuplum ipsius.p.l.et.k.f. ad.f.g. vt duplum ipsius.p.l.ad quintuplum ipsius.p. l. Vnde ex.24.quinti.i.f.se habebit ad.f.g.vt duplũ ipsius. p.l. cum simplo. l. ad quintuplum θ ipsius. p.l. Deinde ex conuersa proportionalitate quintuplum ipsius.p.l. se habebit ad duplum ipsius.p.l.cum simplo.l.vt.f.g.ad. f. i. Sed compositum dupli ipsius . p . l. cum simplo. l.æquale est duplo ipsius.m.n. cum quadruplo ipsius.x.n. cum sexcuplo ipsius.o.n.cum triplo ipsius.n.t.vt per te computare potes.

Superius enim sumpta fuit.i.r.ad quam ita se haberet. f. h. hoc est tres quintæ ipsius.f.g.vt.m.t.ad.t.n.Quare ex conuersa proportionalitate ita se habebit.i.r.ad tres quintas ipsius.f.g.vt.t.n.ad.t.m. Et quia.o.n.sumpta fuit æqualis ipsi.b.g.et.m. n.ipsi b. f. ideo.m.o.ex communi scientia æqualis erit ipsi.g.f. Vnde proportio.r.i.ad tres quintas ipsius. m.o.erit vt.n.t.ad.t.m.vt inquit Archi.

Sed vbi. θ. iam probauimus ita se habere.i.f.ad.f.g. vt duplum ipsi⁹.p. l.cum simplo.l.se habet ad quintuplum ipsius.p.l.hoc est.i.f.ad.m.o.vt duplum ipsius.p.l.cum simplo.l.ad quintuplum ipsius.p.l.

Habemus

Habemus igitur nunc omnes illas conditiones quas Archimedes in praecedenti propositione supponit. Vnde ex rationibus ibi allegatis sequitur. f.r. esse duas quin- tas ipsius. m.n. hoc est ipsius. f. b. Quapropter punctum. r. centrum erit ponderis to- tius sectionis parabolae ex. 8. secundi lib. de ponderibus eiusdem Archimedis.

Inquit nunc Archimedes, quod existente. q. centro ponderis ipsius parabolae. d. b.e. partialis, centrum frusti erit in linea recta. q.r. f. ita remotum à centro. r. quod proportio. q.r. ad partem illam ipsius. r.f. quae reperitur inter centrum. r. & centrum huius frusti aequalis est proportioni totius parabolae ad partialem. Quod quidem ve- rum est ex. 8. primi libri eiusdem.

Inquit etiam punctum. i. illud esse, eo quod cum probatum sit. f. r. duas quintas es- se ipsius. f.b. ideo. b.r. tres quintas erit ipsius. b.f. vt ipse dicit.
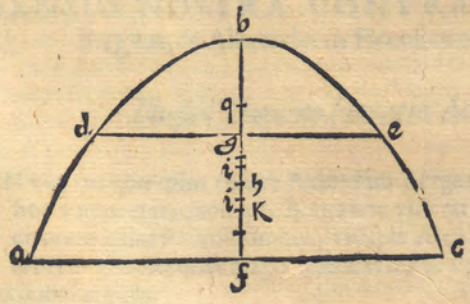
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Sed.q.b. similiter tres quintæ est ipsius.d.b.ex.8.prædicta. Quare.q.r.tres quintæ erit ipsius.f.g.ex. 19. quinti.

Dicamus igitur hoc modo cum.f.b.totum ad totum.b. r. ita se habeat vt abscissum.b.g.ad abscissum.q.b.ex.7.et.8. dicti primi libri eiusdem ideo residuum.f.g.ex f.b.ad residuum.r.q.ex.r.b.erit vt totum.f.b.ad.totum.r.b.ex. 19.quinti Eucli.

Sed iam sub. a. probauimus ita se habere frustum.a.d.e.c.ad parabolam.d.b.e. vt m.t.ad.t.n.sed vt.m.t.ad.t.n.ita assumpta fuit ( vbi. A. ).i.r.ad quam sic se haberet.f. h.hoc est tres quintæ ipsius.f. g.hoc est.q.r.quare ex.11. quinti proportio frusti.a. d.e.c.ad parabolam partialem erit vt.q.r.ad.r.i. Existente igitur.r. centro totius pa rabolæ et.q. centro partialis, ergo, i.centrum erit frusti propositi.

Sed si nullo solido intercedente, voluerimus centrum.i.frusti.a.e. citius inuenire, inueniemus primò centrum.r.totius figuræ ex.8.secundi eiusdem constituendo.b.r. tres quintas totius axis.b.f.& centrum.q.parabolæ.d.b.e.partialis similiter.

Nunc igitur manifestum est nobis, eandem proportionem fore ipsius.q.r. ad. r. i. quæ frusti. a. e. ad portionem. d. b. e. ex.8. dicta. Vnde ex coniuncta pro portionalitate ita se habebit.q.i.ad.i.r.vt.a.b.c.ad.d.b.e. sed vt.a.b.c.ad.d.b.e.ita se habet. m.n.ad.n.t.eo quod vnaquæque harum duarum proportionum sesquialtera est proportioni.f.b.ad.b. g.eo quod.f.b.ad.b.g. ita se habet.vt. m. n. ad. o. n. quare m.n.ad.t.n.ita se habebit vt.g.i.ad.r.i.vnde disiunctim.m.t.ad.t.n. ita se habebit vt q.r.ad.r.i. Iungatur igitur.r.i.quæ quidem.r.i.ita se habeat ad.r.q.vt.t. n. ad. t. m. vt habeatur centrum frusti.

DE.

Solidum maius    $u$    $p$    $n$    $m$    $n$

Cubus maior    $y$    $m$    $n$

Cubus maior    $s$    $t$    $n$    $t$

Solidum minus    $z$    $n.2.o$    $n.1.t$

$\frac{1}{2}$ $p$ $q$    $m.1.u$    $n.2.x$    $n.2.o$    $n.1.t$

$\frac{3}{1}$ $p$ $t$    $m.5.u$    $n.10.x$    $n.10.o$    $n.5.t$

$\frac{2}{1}$ $p$ $t$    $m.2.n$    $n.4.x$    $n.4.o$    $n.2.t$

$\frac{2}{1}$ $p$ $t$    $m.2.n$    $n.4.x$    $n.6.o$    $n.5.t$

$m$    $x$    $o$    $n$



# DEFENSIO NOSTRA CONTRA ANTONIVM
## Bergam, & Alexandrum Piccolhomineum.

### Illuſtri Domino Horatio Muto.

INTER ea quæ olim contra Antonium Bergam, ſermone Italico ſcripſi, hoc vnum erat, quod ipſe Berga non viderat quendam notatu dignum errorem ipſius Piccolhominei, vbi ipſe Alexander arguit quendam authorem in tractatu de magnitudine terræ & aquæ pag. 37. linea. 26. ita dicens, & erit maior aqua.

Quo

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Quo in loco clare videtur ipfum putare eandem proportionem inter diametros, quæ inter fphæras ipfas effe, nec amplius recordari eius quod fcripferat pag.24.

Piccolhom.igitur ibi fupponés centrum. D.effe magnitudinis aquæ,& intra fphæ ram terreftrem, putat omnino caufam effe vt terra fuperet aquam magnitudine, qua fi quod fi punctum. D. vt centrum fphæræ aquæ, vnum idemq; effet cum puncto.E. extremo diametri ipfius terræ, fphæra.A.G.H.fphæræ.A.B.E. dupla effe deberet, quod quidem nullo pacto fieri poteft, quamuis etiam proportio.A.H.ad diametrũ A.E.fuperbipartienfeptimas exifterêt,quæ minor effet quam fefquitertia, ita quod quando etiam. D.E. maior medietate ipfius. D.H. fuiffet, nihilominus tamen terra minor effet aqua, eò quod proportio dupla minor eft,quam tripla ad proportioné fuperbipartienfeptimas, & maior quâ tripla ad proportionem fefquiquartam. Vnde fi Piccolhom. fuppofuiffet proportionem ipfius. D.H. ad. C.E.effe fefquiquartam, rectè profectò dixiffet, fed dicere quod ubicunque exiftat punctũ. D. intra fphæram terreftrem, fequitur ipfam effe maiorem aquea, verum non eft.

Scripfi etiam quòd Piccoloho. decipiebatur vbi loquitur de diaphaneitate aquæ, pag.40.ita dicens.

   „ Et cum rationabiliter aliquis exiftimare non poteft, quod vmbra quæ facit ori
   „ ri e eclipfes Lunæ, producta fit à terra, & ab aqua fimul, vt ab vno corpore aggre
   „ gato ex ijs duobus elementis, & ad vnam communem fphæreceitatem reductis, pro
   „ pterea quod cum vmbra produci debeat à corporibus opacis,quorum opacitas effi
   „ cit illa corpora vmbrofa, aqua autem, fit corpus diaphanum,& tranfparens, nullam
   „ vmbram poterit à fe eminus producere.

Hic enim decipitur Piccolhom. duabus rationibus, quarum prima eft,quod radius luminofus non poteft multum in profundum mergi,vt probaui in.8.epiftola ad Vimercatum, altera verò eft,quod cum fphærica fit aqua maris, fupponatur etiam quod fub ea nulla terræ portio effet, & quod radij folares ipfam,non fecus ac pilam ex criftallo fabrefactam penetrarent, cum autem ipfi radij,tam ab una, quam ab alia parte fuperficiei huiufmodi globi frãgantur, ob diffimilem diaphaneitatem inter ae rem & aquam,ipfi feinuicem interfecarent, vt poft pilam criftallinam videre eft,deinde procedentes,difgregarentur,difciparenturq; quoufque nullam vim illuminatio nis haberent, quod quilibet experiri poterit mediante aliquo vafe uitreo fphærico, aqua pleno, cuiufuis magnitudinis,foli expofito.

Rationes etiam quas eodem loco Piccolho. adducit ad probandum quod fi quis in fundo maris exifteret, nullum uideret lumen, nihil ualent. Quarum prima eft, ubi ita dicit.

   „ Ille qui fe in aquam mergit, cum maiorem lucem, quæ fupra aquam eft, relin
   „ quat, iudicat pro magno temporis fpatio locum illum obfcurum, quemadmodum
   „ accidit quando per multum temporis fpatium fixis oculis in corpore Solis intuiti fu
   „ mus,ab eodem poftea eofdem amouentes,omnia obfcura nobis videntur.

Ipfe autem non confiderat quod talis obfcuritas quæ fequitur vifionem maioris luminis,parum durat, immo cito euanefcit, fed in aqua nunquam reuertimur ad videndum, neque veftigium aliquod luminis ibi videtur, in fundo maris dico, quemadmodum nobis nuntiauerunt hi qui margaritas expifcantur in imis partibus ingen tium æquorum indicorum.

Secunda uerò ratio ipfius Piccolhom.eft ubi ita dicit.

   „ Altera caufa quod nobis obfcurus appareat locus fub aqua, effe poteft obftacu
   „ lum quod aquæ habent ab opacitate terræ fub eorum fundo, etenim ficut chriftallũ

<div align="right">quamuis</div>

,, quamuis perspicuũ siue transparẽs sit, nihilominus propter obstaculum plumbi sub
,, ipso positi, efficit vt radij visuales repercussi reuertantur . ita etiam quamuis aqua sit
,, corpus transparens, nihilominus propter obstaculum terræ opacæ, quæ subsidet in
,, fundum maris efficere potest obscuras partes illas sub aqua , illis hominibus qui in
,, ipsa aqua merguntur .

In hac secunda ratione decipitur Piccolhom. Primum quia si vsque ad imam par
tem maris, Solis radius ferri posset , ille qui ibi esset, attollens oculos sursum Solem
cerneret, deinde aspiciendo ipsum fundum Maris, videret illum , ratione reflexio-
nis luminis ab ipso fundo, & ex eadem ratione speculi ab ipso adducta, quæ contra
ipsum est.

Decipitur etiam cum dicat radios visuales à speculo seu plumbo repercuti, eo ꝙ
non radij visuales sunt hi qui reflectuntur, sed sunt radij luminosi primarij, seu secun
darij qui non ab oculis exeunt sed à corpore lucido .

Scripsi etiam quod si verum esset proportionalitatem continuam quãtitatum ele-
mentorum ex proportione decupla constare, ignem pro maximo, terram verò pro
minimo terminorum sumentes, totum aggregatum ex terra, aqua, aere, & igne , ita
esset maius terra, quemadmodum mille centum & vndecim ad vnum, vnde semidia
meter regionis elementaris esset quasi aut paulo maior decuplo solum semidiame-
tro terræ, vnde inter conuexum ignis, & concauum minimi, seu inferioris orbis lu-
naris, relinqueretur quidam orbis vacuus spissitudinis vnius interualli plus quam vi-
ginti terræ semidiametrorum , quod spatium vacuum orbiculariter, maius exi-
steret ipsa totali regione elementari plus quam trigesies millies, immo si semidia
meter dicti primi orbis lunaris maior esset terrestri vt trigintanouem ad unum, dict?
orbis vacuus maior esset elementari regione plus quam. 58208. ad vnum, proportio
nalitatem igitur continuam quæ ex decupla proportionalitate resultat in elementis
esse putare est maximus error .

Subdit deinde Berga, hoc voluisse Platonem necessario requiri, vt extrema ele-
menta, nempe ignis & terra cum duobus medijs aere, & aqua coniungerentur, cum in
corporibus solidis ( quasi Bergę sint quædam corpora quæ solida non extent ) possit
dari medium æquale in geometrica proportione .

Sed vbi Plato ad sermonem de numero elementorum se confert , postquam ra-
tione creationis ignis, & terrę se proposuisse putat, vt idé de alijs duobus corporibus
medijs probet, comparatione proportionalitatis continuæ geometricæ in tribus ter-
minis, ratione rerum superficialium primò, deinde in quatuor, ratione corporearum
vtitur, ita dicens.

,, Vinculorum verò idest aptissimum atque pulcherrimum quod ex se, & ex ijs quę
,, astringunt, quam maximè vnum efficit, &c.

Quo in loco Plato inferre vult de proportionalitate geometrica trium termino-
rum, in qua ijdem ita se habent, vt medius, primi, vltimiꝗ vice fungatur, ita vt vtrius-
que ipsorum extremorum particeps fiat , cum productum quod à medio termino in
seipso progignitur idem sit ei quod ab extremis fuit, vnde medius, potentia idem est
quod productum ab extremis.

Subdit deinde Plato dicens.

,, Quando enim in tribus numeris, aut molibus, aut viribus, medium ita se habet
,, ad postremum vt primum ad medium, vicissimꝗ vt postremum cum medio, ita me-
,, dium cum primo congruit, tunc quod medium est , & primum sit & postremũ, po-
,, stremum quoque, & primum & media fiunt.

Hic

Hiç animaduertendũ eſt omnes interpretes falli, qui hoc loco Platonem de omni-
bus proportionalitatibus continuis quæ ternario numero ( alia enim Arithmetica,
alia geometrica, alia harmonica dicitur ) continentur, intelligendum eſſe cenſent,
quia de numeris, magnitudinibus, viribusq́; , aut ut dici ſolet, virtutibus mentionem
fecerit. Plato enim nihil aliud inferre voluit, quam eandem paſſionem ( ut ipſe reci-
tat ) inter medium extremaq́; vnius proportionalitatis continuæ geometricæ, tam
in quantitate, quam in qualitate reſultaturã, cum tres termini eiuſdem eſſent ſpe-
ciei, & quia quantitas in duas principes primariasq́; partes, ideſt in continuam, &
diſcretam diuiditur, hanc ob cauſam Plato hoc præcipuè ſignificat numerorum ma-
gnitudinisq́; vocabulis vtens, quibus vniuerſum quantitatis genus complectitur .

Cum verò ait vires, uniuerſum qualitatis genus inferre uult . Quia proportio &
proportionalitas tam continua quam diſcreta, non ſolum inter terminos quanti, ſed
inter eos etiam qui quali attribuuntur elucet.

Sed quod eo loco de harmonica proportionalitate quæ cũ geometrica magis ſim
bola eſt quam cum Arithmetica Plato minime intelligat, ex eiuſdem uerbis cum ita
ſcribit manifeſtè patet .

"  Quando enim medium ita ſe habet ad poſtremum ut primum ad medium, uiciſ-
" ſimq́; ut poſtremum cum medio ita medium cum primo congruit.

Id enim in harmonica proportionalitate non cernitur in qua primus terminus ad
poſtremum, & non ad medium, ita ſe habet geometricè ut differentia inter primum
& medium ad differentiam inter medium & ultimum.

Quod ſi clarum eſt ipſum de harmonica proportionalitate nullo modo intellige-
re, quanto minus de Arithmetica, quæ cum geometrica nihil habet commune.

Cum ucrò Plato ait .

"  Tunc quod medium eſt & primum fit & poſtremum, poſtremum quoque, & pri-
" mum media fiunt, &c.

Nihil aliud oſtendere uult, quam ſimilitudinem quæ inter huiuſmodi medium &
extrema intercedit, cum ipſum medium ad poſtremum, quem primus ad ſeipſum,
eundem reſpectum habeat, in quo eſt ſimilis primo, & contra ad primum terminũ,
eundem reſpectum, quem poſtremum ad ſeipſum habet, unde hac ratione ultimum
repręſentat, uolens Plato inferre de conuenientia quę inter media elementa, & ex-
trema intercedit, ut aquæ inter aerem, & terram, cum aqua, ratione ſuæ frigiditatis,
terrę, ratione uero ſuæ humiditatis aeri ſimilis euadat . Aer uero qui inter ignem,
aquamq́; ponitur quod ad caliditatem attinet cum igne, quod uero ad humidita-
tem ſpectat cum aqua communicet.

Sed quia Plato multis in rebus doctrinam Pythagoricam ſequutus eſt, Pythago-
rici aut em omnia numeris metiebantur, & de omni re ſecundum numerorum ratio
nem diſſerebant, uidensq́; Plato quod inter duos numeros ſuperficiales, inuicẽq́;
ſimiles exiſtentes, unum tantum numerum medium in proportionalitate continua
geometrica cadere poteſt, ideo ſubiungit.

"  Quod ſi uniuerſi corpus latitudinem habere debuiſſet, nullam uerò profundita-
" tem, unum ſanè, tum ad ſeipſum, tum ad extrema uincienda interiectum medium
" ſuffeciſſet .

Sequitur poſtea ſic.

"  Sed cum ſoliditatem mundus requireret, ſolida uerò non uno, ſed duobus ſem-
" per modis copulentur, inter ignem, & terram, Deus, Aerem, Aquamque loca-
" uit, &c.

<div align="right">Volens</div>

## EPISTOLAE.                            401

Volens inferre, quod quemadmodum inter duos numeros folidos, & inuicem fimiles, vn° tâtū medius proportionalis intercedere nó poteft, fed duo neceffariô re quiruntur ( vt ex ijs quæ Euclid.8.lib.16.17.18.et.19.propofitione proponit viden tur)ita dictâte ratione inter igneum, terreumq; corpus duo corpora interiecta efsét, non ratione proportionalitatis continuæ in quantitate eorūdem corporum, fed pro pter fimilitudinem connexionis, cum productum ex duobus medijs proportionali bus æquale fit producto ab extremis, & idem refpectus, quem primum ipforum qua tuor ad fecundum habet, fecundi ad tertium extet, vnde fecundūm primo fimile euadit, & contra, refpectus qui eft quarti ad tertium, fit etiam tertij ad fecundum, vn de ipfum tertium, ratione vltimi fubit, & eius imaginem induit, & hanc ob caufam fic fcribit Plato.

" Propterea ex huiufmodi rebus numero quaternario conclufis, mundi corpus con
" flatum eft, ea connexum comparatione qua dixi. Ex quo feipfum amicitia concor-
" di complectitur, &c.

Vbi Platonem, elementa maiora, minorâue in proportionalitate continua, néc geometrica, nec alterius cuiufuis generis effe noluiffe, clarè perfpicitur, fed huiufmo di fimilitudine, in eo quod media elementa cum extremis conueniunt eft vfus, quæ quidem conuenientia, nullibi maior, quam in proportionalitate continua geome trica reperitur. Sed etiam fi Plato de huiufmodi corporea elementorum magnitu dine feipfum intelligi voluiffet, fi femidiameter regionis elementaris ex æquo vt.39 ad vnum, refpectu femidiametri terræ fuiffet, aqua, ipfam terram, magis quam tri gefies, & octies, non folum decies, & aer quoque eandem magis quam.1500.& ignis magis quam.55000.partibus magnitudine fuperaret.

Subftantia vero rerum quas fcripferam circa finem illius confiderationis talis fuit.

Nunc autem tempus effe videtur, vt ego etiam, ne tantum deftruxiffe, fed etiam conftruxiffe videar aliquid pro veritate differam.

Non eft igitur dubium, folidæ doctrinæ viris, quin præftantiffimus Piccolo. fe cutus fit tutam viam ad explorandum, quod terra maior fit quam aqua, metiendo vtriufque horum corporum fuperficiem detectam. Omittamus autem compenfa tionem illam curuitatis, & concauitatis vallium, & montium, &c. quam ipfe Piccolo. propè finem fexti cap.vellet dare fluminibus, ftagnis, fontibus, & eiufmodi aquis. eo enim in loco labitur Piccolo.vbi non confiderat, quod eiufmodi obliquis fuper ficiebus non refpondent anguli folidi centri fphæræ, qui refpiciunt eorum bafim ad rectos angulos. Sed poftquam Piccolo.comperit fuperficiem terrę detectam, effe maiorem apparente fuperficie fphærica aquæ, proculdubio poterat concludere ter ram effe maiorem aqua, ficuti fecit, etiâ fi aqua profunda effet pyramidaliter vfq; ad mundi centrum, ideft.3500.milliaria, fupponendo tantum effe huius globi femi diametrum.

Verum quia poffet aliquis dubitare circa diligentiam Piccolo.in hifcæ duabus fu perficiebus dimetiendis, vifum eft mihi non alienum fequi aliam viam pro hac veri tate probanda, fupponendo verum effe, quod non vnus folus metitus fuerit, fed mul ti, ideft fupponendo verū effe quod maris profunditas menfurari poffit, & præterea, quod non modo ipfius maris maxima profunditas non perueniat ad quingentos paf fus, ficuti refert Piccolo.in fine fui tractatus, & mihi afferuerunt Hifpani multi, & Lufitani præftantiffimi nautæ, tum Venetijs, tum Parmæ, in Aula Sereniffimæ quon dam Principis, inter quos, Venetijs fuit Illuftris Rodericus Guzmanus, Dominus Francifcus Lopes, Dominus Garzias de Seuilia, multiq; alij. Parmæ autem varij

quos omnes recenfere moleftum effet . Sed etiam fupponendo quod maxima
pelagi profunditas fit, non modo. 500.paffuum, fed etiam. 500.millium paffuum, vt
dixi , & quod mare fit huius profunditatis, non vno in loco tantum, aut multis, fed
quod fupra totam etiam faciem terræ, mare tantę profunditatis ipfam terram vn-
dique operiret, ideft, quod vbicunque nunc terra detecta eft, effet aqua, fpiffitudi-
nis.500.millium paffuum. Atque vt planius intelligar fupponendo quod ficuti to-
tus huius globi femidiameter eft milliariũ. 3500. Terreftris partis femidiameter
effet tñ. 3000. & reliquum femidiametri, id eft quingenta milliaria effet craffitudo
fiue profunditas orbis aquei, in quo nihil neceffe effet laborare in dimetiendis fon-
tibus, fluminibus, lacubus, ftagnis, paludibus,& huiufmodi particulis nullius momen
ti apud peritos, nec curare fubterraneas aquas cauernarum, aut aliorum terræ cauo-
rum, feu terræ porrofitatum, quæ omnia funt circa ipfius terræ fuperficiem. Quia ve
rifimile non eft naturam eiufmodi caua fiue fpong ofitates produxiffe demiffius li-
bramenti maris . Supponendo igitur ea quæ nunc dicta funt , terra tamen effet ferè
duplo maior aqua, hoc eft, vt.12.ad.7. Quod quidem, cuiuis mathematicæ philofo-
phiæ mediocriter perito, fupputatu facillimum eft . Cum proportio diametrorum,
feu femidiametrorum, tertia pars exiftat proportionis eorundem fphærarum . Sed
vt parum periti minore labore fupputare poffint.

Primum fciendum eft, quod fupponendo diametrum globi, ex terra,& aqua com
pofiti,effe.3500.milliarium, & femidiametrum puræ terreftris partis effe.3000.tan
tum, eiufmodi proportio erit ut.7.ad. 6.quia communis maior numerator horum
duum femidiametrorum erit. 500. qui in maiorem ingredietur fepties, in minorem
autem fexies. Et eiufmodi proportio fuperparticularis , vocatur fefquifexta , cuius
triplum erit vt. 57.cum fexta parte ad.36.& idem erit inter dictum globum compo-
fitum, & partem terreftrem fimplicem . Quare fubtrahendo puram, feu fimplicem
partem terreftrem, ex compofito, reliqua pars erit, vt. 21. cum fexta, pro quantitate
aquei orbis, ad quam, terreftris quantitas.36.erit ferè in eadẽ proportione, quæ.12.
ad.7.

Nunc fortaffe alienum non erit videre quanto ferè maior effet terra, quam tota
aqua, non dico autẽ folum de parte illa maximæ e ius profunditatis , quæ nufquam
ad quingentos paffus peruenit , fed de ficto illo orbe aqueo, profunditatis.500. paf-
fuum, qui tẽtum terreftrem orbem circundaret, & tegeret, fupponendo quod per
quingentos paffus profunditatis, quidquid eft terra, effet aqua, ideft fuppofito quod
ex totius orbis compofiti femidiametro exiftente.3500.milliarium, purę terræ femi
diameter effet milliarium. 3499. cum dimidio . Supponendo igitur, vt fupradixi .
Comperietur quod terra effet maior aqua amplius quam.2333. vicibus. Sed quia
partes terræ detectæ rumpunt eiufmodi fictum orbem aqueum, quæ quidem partes,
funt ampliores fuperficię aquæ, vt obferuauit Piccolo.atque alij præftătes viri, ideo
fequetur, vt terra fit maior aqua amplius.4666.vicibus imo amplius quinquies mil-
lecuplo . Si autem quis diceret, in quantitate aquæ computari etiam illam, quæ gi-
gnatur ex vaporibus, qui globum hunc compofitum circundant: refpondeo quod
non modò ei concedo computari eiufmodi aquam, fed fupponendo etiam quod to
tus locus à vaporibus occupatus , qui attolluntur. 52 . milliaria fupra fuperficiem
huius globi, vt iam fupradictum eft, totus effet aqueus,& amplius, fupponendo quod
orbis hic aqueus effet fpiffitudinis, fiue altitudinis quingentorum milliarium fupra
totum ipfum globum compofitum . Tamen terra effet maior ipfa aqua ferè duplo;
qua de re, quifque eiufmodi fupputationum peritus certior fieri poterit. Vnde iti-

dem

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dem affirmare possemus, terram non solum maiorem esse aqua, sed aqua & præterea aere, si aer non tam altè pertingit, quam multi alij præter Piccolo. sentiunt, qui dicunt inde euenire quod aerea humiditas non tam altè ascendere potest, quoniam humiditas ipsa grauitatem secum affert, præterquam quod nubium situs ostendit supra eas materiam esse rariorem quam sint ipsę nubes, infra vero densiorem. Corpora enim eousque ascendunt donec inueniunt constitutionem mediam formæ æqualis ( vt ita dicam ) suis. Quare materia illa quæ impropriè ignis vocatur ( non enim est ignis ) incipit carere humiditate ( qua mediante aer definitur ) circa quinquagesimum secundum milliarium supra superficiem terræ, vt iam supradixi à Vitellione demonstratum fuisse. Aristo. autem affert rationē quare nubes altius nó transcēdāt. Vnde apparet tertiam aeris regionem impropriè aerem appellari, si humiditate caret, vt ait Arist. qua mediante aer definitur, immo potius retinet ignis naturam, vt etiam asserunt interpretes Aristotelis in primum Meteororum. Qui Aristo. in locis supra citatis itidem ostendit se etiam huius modi esse opinionis.

Quod autem attinet ad probandum quod superficies terrę detecta sit altior quam superficies detecta aquæ, id tam clarum est sua sponte philosophis, qui sciunt quid sit altum, quidue demissum, quod superfluum esset quidquid super hoc dicerem præterquam, quòd constat ex demonstratione ab Aristo. facta textu 31. li. 2. de cœlo, in quo agit de corporibus in aqua positis, vnde eiusmodi veritas planissimè aperitur. Omittimus etiam quod præstantes Moderni omnes, eam pro manifestissima ponūt, sicuti apud omnes sani iudicij homines reuera existimatur.

Hæc enim sunt quæ in fine illius considerationis scripseram.

Anno autem præterito editus in lucem fuit tractatus quidam Pulcherrimus, ab Excellentissimo, nec non Doctissimo viro Augustino Michele, Patritio Veneto, ad corroborandam opinionem antiquorum, vbi tot authoritates, totq; rationes adducit, vt nil amplius dici possit. At ego sensum, rationemq;, & non authoritatem aliquā sequutus sum: cum verò dico sensum, de sensu illorum intelligo, qui profunditatem maris metiti sunt, vt non mihi solum, sed, & Piccolo. & alijs permultis retulerunt; de ratione vero à me adducta, aliorum sit iudicium.

Sed iste mirabilis & Excellentissimus vir, verba mea non accepit in eo sensu, vt ego scripsi, ita vt omnino alienas consequentias sibi confingat, quemadmodū pag. 3. sui tractatus inquit, me non concedere naturam produxisse in magna quantitate, atque immensa, id totum, quod bonum, & necessarium est. Hanc enim consequentiam ipse colligit ex eo, quod ego pag. 19. meæ considerationis contra Antonium Bergam scripseram, quod videntur multa corpora alijs nobiliora, nihilominus minora, eo quod quantitas non sequitur nobilitatem, neque ab ea pendet, ita vt res illa quæ nobilior est, necessarium sit vt etiam maior existat. Sed Excellentissimus istæ vir scribit ita me dixisse.

Multa immo infinita corpora sunt nobilia, & necessaria, nihilominus sunt paruę molis.

Vide igitur quantum hoc distat ab illo.

Præterea cap. 12. aliam consequentiam facit, quam ego non tam amplam facio. Ipse enim me inferre vult in alijs terrę partibus cauernas non reperiri, eo quod Montes sint cauernosi. Aspice quæso. pag. 29. meæ considerationis, & clarè videbis me nullo modo negare illas concauitates seu porrositates terræ extra montana loca, circa superficiem terræ, vsque ad æquilibrium, orbiculariter, infimæ profunditatis maris.

Sed putare inferius has porrositates reperiri, cum nulla ratio nobis persuasibilis

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adhuc ab aliquo prodita fit, idoneum nullo pacto effet. Rationes autem ab ipfo Excellentiffimo Augustino adductas circa huiufmodi rem, alij dijudicent, de authoritatibus verò, nihil dicam, quia ab illis petendæ funt, qui profitentur tales facultates, quorum vnius tantummodo authoritas præualere deberet, contra omnes alias eorū qui nunquam attigerunt fummis labris orificia harum fcientiarū. Vt fi exempli gratia non folum authoritas illorum virorum, quos ipfe recenfuit, fufficiens effet vt puta Pioccolo. Naibodæ, Bordini, Clauij, reliquorumq; fautorum verę opinionis, fed Francifci Maurolici tantummodo, qui in primo Dialogo fuæ cofmographiæ ita inquit.

„    Exiftimo autem totum terræ corpus rigidum effe faxum, nam fi arena effet, aut
„ gleba fragilis, ita humorem imbiberet, vt cum eo quafi confunderetur; huc ac-
„ cedit, quod fi mineræ, ac rupes, quæ funt grauiffimæ partes in ipfa plerunque fuper-
„ ficie comperiuntur, multo magis apud centrum effe debent. Videtur ita ratio exi-
„ gere, vt grauiora centro quoque fint propinquiora.

Hæc igitur fola authoritas, inftar reliquarum omnium fufficere poffet. Verum de authoritatibus minime curandum eft, vbi fenfus, ratioq; vera illis opponuntur.

Quod autem numerus animalium aquatilium maior exiftat numero terreftrium, fatis refpondimus pag. 4 1. noftræ confiderationis.

Sed in cap. 1 4. Excellentiffimus Auguftinus ita inquit ( vt etiam fuperius dixerat )
„ quod certiorem cognitionem homo non habet illa, quæ à fenfu prouenit. Et quod
„ nemo eft qui afpiciat terram, & aquam, quod hanc maiorem illa non iudicet, & nó
„ exiftimet.

Quod autem certiorem cognitionem homo non habeat illa, quæ à fenfu prouenit, concedendum non cenfeo. Nam omnis cognitio mathematica ( cum primum gradum certitudinis obtineat ) ab ipfo fenfu fieret, quod omnino alienum eft à veritate. Senfus enim nunquam vidit incommenfurabilitates magnitudinum, vel incoincidentias linearum non tangentium cum curuitate hyperbolica, aut angulum contingentiæ aliquem, nec ( vt vno verbo dicam ) aliquam conclufionem mathematicam, quam volueris. Neque per fenfum eft fcire, inquit Ariftoteles. Cognitio igitur fenfi·iua, certior non eft illa, quæ per habitum fcientificum acquiritur.

Ad reliqua verò, fupponamus nos tunc fuiffe in Arca Noe, cū aquæ cooperiebant omnia cacumina montium, vbi nullum terræ veftigium videbatur, quare proculdubio aquam iudicaremus, atque exiftimaremus maiorem terra, dū nulla alia re vteremur nifi fenfu abfque alio difcurfu intellectuali, ut reliqua illa animalia irrationalia, quæ nobifcum erant in dicta arca. Nó fufficit igitur fuperficiem aquæ tantummodo afpicere, quia neque tunc temporis, aqua erat maior terra, etiam fi non folum tot cubitis attolleretur fupra cacumina montium, fed quingenta milliaria, vt fupradiximus.

Ratio autem illa, ex infinitis, ab ipfo, eo in loco adducta, talis eft.
„    Aqua eft eccentrica ad terram, & pro cêtro habet centrum grauitatis terræ, aqua
„ igitur maioris eft amplitudinis ipfa terra.
„ Hanc etiam confequentiam alijs relinquo Philofophis dijudicandam.

Subfequitur poftea dicens.
„    Præterea proprius locus terræ, eft fuperficies aquæ, igitur terram oportet ab
„ aqua tegi.

Ad hoc etiam aliquis poffet quærere, quis nam erit locus illius partis terræ detectæ ab aqua? nulli dubium erit quin fuperficies aeris, & non aquæ exiftet.

                                                                                    Nunc

Nunc autem si locus terræ est sub aqua, ergo locus aquæ proprius est sub aere, & non sub terra, vnde non erit rationabile putare maiorem copiam aquarum existere in cauernis subterraneis, quam supra superficiem terræ. Adde quod locus illarum aquarum non esset superficies aeris, sed terræ, vnde non minus locus aquç esset terra, quam locus terræ, aqua. Sed missa faciamus hæc.

Cap. verò. 20. ita inquit.

Materia elementorum æqualis est. Ergo aqua maior est terra.

Hæc enim consequentia verissima esset. Sed nullus vnquam Philosophus (vt Philosophus dico ) concedet totam materiam elementarem, in quatuor æquales partes esse diuisam.

Cap. verò. 21. inquit me dixisse non suffecturam paucam spissitudinem. Eo enim in loco pag. 26. mei tractatus contradicens ipsi Bergæ, dixi, quod secundum ipsum Bergam non sufficeret pauca spissitudo.

Similiter etiam dixi, quod secundum ipsum, quanto remotius diffunditur lumen fortasse tantò magis illuminat. Putans ipse Berga quod in propinquo debilius existeret dictum lumen. Et propterea dixi, quod apud ipsum fortasse nihil valet illa propositio, quæ dicit. Agens in propinquo, fortius agit quam in remoto.

Cap. autem. 22. vbi Excellentissimus Augustinus inquit, vnum tantummodo elementum non sufficere ad generationem mistorum. Hoc enim concedo, sed hoc nihil ad me spectat, eo quod meum responsum ad Bergam, erat circa transitum luminis, & non circa generationem elementorum.

Cap. demum. 23. pag. 20. linea. 10. vbi scribit me dixisse, iudicare, oportebat scribere, dubitare.

Puto tamen hoc vocabulum esse errorem Thypographi, quamuis in correctione illud non inuenerim, quia vt ego multoties expertus sum, difficillimum omnes Thypographi errores corrigere, neque (vt fertur) Argi oculi sufficerent.

Hactenus enim in mei defensionem hæc subiungere volui.

Ad defensionem autem Piccolo. aliorumq; virorum meæ opinionis, nec non de proportione duplicata profunditatis maris ad suam amplitudinem, ex consequentia pyramidali: alijsq; similibus rationibus, prodeant alij. Huiusmodi tamen Doctissimi viri ingenium, memoriam, nec non doctrinam valde admiror, atque obseruo.

## DE METHODO PRODVCTIONIS FRACTORVM
### qua vtuntur Pedemontani Agrimensores.

*Anselmo Rosemburg Agrimensori Cæsareo.*

ETHODVS quàm mihi scribis in Prouincia tua maximè in vsu esse, nimis longa atque prolixa est, Pedemontani verò Agrimensores in productione fractorum, valde breui methodo vti solent, quam libenter tibi scribo, eo maxime, vt videas quam rationabiliter operentur.

Scire igitur primum te oportet illos, maximam eorum communem mensuram vocare Trabucum, cuius sextam partem vocant Pedem, duodecimam verò pedis, Vnciam, duodecimã autê vnciæ punctũ, duodecimã demum puncti; Attomum.

Quotiescunque igitur multiplicant trabucum, per trabucum nulli dubium est quin producant trabucum superficialem scilicet. Similiter

Similiter multiplicando pedes, vncias, puncta, & attoma per trabucum, produ-
cunt pedes, vncias, puncta, & attoma superficialia rectangula oblonga, quorum lon-
gitudo est ipsius trabuchi, latitudo vero lineæ dictarum specierum.

Dum vero multiplicant pedem per pedem, nulli dubium est quin producant pe-
dem quadratum, sed apud ipsos non vocatur quadratum, quamuis reuera ita sit, sed
illud vocant duas vncias, quæ quidem sunt rectangula oblonga iam hic supradicta,
quarum vniuscuiusque longitudo sit vnius trabuchi, latitudo vero vnius duodecimæ
partis ipsius pedis linearis.

Productum autem pedis per vnciam, vocant duo puncta, quæ etiam sunt duo re-
ctangula oblonga, vt supra.

Productum deinde vnciæ per vnciam, vocant duos attomos, qui etiã sunt duo re-
ctangula oblonga, vt dictum est, quæ omnia scientificè videbimus.

Pro cuius rei cognitione, sit, exempli gratia. a. e. vnus Trabuchus linearis. e. i. ve-
ro vnus pes. i. o. autem vna vncia, o. u. postea vnum punctum, et . u . t. vnus At-
tomus.

Vnde. e. i. erit sexta pars ipsius. a. e. et. i. o. duodecima ipsius. e. i. et. o. u. duodecima
ipsius. i. o. et. u. t. duodecima ipsius. o. u. Sit etiam. a. b. æqualis. a. e. lineæ & sic. e. d: i.
f: o. g: o. n. &c. terminenturq; parallelogramma. b. e: d: i: f. o: g. u. et. c. t. vnde. b. e. erit
trabuchum quadratum, et. d. i. pes rectangulus oblongus vt supra, et. f. o. vncia rectan-
gula oblonga, et. g. u. punctum rectangulum oblongum, et. c. t. attomus rectangu-
lus oblongus.

De producto igitur trabuchi per trabuchũ, nulli dubium est quin sit quadratum.
a. d. vt superius diximus.

Productum autem trabuchi cum pede erit. d. i. sexta pars ipsius. a. d. cum. e. i. sit sex
ta ipsius. a. e. ex prima sexti vel. 18. aut. 19. septimi, siue etiam ex. 15. quinti Eucli.

Productum autem pedis cum pede erit . e. K. quadratum, quod probandum est
                                                                       duplum
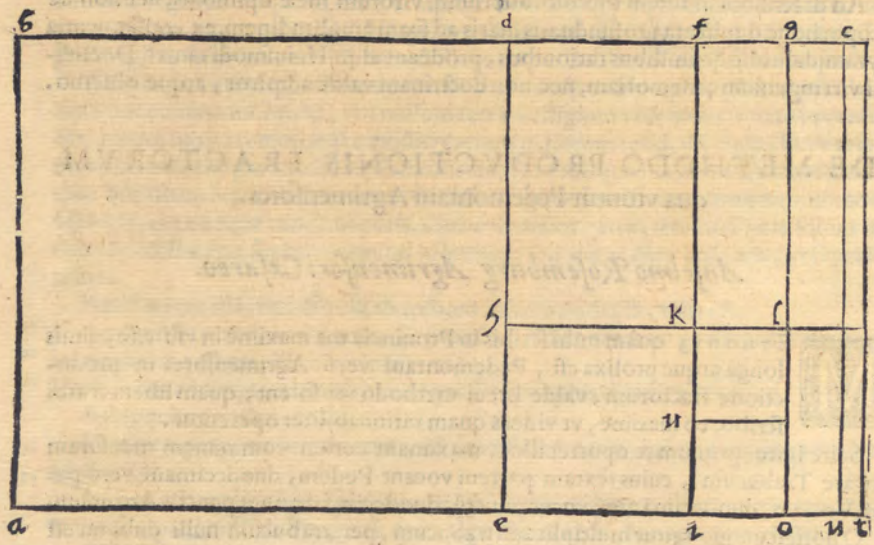
duplum eſſe rectangulo. f.o. Nã.K.i.ſexta pars eſt ipſius.f.i.ex ſuppoſito,et.i.o.duo-
decima ipſius.e.i.proportio igitur.e.i.ad.o.i.dupla eſt proportioni ipſius.f.i.ad.K. i.
quare.K.e.duplo maius eſt ipſius.f.o. eo quod ſi.i.o.vel.f. g. ( quod idem eſt ) duplo
maius eſſet ipſo latere preſenti.o.i.vel.f.g.tunc.f.o.æquale eſſet ipſi.K.e.ex.15.ſexti
vel.20.ſeptimi quod quidem.f.o.duplo maius eſſet ipſo præſenti. f. o. Rectè igitur
inquiunt dicentes productum pedis cum pede eſſe duas vncias, vel ſi mauis, ita dicas
e.K.ſexta pars eſt ipſius.d.i.ex iam dictis propoſitionibus.f.o. autem eſt duodecima
ipſius.d.i.ex ijſdem, cum ex ſuppoſito.i.o.duodecima ſit ipſius.e.i.quare.e.K. duplũ
erit ipſius.f.o.ex communi notione.

Productum verò pedis cum vncia.ſit.K.o.quod probabimus ex ijſdem rationibus
duplum eſſe ipſius.g.u. puncti rectanguli oblongi. Nam.l.o. ſexta pars ſimiliter eſt
ipſius.g.o.et.o.u.duodecima ipſius.o.i.quare proportio.i.o.ad.o.n.dupla eſt propor
tioni.g.o.ad.o.l. ſequitur ergo ex prædictis rationibus.k.o. duplum eſſe ipſius. g . u.
vel ſic, vt in præcedenti, cum.K.o.ſit ſextã pars ipſius.f.o. ex dictis propoſitionibus.
g.u. verò duodecima eiuſdem.f.o.ex ijſdem, nam.o.u.duodecima eſt ipſius.o.i.ergo
K.o.duplo maius eſt ipſo.g.u.

Ex ijſdemmet rationibus productum.l.u.pedis cum puncto duplum eſt ipſius.c.t.
attomi rectanguli oblongi.

Probandum nunc relinquitur productum.o.n.vnciæ cum vncia, quod eſt quadra-
tum, duplum eſſe ipſius.c.t.attomi rectanguli oblongi. Nam.i.n.eſt pars vna ex.72.
ipſius.c.u.et.u.t.pars vna ex.144.ipſius.o.i.ex ſuppoſito, quare proportio.i.o.ad.u.t.
dupla eſt proportioni ipſius.c.u..ad.n.i.ex dictis igitur rationibus.o. n. duplo maius
eſt ipſo.c.t. Vel ſi placet dicas.n. o.eſt vna pars ex.72.ipſius.f.o. ex ſupradictis , eo
quod.n.i.ita ſe habet ad.f.i. vt vnitas ad.72.ſed ex ijſdem rationibus.c.t.pars vna ex.
144.eſt ipſius.f.o.eo quod ita ſe habet.u.t.ad.o.i.quare.o. n. duplo maius erit ipſo .
c.t.

Propoſitum ſit nobis nunc, exercitij gratia , quærere ſuperficiem alicuius rectan
guli, cuius vnum latus ſit trabuchorũ. 3. pedum.2.& vnciarum.3.aliud vero latus ſit
trabuchorum.2.pedum.3. vnciarum vero.2.

Huiuſmodi autem methodo mediante, multiplicando primum latus dictũ.3.2.3.
per numerum trabucorum ſecundi lateris.2.ſcilicet producentur nobis primò trabu
cha ſuperficialia.6.pedes.4.& vnciæ.6.omnia rectagula, vt dictum eſt. Multiplican-
do deinde idem primum latus.3.2.3.per pedes.3. ſecundi lateris . Ex trabuchis. 3.
primi lateris cum. 3. pedibus ſecundi , producentur. 9. pedes rectanguli , hoc eſt
vnus trabuchus cum tribus pedibus rectangulis . Ex pedibus autem huius. 2. cum
ijſdem alterius lateris.3.producentur.12.vnciæ rectangulæ ideſt vnus pes rectangu-
lus. Ex ijſdem pedibus.3.ſecundi lateris, cum.3.vncijs primi lateris producentur.

| Trabucha. | pedes. | vnciæ. | | |
|---|---|---|---|---|
| 3. | 2. | 3. | | |
| 2. | 3. | 2. | | |
| 6. | 4. | 6. | | |
| 1. | 3. | 1. | 6. | |
| | 1. | 6. | 8. | |
| | | | 1. | |
| 8. | 3. | 2. | 3. | |

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18. puncta rectangula, hoc est vna vncia cum. 6. punctis rectangulis. Deinde ex multiplicatione vnciarum. 2. secundi lateris, cum. 3. trabuchis primi lateris, producentur. 6. vnciæ. Ex multiplicatione postea dictarum. 2. vnciarum secundi lateris cum. 2. pedibus primi, producentur. 8. puncta.

Demum ex ijsdem. 2. vncijs secundi lateris cum. 3. primi, producentur. 12. attomi, idest vnum punctum. Quæ omnia collecta facient trabucha. 8. pedes. 3. uncias 2. & attomi. 3. omnes rectanguli oblongi. Pulcherrima profecto operatio.

| Trabucha. | pedes. | vnciæ. | | |
|---|---|---|---|---|
| 3. | 2. | 3. | | |
| 3. | 3. | 2. | | |
| 6. | 4. | 6. | | |
| 1. | 3. | 1. | 6. | |
| | 1. | 6. | 8. | |
| | | | 1. | |
| 8. | 3. | 2. | 3. | |

Videamus nunc exercitij causa, vt dixi, quomodo conueniat calculus iste cum calculo ordinario communi?

Nam quotiescunque dicta latera, fracta fuerint in vncias, primum latus erit vnciarum. 243. secundum autem. 182. productum vero vnius in alterum erit vnciarum quadratarum. 44226. quod quidem productum cum diuisum fuerit per. 5184. vncias quadratas vnius trabuchi quadrati, prouentus erit. 8. trabuchorum, reliquus verò numerus, siue fractus, erit vnciarum quadratarum. 2754. qui cum diuisus fuerit per numerum. 144. vnciarum vnius pedis quadrati, prouenient pedes. 19. quadrati cum vncijs. 18. superabundantibus, dicti autem pedes. 19. significant tres pedes rectangulos oblongos cum vno pede quadrato, hoc est cum duabus vncijs rectangulis oblongis, vt supra.

Videndum nunc est, vtrum illæ. 18. vnciæ æquipolleant tribus punctis rectangulis oblongis: sed hoc manifestè videre est, ex hoc, quia quælibet vncia rectangula oblonga componitur ex. 72. quadratis, punctum autem rectangulum oblongum, cù sit duodecima pars ipsius vnciæ rectangulæ oblongæ, ipsum componetur ex. 6. vncijs quadratis. 18. igitur vncijs quadratis, triplum erit ipsius puncti rectanguli dicti. Vnde clarè patet, quod, quotiescunque voluerimus scire proportionem ipsarum vnciarum quadratarum superabundantium, ad punctum rectangulum oblongum, si dixerimus ex regula de tribus, si. 72. (vncia rectangula oblonga) dat. 18. quid dabùt 12? puncta rectangula oblonga, quarum vnaquæque est duodecima pars ipsius vnciæ rectangulæ oblongæ, in præsenti autem casu prouenient. 3. pro quarto termino quæsito, & habebimus propsitum.

SO-

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## SOLVTIO CVIVSDAM QVÆSITI.

### *Magnifico Ludouico Fauzzoni amico cariſſimo.*

VI quæſiti ſolutio quam neſcio quis te docuit, valde diuerſa eſt à vera. quæſitum enim tale fuit.

Reperiuntur quatuor ſocij, Ludouicus, Hieronymus, Franciſcus, & Laurentius quorum primus, Ludouicus ſcilicet, poſuit aureos. 6000. Hierony mus verò aureos. 5000. Franciſcus autem. 2000. & Laurentius. 1000. quorum ſumma faciebat aureos. 14000. interim tamen de tali ſumma Ludouicus recepit aureos 2000. Hieronymus verò. 1000. Franciſcus autem. 900. & Laurentius. 800. quapropter in ſumma reſidua Ludouicus non habebat niſi aureos. 4000. Hieronymus etiã 4000. Franciſcus. 1100. & Laurentius. 200. quorum ſumma erat. 9300. Nunc autem iſti ſocij cupiunt augere hanc ſummam per aureos. 20000. tali tamen conditione quod quilibet tantum tribuat vt in totali ſumma, tantam partem unus habeat, quantam alter.

Hoc autem problema tam facile eſt, & cum ſuo theoremate ita coniunctum, quod miror amicum noſtrum illud illico non vidiſſe.

Accipe igitur illos aureos. 20000. & eos collige cum ſumma. 9300. vnde habebis aureos. 29300. pro ſũma totali, cuius quarta pars erit. 7325. quã vnuſquiſq; poſtea habebit in dicta ſumma. Sed ut reperias quantitatem aureorum quam quilibet prius debet contribuere, vt poſtea habeat aureos. 7325. in dicta ſocietate. Iubeo, vt Ludouicus demat illos aureos. 4000. quos demum habebat, ex. 7325. reliquum autem erit. 3325. qui quidem numerus erit aureorum nunc contribuendorum ipſius Ludouici. Demptis ſimiliter aureis. 4000. ex dictis. 7325. remancbũt. 3325. pro contributione ipſius Hieronymi. Deinde ſi ex. 7325. extracti fuerint aurei. 1100. relinquent. 6225. pro contributione Franciſci. Demptis demum. 200. ex. 7325. reſidui erunt. 7125. pro contributione Laurentij, & ſic quilibet habebit æqualem portionem in totali ſumma.

### *Speculatio cuiuſdam Methodi reductionis numiſmatum unius ſpeciei in aliam.*

### AD EVNDEM.

Mirum tibi videtur quo pacto verum ſit, quod ſumma medietatis cuiuſuis numeri illorum numiſmatum, quæ hic vocantur Blanci, cum ſexta parte eiuſ dem medietatis, ſemper ſit numerus florenorum huius prouinciæ. Vt exempli gratia, quotieſcunque reducere voluerimus. 48. Blancos in Florenos, ſi medietati ipſius. 48. hoc eſt. 24. adiecta fuerit ſexta pars ipſius medietatis, quæ eſt. 4. tunc habebimus. 28. & ita dicemus quod. 48. Blanci conſtituunt Florenos. 28. quod quidem verum eſt.

Huiuſmodi autem rei ſpeculatio ita ſe habet. Nam vnuſquiſque Blancus diuiditur in. 7. æquales partes, quarum. 12. conſtituunt vnum Florenum, horum verò numiſmatum communis menſura, vocatur Groſſus, vt ſcis, ex quo ſequitur, quod ſi

Fff 28.

28.Floreni æquantur Blancis.48.tot Groſſi erunt in.28.Florenis quot in.48.Blancis. Fingamus igitur, mente,noſtram figuram. 79. Theorematis Arithmetici.x.u.o. e.n.ſupponendo ambo producta.u.x.et.n.e.inuicem æqualia exiſtere,& vnumquodque eſſe groſſorum.336. ſit etiam.o.x.vnus Florenus.12.groſſorum.o.n. verò Blancus. 7.eorundem groſſorum. o. e. autem Blancorum. 48. Nunc certi erimus ex. 15. ſexti vel.20.ſeptimi Euclidis eandem fore proportionem.o.u.ad.o.e.quæ.o.n.ad.o. x. ſed.o.n.eſt ſumma medietatis ipſius.o.x.cum ſexta parte dictæ medietatis,ita igitur erit.o.u.ipſius.o.e.hoc eſt ſumma medietatis.o.e.cũ ſexta parte medietatis eiuſdem , quæ ſumma in præſenti exemplo erit.28.

Hac enim ſpeculatione mediante, poteris methodum inuenire conuertendi Florenos in Blancos.Vt ſi nobis propoſiti fuerint Floreni. 28 . Voluerimuſque inuenire quot Blancos faciant , ſuppoſita menſura communi , iam ſupradicta. Nam duplicabimus numerum Florenorum , à quo duplo detrahemus ſeptimam partem,reliquũ verò erit numerus quæſitus.

Huiuſmodi autem rei ratio eſt , quia, cum in ſupradicta figura, proportio.o.e. ad o.u.æqualis exiſtat ei , quæ.o.x.ad.o.n.atque etiam.o.x. ſit minor duplo ipſius. o. n. per ſeptimam partem ipſius dupli.o.n.minor erit.o.e.duplo ipſius. o. u. per ſeptimã partem eiuſdem dupli ipſius.o.u.

Idem affirmo de quauis conuerſione aliorum numiſmatum, quorum ſemper.o.x. maior ſit.o.n.verò minor. Vt ſi.o.x.æquiualeret.7: et.o.n.valeret.4. et. o. e. valeret 42. quæ quidem.o.e.menſuraretur ab.o.n.

Si cuperemus ſcire quot.o.x.ſint in.o.n. Primo dicemus in. o.n.reperiri ſummam medietatis ſex ſeptimorum ipſius.o.x.collectæ cum vna ſeptima parte ipſius.o.x.ſeu (vt ita dicam ) cum tertia ipſius medietatis . Vnde dempta ſeptima parte ipſius.42. quæ eſt.6. collectaq̃; cum medietate reſidui, quæ eſt.18.habebimus.24.res,quarum vnaquæque æqualis erit ipſi.o.x.

Sed ſi quis cupiat reperire.o.e.dato.o.u.duplicet.o.u.à quo demat quartam partẽ ipſius.o.u.& habebit propoſitum. Nam ita ſe habere oportet.o.e.ad.o.u.quemad modum.o.x. ad.o. n.

## De lucro mercantili.

### AD EVNDEM.

VOD demum ſcire à me deſideras,eſt , quod cum vendideris libram vnam mercis pro.4. ſolidis,& lucratus fueris.2. cum quarta parte vnius pro ſingulis decem libris, ſcire velles quantum lucri facturus eſſes in libris decẽ dando ſingulam libram pro. 6. ſolidis.

Nulli dubium eſt quin decima pars de.2.cum quarta vnius ſit lucrum libræ vnius. Quæ dẽcima pars ſunt noué quadrageſimæ partes, & hæc ſubducta à ſolidis.4. reliqui erunt ſolidi.3. cum.31.quadrageſimis partibus pro ſorte vnius libræ. Quę ſors ſubtracta à ſolidis.6.remanebunt ſol. 2. cum.9. quadrageſimis lucri pro libra, quod multiplicatum per.10.proueniunt ſol.22. cum quarta parte vnius, & tantum aſcenderet lucrum, quod fieri poſſet in libris decem ſi quamlibet , ſol. 3.cum.31.quadrageſimis nobis conſtaret.

Vel ſic multiplicemus ſortem vnius libræ per.10.productum erit. 37. cum tribus

quartis

quartis, iterum multiplicemus per. 10. fortem cum lucro vnius libre quod eſt. 4. pro-
ductum erit. 40. differens à primo ſol. 2. cum quarta parte, multiplicemus pariter
per. 10. precium. 6. ſolidorum proueniens erit. 60. à quo deducendo productum for-
tis librarum. 10. quod erat ſol. 37. cum tribus quartis ſupererunt ſol. 22. cum quar-
ta parte, vt ſupra.

## DE DIGNITATIBVS PLANETARVM.

### Adriano Panetio.

VOD eam diſtinctionem orbium, quæ iam inualuit, non teneas, ſed putes
totum eſſe quoddam continuum excipiens corpora ſtellarum, nouum nõ
eſt, nam nonnulli ſolidæ doctrinæ Philoſophi idem cenſuerunt. Sed
quod attinet ad dignitates planetarum in ſignis zodiaci, ſcias huiuſmo-
di ordinem me comprehendere eſſe deſumptum ab ordine antiquo orbium ipſorũ
planetarum, quiquidem ordo erat, vt ſtatim poſt Lunam ſuccederet Sol, poſt So-
lem Mercurius, tum Venus deinde Mars, poſtea Iupiter, & tandem Saturnus per
eoſdemq; orbes, retro redibant, atque hoc cognoſcitur conſtituendo Cancrum do
micilium Lunæ, Leonem, Solis, Virginem, Mercurij, Libram, Veneris, Scorpio-
nem, Martis, Sagittarium, Iouis, Capricornum, Saturni, Incipientes deinde ab
Aquario, qui ad nos propius accedit eundemq; tribuentes Saturno, Piſces, Ioui,
Arietem, Marti, Taurum, Veneri, & Gemellos, Mercurio, ſeptem Planetas cum
duodecim ſignis zodiaci concordes reddebant.

Quod deinde Ariſtoteles in libris de ſenſu & ijs quæ ſenſibus percipiuntur, dicit
pupillam oculi eſſe nigram, non ita ſe habet, nam idem eſt, ac ſi quis diceret nigrũ
eſſe illud medium, quod permitteret tranſitum lumini per ſuam diaphaneitatem, nul
lum lumen à ſeipſo reflectens, & etiam ac ſi quis diceret nigrum eſſe aerem alicuius
cubiculi vndequaque clauſi tenebroſi.

Quod etiam idem Ariſtoteles volens adducere cauſam, cur oculus magis mate-
riam aquæ, quam aeris participet, dicens id ea ratione fieri, quod aqua magis quam
aer ſeruari poſſit, eodem libro ſcribit, eſt reuera admirandum. Ibi enim clarè de-
monſtrat ſe planè ignorare, & conſtructionem oculi, & cauſam diuerſitatis eorum
humorum tam in ſubſtantia, quam in figura, quæ non aliunde dependet quam quod
diuerſam refractionem radiorum luminoſorum producat, qui per pupillam ingre-
diuntur, vt ad proprios ſibiq; deſtinatos locos dirigantur radij, vt à virtute viſiua per
fectius ſentiantur.

### De ratione Frigiditatis locorum umbroſorum.

#### AD EVNDEM.

VEra ratio vnde fiat, vt quanto magis ſentitur calor in locis expoſitis Soli, tan-
to minus ſentiatur in vmbra, vbi Solis radius non reflectitur, eſt quia cum ra
refactus eſt aer à vehementi calore radij ſolaris, ſeipſum colligit, & condenſatur in
locis, à quibus à calore, ratione rarefactionis, non expellitur, & quia naturaliter ca-
lor ſequitur rarum, rarum calorem, & frigidum denſũ, & denſũ frigidum, vt vnicui
que ſanè mentis patet, hanc ob cauſam ſequitur rem ita ſe habere vt diximus. Poſſu
mus etiam abſque dubio credere huiuſmodi ratione fieri, vt frigus matutini tempo
ris, in crepuſculo maius eſſe eo, quod noctu viguit. Nam materia conſiſtens in co-
no vmbræ terræ, ſemper denſior eſt ea, quæ extra reperitur, imo noua materia cõn
tinuo condenſatur, propter motum vmbræ, quæ ſemper corpori ſolari opponitur. hæc

Fff 2 autem

autem noua condensatio dico semper fit in crepusculo matutino, hoc est in parte co
ni à Sole pulsa , in parte vero contrari a ipsius coni hoc est in parte crepusculi ve-
spertini , contrarium accidit , quia potius aliquantulum in hac parte materia coni ra
rificatur , quia extrinseca condensatur, in parte vero matutina extrinseca rarificatur;
& propterea intrinseca conde nsatur .

## QVOD RECTE ARIST. SENSERIT COELVM
casu non esse productum .

### *Hieronymo Condrumerio.*

Ferunt Aristippū tempestate maris ad incognita littora delatum , cum in are-
na vidisset quasdā figuras geometricas delineatas exultantē lętitia dixisse: Hæc
sunt hominum vestigia . Nam consonum rationi nōn erat, vt huiusmodi figuræ ca-
su essent impressæ: neque etiam credendum est ingentem hanc machinam tanto or
dine constantem fortuitò esse productam, cum nulla quantumuis minima eiusdem
particula, dummodo nitatur ordine , aliquo modo casu effecta fuerit; cūm casus ni-
hil producat, quod regulam & ordinem seruet . Non est igitur producta casu admi
randa correspondentia, quæ est obiectorum cum potentijs , luminis cum oculo ,so-
ni cum auditu , saporis cum gustatu, odoris cum odoratu, qualitatum tangibilium cū
tactu . Si diligenter deinde cuiuslibet rei naturalis operationem considerabimus ,
eas tanta arte constructas videbimus , vt cogamur fateri aliquam prudentissimam ,
& sagacissimam mentem eas formasse, si ergo quælibet mūdi pars tanta cum ratione
& ordine est constructa : quomodo fieri poterit, vt de toto ipso mundo id in dubium
vocemus, certissimeq; non credamus diuinissimam aliquam mētem esse à qua exqui-
sitissima huius vniuersi harmonia , quæ ex tot tantisq; partibus , maximo ordine ni-
tentibus conficitur , non dependeat?

## VARIA RESPONSA.

### *Nicolao Petreio.*

AD ea quæ mihi scribis dico, quod excrementa quæ ex corpore sano prodeunt
in sua ipsorū qualitate sensibili ita se habēt ad facultatē illius partis eiusdem
corporis, ut cā non lędant, quēadmodū efficeret sputū , si esset amarū, aut quod ex
cernitur naso fętidū esset. Imagineris igitur quēadmodū possit esse verū id quod idē
amicᵒ noster ait. Pręterea si aliquid tibi in oculū inciderit, an nescis quomodo statim
affatim affluat humor, vt id foras ppellat, vel abducat?(mirabile opus naturæ.) Dic
etiā eidem non absque mysterio naturam in tot miserijs senectutem posuisse, cum
sæpissimē senex mori desideret, ut huius vitæ calamitatibus liberetur, vnde fit , vt
cum eius aduentum sentiat , minus affligatur . Dicito etiam eidem, naturam non
fuisse tam solicitam dē quibusdam partibus quemadmodum est de toto , vnde ma-
gis rotunda, & polita poterat esse superficies terræ, quam nunc est , quia natura ma
gis respicit totum, quam partes, & magis maiores, quam minores.

Dum tuas legerem, me continere non potui quin riserim, id quod scribis te inter-
rogasse eum Philosophum naturalem, vnde fit , vt ventus sit frigidus, eumq; tibi re
spondisse, quod à remotissimis partibus veniat, genereturq; à vaporibus terræ frigi-
dis . ( cum ipsa sit frigida . ) Cæterum miror quod ab eo non quæsieris, vnde oriatur
frigiditas, quæ percipitur ab agitatione aeris, qui quidem à vaporibus terræ non
prosilit, nec à remotissimis partibus ad nos accedit. Sed quia de eadem re me in-

terro-

terrogas, scito naturã coniunxisse frigiditatẽ cũ densitate, & caliditatẽ, cũ raritate, vt sup.diximus, ita vt cum aliquod corpus densat, frigidũ reddat, & dum rarefit maiorem caliditatem acquirat, & sic econtra fit, vt quanto magis aliquod corpus refrigeratur, tanto densius reddatur, & quanto calidius fit tanto rarius efficiatur. Quoties igitur agitabitur aer, aut aliud corpus, quod ratione suæ subtilitatis, velociter condensari, & rarefieri possit, eius partes densiores semper erunt frigidæ, & hanc ob rem quilibet ventus, qui per calida loca non transeat, natura sua frigidus, calidus autem per accidens erit. Hinc fit vt vasa vitrea, & terrea tam in vehementi frigore, quam in magno æstu frangantur, quia horum vnum fit, ne aliquis locus vacuus remaneat, & aliud ob loci necessitatem, sed hoc non sequeretur, si in materia, qua huius modi vas constat, aliqua aeris portio non contineretur.

### DE LVMINE LVNÆ, DE FINE LVMINIS,
de fine motus corporum cælestium, de albedine,
de sphæra.

#### Clarißimo Antonio Nauaiero.

VMEN Lunæ etiam si sit lumen reflexum Solis ab ipsa Luna, ab ea tamen non ita reflectitur, vt à superficie polita speculi, cũ eius luminis tantã quãtitatem super ipsum corpus lunare videamus, & eo modo terminatã quo conspicimus. per se lumen, causa oculi est effectum, per accidens autem puta quod vis. Terra deinde nunquam lunari lumine (quãuis solaris reflexio existat) omnino destituta est, dico etiam, neque in ipsis ecclipsibus solaribus vel lunaribus, in solaribus enim cum Sol tot millia vices maior sit Luna, Luna verò minor terra, sequitur, vt terra non omnino priuata remaneat lumine Lunæ, in ecclipsibus verò lunaribus Luna semper videtur, gratia luminis solaris, quamuis refracti. Motus corporum cœlestium fit ratione situs, & varietatis virtutis stellæ in diuersis locis, hæc autem varietas absque diuerso situ eiusdem stellæ, nec diuersus hic situs absque motu fieri posset, ita vt motus stellarum sit ratione diuersitatis situum ipsarum, ergo motus, & diuersitas situum, fit, ob diuersam influentiam. Quæ autem de albedine fratri tuo dixeram, erant, quod inter oẽs colores albedo, certo quodam modò, maiorẽ similitudinem habet cum lumine. Primò quia magis coniungitur cum lumine. Secundò quia magis afficit sensum. Tertiò quia absque resistentia magis recipit qualitatem aliorum colorum, quam alij colores. Quartò quia maximus est omnium colorum. Quintò quia simplicior est reliquis. Sextò quia disgregat visum. Septimò quia qualitas quæ in niue alba esse videtur, nihil aliud est quam multitudo quædam luminum reflexorum, & non albedo, similis ei, quæ est lactis, aut panni, quæ quidem septima causa effecit, vt ipsam albedinem, magis quam alium quemuis colorem cum ipso lumine compararem, cum nihil sit, quod esse suum trãs mutans, aut apparenter, aut essentialiter, illud ipsum prius non transmutet in formam sibi propinquiorem, vt manifestè patet. Est etiam huius rei octaua ratio, magni ponderis, quia scilicet nullus sit color, qui magis resistat lumini, aut in quem lumen minorem impressionem faciat, quam albedo. Vnde sequitur, obiecta alba, minus esse combustibilia quam alia, cum quælibet res in suum contrarium quam in

sibi

sibi simile valentius agat, vt rectè vidit Aristoteles cum dixit, omne contrarium à suo contrario pati natum est.

Inter corpora, multum simplicitatis retinet sphæra.

Circa quod, præter rationes adductas ab Aristotele in libris de Cœlo, possumus etiam ratiocinari à facilitate motus vndiq; ab eo quod violentiæ non resistat, ab eo, quod apta nataq; sit quiescere supra quoduis punctum suę superficiei, ab eo quod ab aliqua superficie alterius corporis sese tangi non permittat, quæ curuitate concaua non adæquetur, nisi medio vnius puncti. Verum est, quòd licet hæc vltima ratio nõ sit propria sphæræ, est tamen causa simplicitatis in eo, in quo reperitur, sed proprię passiones sphæræ sunt supradictæ, præter quam quod alia eiusdem sphæræ est proprijssima, quæ est distantia eius termini ab vno tantummodo puncto idest ab eiusdé centro, & etiam posse diuidere corpus aliquod medium, cum æquali resistentia circa punctum, quod prius in motu reperitur.

Aequalitas autem rerum, est etiam valde similis simplicitati, & vnitati.

## Comparatio uisus, & auditus.

### AD EVNDEM.

QVod ad visum & auditum attinet, magis necessarium esse visum, & nobiliorem quam auditum existimo, primò quia si quis visu orbatus esset, contra frigus, & calorem, contra famen, & sitim nil prouidere posset, neque aliud quicquam hoc vocabulum prouidere significat, neque absque periculo vitæ ab vno loco ad alium ferri posset, neque aliquid arte facere.

Sed si quis destitutus esset facultate audiendi, supradictas tamen operationes pręstare posset, neque modo careret, quo animi sui sensa absque beneficio soni, sed ope figurarum & characterum alteri aperiret: neque etiam munere speculandi scientias (excepta musica) destitueretur. Ad scientiam comparandam, longè magis necessarius est visus, quam auditus præterquam, quod visus maiorem numerum obiectorum, & differentiarum rerum percipit, & inter reliquos sensus velocissimè imò in instanti operatur, magis remotè quam alij, & exactius sentit, minusq; quam reliqui afficitur, præterquam quod semper agit, dummodò non dormiat animal. Præterea sese magis patefacit, & prodit anima per oculos, quam per aliud, cuiuslibet sensus, instrumentum. Oculo magis quam alia corporis parte, hominis natura cognoscitur: & si aliquid speculari volumus, quod sine imaginatiua fieri non potest, statim imaginamur nos videre huiusmodi rem, ac si oculo fuisset compræhensa, & ab imagine quæ est vnum ex obiectis oculi, imaginatiua nuncupatur. Beneficio oculorum omnes ferè scientiæ sunt adinuentæ. Auditus nil aliud quam sonum capit, auditus nunquam detulit intellectui figuram, corpus superficiem, aut lineam, materiam, formam, locum, dimensionem, plenum inane, nec innumera alia accidentia, quæ ab oculo compræhenduntur. Quæ verò visui, & auditui sunt communia, sunt etiam tactui communia, vt numerus, motus, maius, & minus, sunt tamen aliqua oculo & tactui communia, quæ auditus non potest capere, vt durum, molle, acutum, obtusum, asperum, lene, planum, curuum, concauum, conuexum, magnum, paruum, & supradicta, idest figura corpus & cętera, vt etiam rectum, obliquum, & similia.

Aristo-

Aristoteles circa finem primi capitis libri de senfu ait mediante visu, magis quā quolibet alio sensu, nos percipere sensibilia communia. Vbi eundem per se, & non per accidens magis necessarium esse quam auditum, tam in ijs quæ ad victum, quam in ijs quæ ad scientiam pertinent esse asserit, quia auditus intellectui confert per accidens. Vide etiam quod idem scribit primo metaphysicorum. Et si ad aliquid perfectè cognoscendum, oculo sese nobis offerrent ea omnia obiecta, quorum species in imaginatiua formamus, ipsa imaginatiua non egeremus. Sed quia hoc fieri non potest, hunc thesaurū imaginatiuē, seu memoriæ ad conseruandam imaginem omnium obiectorum sensibilium nobis dedit natura, vt ope discursus intellec° circa dictas imagines, rerum veritatem venari possimus. Sed vt ad propositum redeamus, beneficio oculi animal liberum est, cum sine ipso locum mutare nequeat, vt sit tutum. tenebræ, priuatioq; visus sunt ferè vnum, & idem. Neque vllus est sensus, qui sit magis similis intellectui quam visus: neque alij sensus habent obiecta viciffim communia, quæ non sint etiam oculo communia, sed inter oculum, & quemlibet alium ex sensibus, inuenientur quidem obiecta communia, quæ cum alijs non communicabunt, vt inter oculum & tactum, figura, acutum, obtusum, & similia, quæ alijs sensibus non percipiuntur. Mediante visu, & auditu etiam, compræhendū tur variæ distantiæ, situsq; obiectorum, nec non proportiones, & alia quę ab alijs sensibus non compræhenduntur. Multa obiecta deinde sunt subiecta gustatui, quę alijs accidentibus prędita sunt, vnde cum fuerint semel degustata, talia, qualia sunt ab oculo percipiuntur, quod nullus ex alijs sensibus præstabit. Idem de obiectis odoratus dico. Sensuum nullus est qui maiorem similitudinem gerat cum vigilia & cum vita, quam visus, neque aliquid est, quod magis repræsentet imaginem somni, & mortis, quàm cęcitas.

Qui sibi oculos eruit vt melius specularetur maxima stultitia prius obcęcatus fuit quia soni magis impediunt speculationem quàm lumina, imò qui commodè vult contemplari, quantum plus potest nititur longius esse ab omni strepitu, magis quàm à locis luminosis, & animal magis lætatur lumine quam sono: & ad speculationem nos magis inuitat harmonia luminum variorum colorum & figurarum, quàm harmonia sonorum, præterquam quod instrumentum visus totius corporis partium est pulcherrima, & in eminentiori loco locata, si de instrumentis sensuum loquamur, & veluti finestræ animæ. Et si Aristoteles dicat oculos & aures in vno eodemq; orbe existere, volens inferre quod in eodem æquilibrio sint æqualiter alta non ita se habet, quia (si de homine loquamur) oculus est altior aure. Beneficio huius sensus, eorum quæ absunt, & longo iam tempore sunt mortui, animi sensa, & conceptus intelligimus, neque alia ratione rerum omnium memoria seruari potest. Si cabala unquam vera fuit, nulla alia ratione est deleta, quam quia alicuius signi visibilis medio conseruata non fuerit, & quæcunque non scribuntur, idest oculo non cōmendantur parum durant cito obliuioni tradunt. In maiori semper pretio fuit pictura quā musica: obiectis visibilibus magis quam ijs quæ sub auditu cadunt, affectus animi, atq; alia quælibet res naturalis exprimi possunt. Aegyptij volentes significare Deum, oculi medio id præstabant.

Oculus, respectu aliorum instrumentorum sensuum, est quasi epicyclus animæ, neque defuit qui crederet oculum esse principem animi partem.

Oculus à Sole, & à Luna ita dependet, vt qui tempore defectus cuiuslibet luminaris nascitur, statim cæcus euadat, neque aliqua est corporis pars in qua magis appareat

pareat differentia vitæ à morte; quam in oculo. Aristoteles ad finém cap.15.lib.pri
mi posteriorum ait, clarum esse quod si aliquis sensus deficiat, futurum vt aliqua
quoque scientia desit. Considera, quot scientijs careret homo, si visu orbaretur.

Et in tertio de anima ait, eum qui non sentit, nihil intelligere posse; id quod in-
de confirmat, quia nihil sit in intellectu, quod prius non fuerit in sensu. Plato in ti
meo ait, oculos nobis attulisse rerum optimarum notitiam, & si oculus non fuisset ni
hil eorum, quæ ad cœlum spectant inueniri potuisse, & cognitioné diei ac noctis ab
oculis ortum duxisse, vt reuolutiones mensium, & annorum metiri, & tempus co-
gnoscere, & inuestigare ordinem naturæ vniuersalis possemus; quibus philosophiã
nobis comparauimus, ut alia multa omittam, quæ ibi à Platone dicuntur. Addam
hic & aliam specialem differentiam inter auditum & visum, quæ est, vt obiectum vi
sus sit permanens, & obiectum auditus transitorium siue successiuum aut, vt alio mo
do idem dicamus, obiectum visus particpes sit æternitatis, illud autem quod est au-
ditus non item, nam auditus tempori subiectus est, visus autem minimè. Vel si di-
camus operationem auditus absque tempore fieri non posse cum sit motio, operatio
verò visus, nullo indiget tempore, cum ipsa sit momentanea, & propterea instan-
tanea. Nam momentum non est motus, nec instans tempus.

## QVARE HYEME VIDEATVR HALITVS
animalium non autem æstate, & de vento.

### Pancratio Mellano.

VNDE fiat vt hyeme halitum nostrum videamus, & non æstate, ratio est ab
eiusdem halitus congelatione, quæ ab extrinseco frigore fit. Prius enim
scire debes aerem attractu in pulmone, foras deinde erumpere cum alio
vapore aliquantulum crassiore humido, & excrementitio expulso à natu-
ra, quæ continuò nostrum corpus euaporare facit, vnde sequitur dum aer foras à pul
mone pellitur, maiorem semper materiæ portionem, ea quæ intus attracta est exire:
vnde statim vt dicta materia foras expulsa, frigidum aerem offendit, cum constet ex
partibus crassis, & obnoxiis congelationi, condensatur in formam vaporis, ad dif-
ferentiam aeris ambientis qui in se eas partes crassas non habet, à quibus quidé par-
tibus condensatis, & redditis opacis reflectitur lumen, atque hanc ob causam æstate
hoc non fit, quia calor vim condensandi non habet.

Ventus nihil aliud est quam quidam aeris motus, cum condensatur, ob defectum
caloris, neque (pace Aristotelis dicam) est exhalatio sicca. Exemplum à Vitruuio
allatum nil planè valet, quantum spectat ad venti naturam, cuius rationem à me re-
quiris. Exemplum etiam ventilabri quo tempore æstate vtimur negligendum pe-
nitus non est, quia eius beneficio, non solum arcemus à nobis aerem ambientem
calidum, sed alium etiam aerem circa nos condensamus: & quia ordo naturæ est hu
iusmodi quod quemadmodum calor sequitur raritatem corporú, sic etiam frigus
eorundem densitatem sequatur. Quod si vis vt exemplo illustrem, diligenter ob-
seruato tempore æstatis cum aliqua nubes nobis Solem adimit, vbi aer qui in eius
                                                                            vmbra

vmbra reperitur, tantum quantum defectus caloris radij folaris fert, qui per vim, dictum aerem rarefactum conseruabat, statim dictum aerem condensari cognosces: & quia ea condensatio homogenea non est, ob diuersas rationes, hanc ob causam percipimus eam aeris impulsionem, & inæqualiter, dum verò eadem vmbra discedit, ventus, ferè, statim cessat, & sæpe ante quam dicta vmbra discedat; cuius rei causa est longa mora quam trahit vmbra, ita vt prius absoluatur reditus aeris ad formã, quæ ei conuenit in huiusmodi vmbra, quam faciet nubes dum Sol detegitur.

Vera non funt ea, quæ tibi Arnoldus dixit, vt mihi tuis literis significasti. Nam ego ita dixi, videlicet, quod quotiescunque aliquis aspexerit aliquod punctum in superficie speculi, tunc imaginem ipsius post dictam superficiem videbit duplicatam, si verò aspexerit imaginem intra speculum, tunc illud punctum videbit duplicatum, huiusmodi autem rei ratio pendet ab hijs quæ ad Franciscum Vimercatum scripsi, quæ si memoria tenes, nullum tibi dubium remanebit. Nam ea tibi omnia ostendi.

Dum verò dicis omnem proportionem rationalem diuidi posse duobus numeris mediantibus in tres æquas partes, mihi ad memoriam reuocas id quod quidam Vitruuij commentator asserit super primum cap. noni lib. eiusdem Authoris, ita dicens.

,, Benè esse potest vt diagonalis ( quadrati scilicet ) numerorum via reperiatur, sed
,, fortasse intercedent fracta.

Miror te non memoria tenere quid sint numeri rationales quidúe surdi, neq; consideras, non solum non esse diuisibilem in tres æquas partes omnem proportionem rationabilem, sed neque in duas, vt sunt superparticulares proportiones, necnon aliæ innumeræ, sed cum talia scribis te nimis parum versatum in istis rebus ostendis.

Id verò quod tibi dicere volebam nudiustertius de Mercurio erat, quod nullo pacto confidendum est calculis qui fiunt de cursu Mercurij, eo quod eius situs nullo modo obseruabilis est, nam ipse nunquam nec vbiuis locorum orbis terrarum visibilis est altior. 18. gradibus supra orizontem, sed neque confidendum esset si etiã ipsum videremus altum. 20. gradibus, propterea quod magna refractio radiorũ infra hos gradus nos valde fallit, quæ quidem refractio, nec vbiq;, nec omni tempore vniformis est, propter diformem seu inæqual em crassiciem vaporum quæ continuò mutatur. Imo multoties eum videre putabimus supra orizontem, existente ipso sub orizonte.

## Quod Ouidius transcurrit à motu diurno, ad motum annuum præter rem.

### AD EVNDEM.

TVus etiam Ouidius cespitauit, cum pro itinere vnius diei efficiat, vt Phaeton à patre edoctus sit etiam de itinere annuali.

Nam, quod Phaeton petat pro cursu vnius diei, clarè patet ex diuersis locis, & primò vbi ita scribit Ouidius.

,, Currus petit ille paternos.
,, Inq; diem alipedum ius & moderamen equorum.

Deinde vbi Pater ita loquitur.

,, Ardua prima via est, & qua vix mané recentes.

Ggg Enitun-

,, Enituntur equi medio est altissima cælo.
,, Vnde mare, & terras ipsi mihi sæpe videre.
,, Fit timor & pauida trepidat formidine pectus.
,, Vltima prona via est & eget moderamine certo.
　　Etiam vbi dicit.
,, Dumq; ea magnanimus Phaëton miratur, opusq;
,, Perspicit, ecce vigil nitido patefecit ab ortu.
,, Purpureas aurora fores, & plena rosarum.
,, Atria, diffugiunt stellæ, quarum agmina cogit.
,, Lucifer, & cæli statione nouissimus exit.
　　Nec non vbi ita inquit.
,, Et si ( modo credimus ) vnum
,, Isse diem sine Sole ferunt, incendia lumen
,, Præbebant.
　　Quod autem à Patre instruatur etiam de cursu annuali,
　　videbitur vbi ita dicit.
,, Nitor in aduersum, nec me, qui cætera vincit.
,, Impetus, & rapido contrarius euehor orbi.
　　Et vbi ita loquitur.
,, Forsitan & lucos illic, vrbesq; Deorum.
,, Concipias animo delubraq; ditia donis
,, Esse per insidias iter est, formasq; ferarum.
,, Vtq; viam teneas, nulloq; errore traharis.
,, Per tamen aduersi, gradieris cornua Tauri.
,, Aemoniosq; arcus, violentiq; ora Leonis.
,, Sæuaq; circuitu curuantem brachia longo.
,, Scorpion atque aliter curuantem brachia cancrum.
,, Nec tibi quadrupedes animosos ignibus illis.
,, Quos in pectore habent quos ore & naribus efflant, &c.
　　Sed lucidius etiam hoc videre est inferius vbi ita loquitur.
,, Nec tibi directos placeat via quinque per arcus.
,, Sectus in obliquum est lato curuamine limes.
,, Zonarumq; trium contentus fine, polumq;
,, Effugit australem iunctamq; aquilonibus arcton.
,, Hac sit iter, manifesta rotæ vestigia cernes.
　　Et vbi etiam dicit.
,, Neu te dexterior tortum declinet ad anguem.
,, Ne ve sinisterior pressam rota ducat ad aram.

*De supputatione quinque corporum regularium.*
*De aliquibus etiam eorum sympathijs.*

### AD EVNDEM.

ID quod à me desideras, ab alijs etiam factum est, sed ne me putes laborem euita
re, non prætermittam aliquid tibi scribere, earum rerum quæ ab Euclide colle
gi,

gi, methodo etiam qua vtebar dum in iftis rebus me aliquo modo exercebam.

Quotiefcunque igitur fcire volueris quantitatem corpulentiæ cuiufq; quinq; corporum regularium ab vna eademq; fphæra terminatorum feu circunfcriptibiliũ curabis primum, cognofcere quantitatem lateris cuiusq; eorum, talium partium, qualium femidiameter dictæ fphæræ fit. 100000. ex tabulis finuum Nicolai Copernici. Propone igitur tibi ante oculos figuram femicircularem vltimæ propofitionis. 13. lib. Eucli. & inuenies.c.d.tertiam partem femidiametri.d.b. effe partium.33333. æqualem finui arcus.f.e.graduum. 19. mi.28.qui quidem arcus dẽptus cũ fuerit à tota quarta.b.f.remanebit arcus.e.b.gra.70.mi.32.cuius corda erit latus exaedri, quod latus ita cognofces, fumendo fcilicet finum medietatis.b.e.hoc eft finum gra.35.mi.16.qui erit partium.57738.cuius duplum erit partium. 115476. pro latere cubi.

Dempto poftea quadrato lateris exaedri, & quadrato totius diametri. a. b. refidui radix quadrata, erit.a.e.latus Tetraedri. Vel fi duplicaueris finum dimidij arcus.a.e.qui quidem arcus, componitur ex quarta.a.f.& ex arcu.f.e.iam inuento, fiue, vt refiduus totius dimidij circuli, dempto.b.e.iam fupra inuento, habebimus idem latus.a.e.partium.163294.

Pro latere verò Octaedri accipere potes radicem quadratam dupli quadrati ipfius.d.b.& habebis.f.b.latus quæfitum. Vel fi malis accipe duplum finus medietatis arcus.b.f.quod duplum erit.f.b.partium.14142.

Pro latere verò Duodecaedri, diuide latus Exaedri ex methodo. 11. fecundi Eucli.cuius maior pars erit latus quæfitum, partium.71368.

Sed pro latere Icofaedri, te primum oportebit inuenire quantitatem anguli g. d. a.hoc eft ipfius arcus.b.n.qui tali angulo fubiacet, quod cum pluribus modis inueniri poffit, nihilominus, hunc feruabis, inuenies primò quantitatem.d.g. quæ eft radix quadrata fummæ duorum quadratorum hoc eft.d.a.et.a.g. quæ. a. g. æqualis eft diametro.a.b.vt fcis, dices poftea, fi.d.g.correfpondet ipfi. g.a.cui correfpondet.d. h.femidiametro fphæræ? tibi veniet.h.k.finus arcus.a.h.hoc eft. b. n. graduum. 63. min.26.cuius medietas gra.31.mi.43.pro finu fuo habet partes.52571.cuius finus duplum eft partium.105142.pro latere Icofaedri.

Incipiendo nunc à Tetraedro, fcire debes, quod pars.a.c.totius diametri.a.b. æqualis eft axi ipfius Tetraedri, quæ quidem.a.c.vt fubfefquialtera ipfius.a.b.erit partium.13333.

Quæres poftea quantitatem fuperficialem vnius faciei ipfius Tetraedri, hac methodo, inueniendo primum radicem quadratam trium quartarum quadrati ipfius.a.e.lateris Tetraedri, eo quod latus hoc, fefquitertium in potentia eft ipfi perpendiculari terminatẹ ab vno angulorum trianguli æquilateris & à latere ei oppofito ex.11.tertijdecimi ipfius Eucli. quẹ quidem perpendicularis, erit partiũ.141416. & hæc multiplicata cum medietate lateris trianguli, hoc eft cum. 81647. tibi dabit fuperficiem quæfitam, hoc eft bafim Tetraedri partiũ fuperficialiũ. 11546192152. Hãc demum bafim multiplicando cum tertia parte axis Tetraedri habebis corpulentiam totius Tetraedri, quæ erit.513158964003488.

Neque tibi hoc loco occultare volo quandam meam animaduerfionem, quæ eft, quod diameter feu perpendicularis ( fupradicta ) faciei ipfius Tetraedri, femper æqualis eft lateri ipfius Octaedri circunfcriptibilis ab eadem fphæra, hoc eft ipfi. b. f. quapropter quotiefcunque ipfam perpendicularem habere voluerimus accipiendo b.f.habebimus intentum. Et quod hoc verum fit poffumus ita demonftrare.

Primum, notum nobis eft, ipfam perpendicularem, triplam effe eius parti, quæ

Ggg 2          à cen-

à centro circuli, ipſum triangulum circunſcribentis, terminatur, & à baſi, vt in tertio
propoſito decimæſeptimæ quartidecimi Eucli. probatur, ex quo ſequitur proportio
nem huiuſmodi perpendicularis ad axem Tetraedri, hoc eſt ad.a. c. ſeſquioctauam
eſſe in potentia , ex penultima primi Eucli. Sed cum. d. c. tertia pars ſit ipſius. d. a. vt
etiam ex. 2. propoſito, ſeu corollario decimæſeptimæ. 14. lib. diſcurrere licet, cum ex
dicto corollario. d. c. ſit ſexta pars ipſius. a. b. Quare. d. c. quarta pars erit ipſius. a. c. vn
de. a. c. ſeſquitertia erit ipſi. a. d. in longitudine, ideoq́; quadratum ipſius. a. d. ad qua-
dratum ipſius. a. c. erit vt. 9. ad. 16: & ita duplum quadrati ipſius. a. d. hoc eſt  quadra-
tum ipſius. b. f. ad quadratum ipſius. a. c. erit, vt. 18. ad. 16. hoc eſt ſeſquioctauum, er-
go. b. f. æqualis erit dictæ perpendiculari, ex. 9. quinti.

Cubus poſtea ipſius. b. c. erit partium. 1539838576570176.

Pro Octaedro deinde , accipies productum diametri in ſemidiametrum , quod
productum, æquale erit quadrato diuidenti per æqualia Octaedron, hoc igitur pro-
ductum , multiplicando per. 100000. ſemidiametrum ſphæræ, tibi dabit columnam
quadrilateram cuius tertia pars, erit partium .666666666666666. cuius duplum
erit ipſum Octaedron partium. 1333333333333.

Pro Icoſaedro autem , oportet prius quantitatem perpendicularis inuenire, quæ
perpendicularis, per æqualia diuidit baſim ipſius Icoſaedri , quæ vt radix quadrata
trium quartarum quadrati lateris ipſius baſis, erit partium. 91055. talium , qualium
dictum latus erit partium. 105142. cuius medietas eſt. 52571. quæ medietas ſi mul-
tiplicata fuerit cum dicta perpendiculari , dabit totam baſim ſuperficialem , hoc eſt
ſuperficiem vnius trianguli æquilateris partium ſuperficialium. 4786852405. quo
facto, accipe quadratum duarum tertiarum ipſius, hic ſupra dictæ perpendicularis ,
ipſumq́; denie ex quadrato ſemidiametri ſphæræ, hoc eſt, ex quadrato ipſi°. 100000
radix poſtea quadrata reſidui, erit partium. 79468. & hæc erit perpendicularis à cen
tro ſphæræ ad, vnam baſim ipſius Icoſaedri , quam volueris, quam perpendicularem
ſi multiplicaueris cum quantitate ſuperficiali, hic ſuperius reperta, vnius baſis , con-
ſequeris columnam trilateram partium. 38040158692054o. cuius tertia pars, erit
partium. 12680052897351 3. pro vna ex. 20. Pyramidibus ipſum corpus compo-
nentibus . Breuius tamen hoc efficiens, ſi multiplicaueris baſim dictam, cum tertia
parte ipſius perpendicularis, hanc poſtea pyramidem multiplicando per. 20. habebis
totam corpulentiam ipſius Icoſaedri partium. 2536010579470260.

Pro Duodecaedro demum, accipe ſinum gra. 36. qui grad° ſunt pro dimidio quin
tæ partis totius gyri circularis, q quidé ſinus, erit partium. 58778 . cuius quadratum
ſi dé pſeris ex quadrato ipſi°. 100000. ſemidiametri circuli circûſcribentis aliqué pé-
tago num æquilaterum, & æquiangulum, tunc radix reſidui, erit perpendicularis du-
cta à centro dicti circuli ad medium vnius lateris ipſius pentagoni , quæ perpendicu
laris, erit partium. 80902. talium qualium medietas lateris dicti fuerit. 58778 .
Nunc verò dicendo ſi. 58778. dat. 80902. quid nobis dabit. 35684? medietas lateris
ipſius Duodecaedri, vnde dabit. 49116. pro perpendiculari, à centro ipſius penta-
goni , ad latus ipſius Duodecaedri, quæ multiplicata cum medietate ſupradicta ip-
ſius lat eris, hoc eſt cum. 35684. producet vnum ex quinque triangulis componenti-
bus vn um pentagonum, ſeu vnam baſim ipſius Duodecaedri, quod quidem triangu
lum, e rit partium. 1752655344. ſu perficialium, quas ſi per quinque multiplicaueris
habeb is vnam baſim pentagonam dicti corporis partium. 8763276720. Dicendum
poſtea eſt, ſi ad. 80901. conuenit ſemidiameter circularis partium. 100000. quid có
ueniet partibus. 49116. dabit. 60711. pro tali ſemidiametro circulari , cuius quadra-
tum,

tium, si dempseris ex quadrato ipsius. 100000. semidiametro sphæræ, tunc radix quadrata residui, erit perpendicularis à centro sphæræ ad centrum pentagoni partium. 79461. cuius tertia pars, si multiplicata fuerit cum pentagono supra reperto dicti corporis producet vnam ex. 12. pyramidibus componentibus dictum Duodecaedron, quæ pyramis, demum, multiplicata per. 12. dabit totam corpulentiam ipsius Duodecaedri partium. 278 53 549 25791680.

Nunc verò si experiri voluerimus vtrum isti calculi duorum corporum maiorum sint rectè supputati, dicem° si ad corp°. 12. basiū, q° est partiū. 2785354925791680 conuenit numerus partium. 253601057 9470260. ipsius Icosaedri, quid conueniet lateri cubi partium. 115476. & inueniemus conuenire latus ipsius Icosaedri partium 105138. eo quod probatum sit in. 10. propositione. 14. li. Eucl. eandem proportionē esse corpulentiæ ipsius Duodecaedri ad corpulentiam ipsius Icosaedri, quæ lateris cubi ad latus Icosaedri.

Hæc autem corpora, ita sibi inuicem, & cum eorum sphæra harmonicè conueniūt quemadmodum antiqui philosophi inuenerunt, vt mirandū non sit, ipsos credidisse omnia quæ natura constant, aliquo pacto ex istis corporibus fieri. Considera quæso quomodo conueniant inuicem Tetraedron, Octaedron, & Icosaedron, cum uniuscuiusque bases sint triangulares æquilateræ intelligendo semper hæc corpora ab eadem sphæra circunscriptibilia.

Octaedron, cum Tetraedro etiam in hoc conuenit, quod latus Octaedri æquale sit ei perpendiculari, quæ diuidit basim Tetraedri per æqualia, vt supra demonstrauimus.

Harmonicis etiam interua llis hæc duo corpora inuicem concordantur, cum basis Tetraedri ad basim Octaedri seruet proportionem sesquitertiam, consonantiæ diatessaron. Et proportio omnium superficierum siue basium Octaedri simul sumptarum, ad omnes bases ipsius Tetraedri simul sumptas sit sesquialtera, consonantiæ diapentis. Neque omittendum est, quod proportio Octaedri ad triplum Tetraedri sit, vt latus Octaedri ad latus Tetraedri.

Proportio verò lateris Octaedri, ad axem Tetraedri, potentia est sesquioctaua, vt supra vidimus interuallum scilicet harmonicum toni maioris.

Harmonia verò Tetraedri, & Exaedri cū eorum sphæra, talis est, vt proportio diametri sphæræ, potentia, tripla sit lateri Exaedri, & sesquialtera lateri Tetraedri, ex quo sequitur latus Tetraedri potentia duplum existere lateri Exaedri. Interuallum enim triplum in harmonicis, componitur ex diapason, & diapente, & sonat speciem diapentis. Duplum verò est diapason, sesquialterum autem est diapente, quæ consonantiæ perfectissimæ sunt.

Proportio verò diametri sphæræ, potentia dupla est lateri Octaedri, consonantiæ diapason. Ex quo sequitur proportionem lateris Tetraedri ad latus Octaedri, potentia, sesquitertiam esse, hoc est consonantiæ diatessaron, & proportionem lateris Octaedri ad latus Exaedri, potentia, sesquialteram esse, ita quod quatuor istæ potentiæ, idest diametri sphæræ, lateris Tetraedri, lateris Octaedri, & lateris Exaedri constituunt harmoniam ferè perfectissimam, ijs terminis comprehensam. 6.4.3. 2. (dixi ferè, quia ditonus supra terminum. 3. vel semiditonus sub termino. 2. hoc loco non reperitur, cuius quidem terminus esset. 2. cum duabus quintis.)

Adde quod diameter sphæræ triplus est longitudine ad perpendiculare ductam à centro sphæræ ad basim Octaedri, quæ proportio, vt supra dictum est, dicitur diapasondiapente, practici verò eam vocant duodecimam.

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Diameter verò fphæræ fefquialter eft longitudine axi Tetraedri, confonantiæ diapentis. Axis autem Tetraedri fefquitertius eft longitudinis femidiametro fphæræ confonantiæ diateffaron. Ita quod ifti tres termini, qui funt, diameter fphæræ, axis Tetraedri,& femidiameter fphæræ conftituunt etiam valde perfectam harmoniam huiufmodi numeris contentam. 6. 4. 3. corpulentia verò Exaedri ad corpulentiam Tetraedri tripla eft, confonantiæ iam fupradictæ diapafondiapente. Si verò de vnifono aliquid videre defideras, confidera æqualitatem dupli quadrati diametri ipfius fphæræ, cum omnibus bafibus Exaedri, vel potentia diametri fphæræ cum duabus potentijs fimul fumptis, quarum vna eft lateris Tetraedri,reliqua verò lateris Exaedri, vel æqualitatem numerorum laterum Tetraedri,cum bafibus Exaedri. Nec mihi videtur filentio inuoluendum effe,antequam vlterius progrediar notabilem fympatiam inter triangulum æquilaterum, & Tetraedron (quauis triangulū corpus non fit)non folum ob inalterabilitatē harum duarum figurarum.(nam omnes aliæ alterabiles effe poffunt, ijfdem lateribns exiftentibus, cum ex quadrato rombus, vel ex pentagono equiangulo, pentagonum non æquiangulum &c. efficiatur) fed quod quemadmodum latus trianguli æquilateri fefquitertium potentia eft perpendiculari ipfum per æqualia diuidenti, ita latus Tetraedri, fefquialterum eft potentia axi ipfius Tetraedri, vnde cum dempta fuerit illa proportio fefquitertia, ex hac fefquialtera relinquetur nobis proportio fefquioctaua, inter perpendicularem trianguli, & axēm Tetraedri(quod etiam fupra demonftrauimus.)Tranfeamus nunc hęc,nec omittamus tamen fympatias quafdam inter Exaedron, Octaedron,& Tetraedron, hoc eft quod eadem proportio fit inter corpulentias Exaedri, & Octaedri, quæ inter eorum fuperficies,nec non,vt latus Exaedri ad femidiametrum fphæræ. Proportio verò bafis Exaedri ad bafim Tetraedri, vt latus Tetraedri ad perpendicū larem diuidentēm per æqualia eius bafim.

Hactenus fatis dictum fit de Tetraedro, Exaedro, & Octaedro cum fphæra. Dicē dum nunc cenfeo aliquid de reliquis duobus mirabilibus corporibus, quamuis ferè omnia hæc ab antiquis philofophis inuenta fint, quorum primum eft, quod tam bafis Duodecaedri, quàm Icofaedri,ab vno eodemq; circulo circunfcriptibiles funt,verùm, talis paffio accidit etiam bafibus Exaedri & Octaedri. Præterea quemadmodum in Duodecaedro, quilibet angulus folidus terminatur tribus angulis pentagonorum æquiangulorum ita in Icofaedro, quilibet angulus folidus viceuerfa terminatur quinque angulis triangulorum æquiangulorum. Et tam vnum, quam alterum horum corporum, triginta lateribus continetur. Et tot folidos angulos triangulares, habet Duodecaedron, quot bafes triangulares continet Icofaedron.

Et Icofaedron,tot folidos angulos pétagonos, quot bafes pétagonas habet Duodecaedron. Et tam vnum quam alterum habet.60.angulos fuperficiales. Eadéq; proportio eft omnium bafium fimul fumptarū Duodecaedri ad omnes bafes fimul fumptas ipfius Icofaedri, quæ corpulentiæ ipfius Duodecaedri ad corpulentiam Icofaedri(quamuis hęc paffio accidat Exaedro cum Octaedro,vt fpra diximus)quę quidem proportio,eadem etiam eft, quę lateris Exaedri ad latus Icofaedri, vt fupra iam dictum fuit.

NOVA

## EPISTOLAE. 423

# NOVA INVENTIO COMPONENDI ASTROLABIA
cum Horologijs artificialibus.

*Jacobo Mayeto Ingeniosissimo Horologiorum Serenissimi*
*Sabaudiæ Ducis Artifici.*

ONNVNQVAM confideraui mirabilem pulchritudinem, fimul cum vtili-
tate coniunctam, illorum horologiorum, quæ in Germania conftruuntur
cũ mobili Rete, feu Aranea Aftrolabij fup Tabulã regionis, in qbus cóti
nuo videñ oriri, occidereq; cæleftia figna, cælum mediare fupra orizon
tẽ, necnon fub eo, & vt vno verbo dicam, continuo erecta videtur tota cœli figura.
Sed quia talia horologia omnia eorum limbum diftinctum habent in. 2 4. horas, qua
propter diametrum limbi, minorem duobus palmis, feu femipede effe non oportet
ne interftitia horarum juftò breuiora feu anguftiora efficiantur, etiam ne interualla
dentium rotæ indicis nimis angufta fint. Sed quia talis magnitudo vt plurimum in-
commoda exiftit. Ideo non inutile fore cogitaui, fi modus aliquis inuentus fuerit,
vt ea omnia efficiantur in limbo diuifo tantummodo in. 1 2. horas æquales, ipfumq;
inueni, qui quidem erit, efficiendo vt Tabula ( in qua defignantur cęleftes domus,
cum almicantarat, atque azimut) Reti fubiectæ, mobilis fit, tardior tamen ipfo Re-
te cum indice, pro duplo temporis, hoc eft, quod eo tempore, quo Aranea cum in
dice circunuoluetur fpacio. 1 2. horarum vno gyro perfecto, ipfa Tabula efficiat tan
tummodo fex interftitia horarum. Ideft dum Tabula dicta efficit vnam integram re
uolutionem, Aranea, feu Zodiacus cum indice, duas efficiat reuolutiones. Ita quod
Aranea cum indice perficiet vnam reuolutionem fpacio temporis, 1 2. horarum, Ta-
bula verò perficiet eam fpacio temporis. 2 4. horarum. Vnde fequetur quod Ara-
nea feu Zodiacus cum indice, fpacio. 2 4. horarum perfectè circunuoluetur fupra Ta-
bulam, & ita huiufmodi horologia, in hoc nihil differrent ab illis fupradictis. Vt au
tem facias dictam tabulam tardiorem duplo temporis Araneæ cum indice, quamuis
diuerfis modis hoc fieri poffit, pręftantiorem tamen iudico, fi cum Rota indicis, aliã
Rotam concentricã coniunxeris, ita tamen, vt vnaquęq; liberè poffit volui, fimiliter
fi cum ea horologii particula ( quę circũagit Rotam indicis, quæ Italicè R o c н e т т o.
Germanicè verò T r 1 в vocatur, Latinè aũt ipfum vocabo, C o l i n v m, qui fub ro-
ta fufi reperitur ) coniunxeris alium colinum quem, fecundum vocabo, concentricũ
verò cum primo, cum eoq; confolidato, numerum verò dentium, tam Rotę adiunctę
quam fecundi colini, varijs modis poteris inuenire, quorum primus erit, vt numerus
dentium fecundæ Rotę duplus exiftat numero dentium primę, efficiendo fecundum
colinum eiufdem numeri dentium quo primum, fed quia interualla dentium huiuf-
modi Rotę, nimis angufta fortaffe refultabunt, propterea alios etiam modos inue-
ni, quorum vnus erit ( dum numerus dentium primi colini par fuerit) efficiédo fecun
dam Rotã eiufdẽ numeri dentiũ cuius eft prima. fecũdũ vero colinum, medietatis
ñumeri dentium cuius erit primus. Attamen fi primus colinus effet. 4. dentium, fecun
dum oporteret effe duorum dentium, vnde motus fecundę Rotę non effet ita conti-
nuus. Quapropter alium etiam modum excogitaui, hoc eft, cupiendo vt fecundus
colinus, ex tribus dentibus exiftat, fi primus ex. 4. repertus fuerit, oportebit prius ex
regula de tribus, numerum quendam inuenire quo inuento ipfum duplicare, & hunc
duplicatum numerum conueniet fecundam Rotam habere, vt ipfa poffit ab illo co-
lino triũ dentiũ circunuolui in duplo temporis, quo prima à fuo colino quatuor den
tium.

tium. Exempli gratia, si prima Rota constaret ex.36.dentibus, dicendum esset, si
4.conuenit cum.36. cum quibus conuenient. 3.& inueniemus.27.cum quo numero
dicta secunda Rota circunuolueretur eodem tempore à suo colino trium dentium,
quo prima à suo quatuor dentium,quare duplicando.27.haberemus.54. pro nume-
ro dentium dictę secundæ Rotæ, vt duplo temporis circunuoluatur quo prima. Sed
si primus colinus constaret ex.6. dentibus, existente sua Rota ex. 36. vellemusq́; ꝗ
secundus existeret ex.4.tunc suam Rotam oporteret habere dentes. 48. ex dicta re-
gula. Si autem primus colinus constaret ex numero impari,nihil referret, dummo-
do huiusmodi numerus impar, seu par, existeret pars propria numeri dentium, vel
ipsius dupli primæ Rotę, hoc est, esset pars aliquota numeri dentium ipsius primæ
Rotæ vel ipsius dupli. In ijs verò horologiis in quibus duplum numeri dentium di-
ctę primę Rotę non erit multiplex numero dentium primi colini, hoc fieri non pote
rit. Ratio enim tam clarè, tibi consideranti, patebit, vt nullis verbis indigeat cum
semper numerus dentium secundę Rotę multiplex esse debeat numero dentium se-
cundi colini. Idem autem non dico de prima Rota cum suo colino, hoc est; vt nu-
merus primę multiplex sit numero sui colini,nam hoc necessarium non est. Pona-
mus exempli gratia primum colinum constare sex dentibus, suam vero Rotam den-
tibus.21.cuius quidem numeri, 6.non est pars aliquota, sed dupli ipsius. 21.ipse.6.
est pars aliquota. Nunc verò si voluerimus numerum dentium secundę Rotę inue-
nire, cuius colinus ex quinque dentibus existat (supposito primo ex.6.constare) tunc
ex regula de tribus, diuiso producto, quod sit ex.21.in.5.per. 6. exibit. 17. cum di-
midio, cuius duplum esset.35.qui multiplex est ipsi quinque. Reperto igitur nume
ro secundę Rotę, cum numero ipsius colini, oportet nunc scire modum compositio-
nis, seu coniunctionis harum rerum, hoc est duorum colinorum concentricorū (sed
dē ijs satis iam superius dictum fuit) duarum Rotarum concentricarum cum Tabula,
cum Zodiaco,& cum indice, seu Ostensore, cuius quidem Ostensoris medietas tan
tummodo nobis sufficiet. Sciendum igitur nunc est quod cum primus colinus re-
uoluat totam primam Rotam,spacio temporis.12.horarum,oportet vt eius axis,seu
arbor voluat ostensorem, Zodiacumq́;, eodem temporis spacio, & quia Rota hęc
inalterabis est, propter eius coniunctionem cum suo colino,& nos oporteat indicem
Zodiacumq́;, quotidie ferè, dirigere, suisq́; locis collocare, ideo nos oportet, indi
cem,Zodiacum,& primam Rotam, ita cum axe,seu arbore coniungere, vt possimus
dicta omnia efficere. Pars igitur Arboris, seu axis dicti, quæ ingredi debet in prima
Rota,sit rotunda,& contigua ipsi Rotæ, non autem continua, vel cum Rota consoli-
data. Pars verò quę per foramen Zodiaci, seu Araneę transibit, sit quadrata vsque
ad summitatē ipsius axis (tali spissitudine, vt in claui ipsius horologij ingredi pos-
sit) & ita foramen ipsius Araneę, quadratum sit, Ostensor autem circa axem, com
positus sit tali ordine, vt circa paruum circulum volui possit, qui paruus circulus ha-
beat quadratum foramen, per quod transeat axis, qui axis aliquantulum emineat
supra ostensorē. Sub Aranea vero vel Zodiaco,locata erit Tabula,vt nūc dicemus,
sed sciendum est prius, quod inter Tabulam,& suam secundam Rotam, aliam lami-
nam immobilem interpositam esse oportet, quę circulare foramen habeat, per qď
quędam breuis fistula transeat circundans axem & coniungens Tabulā cum sua Ro-
ta, cuius quidem fistulæ superficies concaua, rotunda sit, superficies verò extrinse-
ca,non tota, nisi ea pars, quę secundam Rotam ingreditur, vt in rotundo foramine
ipsius Rotę, dicta fistula volui possit, pars vero extrinseca quę Tabulam ingredi de-
bet, sit quadrata. Tabula vero quatuor paruissima foramina habeat in extremitati-
                                                                                        bus

bus linearum, meridianę, & verticalis, vt acu mediante volui poſſit, prout oportebit.

Perfectum igitur cum fuerit opus hoc, te oportet ſcire modum ipſo vtendi. Quapropter quotieſcunque volueris, aſpice Solis locum in Zodiaco, Ephemeridibus me diantibus, idem dico de vnoquoque reliquorum planetarum. Inuento poſtea Solis loco in noſtro Zodiaco horologij, manu mediante, volue oſtenſorem, ita, vt linea fiduciæ tranſeat per gradum Solis, deinde, claui ipſius horologij mediante, volue indicem, ita cum Zodiaco coniunctum, vt linea fiducię, punctum, ſeu partem horę oſtendat in limbo horologii, quę quidem hora notanda eſt ſi fuerit ex ijs quę incipiunt à meridie vſque ad mediam noctem, vel à media nocte vſque ad meridiem, tunc acu ſupradicta mediante, poſita in aliquo illorum quatuor foraminum, circunuoluenda eſt Tabula, ita, vt extremitas lineę meridianę ſupra orizontem, ex ęquo incidat inter duodecimam horam, & lineam fiducię, computum incipiendo à duodecima hora, ſi vero dicta indicis hora fuerit ex ijs quę incipiūt à media nocte & deſinunt poſtea in meridie, oportebit, acu mediante, circunuoluere Tabulam, quouſque punctum extremum meridianæ ſub terra, medio loco exiſtat inter duodecimā horam, & horam oſtenſam à linea fiducię. Quo facto continuo videbis erectam cęli figuram. & quia vidiſti loca planetarum in Ephemeridibus, videbis etiam eorum loca accidentalia in domibus ſcilicet accidentalibus, ſi aliquas fixarum in Aranea deſiderabis, accipere poteris Ocu. ♉, cor. ♌, ſpi. ♍, Liram, Aquilam, & Arcturum, dum locus fuerit capax. Nec te moueat, quod oportebit lineam fiducię ſupra gra. Solis quotidie collocare, quod nihil refert. Nam oportet etiam quotidie cordam fuſo circunuoluere.

## DE DEMONSTRATIONIBVS PROPOSITIONVM
### Mathematicarum, nec non de Aſtrologia Iudiciaria.

*Illuſtriſſ. D. Volfardo Aiſeſtain.*

NIHIL mihi gratius & iucundius afferri potuit tuis literis, quibus te cupidum oſtendis ſciendi rationem, quare ego non vna methodo ad omnes propoſitiones demonſtrandas vſus ſim, hoc eſt, quare non omnia ea Eucl. Theoremata citem in vnaquaque propoſitione, quę ad eā demonſtrandam faciūt, quemadmodum in mea Gnomonica vidiſti me aliquando omiſiſſe. Reſpondeo ꝙ mathematicę demonſtrationes, hominibus Euclidis Elementa poſſidentibus, non indigent aliqua citatione numerorum Theorematum ipſius Euclidis, & ſi aliquando vſus ſum aliqua citatione eorundem, hoc feci propter conſuetudinem noſtri tempo ris, vel etiam ad faciliorem intelligentiam illorum, quibus ſcribebam. Sed omnia quamuis minima citare, vt faciūt nonnulli, mihi, nimis laborioſum, ſuperfluumꝗ; videtur, preſertim ijs (vt dixi) qui memoria tenent prima Elementa. Hęc igitur eſt vna ratio. Alia, quia multoties, ita coniuncta eſt ſpeculatio cum ipſa concluſio ne, vt mihi ſepius viſum ſit ſuperfluum, aliquid de ipſa theoria ſcribere. In iis enim, quę dum puer eram ſcripſi, videbis ſcrupuloſam illam methodum, ſed poſtea, non niſi in arduis propoſitionibus me nihil eſſentiale pretermittere.

Circa vero id de quo me interrogas, ſcilicet, vtrum putem omnia vera eſſe, ea quę ſcripta reperiuntur in libris Aſtrologiæ iudiciarię. Reſpondeo quod non, imo

puto

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puto plurima falfa effe . Nam illa multitudo partium,vt pars vitę, pars Hylech, pars
futurorum , & relique omnium domorum cœleftium , falua parte fortunę,funt merę
nugę . Idem dico de faciebus, fiue decanis, de terminis, & de gradibus ipfis , vt pu-
ta azemenis, puteis ,vacuis, fumofis, & de reliquis. De Domibus vero , Exaltationi
bus, nec non triplicitatibus, experientia côfirmat ea vera effe.Idê affirmo de Domi
bus accidentalibus, rationalibus tamen, non autê de Domibus Campani,& Gazuli.
Obferuationes etiam complexionum feu influentiarum ipforum Planetarum rectè
factæ funt, quę etiam à coloribus ipforum Planetarum ferè iudicari poffunt . Con-
iunctiones afpectusq; ipforum inuicem, fimiliter mirabilia faciunt, &ex maiori par
te,ea,quę de iftis fcribuntur vera funt. Reuolutiones annuę fimiliter,cum Domino
anni.Dominum verò orbis Diuiforemq; non approbo , nam hic pendet à termino,
ille verò ab hora. Nouenarias autem Dodecathemoria , Alfridarias, & multa iis fi
milia omnia nego . Antifcia, vera funt , ideft influunt, malos tamen effectus , alia
plus alia verò minus, prout aliqua eorum funt tetragona,alia verò trigona,alia ma-
gna,alia parua,magna funt,vt Arietis cum Virgine, & Librę cum Pifcibus, parua ve
rò, debiliaq; Geminorum cum Cancro , & Sagittarij cũ Capricorno . Sed difufius
hęc oĩa videbis in meo illo particulari tractatu,de quo tibi aliàs dixi,in quo multa
videbis , quę omnia ab experientia,ex multis à me obferuatis, comprobata funt ,
quem quidem tractatum cum quibufdam alijs meis fpeculationibus in lucem prode
re cupio , fi fieri poterit,antequam ad directionem mei Horofcopi cum corpore
Martis Anęretę perueniam,quę quidem directio circa annum millefimum quin-
gentefimum nonagefimum fecundum eueniet.

F I N I S.

# ERRATA CORRIGITO IVXTA INFRASCRIPTAM TABVLAM

Reliquos verò errores, qui orthographiam respiciunt, benignus lector corrigat, qui sciat multos errores in editione irrepsisse, quod non paucos dies morbo fuerim detentus dum præsens opus excuderetur.

| Pag. | Lin. | Errata | Correcta | Pag. | Lin. | Errata | Correcta |
|---|---|---|---|---|---|---|---|
| 3 | 29 | æqualis | æquali | 158 | 26 | versa | versam |
| 8 | 35 | mains | maior | 158 | 26 | sit | sint |
| 9 | 15 | in vnitate superficialis, erit ac | inunitate superficialis erit, ac | 162 | 22 | cindenda | scindendæ |
| 11 | 1 | prouecuens | prouenientem | 163 | 7 | oppesitus | oppositum |
| 11 | 8 | futurum | futurus | 164 | 24 | tanta | tantæ |
| 11 | 31 | illa nihil aliud sunt | illud nihil aliud est | 164 | 37 | adiuncta nobis essent duæ aliæ | adiuncti nobis essent duo alii |
| 11 | 38 | diuidemus | diuidamus | 164 | 39 | sestineret | sustineret |
| 12 | 1 | tertiæ, sine | tertiæ sint | 164 | 40 | dicta | dicti |
| 12 | 2 | productum | productus | 164 | 41 | aliquam | aliquem |
| 19 | 30 | prouveniens | prouenientem | 165 | 3 | sufficiant | sufficiat |
| 19 | 30 | productum æquale | productus æqualis | 165 | 4 | subsesquialter | subsesquialterüm |
| 21 | 27 | eadem | eædem | 186 | 32 | finit | finitam |
| 24 | 18 | est | esset | 187 | 29 | propinquæ | propinqui |
| 26 | 39 | est | esset | 148 | 30 | philosophi supra | philosophi, proximè supra |
| 41 | 24 | quæ | quæ | 148 | 31 | contingeret mense | contingeret vt mense |
| 41 | 31 | distinguendæ | distinguendo | 187 | 34 | regionis)non considerans | regionis)in quo nõ considerauit |
| 59 | 5 | subsequens | subsequentem | 190 | 11 | lenas | lenas |
| 61 | 3 | hæc via tenendæ | hanc viam tenere | 190 | 23 | lenis | lenis |
| 61 | 3 | fuit | fuerit | 191 | 10 | aliud quadratum | alius quadratus |
| 61 | 45 | numerum quæsitum | numerus quæsitus | 191 | 15 | qualis | qui |
| 62 | 30 | quantum est dimidiã occupatam | quanta est dimidia occupata | 193 | 17 | cum | tum |
| 64 | 4 | speculari | considerari | 193 | 36 | sit | facit |
| 64 | 15 | totq; sunt termini | totq; esse terminos | 193 | 40 | pleno vbi | pleno posuerimus, vbi |
| 64 | 18 | hoc est numerum | hoc est per numerum | 195 | 10 | valent | valent apud ipsos |
| 64 | 19 | primo quod vnus est | primo qui vnus est | 196 | 19 | phylosophia | philosophia |
| 66 | 18 | minimum | minimus | 196 | 20 | pellendo | plendo |
| 67 | 13 | maximum terminum addendü | maximus terminus addendus | 198 | 18 | sunt etiam | esse etiam |
| 72 | 1 | numerum | numerus | 204 | 12 | principium | principium |
| 72 | 8 | singulos itinere | singulos in itinere | 204 | 18 | indigna | indignas |
| 74 | 7 | itinerarium | itinerantium | 204 | 21 | diffundetur, crescat | diffundatur, & crescat |
| 78 | 21 | morüm | modum | 205 | 16 | aliquando ser | aliquando me ser |
| 78 | 24 | noueratius | nouenarius | 205 | 19 | anno præter, necessitatē, gignitur | anno, præter necessitatē gignitur |
| 81 | 10 | iuncta | iuncta | 206 | 21 | plenilunium, quod | plenilunium fieri, quod |
| 88 | 35 | armonica | harmonica | 207 | 26 | inchoet annus | inchoetur annus |
| 91 | 6 | Quare argumétãdo permutando | Quare permutando | 209 | 9 | soli, quod punctum | solis, cuius punctum |
| 95 | 27 | vitium | vicium | 210 | 10 | calebrandi | celebrandis |
| 97 | 14 | summa | summam | 212 | 33 | Inuenta | Inuenti |
| 97 | 30 | illud vero quod con | illud vero con | 212 | 33 | duæ | duo |
| 98 | 22 | desideraremus | desiderauerimus | 214 | 9 | classi | classis |
| 103 | 23 | dispositis facit | dispositis, tantum facit | 214 | 10 | Inter Eximias | Quia inter Eximias |
| 104 | 39 | dispositis, facit | dispositis, tantum facit | 214 | 26 | reperiretur | reperietur |
| 107 | 21 | eccedit | excedit | 214 | 34 | necessario sit futurum, vt | necessarium sit, vt |
| 110 | 41 | habuerit eius cerebrum | habuerit cerebrum | 215 | 37 | quod si velimus | si velimus |
| 113 | 46 | sufficeret | suffecisset | 215 | 39 | potest vno eodemq; | vno eodemq; |
| 116 | 8 | quarum secundam | quorum secundum | 215 | 45 | sit circulus | sit vere circulus |
| 116 | 8 | prima, tertiam | primi, tertium | 217 | 2 | Quod cum verum | Quod si verum |
| 116 | 9 | secunda | secundü | 217 | 4 | nulla est ratio | nulla esset ratio |
| 130 | 23 | ducendæ | ducendus | 221 | 31 | inueniemus | designabimus |
| 133 | 10 | lineam quæ | lineam, quæ | 2 | 20 | falli | fallat |
| 133 | 11 | dratam | dratum | 225 | 20 | si quæ | siq; |
| 134 | 16 | inueniet | inuenimus | 225 | 25 | chuum | chuium |
| 137 | 22 | distams | distantius | 226 | 14 | oleum effundebat | effundebatur |
| 137 | 45 | Notissimum igitur primum | Notissimum primum | 227 | 5 | euadit | euadet |
| 139 | 15 | est | sit | 231 | 36 | supradicta, minuta | supradicta minuta |
| 139 | 28 | duas | duos | 232 | 21 | progrediuntur | progredi |
| 141 | 5 | comperuisse | comperisse | 234 | 15 | altiori cœlo | altius cœlum |
| 142 | 19 | quã | ac | 239 | 12 | dum | cum |
| 143 | 17 | linea | lineam | 241 | 41 | offerens | cogitans |
| 144 | 23 | patebit, si quis | patebit, quòd si quis | 245 | 25 | diametri | diametros |
| 145 | 18 | constante | constantem | 248 | 42 | sit, & ob id | sit, ob id |
| 146 | 20 | pastam | massam | 249 | 42 | alium numerum | alio numero |
| 146 | 22 | pastam | massam | 250 | 13 | um voluerimus | cum inuestigare voluerimus |
| 148 | 13 | pponit nõ cõcludit melius aũt | proponit .nõ concludit, melius aũt | 251 | 1 | inuenerimus | inuenimus |
| 14 | 41 | prætergradiatur | prætergradietur | 252 | 6 | tros | ter |
| 152 | 6 | videtur | videri | 252 | 6 | alia verò diametro | alius verò diameter |
| 153 | 22,23 | quia libra | quia cum libra | 252 | 8 | te id non | te non |
| 153 | 23 | materiales, cum sustineantur | materiales sustineantur | 252 | 12 | duabus | duobus |
| 153 | 24 | existente, vnde aliqua | existente, aliqua | 255 | 16 | in numero | numero |
| 154 | 23 | futurum | effecturam | 255 | 25 | cum non videat si | cum non videat quod si |
| 155 | 25 | carta | charta | 255 | 36 | suo centro | suis centris |
| 56 | 23 | sufficere | sufficeret | 255 | 39 | mille milliaria | mille milliariorum |
|  |  |  |  | 255 | 41 | recenta mille | tercenties mille |

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|---|---|---|---|
| 257 | 5 | quem | quod |
| 257 | 11 | quem | quod |
| 257 | 18 | quod | nam |
| 258 | 15 | ictum | ichus |
| 258 | 15 | eundem esse futurum | idem est et |
| 258 | 27 | debet, quanto | debet in huiusmodi situ, quanto |
| 258 | 41 | quod | qui |
| 262 | 17 | erunt | supponuntur |
| 263 | 22 | a K facta sit | .a. K. sit |
| 263 | 22 | semidiameter esse vnius | semidiameter vnius |
| 265 | 4 | cognitæ | cogniti |
| 272 | | in figura vbi est.P. | ponatur.R |
| 272 | 18 | E.i. ad. E.i. | E.I.ad.E.i. |
| 273 | 3 | licet | liceret |
| 274 | 7 | te | tibi |
| 275 | 10 | dividere | dividendo |
| 276 | 47 | accipiemus | accipiamus |
| 278 | 8 | Moretæ | Moretæ |
| 280 | 25 | diminutæ, quartæ | diminutæ seu defectiuæ, quartæ |
| 281 | 36 | gravissimum | gravissimo |
| 281 | | in fine pag. vnamquanque Zifram accommoda sub vnoquoque alphabeti characterem ad priores Zifras prius addendo. 1. Idem dico in pag. 282. & inter, D. et E. pone.b. | |
| 285 | 24 | antea | ante |
| 289 | 11 | quanta pars | quantam partem |
| 289 | 25 | quanta pars | quantam partem |
| 289 | 42 | est | erit |
| 290 | 10 | fatile | fatilis |
| 290 | 19 | æqualis | æquale |
| 290 | 43 | circumscriptibilis | circumscribentis |
| 291 | 1 | od | Quod |
| 294 | 33 | detractis | detracti |
| 294 | 34 | angulis contingentia solidisq; | anguli contingentiæ solidiq; |
| 295 | 24 | chordæ | chordam |
| 295 | 32 | lixum | lixus |
| 297 | 1 | distincta, procedendo | distincta sint, procedendo |
| 299 | 32 | reflessum | reflexum |
| 299 | 34 | flossum | flexum |
| 302 | 14 | eosdem | eosdem |
| 304 | 3 | tæ | tus |
| 305 | 21 | Idem facere | Idem possumus facere |
| 312 | 37 | dupla | duplo |
| 313 | 6 | quotiescunque | qui |
| 315 | 23 | retrogradiendum | retrogradiendum |
| 316 | 22 | vbi | tibi |
| 324 | 41 | si constitueremus | si nos constitueremus |
| 325 | 12 | simili ad | simili. Ad |
| 326 | 9 | longare | ducere |
| 329 | 5 | ipsi ellipsis | ipsius ellipsis |
| 333 | 15 | cogitetur | cogitemus |
| 333 | 17 | vnde pro genera | vnde ex genera |
| 333 | 20 | æqualem esse longitudini | æqualem longitudini |
| 333 | 21 | ei | eam |
| 333 | 21 | minor | minorem |
| 333 | 23 | nor | norem |
| 333 | 24 | maior | maiorem |
| 333 | 25 | maior | maiorem |
| 333 | 37 | communeq; | communiq; |
| 335 | 26 | reflectit | reflectitur |
| 335 | 47 | & remotiori | vel remotiori |
| 336 | 19 | speculi superficiem | speculi planam superficiem |
| 336 | 22 | reflectit | reflectitur |
| 337 | 8 | mam tantummodo imaginem | vna tantummodo imago |
| 337 | 12 | ipsæ | ipsi |
| 337 | 42 | quæ | qui |
| 338 | 14 | protractæ | protracto |
| 338 | 26 | ratioinare | apta |
| 340 | 23 | Alius modus | Alium modum |
| 341 | 9 | æquale | æqualem |
| 343 | 23 | videbitur | videri |
| 344 | 5 | hoc est | hoc est quod |
| 344 | 1 | punctus | punctum |
| 345 | 6 | ei | cum altitudine |

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|---|---|---|---|
| 345 | 19 | ipsum. repertum | ipsum, vt repertum |
| 347 | 16 | reflectit | reflectitur |
| 352 | 38 | opinauerunt | opinati sunt |
| 353 | 8 | parabolem | parabolam |
| 353 | 15 | hæc | hic |
| 353 | 18 | parallelam | parallelæ |
| 356 | 15 | muri, & | muri, in oriZontali.q p et |
| 356 | 33 | mediante. cognoscere | mediate. cuius axis ad oriZontem erectus sit.o.n; cognoscere |
| 360 | 13 | duplum | duplus |
| 360 | 28 | vnde iste circulus | iste circulus |
| 361 | 36 | quæ | qui |
| 364 | 23 | diameter | diametrum |
| 367 | 25 | prolungando | producenda |
| 369 | 2 | æqualem | æquale |
| 370 | 4 | sians | sinum |
| 370 | 30 | summæ | summam |
| 372 | 1 | cæterarum | cæteris |
| 372 | 36 | qui qui | qui |
| 378 | 11 | hæc | hæ |
| 382 | 41 | prolungatis | productis |
| 383 | 3 | quæ | qui |
| 383 | 13 | ginabis | ginaberis |
| 383 | 20 | trianguliq; | triangulaq; |
| 383 384 | | in prima parabola obliqua extremitatem inferiorem diametri signabis charactere. d. in secunda vero charactere.r. | |
| 384 | 29 | maior proportio | maiorem proportionem |
| 384 | 37 | proportio habebit | proportionem habebit |
| 385 | 5 | c.b. | .e.b. |
| 386 | 22 | b.e. | .b c. |
| 386 | 29 | positum | positus |
| 385 | 31 | b.c. | .b.e. |
| 386 | 33 | & proportionalitate | & ex proportionalitate |
| 386 | 34 | antecedens in | antecedens. A B. in |
| 386 | 40 | quod | qua |
| 388 | 2 | B.) | (β ) |
| 388 | 8 | b.cum duplo.b.o. | .b.d.cum duplo.b.e. |
| 388 | 45 | b.e. | .b.c. |
| 388 | 45 | b e | .b.c. |
| 388 | 46 | ad.e.b. | ad.d.b cum .e.b. |
| 390 | 25 | diuisa | diuisas |
| 392 | 3 | sit paraboles.a. | sit paraboles.a.b.c. |
| 392 | 27 | aliqua reliquarum | aliquus reliquorum |
| 393 395 397 | | in parabola vbi sub g est.i.pone.r. & supra solidum minus dicatur cubus minor (leantur duo vero lineæ sine characterib. de | |
| 394 | 18 | w.y.ad.m n. | vt p.ad.m.n. |
| 394 | 33 | x.o. | n.o. |
| 394 | 34 | i.k.ad.i.k.ad. fg. | i.k.ad fg. |
| 395 | 11 | quintas | quintæ |
| 396 | 1 | d b. | g.b. |
| 397 | | Vbi rubrica dicit.Defensio nostra contra Anto. Bergam & contra Antonium Bergam, & Alexandrum Piccol. | dicatur. contra Anto. Bergam & Alex. Piccol. atq; defensio nostra contra Excell. August. Michelem |
| 399 | 32 | tione | tionem |
| 400 | 39 | vnum tantum numerum medium | vnus tantum numerus medius |
| 402 | 19 | diametrum | semidiametrum |
| 403 | 7 | suis | suæ |
| 403 | 9 | iam supra | eo tractatu |
| 403 | 17 | quidquid | quidquam |
| 404 | 37 | milliaria | milliaribus |
| 406 | 19 | trabuchum quadratum | trabuchus quadratus |
| 410 | 38 | Quæ sors | quæ sorte |
| 410 | 41 | quamlibet | quælibet |
| 411 | 27 | dicens | dicat |
| 411 | 28 | eodem libro | vt eodem libro |
| 412 | 24 | non dependeat? | dependeat? |
| 413 | 2 | vt supra | vt alias |
| 413 | 22 | millia vices | millibus viciam |
| 415 | 32 | finestra | fenestra |
| 416 | 15 | operatio | operationem |

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