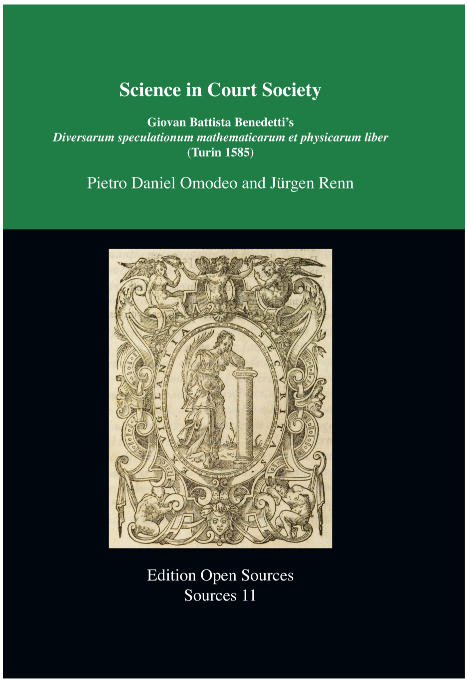
Edition Open Sources
Sources 11

Pietro Daniel Omodeo and Jürgen Renn:

Mechanics

DOI: 10.34663/9783945561485-06



In: Pietro Daniel Omodeo and Jürgen Renn: *Science in Court Society : Giovan Battista Benedetti's <i>Diversarum speculationum mathematicarum et physicarum liber</i> (Turin, 1585)*
Online version at https://edition-open-sources.org/sources/11/

ISBN 978-3-945561-16-4, DOI 10.34663/9783945561485-00

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de

Chapter 5
Mechanics

The book on mechanics, *De mechanicis*, the third of the *Diversae speculationes*, is divided into twenty-five chapters. Mechanical issues and references to mechanics can also be found in the epistles. As to the discussion of the motion of fall through media and of hydraulic problems, these are not part of this book. *De mechanicis* begins with a brief preamble in which Benedetti claims that he treats topics that have never been dealt with before or have not been sufficiently explained. In this section we will discuss the positioning and controversies implicit in this strong statement in an age when mechanical studies were very lively in the Italian peninsula and abroad. We will first offer an overview of Book 3 of the *Diversae speculationes*. Second, we are going to look more closely at the first foundational chapters of the treatise. Third, we will consider the rivalry with Del Monte, emerging from the latter's harsh criticism of Benedetti and, in part, his misunderstanding of some crucial elements of Benedetti's theory. The context of these lively disputes is the reaction to the publication of Tartaglia's eclectic work on this subject, the *Quesiti, et inventioni diverse* (1546), and his re-issue of the medieval classic on the science of weights. Benedetti, as a critical pupil of Tartaglia, could not sympathize with the absolute rejection of Tartaglia and the medieval tradition his approach rested upon. At the same time, he felt the need to distance himself from several aspects of Tartaglia's treatment, as we will reconstruct in detail in this section. The debates between Benedetti and Del Monte arguably culminated with Galileo's work, which stands out as a sort of synthesis of earlier positions. Understanding these historical developments, as well as the intellectual triangle Benedetti-Del Monte-Galileo, is fundamental in order to trace Benedetti's influence on his contemporaries and on the young Galileo.[1]

5.1 An Overview of *De mechanicis*

5.1.1 The Foundations of the Theory of the Balance

Chapters 1 to 6 of *De mechanicis* contain a systematic account of the foundation on which Benedetti built his mechanics. Chapter 1 clarifies qualitatively how the variable weight changes depending on the obliqueness of the balance beam. While a body attached to the end of the beam has a maximum weight if the beam is in a horizontal position, it vanishes when the beam is in a vertical position. Benedetti explained this behavior as a consequence of the different extent to which the attached weight rests on the center of the balance. If the position of the beam is close to the vertical, the weight of a body attached to the end of the beam is close to zero since it rests nearly completely on the center of the balance.

Chapter 2 clarifies the positional changing of the weight quantitatively. Benedetti related the balance with an oblique position of the beam to a bent lever with one horizontal and one oblique arm, thus providing the precondition for a generalization of his result. A

[1]Section 5.1 is derived from Renn and Damerow 2012, chap. 6.1–6.3 and section 5.2 from Renn and Omodeo 2013.

generalization of this kind is indeed required if the lines of inclination of the bodies at the end of a balance are conceived as being directed to the center of the earth and hence no longer as being parallel to each other. Benedetti mentioned this possibility at the end of this chapter, but considered the angle between the two directions as being too small to be measured and thus not necessary to be taken into account.

In chapter 3 Benedetti generalized from the downward inclination of a body attached to the balance beam to forces acting upon the body not vertically but making an acute or obtuse angle with the horizontal beam. Accordingly, he replaced the bodies at the end of the balance beam with two weights or two moving forces (*duo pondera, aut duae virtutes moventes*), as he formulated somewhat ambiguously. His derivation of their quantities was based on a reinterpretation of the horizontal distances between the center of the balance and the vertical projections of the bodies at the end of a beam in an oblique position (Figure 5.1). He interpreted these distances as perpendicular distances from the center of the balance to the lines of inclination, and was thus also able to apply the result he achieved for vertically descending weights to lines of inclination caused by forces that are not vertical.

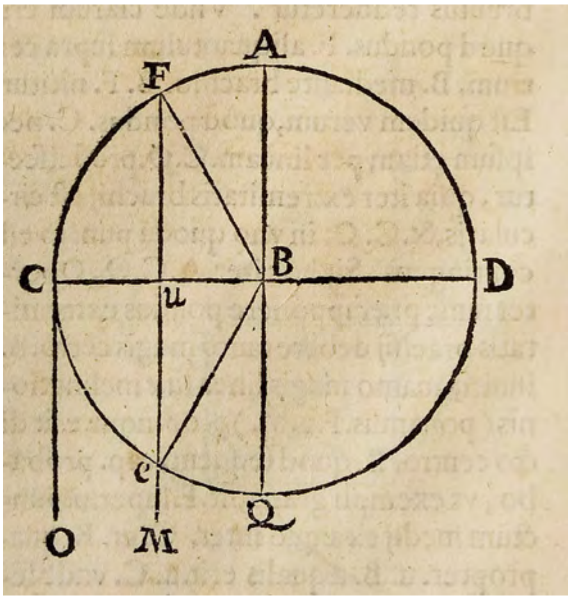


Figure 5.1: Benedetti's diagram showing a balance CBD or FBD. The lines CO and FUEM are the so-called lines of inclination connecting the weights C and F with the center of the elements. The length of the projection on the horizontal is proportional to the positional heaviness. (Max Planck Institute for the History of Science, Library)

Benedetti maintained that his arguments in chapters 1 to 3 clarify all the causes operating on balances and levers. To demonstrate this, he discussed in chapters 4 and 5 the validity of his results if applied to material balances and levers, taking into account that they have a beam with finite extension. This, however, does not imply that he calculated the influence of the weight of the beam itself. His discussion was rather restricted to a justification of his claim that the geometry of a rectangular beam does not require a modification of his propositions. In chapter 5 he treated the case of a lever whose fulcrum is at one of its ends.

Finally, in chapter 6, Benedetti added the description of an instrument used in bakeries for treating the dough. He explained the function of the instrument by applying his proposition from chapter 3.

The systematic approach used by Benedetti in this first part of his treatise is complemented by chapter 9, in which he explained the division of the scale of a steelyard into equal intervals.

5.1.2 Criticism of Tartaglia and Nemorarius

In chapters 7 and 8 Benedetti criticized the theorems of his former teacher Tartaglia, in particular those that Tartaglia adapted from Jordanus Nemorarius. Both chapters deal exclusively with some propositions of Book 8 of Tartaglia's *Quesiti, et inventioni diverse*,[2] which is concerned with the science of weights and is entitled, accordingly, *Sopra la scientia di pesi*. In those cases in which Tartaglia's propositions are adapted from Nemorarius, Benedetti mentioned explicitly the corresponding proposition in the edition of Nemorarius' *De ratione ponderis*, corrected and illustrated by Tartaglia, and published under the title *Iordani opusculum de ponderositate Nicolai Tartaleae studio correctum novisque figuris auctum*.[3]

Chapter 7 starts with some brief critical remarks on Tartaglia's propositions 2 to 5. Tartaglia's proposition 2 essentially paraphrases and modifies the Aristotelian claim that the speed of moving bodies is proportional to the driving force. Following Nemorarius, Tartaglia maintained that the velocities of descending heavy bodies of the same kind are proportional to their power (*potentia*), while in the case of ascending bodies their velocities are inversely proportional to their power. For bodies of the same kind their power is conceived here as proportional to their sizes, that is, to their weights. Descending bodies are thus simply falling bodies with velocities proportional to their weights, while in the case of ascending bodies their weight acts as a resistance. Tartaglia's proposition 3 generalizes proposition 2 for bodies with equal weights but unequal positional heaviness. His proposition 4 maintains that in the latter case the power of bodies attached to a balance is proportional to the distances from the center.

Benedetti's critical remarks are somewhat eclectic. He argues that Tartaglia, in his second proposition, does not take into account the quantity of external resistance (*quanti momenti sint extrinsecae resistentiae*). With regard to Tartaglia's third proposition, Benedetti points to its assumptions, namely that the bodies have to be homogenous and must have the same shape. He criticizes Tartaglia's proof as it does not actually require these assumptions, but would also be true for heterogeneous bodies or for bodies with differing shapes. Concerning the fourth proposition, he criticizes Tartaglia for not proving what he claimed to prove. Instead, he should have followed Archimedes's proof of the law of the lever.

Benedetti's chapter 7 continues with a detailed discussion of the second part of Tartaglia's proposition 5 and the following two corollaries and is thus directly concerned with the equilibrium controversy, that is, the controversy about whether or not a balance in equilibrium removed from its horizontal position will automatically return to this position. Tartaglia maintained in this proposition that a balance that is in equilibrium in a horizontal position will necessarily return to this horizontal position when moved into an oblique position. In a first corollary, he claimed that the more the balance beam is brought into an oblique position, the more the bodies attached to it become positionally lighter. In a second corollary, he claimed that while both bodies in this case become positionally lighter, the lifted body loses less of its positional heaviness than the body moving down. He concluded that the beam will return to a horizontal position. Benedetti

[2] Tartaglia 1546.
[3] Nemore 1565.

questioned Tartaglia's approach by referring to the first three chapters of his own treatise, arguing in particular that Tartaglia's second corollary must be wrong. He discussed once more the balance beam in an oblique position, but now without the assumption that the lines of inclination of bodies attached to the balance beam are parallel. Rather, he considered the case that these lines are directed to the center of the world, showing, as we have discussed, that it is not the lifted body, but rather the body that is moved down, which loses less of its positional heaviness.

Benedetti continued in chapter 8 with critical comments on Tartaglia's propositions 6, 7, 8, and 14. Tartaglia's proposition 6 contains the proof of his fallacious claim that the lifted body of an oblique balance beam loses less of its positional heaviness than the body moving down, now modified by the further claim that the difference is smaller than any finite quantity. Tartaglia claimed:

> […] that the differences between the heaviness of these two bodies is impossible to give or find between two unequal quantities.[4]

Like Del Monte had done before him, but with different results, Benedetti criticized Tartaglia for not taking into account that the lines of inclination are not parallel.

Tartaglia's proposition 7 contains the simple statement that if the arms of a balance are unequal and bodies with equal weights are attached to the ends of the beam the balance will tilt on the side with the longer arm. Benedetti criticized Tartaglia again for not taking into account that the lines of inclination are not parallel, and claimed that in any case Tartaglia did not give the correct cause of the effect.[5]

Tartaglia's proposition 8 formulates, following Nemorarius, the law of the lever in terms of positional heaviness, stating that if the lengths of the parts of the balance beam with unequal arms are inversely proportional to the weights of the bodies attached to them, their positional heaviness will be equal. Benedetti criticized that this proposition is much better demonstrated by Archimedes.

Finally, Tartaglia's propositions 14 and 15 concern Nemorarius's proof of the law of the inclined plane, which from a modern perspective is essentially correct. Benedetti criticized Tartaglia's argument by attributing to it an interpretation of the inclined plane as a balance, with the top of the plane being its center. His criticism, based on the propositions of his chapters 1 to 3, thus completely missed the point of Tartaglia's argument.

5.1.3 Criticism of Aristotle's Mechanics

Benedetti's treatise on mechanics continues mainly with critical notes on the Aristotelian *Mechanical Problems*.[6] His notes are as diverse as the Aristotelian *Mechanical Problems* themselves.

Before he embarked on this criticism, Benedetti dealt with the problem of why a steelyard carries a linear gradation in chapter 9.[7] He took into account the weight of the beam and that of the scale by postulating the equilibrium of the balance when no extra weight is added. Then he added weights of one pound on both sides, arguing that, by

[4] Tartaglia 1546, 91*r*: "[…] che la differenzia ch'è fra le gravità de questi dui corpi egli è impossibile a poterla dar, over trovar' fra due quantità inequali." Translation in Drake and Drabkin 1969, 130.
[5] We will discuss Benedetti's criticism in more detail later.
[6] Aristotle 1980. See Rose and Drake 1971 and also the introduction to Nenci 2011.
[7] Benedetti 1585, 152. See Drake and Drabkin 1969, 178.

common science (*scientia communis*),[8] the balance stays in equilibrium if they are placed at equal distances from the fulcrum. He had thus found the mark on the beam that indicates a magnitude of one pound. He then successively placed further weights onto the scale, now arguing from the law of the lever that they must be compensated by distances proportional to their number. He thus avoided the problem of applying the law of the lever directly to a material steelyard, just as one does in practice when gauging such a balance.[9]

In chapters 10 and 11 Benedetti started with critical remarks on Aristotle's first problem. Aristotle asked why larger balances are more accurate than smaller ones.[10] Actually, this concrete physical question is not the focus of the extensive answer the author gave to this problem. Instead, he provided a long proof of the basic explanatory principle which plays a major role in the whole treatise. At the end of the proof Aristotle argued that the same load will move faster on a larger balance, thus making such balances more accurate.[11]

The criticism Benedetti applied to Aristotle's argument has two parts. In chapter 10 Benedetti began by rejecting Aristotle's claim that the circumference of a circle combines concavity with convexity. He then argued against a specific part of Aristotle's proof of his principle which involves the superposition of motions. In this part Aristotle showed that:

> [...] whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line.[12]

He concluded:

> [...] if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line.[13]

For the Aristotelian author this proposition served as a means to describe circular motion as a result of two movements with no fixed ratio. Benedetti, however, did not relate his criticism to this context. He argued only that Aristotle's inference concerning movements in two directions is not sufficient since a straight movement can result from two quite different motions. This criticism does not really relate to the Aristotelian argument, other than by showing that his entire attempt to derive the behavior of a balance from a principle of circular motion is misguided.

In the same vein, Benedetti's criticism in chapter 11 then deals directly with Aristotle's answer to the question of why larger balances are more accurate than smaller ones. He argued that Aristotle's argument is not well founded since the greater accuracy has nothing to do with the motion of the balance beam but only with the geometrical constellation.[14] To conclude he added a consideration of material balances, arguing according to his own principles that a weight on the larger balance will be positionally more effective.

---

[8] In the sixteenth century the term *scientia communis* was used to designate knowledge common to all mathematical sciences, its core being the Euclidean theory of proportions. See Sepper 1996, 153–154.

[9] See the discussion in Damerow, Renn, et al. 2002.

[10] Aristotle 1980, 1, 848 b 1–850 a 2 (337–347).

[11] Aristotle 1980, 1 (347).

[12] Aristotle 1585, 507: "Quandoquidem igitur in proportione fertur aliqua id, quod fertur, super rectam ferri necesse." Translation in Aristotle 1980, 1, 848 b 11–848 b 13 (337).

[13] Aristotle 1585, 508: "Si autem in nulla fertur proportione secundum duas lationes nullo in tempore, rectam esse lationem est impossibile." Translation in Aristotle 1980, 339.

[14] Benedetti 1585, 153; Drake and Drabkin 1969, 180–182.

Benedetti's chapter 12 concerns problems 2 and 3 of the Aristotelian *Mechanical Problems*.[15] Problem 2 raises the question that forms the starting point of the equilibrium controversy:

> If the cord supporting a balance is fixed from above, when after the beam has inclined the weight is removed, the balance returns to its original position. If, however, it is supported from below, then it does not return to its original position. Why is this?[16]

Aristotle implicitly assumed that the balance beam has a certain thickness and weight. It follows as a result of the geometry of the balance in an oblique position that if the beam is fixed from above, a greater part of the beam is on the lifted side of the perpendicular line across the suspension point. Consequently the beam will move back by itself into the horizontal position. The opposite is true for a beam fixed from below. In this case, the greater part of the beam is on the lower side so that it cannot move back into a horizontal position by itself.

Benedetti criticized the first case by arguing that it is not only the weight of the beam that causes it to return to the horizontal position, but also the different distances of the weights in an oblique position from the vertical through the point where the beam is fixed. According to his theory of the dependency of the weight on the obliqueness of the beam, the weights must be different on both sides. Benedetti thus generalized Aristotle's argument to the case of a balance without a material beam carrying weight itself.

In the second case of a beam supported from below, he argued that Aristotle is completely mistaken. Benedetti maintained that the beam will not remain in its oblique position, but that the lower part will move down until the beam is in the vertical position.

Problem 3 of the Aristotelian *Mechanical Problems*[17] concerning an explanation of the effect of a lever is, for Benedetti, not worth the effort of a detailed criticism. He only briefly notes that Aristotle did not give the true cause, which one will find in his own theory presented in chapters 4 and 5.[18]

In the very short chapter 13, Benedetti criticized problem 6 of the Aristotelian *Mechanical Problems*:

> Why is it that the higher the yard-arm, the faster the ship travels with the same sail and the same wind?[19]

The Aristotelian answer provided in the *Mechanical Problems* is based on an interpretation of the yard-arm as a lever that has its base at the point where the yard-arm is fixed as the fulcrum. Benedetti maintained that this interpretation of the yard-arm as a lever:

> […] does not give the true explanation. For on this kind of explanation the ship would have to move more slowly rather than more swiftly. For the higher

---

[15] Aristotle 1980, 347–355; Drake and Drabkin 1969, 182–183.

[16] Aristotle 1585, 511: "Cur siquidem sursum fuerit spartum, quando deorsum lato pondere, quispiam id admovet, rursum ascendit libra: si autem deorsum constitutum fuerit, non ascendit, sed manet?" Translation in Aristotle 1980, 347–349.

[17] Aristotle 1980, 353–355.

[18] Benedetti 1585, 154; Drake and Drabkin 1969, 183.

[19] Aristotle 1585, 515: "Cur quando antenna sublimior fuerit, iisdem velis, et vento eodem celerius feruntur navigia?" Translation in Aristotle 1980, 361.

> the sail that is struck by the force of the wind, the more the ship's prow will be submerged in the water.[20]

Benedetti added one sentence with his own explanation, according to which the ship with a higher sail moves more swiftly because the wind blows more strongly in the higher region.

Chapter 14 provides a long discussion of problem 8 of the Aristotelian *Mechanical Problems*. The question posed in this problem is why round and circular bodies are easiest to move. Three examples are mentioned and later discussed: the wheels of a carriage, the wheels of a pulley, and the potter's wheel. Benedetti claimed that Aristotle's answer to the question he posed is not sufficient. Nevertheless, Benedetti himself argued essentially in a similar manner, only somewhat more extensively. Both of them argued that the circle, contrary to differently shaped bodies, touches a plane only at one point which can be considered as the fulcrum of a lever. But Benedetti added a further argument which is not given by Aristotle. He argued that a circle can be pulled along a plane without difficulty and resistance:

> […] because in such a case the center will never change its position by moving upward from below, i.e., will never change its position with respect to the distance or interval which lies beween it and line *AD*.[21]

At the end of the chapter, Benedetti discussed the question of why a potter's wheel set into motion by an external force will continue to rotate for a time but not forever. In his response he took into account the friction with the support of the wheel and with the surrounding air. But he also discussed reasons that are more deeply concerned with the nature of such motion, as we have discussed above. He claimed, in particular, that the rotational motion is not a *natural motion* of the wheel, evidently making reference to the Aristotelian distinction between natural and violent motions. He also claimed that a body moving by itself because an *impetus* has been impressed upon it by an external force has a natural tendency to move along a rectilinear path. This statement seems to comes close to the principle of inertia of classical physics, but it actually deals with rectilinear motion as a forced motion and does not involve any assertion about its uniformity. Benedetti seems to suggest, in any case, that this natural tendency is in conflict with the forced rotational motion of the wheel, which in turn slows it down. The smaller the wheel and the more its parts are constrained to deviate from the rectilinear path, the greater the decrease in speed will be.[22]

In chapters 15 and 16 Benedetti dealt with issues of scale as they are brought up by the Aristotelian *Mechanical Problems*. In chapter 15, consisting merely of one short sentence, Benedetti referred to his own earlier treatment of Aristotle's question of why larger balances are more exact (erroneously citing chapter 10 instead of chapter 11 of his treatise) in order to deal with the ninth problem of the Aristotelian *Mechanical Problems*, which reads:

[20] Benedetti 1585, 155: "[…] verum non est. Huiusmodi enim ratione navis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsum, tanto magis proram ipsius navis in aquam demergit." Translation in Drake and Drabkin 1969, 183.

[21] Benedetti 1585, 155: "[…] quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiae seu intervalli, quae inter ipsum lineamque *AD* intercedit." Translation in Drake and Drabkin 1969, 184.

[22] For the historical context, see Büttner 2008.

> Why is it that we can move things raised and drawn more easily and more quickly by means of greater circles?[23]

In chapter 16 he discussed the tenth problem of the Aristotelian *Mechanical Problems*, which reads:

> Why is a balance moved more easily when it is without a weight than when it has one?[24]

In his detailed response to this problem—indeed much more detailed than the one found in the Aristotelian text—Benedetti compared balances that are alike with different sets of weights on their scales, one with two weights of one ounce, the other with two weights of one pound. He then added a half-ounce weight on one side of each balance and observed that the balance with the smaller weights moves more rapidly. He explained this effect by referring to the dynamical assumption that one always has to consider *the ratio of the moving force to the body moved*.

In chapter 17 Benedetti addressed the twelfth problem of the Aristotelian *Mechanical Problems*, which reads:

> Why does a missile travel further from the sling than from the hand?[25]

Benedetti's response is based on the concept of *impetus*, conceived as an intrinsic cause of motion originally acquired by the action of an external force that then gradually decreases after separation from the original mover. He argued that a greater impetus can be impressed by the sling due to the repeated revolutions which evidently lead to an accumulation of this intrinsic force. He observed that the impetus would lead, if not impeded by the sling or the hand, to a straight motion of the projectile along the tangent to the circle of its forced motion. He also noted—distancing himself from a claim made by Tartaglia—that the motion due to the impressed force can mingle with the projectile's natural motion downward, thus leading to a curved trajectory. It may well be the case that it was this claim that later convinced Galileo and Del Monte to perform their experiment on projectile motion from which they drew the conclusion that such a mixture of motions indeed takes place.[26]

In chapter 18 Benedetti considered problem 13 of the Aristotelian *Mechanical Problems* dealing with the question of why larger handles can be moved more easily around a spindle than smaller ones.[27] In his short response Benedetti simply referred to the fourth and fifth chapters of his own treatise, stressing that everything depends on the lever. He was evidently convinced that the Aristotelian reduction of such problems to properties of the circle is superfluous, if not misguided.

In chapter 19 he handled problem 14 of the Aristotelian *Mechanical Problems* in the same way. It reads:

---

[23] Aristotle 1585, 517: "Cur ea, quae per maiores circulos tolluntur et trahuntur, facilius et citius moveri contingit […]?" Translation in Aristotle 1980, 365.

[24] Aristotle 1585, 517: "Cur facilius quando sine pondere est, movetur libra, quam cum pondus habet?" Translation in Aristotle 1980, 365.

[25] Aristotle 1585, 518: "Cur longius feruntur missilia funda, quam manu missa […]?" Translation in Aristotle 1980, 367.

[26] See the discussion in Renn, Damerow, and Rieger 2001.

[27] Aristotle 1980, 367.

> Why is a piece of wood of equal size more easily broken over the knee, if one holds it at equal distance far away from the knee to break it, than if one holds it by the knee and quite close to it?[28]

Again, Benedetti just referred to the earlier chapters of his treatise.

In chapter 20 Benedetti reconsidered problem 17 of the Aristotelian *Mechanical Problems*, which reads:

> Why are great weights and bodies of considerable size split by a small wedge, and why does it exert great pressure?[29]

In the Aristotelian text, the answer is based on interpreting the wedge as two levers opposite to each other, their fulcra being placed at the entry points of the wedge into the wood. Benedetti, however, disagreed with the identification of the two levers allowing the action of the wedge to be interpreted in terms of force, fulcrum, and resistance. He claimed that the fulcrum is actually placed just underneath the deepest point of the opening produced by the wedge entering a block of wood.

In chapter 21 Benedetti claimed to provide the true explanation of compound pulleys. He reduced a compound pulley to a chain of balances by appropriately identifying forces and fulcra, each wheel of the pulley corresponding to one balance.

In chapter 22 Benedetti discussed Aristotle's wheel, i.e., problem 24 of the Aristotelian *Mechanical Problems*, which reads:

> A difficulty arises as to how it is that a greater circle, when it revolves, traces out a path of the same length as a smaller circle, if the two are concentric.[30]

While the author of the *Mechanical Problems* referred to dynamical reasons in explaining this apparent paradox, Benedetti resorted to a kinematic argument, a pointwise reconstruction of the trajectory of the motion of a point on the circumference, arguing that it results from a superposition of two motions. In the case in which the motion is controlled by the larger circle, a point on the circumference of the smaller circle traverses a path resulting from an *addition* of two motions. In the case in which the motion is controlled by the smaller circle, a point on the circumference of the larger circle traverses a path resulting from a *subtraction* of two motions.

Chapter 23 of Benedetti's treatise does not exist.[31] In chapter 24 Benedetti discussed problem 30 of the Aristotelian *Mechanical Problems*, which reads:

> Why is it that when men stand up, they rise by making an acute angle between the lower leg and the thigh, and between the trunk and the thigh?[32]

In his response Benedetti suggested that the reason for this behavior is to create an equilibrium of the body with regard to the line that serves as support underfoot.

In chapter 25 Benedetti addressed the last problem, problem 35 of the Aristotelian *Mechanical Problems*, which reads:

---

[28] Aristotle 1585, 518: "Cur eiusdem magnitudinis lignum facilius genus frangitur, si quispiam aequi diductis manibus extrema comprehendens fregerit, quam si iuxta genu?" Translation in Aristotle 1980, 369.

[29] Aristotle 1585, 520: "Cur a parvo existente cuneo magna scinduntur pondera, et corporum moles, validaque sit impressio?" Translation in Aristotle 1980, 371.

[30] Aristotle 1585, 525: "Dubitatur quam ob causam maior circulus aequalem minori circulo convolvitur lineam, quando circa idem centrum fuerint positi." Translation in Aristotle 1980, 387.

[31] In Drake and Drabkin 1969, 193; chapter 22 is erroneously numbered as chapter 23.

[32] Aristotle 1585, 532: "Cur surgentes omnes, femori crus ad acutum constituentes angulum, et thoraci similiter femur, surgunt?" Translation in Aristotle 1980, 403–405.

> Why do objects which are travelling in eddying water all finish their movement in the middle?[33]

Benedetti's answer simply referred to the fact that whirlpools are depressed in their middle without giving an explanation of this phenomenon. He could thus restrict himself to arguing that the motion of an object to the center of such a whirlpool is simply its natural downward motion. The final comment by Benedetti is a remarkable conclusion to his criticism of Aristotle as well as his treatise on mechanics:

> But in the case of all those other problems that I have omitted, Aristotle's explanations are correct.[34]

5.2 The Beginning of Benedetti's Mechanics

After our overview of Benedetti's book on mechanics, we concentrate on the theses he expounded in the first chapters because they have a foundational character and proved particularly controversial, at least in light of Del Monte's criticism, which we are aware of from the comments he made in one of his notebooks and from marginal notes in his own copy of Benedetti's book.

5.2.1 *De mechanicis* I: "On the different positions of balance beams"

In chapter 1, Benedetti notes that "a body (*pondus*) [...] acquires a larger or smaller weight (*gravitas*) depending on the different ratio of the beam's position" (*pondus... maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii*). According to him, a body has the greatest heaviness when the beam at whose extremity it is loaded is in the horizontal position. His idea is based on a simple common-sense intuition: if one considers an equal-arms balance suspended at its center, the weight of a loaded body is:

- borne entirely by the fulcrum when resting vertically upon it,
- entirely hanging on the fulcrum when suspended vertically below it,
- not supported in any way by the fulcrum when the beam is in the horizontal position.

In the first case, the body completely rests or leans on the center (*nititur*), and the center in turn hinders (*impellet*) the downward tendency of the weight. In the second case, the body is suspended vertically (*pendet*) and the center "attracts" it (*attrahet*), in the sense that it hinders its natural tendency to fall down (*inclinatio*). Hence, the body attains its maximum weight in the third case. If the balance beam moves upward, departing from the horizontal position, the weight slowly decreases and reaches its minimum at the top when the beam is in the vertical position. If the rotatory motion around the fulcrum continues, now downward, the weight increases again until it reaches its maximum in the horizontal position. It then diminishes until it is suspended entirely below the fulcrum. Benedetti visualizes these variations of weight depending on the position (*situs*) in a diagram comparing the lines connecting the weight to the center of the world in different cases, more specifically if the beam is:

- horizontal,

[33] Aristotle 1585, 533: "Cur ea quae in vorticosis feruntur aquis, ad medium tandem aguntur omnia?" Translation in Aristotle 1980, 409.
[34] Benedetti 1585, 167: "[...] a quo aliarum omnium quaestionum, quas ego omisi rationes sunt bene propositae." Translation in Drake and Drabkin 1969, 196.

- raised upward, or
- moved downward with the same angle as in the second case (which is equivalent to 2).

The parallel lines, called *lineae inclinationis* or *lineae itineris*, indicate the direction in which a body would fall if it were free. The closer these lines are to the center of the beam, Benedetti says, the "less heavy" the body becomes.

In his own copy of Benedetti's book, Del Monte wrote a brief annotation in the margin of chapter 1: "this first chapter is derived entirely from our treatise on the balance in the *Mechanicorum liber*."[35] Clearly, he sought to assert the relevance of his treatise for Benedetti's speculations, in spite of the latter's claims of originality. It should be remarked, however, that Del Monte's treatment of the balance, based on the concept of center of gravity, was significantly different from Benedetti's, which was based on an original reworking of *positional heaviness*. Del Monte merely reassessed a concept received from authors such as Jordanus Nemorarius, Tartaglia, and Cardano, all of whom he personally opposed. In his book on mechanics, Del Monte had in fact criticized the concept of *positional heaviness*. Downplaying Benedetti's theory as a repetition of his predecessor's theories, he could therefore claim that his own treatment already included a summary (as well as a criticism) of Benedetti's approach.

5.2.2 *De mechanicis* II: On the proportion of weights at the extremities of a balance beam in a position other than the horizontal

In chapter 2, Benedetti deals with the proportions of a weight placed at the extremity of a balance beam if its position is not horizontal (*De proportione ponderis extremitatis brachii librae in diverso situ ab orizontalis*). The thesis to be demonstrated is the following: "The proportion between [the weight of] a body (*pondus*) at *C* and [the weight of] the same body (*pondus*) at *F* corresponds to that between the whole beam *BC* and its part *BU*, which is [set on the beam *BC* and is] delimitated by the fulcrum and the [intersection between the beam and the] inclination line *FUM* that connects the weight at *F* to the center of the world" (Benedetti 1585, 142). For the sake of simplicity, we will represent these relations symbolically in modern terms:

$$C : F = BC : BU$$

where *C* is the weight in the horizontal position and *F* in the inclined position; *BC* is the beam and *BU* the part of the beam *BC* between the center *B* and the perpendicular line drawn from *F*.

Benedetti's demonstration is as follows. He imagines placing a weight *D* on the other extremity of the balance that has the same proportion to *C* as *F*, that is, the following proportion expressed in modern terms:

$$D : C = BU : BC.$$

In accordance with Archimedes's *De ponderibus* I. 6, the balance will be stable if the weight *C* is loaded at *U*, since weights and distances from the fulcrum are proportional by supposition.

[35] "Hoc primum caput to[tum] desumptum est a n[ostro] *Mechanicorum libri* tractatu de lib[ra]."

The next step is to show that $F : C = BF : BU$ (where BF is the beam, hence $BF = BC$). In order to demonstrate this, Benedetti resorts to the mental model (*imaginemur*) of a string hanging vertically from *F*, to which a weight equal to *C* is suspended. He claims that it is visually evident that the weight has the same effect at *F* as at *U*. The same is valid for the case in which the weight is suspended from *U* and intersects the circumference described by the rotation of the beam at a point *E*. In both cases, the balance would remain horizontal since the weight *C* at *F*, *U*, or *E* would balance the weight at *D*. Benedetti further argues that the balance under consideration can be treated like a bent lever with a horizontal and an inclined arm (*FBD* or *EBD*): "si brachium BE consolidatum fuisset […]" (If the beam *BE* was made solid […]).

The author concluded that his reasoning has satisfactorily demonstrated his thesis: "A body (*pondus*) is more or less heavy (*grave*) the more or less it hangs from (*pendet*) or rests on (*nititur*) the fulcrum" (Benedetti 1585, 142). And he deems this resting on or hanging from the fulcrum to be the most direct cause (*haec est causa proxima, et per se*) of the positional changing of a weight.

As an additional commentary, Benedetti remarks that in his diagram he supposes the inclination line *CO* to be perpendicular to *CB* and parallel to *BQ*, whereas *CO* and *BQ* in fact converge at the center of the sphere of the elements (*centrum regionis elementaris*), that is, the earth. But for the sake of his present argumentation, this angle is negligible and one may simply assume perpendicularity and parallelism. Benedetti thus developed a method to quantify positional heaviness that corresponds to the modern concept of "torque."

5.3 Del Monte's Criticism Concerning the Non-Negligibility of the World's Center

As will be shown in the following section, it was only in his initial treatment of the inclined balance, in chapter 1 of *De mechanicis*, that Benedetti neglected to consider the convergence of the inclination lines to the center of the elements. This omission gave rise to criticism. Del Monte severely criticized both this assumption and Benedetti's reasoning in general in *De mechanicis*, in his handwritten notes on scientific and technical matters known as *Meditatiunculae de rebus mathematicis*. In his notes he assessed Benedetti's arguments from his perspective, relying on the concept of the center of gravity as it was developed in his own book on mechanics.

In a marginal note to the *Diversae speculationes* (Figure 5.2), Del Monte expressed his disagreement with Benedetti's conclusion: "Thus, in this manner, a weight (*pondus*) more or less hangs from or rests on the center; this is the next cause and the [cause] in itself [of the variation in heaviness]."[36] His disagreement reads as follows:

> because that [that is, the greater or smaller extent to which a weight rests at the center] is neither the next [cause] nor the [cause] in itself. For the weight at *F* of the arm *BF* is not equally heavy as the weight *U* of the arm *BU*; nor is the weight at *E* of the arm *BE* equally heavy as the weight at *U* of the arm *BU*. Thus, this entire demonstration is false.[37]

[36] "[…] unde fit ut hoc modo pondus magis aut minus a centro pendet aut eidem nititur: atque haec est cause proxima, et per se […]."

[37] See Renn and Damerow 2012, 207: "non est neque proxima neque per se; nam [pond]us in *F* brachii [*BF*] non est equegrave ut pondus in *U* brachii *BU*; [nec] pondus in *E* brachii *BE* est equegrave ut pondus [in] *U* brachii *BU*. Unde tota haec demonstratio falsa est."

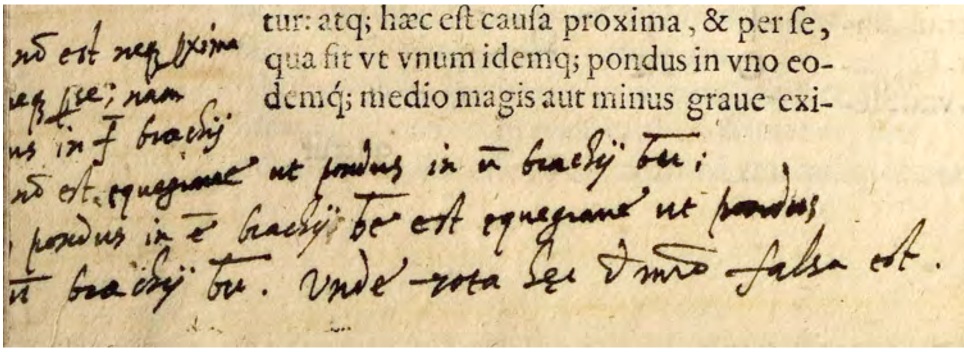


Figure 5.2: Del Monte's marginal note to *De mechanicis*, II. (Max Planck Institute for the History of Science, Library)

This means that Del Monte did not accept the claim that a weight is equally heavy in different positions on the balance beam, provided the projections of the beam along the horizontal are the same length or rather, as Benedetti writes, the distances between the projections of the beam on the horizontal and the center have the same lengths.

To find Del Monte's counter-arguments, one must look to the *Meditatiunculae*, f. 145, *Contra Cap. 2 Jo. de Benedicti de Mechanicis*. As mentioned, he basically rejected Benedetti's perspective by objecting that he did not take into due account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other, as Benedetti assumed in this part of his treatise.

In his diagram (Figure 5.3) Del Monte compared the line *LUS* (parallel to the line *AQ*, connecting the fulcrum *B* of the balance with the center of the world *M*) with the line *FM* (connecting the upper weight *F* and the lower weight *E* with the center of the world *M*). *S* is the point at which the line *LUS* meets the circle that the beam makes around the fulcrum, which is above the position of the lower weight *E*.

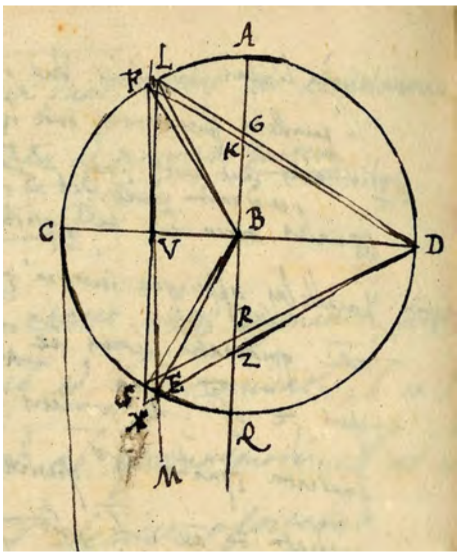


Figure 5.3: Del Monte's critical reworking of Benedetti's diagram in *Meditatiunculae*, f. 145. (Bibliothèque Nationale de France)

Next, he considered a bent lever made of the oblique arm *BS*, rigidly connected to the straight arm *BD*, assuming that *BU* is half *BD*. If a weight is now placed at *S* that is double the weight at *D*, the bent lever will be in equilibrium, as Del Monte showed with reference to his book, because the center of gravity of the weights at *S* and at *D* will be at the point *R*, which will be in its lowest place on the vertical line *BQ*. He therefore concluded that it is the weight at *S*, but not the lower weight *E*, that will be equally heavy as the weight at *U*.

He proceeded to demonstrate this in greater detail by considering the proportions into which the line connecting the two weights is cut by the perpendicular *BQ* for the two cases, that is, the weight placed at *S* and the weight placed at *E*. Del Monte concluded that the same weight is heavier at *S* than at *E*. He then turned to a closer consideration of the upper weight *F*. Again he constructed a bent lever *LBD* in equilibrium in order to compare it with the bent lever formed with the upper weight *F*. Again he showed that the weight is heavier at *L* than at *F*.

Del Monte concluded by summarizing that the entire fallacy is due to Benedetti assumption that the weight at *F* would gravitate in the same way as at *U*, which would only be the case, according to Del Monte, if it were to hang freely.

5.4 Benedetti on Weights and Forces Acting on a Balance

Chapter 3 of Benedetti's *De Mechanicis* contains a generalization of the results of chapter 2 or, rather, presents a general rule concerning the action of forces (*virtutes*) on balance beams, including in the case that they do not act vertically downward but also with an acute or obtuse angle. Benedetti moves forward from the result of the previous chapter as follows: the length of the line perpendicularly connecting the center to the line of inclination (the line *BU* in the diagram) allows the quantity of the positional force (*quantitas virtutis... in... situ*) of a weight (*F* in the diagram) to be established. Thus, Benedetti calls the positional weight a force, and this is the presupposition that allows him to generalize from *gravitas* the action which he calls *virtutes moventes*, or "moving forces." The thesis of this chapter is summarized in its title: "That the quantity of any given weight (*pondus*) or moving force in relation to another quantity can be determined thanks to the perpendicular projections connecting the center of the balance to the line of inclination."

Benedetti draws two diagrams showing a balance at whose extremities two weights or forces act in different directions (Figure 5.4). At the left extremity *B*, a weight *E* has a downward tendency, while at the right extremity, a weight *C* acts making an acute or an obtuse angle. According to Benedetti, the length of the perpendicular projection drawn from the center to the inclination line, *OT*, permits the determination of the distance *OI* on the beam at which the same force acting vertically downward produces the same effect. Given this equation, Benedetti can determine how much the force acting in a non-perpendicular direction has to be augmented in order to balance an equal weight acting perpendicularly on the opposite beam. This measure is given according to the following proportion (expressed in modern terms):

$$E : C = BO : OI$$

where *E* is the weight acting vertically on the extremity *B*; *C* is the *virtus movens* acting on the opposite extremity *A* at an angle; *BO* is the left beam and *OI* the part of the right beam *OA* determined as explained above.

In his argumentation, Benedetti thus equates a balance (*BOI*) with a bent lever (*BOT*). Accepting this equation, he concluded that, according to commonly shared knowledge (*communi quadam scientia*), the weights or forces that are required to obtain a perfect balance can easily be calculated.

The chapter ends with a cosmological corollary: "The closer the center *O* of the balance is to the center of the elementary sphere, the less heavy (*minus grave*) it becomes." In fact, the angles between the beam and the inclination lines become progressively smaller.

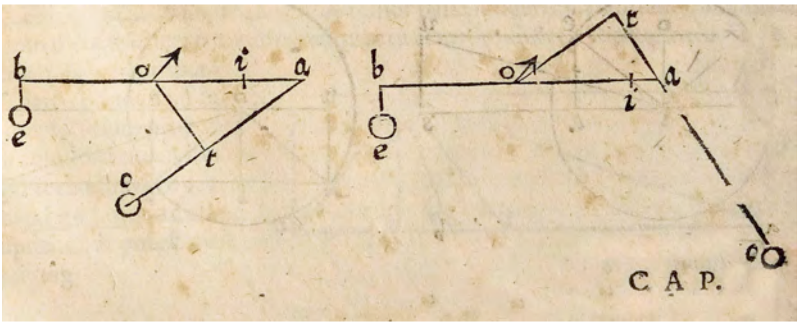


Figure 5.4: Benedetti's representation of forces acting on a balance in arbitrary directions. (Max Planck Institute for the History of Science, Library)

5.5 Del Monte's Misunderstanding

In his notes on folio 146 of the *Meditatiunculae*, Del Monte grappled with Benedetti's instructions on how to determine positional heaviness in the case of forces acting in an arbitrary direction. These he refuted at length under the erroneous assumption that Benedetti had claimed forces can be indiscriminately replaced by weights. Like Benedetti, Del Monte considered a bent lever *BOAC* with fulcrum *O*, weights *E* and *C*, a straight arm *BO*, and a bent arm *OAC* to discuss the two cases of an acute and an obtuse angle *BAC* (Figure 5.5).

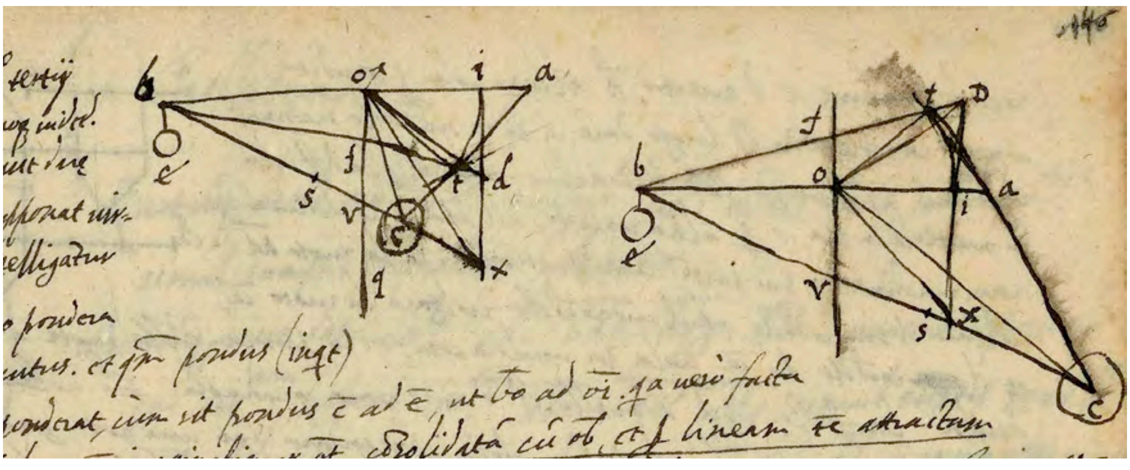


Figure 5.5: Del Monte's critical reworking of Benedetti's representation of forces acting on a balance in arbitrary directions. (Bibliothèque Nationale de France)

He first recapitulated Benedetti's procedure, assuming that a vertical line *OT* drawn from the fulcrum to the line *AC* represented the oblique arm of the bent lever. He stated that when the weight *C* is placed at the end of the horizontal line *OI*, whose length is the same as that of the perpendicular *OT*, according to Benedetti it will be in equilibrium with the weight *E* if the weight *C* is to the weight *E* as is *BO* to *OT* or *OI*. Del Monte then summarized Benedetti's claim that when a force represented by the weight *C* acts along the line *TC*, the bent lever formed by the straight arm *BO* and the oblique arm *OTC* will also be in equilibrium, which he doubted.

Del Monte reformulated this claim by stating that the same weight *C* will be in equilibrium with the weight *E* whether it is placed on the straight balance *BOI* or on the broken bent lever *BOTC*. He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and as a result arrived at absurd conclusions.

Del Monte then showed that the same weight will be heavier on the horizontal at point *I* than along the bent lever at *T*, demonstrating that the bent lever *TOB* will not be in equilibrium if the straight lever *BOI* is in equilibrium. To show this, Del Monte again proceeded by finding the center of gravity of the weights *E* and *C* placed at *T*. More precisely, Del Monte determined a position for the weight *C* where the bent lever is in equilibrium, a position, however, that is distinct from *T*. Thus it follows that *T* cannot be the position of equilibrium. For this purpose, he extended the line *BT* to *D*, just beneath *I*, so that it is immediately evident that if the weight *C* is placed at *D*, the center of gravity of the two weights will be just beneath the fulcrum.

Using the same pattern, he continued by showing that the bent lever *BOC* cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum. Finally, he applied this argument to the broken bent lever *BOTC*. Del Monte next addressed the case in which the bent lever is characterized by an obtuse angle *BAC*, showing that the weight at *T* is lighter than the weight at *I*. In his concluding remarks, however, he began to waver. Once again, he stated that Benedetti is completely mistaken when applying his procedure to weights. But he did admit that this may be true when dealing with a force.

As an afterthought, Del Monte once again criticized Benedetti's appeal to common sense: he did not feel this to be worthy of an expert mathematician. And as a second afterthought, he constructed an extreme case in which it is immediately clear that the broken bent lever cannot be in equilibrium if weights are attached to it rather than forces.

The following considerations enable Del Monte's marginal annotations to Benedetti's *De Mechanicis* III to be understood. These are not perfectly legible, but nonetheless their meaning becomes clear in light of the *Meditatiunculae*:

> If we understand that a weight is at *C*, as we can assume from his own words, then *CT* must also be understood as being solid [and connected with] the solid lines *TO* […] If we hence understand that *C* is a weight and not moving, [the proposition] is false. If it is understood that *C* moves as […] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing […] therefore as is evident in chapter 7. All demonstrations of the author are founded on these two chapters inasmuch as they are the first fundaments of mechanics; once their falsity is recognized, everything is rejected.[38]

[38]See Renn and Damerow 2012, 213: "si intelligamus p[ondus] in *C*, ut supponi p[otest] ex verbis ipsius, intelligendum est *C*[*T*] quoque consolidatam consolidatis *TO* […]. Unde si intelligamus *C* pondus et non

5.6 Diverging Approaches to Tartaglia

Del Monte's and Benedetti's criticisms of Tartaglia's conception of positional heaviness help us to understand where these two scholars converge and diverge on the issue of the equilibrium (or lack of equilibrium) of a balance deflected from its horizontal position, and also the reasons for the presumed equilibrium or tendency to restore it. Moreover, their arguments reveal a different attitude toward the medieval tradition of the *scientia de ponderibus* and the *gravitas secundum situm*.

5.6.1 The Tradition of Nemorarius, Tartaglia, and Cardano

The concept of *gravitas secundum situm*, or positional heaviness, was extensively employed in Jordanus Nemorarius's *Liber de ponderibus*. Del Monte owned and annotated a sixteenth-century Nuremberg edition of the book, commented, and illustrated by Petrus Apianus. Del Monte's handwritten annotations document his general disagreement with the approach of this medieval scholar, who did not know the Archimedean concept of the center of gravity and therefore tried to develop a deductive science of weights relying solely on the Aristotelian theory of motion and its development in the Arabic tradition of the science of weights. We have already hinted at the Aristotelian framework underlying the concept of *gravitas secundum situm*. In his book, Jordanus stated that a deflected balance would return to the horizontal position (his second proposition) (Nemore 1565, B2 *r*). According to Jordanus, the upper weight acquires more positional heaviness than the lower one due to the fact that its descent is less oblique. In fact, he postulated that positional heaviness depends on the obliqueness of descent of a weight (his fourth postulate) and that "a more oblique descent partakes less of the straight [descent] for the same quantity [of path]" (fifth postulate) (Nemore 1533, A4 *r*). The determination and possibly the quantification of obliqueness was therefore essential to establish the behavior of a deflected balance.

In the sixteenth century, Tartaglia in *Quesiti, et inventioni diverse* (1546), and Cardano in Book 1 of *De subtilitate* (first edition, 1550) and in *Opus novum de proportionibus* (1570), expounded their own versions for determining descent and reinforced Jordanus's second proposition (that the deflected balance returns to the horizontal position). A brief account of three ways to determine positional heaviness is given in the following pages. The first two are derived from Tartaglia and the last from Cardano.

Descent: A first method of dealing with positional heaviness consisted in comparing the lengths of the projections of the equal arcs described by the motion of opposite balance beams—one ascending and one descending—on the vertical line of descent to the center of the world.

As Tartaglia's diagram in Figure 5.6 shows, the vertical component of descent of the upper weight is always larger than that of the lower. Thus, the former acquires more heaviness (*secundum situm*) than the latter and the balance returns to the horizontal position.

---

movens, falsa est i[ta]que si intelligatur *C* movens ut homi[…] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i…] ipse [vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil […] ut patet in 7 cap. In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur]."

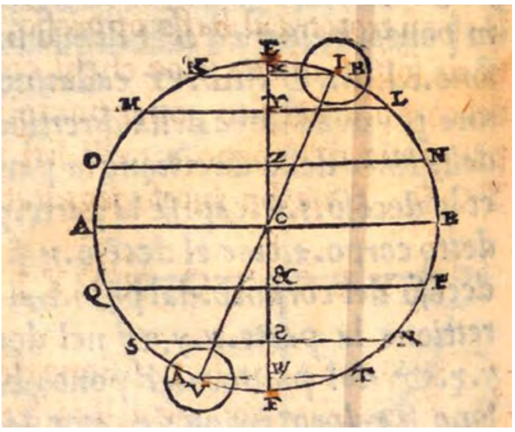


Figure 5.6: According to Tartaglia, the body at *I* is positionally heavier than the body at *V*, since the projection of the arc *IL* on the vertical *XY* is greater than the projection of *VF*, *WF*. (Max Planck Institute for the History of Science, Library)

Angle of contact: Tartaglia's second method of determining positional heaviness consists in comparing the angles between the circular path of the beams and the perpendicular lines connecting the weights to the center of the elements (as already mentioned in chapter 4). These angles "of contact" are also called "curvilinear angles" or "mixed angles" since they result from the intersection of a straight line downward and a curved line, that of the circle circumscribing the balance (Figure 5.7).

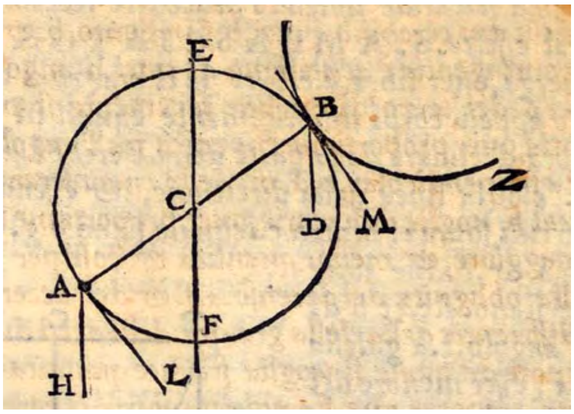


Figure 5.7: Tartaglia's representation of the angle of contact for the determination of positional heaviness. (Max Planck Institute for the History of Science, Library)

By comparing the angles of contact of the two weights, Tartaglia could establish that the higher angle is always smaller than the lower; therefore, the higher weight has a straighter descent and is positionally heavier. The inclined balance would therefore return to the horizontal position. It should be noted that Tartaglia perceived the comparison of curvilinear angles as problematic. He considered the ratio of two such angles to be less than any ratio between determined quantities. As a consequence, no weight placed on the positionally lighter side of the deflected balance could compensate for the other weight and keep the balance inclined. On the contrary, any additional weight—no matter how small—would have produced an opposite displacement of the balance beam toward the vertical.

The angle between the support and the beams: We have so far considered two ways of determining positional heaviness on the basis of Tartaglia's *Quesiti*. Assuming that positional heaviness depends on the obliquity and straightness of descent, positional heaviness can be determined either from the projections of the descents on the vertical, or the curvilinear angles that are produced by the intersection of the descent arcs and the lines connecting the weights to the center of gravity. Cardano considered three criteria for establishing positional heaviness which he mistakenly regarded as equivalent: first, the distance of the beam from the vertical; second, its distance from the horizontal; and third, an angle that he called *meta*. This was the angle between the support of the balance and the beam. Commenting on the diagram that is reproduced here as Figure 5.8, he explained:

> Aristotle says that this happens when the support is above the balance, because the angle *QBF* of the *meta* is larger than the angle *QBR*. And similarly, when the support is *QB*, the *meta* will be *AB*, and thus the *RBA* will be larger than the angle *FBA*, but the larger angle will render the weight heavier. […] The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.[39]

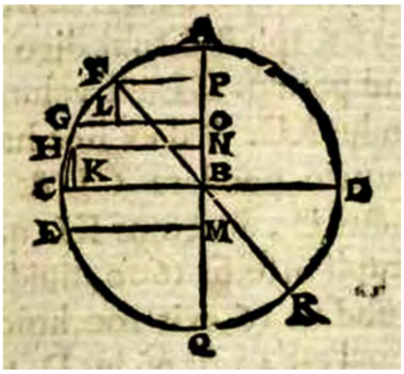


Figure 5.8: According to Cardano, there are three ways to determine positional heaviness. The positional heaviness at point *F*, for instance, may be determined by the horizontal *FP*, by the vertical *FL*, or by the angle *QBF*. (Max Planck Institute for the History of Science, Library)

Given these premises, Cardano contended that a weight will reach its maximum positional heaviness in the horizontal position. He therefore shared Nemorarius's and Tartaglia's opinion about the return of an inclined balance to the horizontal position.

5.6.2 Del Monte's Critical Remarks on Positional Heaviness

Del Monte's criticism of Benedetti, in the *Meditatiunculae* as well as in the marginal remarks of his copy of *Diversae speculationes*, are closely related to his criticism of Nemorarius, Cardano, and Tartaglia in his *Mechanicorum liber* (1577). Here he dealt

[39] Cardano 1550, 17–18: "Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus *QBF* metae, maior est angulo *QBR*. Et similiter quum trutina fuerit *QB*, erit meta *AB*, et tunc angulus *RBA*, maior erit angulo *FBA*, sed maior angulus reddit gravius pondus. […] Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora."

extensively with the balance and provided a detailed discussion of the theories of these scholars which he judged to be irremediable. These theories supported the idea that an inclined balance returns to the horizontal and were thus at odds with his own treatment of the matter, which he based on the Archimedean concept of center of gravity. Del Monte believed that an ideal balance would remain in any position as long as it had equal arms, was hinged on its fulcrum and was loaded with equal weights. The only difficulty in testing this theory, he asserted, was the technical difficulty in constructing a perfect balance. It should be noted, moreover, that he assumed that a center of gravity meeting the requirement of his (and Pappus's) definition of the center of gravity always exists:

> The center of gravity is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotation by that motion.[40]

Apart from the conceptual irreconcilability between his own approach and that of the Nemorarius school, Del Monte tried to demonstrate the inconsistencies of positional heaviness also within the conceptual framework of his adversaries. One of his main objections was based on a consideration of the cosmological context, which he considered relevant to correctly treat the inclined balance, at least with regard to positional heaviness. Of course, this aspect indeed matters when considering Tartaglia's remark that the difference in positional heaviness is infinitesimally small and cannot be compensated by any finite weight resulting from the infinitesimal difference between curvilinear angles.

Contrary to the assumptions of Nemorarius and his successors, Del Monte noted that the downward tendencies of the weights are not parallel but converge at the center of the world. Since the directions toward the center of the world from different points on the circular path of the end of the beam cannot be parallel, they are inappropriate for representing positional heaviness. From the fact that those lines converge, he argued further that the lower weight should actually become positionally heavier than the higher one. His idea is clearly illustrated by the diagram in Figure 5.9.

Del Monte objected that, from the point of view of positional heaviness, it is not in the horizontal position that a body weighs the most but at that point where a straight line drawn from the center of the world touches the circle described by the balance arm. Certainly, if the center of the world were infinitely distant and all lines of direction converging at it were perpendicular and parallel to each other, then the extreme point would mark the horizontal position of the balance arm. Still, for a finite distance from the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum. Del Monte even demonstrated that the closer the balance is to the center of the world, the further this "extreme point" (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum).

Del Monte's crucial objection to the Nemorarius school was that one should not consider both weights separately, but rather in terms of their connection by the balance beam. He drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. With regard to their positional heaviness the two weights are then equal. Thus Del Monte could claim, using the premises of his adversaries, that the deflected balance does not return to the horizontal.

[40] Del Monte 1577, 1*r*: "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latione circumvertitur." Translation in Drake and Drabkin 1969, 259, revised in Damerow and Renn 2010, 57.

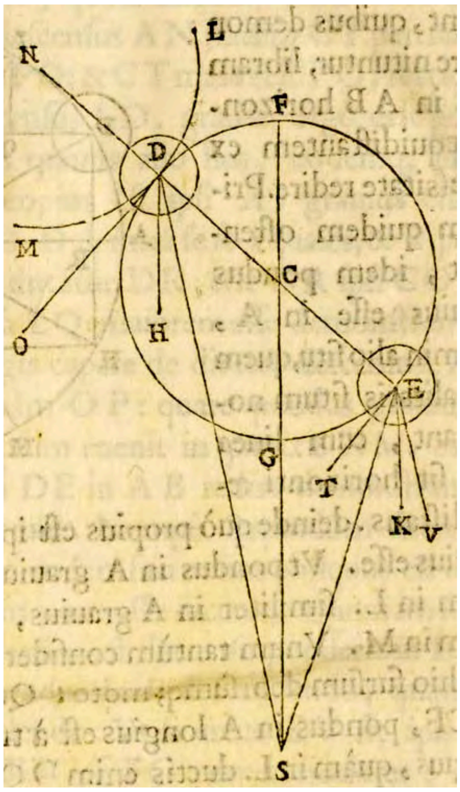


Figure 5.9: According to Del Monte, if *S* represents the center of the world, then the mixed angle *SEG* between the circular path of the weight at *E* and the direction to the center of the world is less than the mixed angle *SDG*. Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at *E* must be heavier than that at *D*. (Max Planck Institute for the History of Science, Library)

5.6.3 Benedetti on Tartaglia's and Nemorarius's Shortcomings

Benedetti addressed the ideas of Tartaglia and Nemorarius on positional heaviness in section seven of his *De mechanicis*. There, Benedetti stressed that his approach to positional heaviness, focusing on the distance from the fulcrum to the line of inclination, was distinct from and superior to Tartaglia's approach in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central thesis that when a balance is moved from its horizontal position, it will return to this position because the body that has moved upward will attain greater positional heaviness than the body which has moved downward. As we have seen above, Jordanus's and Tartaglia's arguments were based on a comparison of the descents of the two weights. In other words, the balance would have to break in the middle to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by Del Monte, that when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and positioned in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he came to the conclusion that when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

> Therefore the weight of *A* in this [lower] position will be heavier than the weight of *B*.[41]

[41] Benedetti 1585, 148: "Pondus igitur ipsius *A* in huiusmodi situ, pondere ipsius *B* gravius erit." Translation in Drake and Drabkin 1969, 176.

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line corresponding to the weight that had been lowered is longer than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally, so that one would expect the balance to tilt into a vertical position.

Benedetti added some more critical remarks on Tartaglia's consideration of positional heaviness. As we have seen, Tartaglia had argued in *Quesiti* that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight. This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument, Benedetti again emphasized that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just as Del Monte had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even such a small deviation from the parallel must be relevant. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as being parallel to each other.[42]

In summary, Benedetti introduced a way of determining the positional effect of a weight or a force that, in the cases he considered, essentially produces the same results as the application of the modern concept of torque. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

5.7 The Triangulation Benedetti-Del Monte-Galileo

In this chapter, we have dealt with Del Monte's and Benedetti's different approaches to mechanics emerging from their reflection on the balance and their treatment of earlier authors. Relative to the issue of positional heaviness, Del Monte's self-positioning was essentially external whereas Benedetti positioned himself (albeit critically) within the tradition of the Nemorarius school. He explicitly mentioned Tartaglia and Cardano as relevant sources for his treatment, whereas he omitted any mention of Del Monte.[43] In spite of their opposite intentions and mutual suspicion, Benedetti and Del Monte shared several opinions and sometimes reached the same conclusions, albeit following different paths: both considered the cosmological center of gravity relevant for an evaluation (and criticism) of Tartaglia's concept of positional heaviness, and both remarked that one cannot treat the two balance beams separately, but rather emphasized that they must be considered simultaneously. Moreover, both stressed the ambiguity of the concept of a mixed angle

[42] Benedetti 1585, 150: "Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit." Translation in Drake and Drabkin 1969, 177.
[43] Benedetti 1585, f. A3*r*.

and the difficulty of its determination. Nevertheless, their approaches were quite different. As mentioned, Benedetti still worked within the framework of the *gravitas secundum situm*, while Del Monte renounced it in favor of the concept of *centrum gravitatis*. For Del Monte, the displacement of the balance toward the vertical position was an absurdity that revealed the untenability of Tartaglia's premises. Benedetti deemed this vertical tilt to be the consequence of a correct analysis of the balance based on a concept close to the modern idea of torque, in consideration of the cosmological context. Furthermore, one should stress the importance of Benedetti's attempt to determine the quantity of positional heaviness, a fact that distinguishes him from his predecessors. Additionally, unlike Del Monte, he treated the balance by also taking into consideration the general case of forces acting arbitrarily on the beams.

In conclusion, it may be useful to recall the problems linked to the triangulation Benedetti-Del Monte-Galileo, on which the equilibrium controversy sheds new light. The remarkable proximity of these authors on several issues is well known in the history of mechanics. Nevertheless, recent accounts tend to neglect or even deny a possible influence of Benedetti on Galileo.[44] By contrast, the influence of Benedetti on Galileo was assumed and underscored by earlier scholars like Raffaello Caverni, Pierre Duhem, Emil Wohlwill, and Ernst Mach.[45] It is helpful to mention the most important issues common to these authors: the attempt at a theory of motion based on Archimedean hydrostatics, the treatment of the acceleration of fall and its causes, the formulation of what in hindsight appear as proto-inertial principles, a similar treatment of the bent lever, the analysis of the relation between vibrating strings and musical tones, their views on the irradiation of surfaces and on thermal and hydrostatic phenomena, and, last but not least, their support of the Copernican world system.[46] Although many of these themes and ideas belonged to the shared knowledge of preclassical mechanics, in some respects the agreement of their approaches is so striking that one may suspect that this is not mere coincidence.[47] Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*. In any case, the strongest evidence of Galileo's acquaintance with Benedetti's insights is provided by Del Monte's *Meditatiunculae*.

---

[44] See the discussion by Ventrice in Bordiga 1985, 732–736. He mentions Drake, Drabkin, Fredette, and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. Notable exceptions are the commentaries by Carugo and Geymonat in their edition of Galileo's *Discorsi*, see Carugo and Geymonat 1958. Bertoloni Meli even considers the possibility of Del Monte and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on Galileo's thinking because that influence supposedly would have arrived too late, see Bertoloni Meli 2006, 61–65.

[45] Cozzi and Sosio 1996. For an overview of such potential connections, see the discussion in Bordiga 1985, 732–736 who also mentions Mersenne, Clavius, and Cardinal Michelangelo Ricci as possible intermediaries.

[46] For an overview, see Bordiga 1985.

[47] See, for instance, Drake and Drabkin 1969, 36. Yet, the question of Benedetti's direct impact on Galileo remains unclear, in particular as Benedetti's work was never mentioned by Galileo.

There are several possible connections between Benedetti and Galileo that have been considered in the past. For instance, Benedetti is referred to by Galileo's Pisan colleague Jacopo Mazzoni in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597. See Mazzoni 1597. He is often mentioned in the Galileo Studies as the addressee of a famous letter by Galileo arguing for the Copernican system (May 30, 1597). See Galilei 1968, vol. 2, 194–202. In his book Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous. See Benedetti 1585, 183–184. For a historical discussion of the context of this argument in contemporary technology, see Freudenthal 2005, a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also refers explicitly to Copernicus. See Mazzoni 1597, 193 and Galilei 1960, 326. It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni, and Del Monte during Del Monte's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.

An important clue is page 145*bis* of the *Meditatiunculae* (Figure 5.10), which is the page opposite the one containing the detailed criticism of Benedetti dealt with in this chapter. This page shows Galileo's construction of the inclined plane, reducing it to a bent lever.

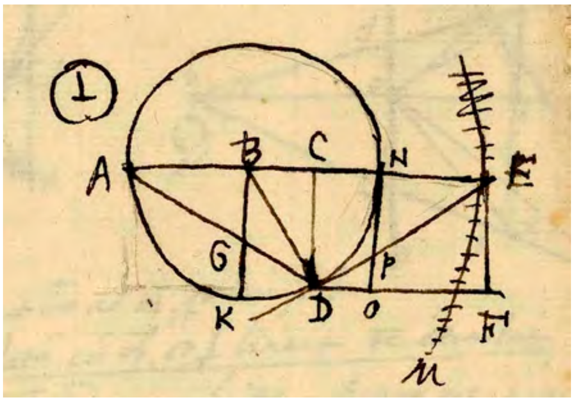


Figure 5.10: Del Monte, *Meditatiunculae*, p. 145*bis* showing Galileo's construction relating the bent lever to the inclined plane. (Bibliothèque Nationale de France)

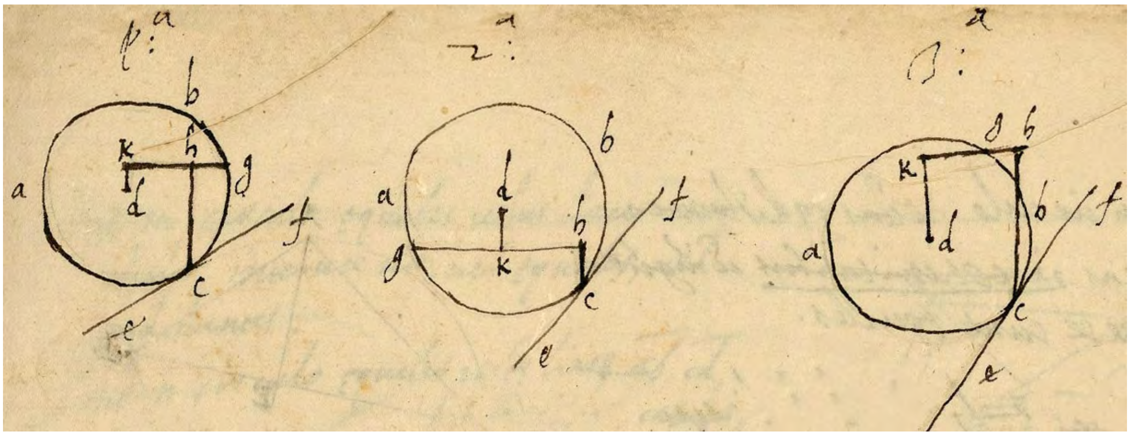


Figure 5.11: Del Monte's construction related to the inclined plane on p. 64 of his notebook. The construction was adapted from Pappus's erroneous solution. (Bibliothèque Nationale de France)

This fact is all the more noteworthy since Del Monte's notebook, on an earlier page, also contains his own problematic adoption of Pappus's analysis of the inclined plane (Figure 5.11).[48] In his writings, Galileo had criticized this analysis, substituting it with his own solution of the problem, which makes use of the bent lever conceptualized in the same way as Benedetti had done.[49] Del Monte therefore must have learned about this proof from Galileo, and he must also have seen the connection to Benedetti's methods. In any case, it is likely that the two scientists discussed this connection and quite plausible that Galileo became familiar with Benedetti's work through Del Monte. Galileo began to correspond

48 Del Monte 1587, 64.
49 Galilei 1960, 172.

with Del Monte in 1588, three years after the publication of Benedetti's *Diversae speculationes* and shortly before he embarked on the writings that later became known as *De Motu*.[50] Galileo first wrote a dialogue version of *De Motu* and then an essay in twenty-three chapters. Only the second essay version of these writings contains his proof of the law of the inclined plane, the argument about continuity of motion along a straight line, and a mention of Copernicus. This version was most likely written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time discussed explicitly the problem of the effective lever arm, was written much later, certainly after he had visited Del Monte in 1592 during his journey to Padua. Hence, it seems most likely that Galileo was already familiar with Benedetti's key ideas at the time of writing these works.

Recent research into Del Monte's biography has shown that Del Monte and Galileo must have met as early as 1589 in Tuscany.[51] They might even have met jointly with Galileo's teacher, Mazzoni, who, as mentioned earlier, cited Benedetti in his work. Thus, Del Monte, Mazzoni, and Galileo may have discussed Benedetti's *Diversae speculationes*, leading Galileo to reconsider his work in progress on motion and, in particular, his treatment of motion along inclined planes, making use of Benedetti's theory of the bent lever that was mentioned in Del Monte's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may have started taking the Copernican hypothesis much more seriously after his encounter with Benedetti's work, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597, Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they had meanwhile reached an agreement, and also tried to press him on the Copernican hypothesis.

In particular, Galileo's concept of *momento*[52] and his analysis of the bent lever—crucial to both his mechanics and his theory of motion—evidently emerged from the midst of the controversy about positional heaviness. In that debate, Galileo took a position much closer to Benedetti than to Del Monte. Rather than *gravitas secundum situm*, Galileo used the concept of *momento* or *momentum* that Del Monte had introduced in his book by quoting Commandino's definition of the center of gravity. But while Del Monte made no further use of this in his mechanics, Galileo took this concept from the respected Urbino school, gave it a new meaning that was taken from Benedetti, and made it a pillar of his own conception, which included Commandino's definition of the center of gravity:

> Center of gravity is defined as that point in every heavy body around which parts of equal moments are arranged.[53]

The evidence for this claim concerning Benedetti's legacy in Galileo's work derives from the marginal notes Del Monte made in his copy of Benedetti's book, as well as from his entries in the *Meditatiunculae* which contain traces of Galileo's intervention in this controversy.[54]

According to Benedetti and Galileo (and contrary to Tartaglia and Del Monte), the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum

---

[50] Galilei 1960. For a thorough discussion of the chronology of these writings, see Giusti 1998.

[51] Menchetti 2012.

[52] See the extensive discussion in Galluzzi 1979.

[53] Galilei 1968, vol. 2, 159: "Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti." Translation in Galilei 1960, 151. See also Galilei 2002.

[54] Del Monte 1587.

of the balance to the line of inclination, determines the effectiveness of a weight or a mechanical constellation. In his *Mechanics*, Galileo later stressed how important it is to carefully define the effective distances of weights from their support:

> There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.[55]

In his analysis of the inclined plane using the bent lever, Galileo also made clear that this procedure is critical for determining the *momento* of a given weight.[56] As discussed earlier, in his *Diversarum speculationum [...] liber*, Benedetti convincingly demonstrated the efficacy of this method for determining the magnitude of a force or weight according to its position.

In conclusion, the very existence of Del Monte's annotations on his copy of Benedetti's *Diversae speculationes* provides a definitive answer to the question of whether Del Monte had read this book or not.[57] It is also difficult to imagine that he did not discuss his views on Benedetti's mechanics with Galileo, views that he considered both misguided and profoundly challenging, as is made evident in his handwritten notes. It was most probably Del Monte, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo. At the same time, he also made it virtually impossible for Galileo to openly admit to Benedetti's influence if he did not also want to jeopardize the protection of the most important patron of his early career.

---

[55] Galilei 1968, vol. 2, 164: "Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi." Translation in Galilei 1960, 156–157.

[56] See Galilei 1968, vol. 2, 181. Translation in Galilei 1960, 173.

[57] The knowledge that he had read it, however, is not entirely new. See Renn, Damerow, and Rieger 2001, 74.