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Pietro Daniel Omodeo and Jürgen Renn:

Epistemology

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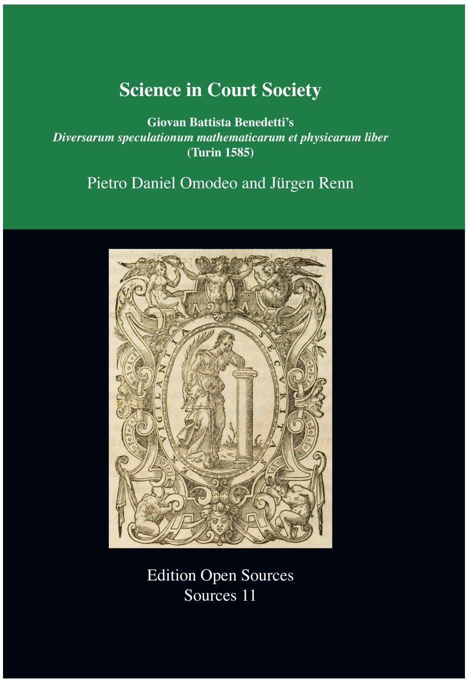

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Chapter 4
Epistemology

One of the most challenging aspects of Benedetti's endeavor was his attempt to merge mathematical and physical speculations, as is clearly stated in the title of the *Diversae speculationes mathematicae et physicae*. In order to understand his way to "physico-mathematics," we will discuss his mathematical epistemology starting from some statements scattered in his major work and then look at the premises implicit in his treatment of nature. We will briefly review the Renaissance reflections on mathematics linked to practical developments in technological fields as well as to eclectic reassessments of Pythagorean and Aristotelian debates on the certainty of mathematics and their applicability to natural philosophy. Focusing on the epistemological premises underlying Benedetti's mechanics, we will discuss medieval and early modern approaches to natural knowledge, which, in spite of their mathematical rigor, rested on a physics and metaphysics of contingency. For many centuries, it was assumed that the mathematical regularity of the phenomena does not imply their causal necessity.

4.1 The Certainty of Mathematics

In the letter to the Venetian patrician Domenico Pisani included in the collection of the *Diversae speculationes* and entitled *De philosophia mathematica* (On Mathematical Philosophy), Benedetti emphasized the philosophical dignity of his discipline, placing it at the same rank as physics, metaphysics, and ethics—if not higher than them, considering the certainty of its demonstrations (*certitudo suarum conclusionum*):

> I am surprised that, although you are well-versed in Aristotelian philosophy, nonetheless you make a distinction between the philosopher and the mathematician in your writings, as if the mathematician were not as much a philosopher as the naturalist and the metaphysician. In fact, as far as the certainty of his conclusions is concerned, he deserves the title of philosopher much more than them.[1]

This reference to mathematical *conclusiones* reveals Benedetti's methodological focus on the dignity and validity of his discipline. In his connection of mathematical and physical speculations, he seems to put the emphasis on the method rather than on ontology and to seek for the certainty of mathematics and its applications by way of its specific logic. This was the position of his correspondent, the Paduan professor Pietro Catena.[2] Along with him, Benedetti maintained that the certainty of mathematics has an extra-sensible and intelligible character.[3] As Benedetti added in his letter to Pisani:

[1] Benedetti 1585, 298: "Miror quod cum in Aristotele sis versatus, in tuis tamen scriptis philosophum a Mathematico separes, quasi mathematicus non sit adeo philosophus, ut est naturalis, et metaphysicus, cum multo magis quam ii philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus."

[2] Benedetti includes a letter to Catena in Benedetti 1585, 371.

[3] See on this De Pace 1993, 228–229.

> Actually, you are not the only one who makes this mistake, but this is more grave in consideration of the fact that, although you [Aristotelians] even label ethics as a philosophical discipline, you do not acknowledge that the divine mathematical sciences also should be adorned with the name of philosophy. In fact, if we consider this name more attentively we will clearly see that it is in itself more suited to the mathematician than to anyone else, since none of the others is more certain in his affirmations than the mathematician. And no one is more driven by the love of science in his cognition. This is evident. In fact, [the mathematician] does not rely on the senses nor accepts any presupposition that is not so true and evident to the intellect that no power whatsoever could show that it is false.[4]

Benedetti was acquainted with scholars quarreling over the status of mathematics, its demonstrative methods, and its legitimacy in the treatment of natural issues.

In his time such debates on the foundations and status of mathematics were intense. As an instance of epistemological reflections on the philosophy of mathematics, historians often mention the controversial theses by the Paduan professor of philosophy Alessandro Piccolomini, with whose work Benedetti was familiar. Piccolomini authored, among other writings, a treatise *De certitudine mathematicarum* (On the Certainty of Mathematics, 1547) affixed to his paraphrases of pseudo-Aristotelian mechanics, *In mechanicas questions Aristotelis paraphrasis*. As one reads in this sort of appendix, one ought not to cast into doubt the certainty of mathematics. However, this does not depend on demonstrative methods but rather on the subject of inquiry: "Mathematical disciplines are certain not due to the force of their demonstrations but rather to their subject matter itself."[5] Their special subject is quantity, connected to matter. Hence, the certainty of mathematics, for an Aristotelian such as Piccolomini, rests on the fact that it deals with universal properties of nature that can be extracted from concrete reality by means of abstraction (*res mathematicae sunt ex abstractione*).

> The cause of the certainty of mathematics is evident from Aristotle's statements. Simplicius is of the same opinion when he states (in *De anima* I 11) that the cause of the certainty of mathematics is due to the fact that they refer to quantity. In fact, as he argues, quantities are sensible things, they have sensible causes and they are known to us as such.[6]

This consideration led Piccolomini to argue that motion can become a mathematical object, if one abstracts from materiality:

[4]Benedetti 1585, 298: "Verum quidem est, te in huiusmodi errore solum non versari; sed gravius est, quod cum vos videatis etiam res morales sub philosophiae appellationem cadere, non animadvertatis divinas scientias mathematicas etiam philosophiae nomine ornandas esse. Quod si eiusdem nomen penitius considerare velimus, inveniemus aperte, mathematico magis illud ipsum quam cuilibet alio convenire, cum nullus ex aliis tam certo sciat id quem affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, et scientiae cupiditatem magis ducantur, ut aperte patet, cum nec etiam ipsi sensui det locum, neque aliquid praesupponat, quem non sit ita verum et intellectui notum, ut nulla quaevis potentia, illud esse falsum ostendere queat."

[5]Piccolomini 1565, f. 107*v*: "Mathematics disciplines esse certas non vi demonstrationis, sed ex subjecti ipsius ratione."

[6]Piccolomini 1565, 106*v*: "Patet igitur ex dictis Aristotelis causa certitudinis mathematicae. Hoc idem sensit Simplicius, qui primo de Anima 11. dicit causam certitudinis mathematicarum esse, quia versantur circa quantum. Quantitates enim ut dicit ipse, sunt res sensatae, et causas sensatas habent, et ideo nobis notas."

> One could argue that, just like magnitude, motion is a common sensible, too. Moreover, it has its effects and causes (see *Physics* V and VI). Thus, there can be a science of motion (a natural one), which is certain, similar to the science of quantity, that is, mathematics.
>
> We can answer to this [apparent objection], that if we consider motion in general, as separated from matter and insofar as it is a continuum […], our consideration will be mathematical. This is not in contrast with our principles.[7]

The "ontological" and not only "epistemological" dimension of mathematical physics would concern later scholars such as Kepler and Galileo, going beyond the shared Aristotelian discourse in their investigations of the mathematical properties of material processes.[8] Benedetti was rather concerned with mathematics as an intellectual tool, a sort of "logic of scientific inquiry." In the above-mentioned letter to Pisani on his mathematical philosophy, he stressed the certainty of mathematical reasoning rather than that of its "objects." Nonetheless, he was interested in the question raised by Piccolomini as to the usefulness of mathematics in the study of motion. As we will discuss, Benedetti's insight concerning the generalization of the methods already in use in mechanics, in the science of weights, established the premises for the conceptualization of problems in dynamics.

Benedetti's interest in mathematics as a conceptual instrument accords with the interest in the demonstrative power of mathematics shown by many scholars entering the debates about mathematical certainty. The publication of Piccolomini's *De certitudine mathematicarum* led to a series of negative or sympathetic reactions, among them the criticism made by the translator of Proclus's *Commentary* on Euclid, Francesco Barozzi, as well as those by the Paduan professors Pietro Catena and Giuseppe Moletti. Barozzi, in his 1560 *Quaestio de certitude mathematicarum*, and Catena, in his 1563 *Oratio pro idea methodi*, argued in favor of the demonstrative certainty of mathematics, contra Piccolomini's exclusive focus on mathematical objects. The theoretical discussion regarding the status of mathematics, the certainty of its demonstrations, their applicability to the investigation of nature, and the hierarchy between natural philosophy and mathematics continued for a while. It also produced frictions among Jesuit scholars such as the philosopher Benito Pereira and the mathematician Clavius, who were inclined to assign different levels of importance to the study and teaching of mathematics in the colleges of their order.[9]

As far as the institutional side of the defence of mathematics is concerned, it opposed scholars and intellectuals benefiting from varying social status, such as mathematicians, philosophers, and theologians. Benedetti's self-perception and, later, Galileo's self-presentation as "philosophers" involved polemical stances. They claimed for their math-

[7]Piccolomini 1565, 107*r*: "Si vero adhuc replicaretur, quod motus etiam est sensibile quoddam commune, sicut magnitudo; habet autem motus suas passionet, et suas causas, ut patet 5. et 6. Phys. ergo ita erit certa de motu scientia, naturalis scilicet, sicut scientia de quantitate, quae Mathematica est. Ad hoc respondere possumus, quod si motum consyderabimus, in communi, abstractu a materia quatenus continuum quoddam est, […] tunc consyderatio erit mathematica, et nihil contra nos."

[8]As Ofer Gal and Raz Chen-Morris recently stressed: "It is not epistemology that worries the two court mathematicians here, but ontology. Neither of them questions the power of mathematics to provide the knowledge they seek; it is the objects that mathematics can be true about that they both feel forced to establish." See Gal and Chen-Morris 2013, 118–119.

[9]The literature on the Renaissance debates on the philosophical status of mathematics is wide. Among other sources, see Giacobbe 1972, Giacobbe 1973, Carugo 1983, Jardine 1990, 693–697, De Pace 1993, Cozzoli 2007, and Axworthy 2016, chap. 2. For the Jesuit debates on mathematics, see Romano 1999. For the seventeenth century, cf. Mancosu 1996, 8–33.

ematical and physical investigations a wide cultural meaning against critics who downplayed such investigations as merely technical and specialistic.

Early polemics over the viability of the *mos geometricus* were not purely intellectual and academic but were also rooted in the rising recognition of the practical import of mathematics in engineering, architecture, mechanics, and warfare. A new class of intellectuals was emerging composed of "scientist-engineers," so to speak, both expert in practical disciplines and trained in letters.[10] Edgar Zilsel already remarked that the Renaissance exaltation of mathematics went far beyond purely Platonic and Pythagorean influences. At that time new mathematical writings were composed and published dealing with the practical problems of commerce, topography, architecture, and the arts.[11] Moreover, the emergence of mathematical and natural conceptions dependent on the advance of technology was reinforced by the growing self-consciousness of new social groups.[12] As an example of the awareness of the status of the practical arts one could mention Filippo Pigafetta's introduction to the Italian edition of Del Monte's work on mechanics. Here he reversed the assessment of craftsmen and practical knowledge, which had been marked by the contempt of aristocrats and traditional intellectuals, as follows:

> 'Mechanic' is a very honored title. According to Plutarch it refers to a profession linked with warfare. It is suited to a man of high rank who is also capable of using his hands and his intelligence to realize wonderful works of rare usefulness and pleasure for human life.[13]

This judgment well expresses the shifting opinion on practical knowledge which also marked Benedetti's environment. We have already stressed the centrality of practical mathematics for the Savoy dukes, in particular Emanuele Filiberto, in their construction of the new capital, Turin.

4.2 Physico-Mathematics

As a direct consequence of this mathematizing epistemology Benedetti dismissed the well-established separation between physics and mathematics in cosmology, that is, he refused to separate the investigation of "causes" and calculation.[14] This anti-fictionalist perspective implied a realist commitment related to the Copernican system and its embedding within a renewed cosmology. As we will discuss in the section on Benedetti's views on the universe, he praised the system "of Aristarchus and Copernicus" as it avoided the absurdities of an anthropocentric conception according to which the immensity of the firmament was created only for us. Rather, all planets are like Earth or, better, like moons reflecting the solar light. Among the direct consequences of the Copernican view was accepting that the fixed stars do not rotate around the center of the world within one day; rather, they are immobile.[15]

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[10]See Valleriani 2010 and Valleriani 2013.

[11]Zilsel 1942.

[12]See Lefèvre 1978.

[13]Pigafetta in Del Monte 1581, *Ai lettori*: "Mechanico è vocabolo honoratissimo, dimostrante, secondo Plutarco, mestiero alla Militia pertinente, et convenevole ad huomo di alto affare, et che sappia con le sue mani et co'l senno mandare ad esecutione opre maravigliose a singulare utilità et diletto del vivere humano."

[14]Hypotheses on conventionalism already emerged from the debate on the conflict between Ptolemy's geometrical models and Aristotle's homocentric cosmology. See Di Bono 1990 and Granada and Tessicini 2005.

[15]Most of these cosmological views are discussed in Benedetti 1585, Book 4. We deal with the details in chapter 6 as well as, partly, in chapter 7.

From this viewpoint, Benedetti's understanding of mathematics is not too removed from that of a mathematician such as Copernicus, who, in Book 1 of *De revolutionibus*, indicated that the mathematical superiority (simplicity and intelligibility) of his own planetary system was such that natural philosophy had to be subordinated to mathematical astronomy and not vice versa. The theologian who wrote the anonymous introduction to Copernicus's work, Andreas Osiander, tried to reaffirm the hypothetical character of mathematical astronomy, and its subordinate position as a discipline relative to physics and theology. By contrast, Renaissance scholars who appreciated the physical meaning of the Copernican system called it "Pythagorean" to underscore at once its natural philosophical and mathematical character.[16] As an extreme case one could mention Bruno's declarations during his Inquisition trial. In order to defend his cosmological views, and in particular the motion of Earth, he did not mention Copernicus but the ancient philosophical school of Pythagoras: "I affirmed [the existence of] infinite individual worlds [i.e., planetary systems] similar to that of the Earth. Following Pythagoras, I regard the latter as a celestial body. The Moon is similar to it, as well as other planets and stars, which are infinite [in number]."[17] Pythagorean cosmology was regarded with suspicion by the Inquisitors and the doctrine of the plurality of worlds became one of the allegations against Bruno, who would be eventually executed as a heretic in Rome. In the same years in which Bruno was a prisoner of the Holy Office in Rome and his works were examined for censure, the censors also attacked Patrizi for his natural views, including the doctrine of terrestrial motion. Although Benedetti shared similar views about the plurality of worlds and the possibility of terrestrial motion, he did not incur any censure. We dare say that he was one of the last Renaissance authors who could freely speculate on nature in Italy before natural philosophy became a highly ideological issue in the religious repression escalating in the 1590s.

Benefiting from his subalpine freedom, Benedetti reflected on Pythagorean cosmology in a section entitled *Pythagoreorum opinionem de sonitu corporum coelestium non fuisse ab Aristotele sublatam*, where he excluded the possibility that the "sound of celestial bodies" is the production of any physical sounds. Rather, he identified the Pythagorean doctrine of the world harmony with divine providence:

> As to motions, dimensions, distances, and influences there is nothing that corresponds to such proportions, but, since all of them depend upon the infinite Divine Providence of God, these velocities, those dimensions, distances, and influences must have the most perfect order and relations among them and relative to the universe.[18]

According to Benedetti's outlook, the harmony of the heavens does not correspond one to one to musical harmony in the strict sense. From this viewpoint, Kepler's later effort to translate heavenly geometries into musical melodies in the *Harmonices mundi libri V* (1619) can be seen as a radicalization of similar "Pythagorean premises."

Most significantly, Benedetti and Kepler shared a commitment in favor of the fusion of mathematical and physical accounts of nature in the frame of an early modern transfor-

[16]Omodeo 2014a, 167–170.

[17]Bruno 2000b, doc. 13, 67: "Ho dechiarato infiniti mondi particulari simili a questo della Terra; la quale con Pittagora intendo uno astro, simile alla quale è la Luna, altri pianeti et altre stelle, le qual sono infinite."

[18]Benedetti 1585, 191: "Quod autem attinet ad motus, ad magnitudines, ad distantias et ad influxus, nihil est, quod hisce proportionibus conveniat, sed quia haec omnia dependent ab infinita et divina providentia Dei, necessario sit ut istae velocitates, eae magnitudines, distantiae et influxus, talem ordinem et respectum inter se ipsa et universo habeant, qualis perfectissimus sit."

mation of natural science in which the methods of the physico-mathematical disciplines gained a paradigmatic status. The epistemological shift also involved well-established disciplines such as astronomy. Kepler's astrophysics, first illustrated in the *Astronomia nova* (1609), was a significant step toward the derivation of celestial geometries from physical forces. Kepler translated a geometrical discipline (Ptolemaic and Copernican mathematical astronomy) into a physico-mathematical one. In fact, he explained the elliptical path of planetary orbits as the effect of interactions of forces. He emphasized the double bound of his astronomy, inseparably intertwining physics and mathematics, in the title of the work: *Astronomia nova αἰτιολογιτός seu physica coelestis de motibus stellae Martis* (New Astronomy Investigating the Causes, or Celestial Physics Concerning the Motions of Mars). As Kepler announced in the introduction: "In this work I mixed celestial physics with astronomy."[19] He meant to launch a new discipline, "celestial physics," that merged mathematical modeling with causal physics.[20] Kepler remarked that the ignorance of physical causes compels scholars to settle for conjectures since no choice can be made between mathematically equivalent hypotheses. By contrast, physical arguments are decisive in deciding between mathematically equivalent models. Therefore, celestial physics and astronomy should be unified. The result was a mixed science (*scientia mixta*) whose data came from the senses and whose demonstrations are expressed in mathematical terms. This physicalization is well shown in Kepler's physico-mathematical concept of "orbit" (*orbitae*) substituting that of orbs (*orbes*) (that is, the material spheres transporting celestial bodies). According to him orbit is "the path together with its physical causes—expressed as physical laws."[21] Shape and velocity of astronomical orbits depend on the force (*vis*) emanating from the sun, that is, on a physical cause of geometrical effects.[22]

Descartes's *Traité du monde et de la lumière* (completed in 1632–1633, but printed posthumously, in 1664) and the *Principia philosophiae* (1644) marked a culminating point in the move toward the reduction of natural disciplines (such as optics and astronomy but also physiology) to material interactions of corpuscles in motion. Descartes's philosophy was particularly influential as it legitimized a mathematical treatment of nature with the advances of physics in his time. At the same time, he connected his explanations to views on matter and causality irreconcilable with the qualitative, essentialist, and teleological accounts of the Scholastic tradition. In particular, his mechanization elevated the results of Renaissance mechanics to a higher and more generalized level.

Benedetti's place is rather at the beginning than at the end of this process. As the title of his major work hints, he was committed to a mathematical-physical investigation of nature. He did not limit his application of a mathematical method to those fields where this approach was already established, but extended it to the treatment of all realms of natural inquiry.

4.3 The Contingency of Nature and Mechanics

Benedetti's mathematical approach to nature did not lead him to the belief that physical phenomena are ruled by necessity. Rather, he shared a medieval and early-modern ontology and epistemology of contingency enabling a particular cohabitation of mathematized physics and indeterminism (in other words, formal determination without causal neces-

[19] Kepler 1937–2001, vol. 3, 19.
[20] Gingerich 1975, 261–278.
[21] Goldstein and Hon 2005, 76.
[22] On Kepler's discovery, see Donahue 1988, Donahue 1993 and Wilson 1968.

sity). In order to better understand it one has to look at Scholastic motives informing his physics, in particular his mechanics, and the scientific and philosophical work of his successors. This will require a short excursus.[23]

4.3.1 Scholastic Treatments of Nature as the Realm of Contingency

It would be misguided to think that a mathematical approach to nature in Renaissance science implies the assumption that natural causation is ruled by necessity. This was indeed not the case for well-established medieval and Renaissance views. Only in the course of the seventeenth century would contingency be banned from the realm of natural causation in the developments of post-Cartesian mechanism. For philosophers such as Baruch Spinoza and Gottfried Wilhelm Leibniz contingency marked the limitations of our knowledge and not an ontological limitation of nature. As one reads for instance in Spinoza's *Ethica ordine geometrico demonstrata* (Ethics, demonstrated in geometrical order) I 29: "There is no contingency in nature. All natural beings are determined by divine necessity to exist and operate in a special manner." (*In rerum natura nullum datur contingens, sed omnia ex necessitate divinae naturae determinata sunt ad certo modo existendum et operandum*). By contrast, in the Renaissance a mathematical treatment of natural phenomena underlaid no principle of sufficient reason, hence it did not imply the necessity of natural causation. In particular, mixed mathematical disciplines that had received a Scholastic embedment or systematization rested on a well-established Aristotelian conception, according to which sublunary phenomena are determined without necessity.

Historically, *contingentia* is the Latin variant translation of the Aristotelian concept of "possibility," both as modal logical *endechomenon* as well as physical-metaphysical *dynamis* within a hylemorphic framework. In the context of the Christian reception, this terminus received an onto-theological connotation in a frame of creationist theology. In late Scholasticism, *contingentia* came to signify the worldly reality, or nature as Creation. Nature was deemed to be contingent. It exists *de facto* but could also not exist because it depends on God's will. As John Duns Scotus put it,

> So then, the first issue has become clear: how there is contingency in things—because it comes from God—and what is in God which is the cause of this contingency—because it is his will.[24]

In Aristotle, there was a tension between two meanings of "possibility." According to *Analytica Priora* (13: 32 a 18–20) the possible is that which is "neither necessary nor impossible," whereas according to *De interpretatione* (13: 22 a 14–13 a 26) possibility is exclusively that which is opposed to "impossibility" and therefore includes also that which is necessary. As a reminiscence of this original tension, one can find in Scholastic philosophy two different definitions of contingency either as "quod est nec impossibile nec necessarium" (that which is neither impossible nor necessary) or "quod non est impossibile" (that which is not impossible).[25] Both meanings were kept in the Latin rendering of the Aristotelian possibility as *contingentia* by Gaius Marius Victorinus (III–IV cent.

[23] We have first discussed contingency and mechanics in the Renaissance in Omodeo and Renn 2015. A volume entirely devoted to ontological and epistemological contingency in the natural debates of early modernity is Omodeo and Garau 2019.

[24] Duns Scotus 1994, 140: "Sic igitur apparet primum, quomodo est contingentia in rebus, quia a Deo, – et quid est in Deo quod est causa huius contingentiae, quia voluntas eius."

[25] Cf. Vogt 2011, 52. The entire first chapter is relevant for a historical overview of the reception and transformation of the Aristotelian concept of "possibility" as "contingency" in the Latin tradition.

CE) and Boethius (IV–V cent. CE), but the Latin expression also suggested affinity between that which is contingent (*contingit*) and that which occurs (*evenit* or *accidit*).[26] This third connotation would eventually prevail through the late-Scholastic differentiation between *contingentia* and *possibilitas* and its reception in the philosophical systems of the seventeenth century (and most notably by Leibniz).[27] Unlike abstract (purely logical) possibility, contingency referred only to that which is real but not so by necessity: "id, quod [est sed] potest non esse" (that which [is but] could not be). In the Christian perspective of the Almighty's Creation, contingency happened to include all that is not God himself, that is to say, nature, or the universe.

This background is fundamental to understand not only theological disputes but also natural philosophical and scientific developments during the Middle Ages and the Early Modern Period. The connotation of nature as contingent—as that "which could not be"—is theological and metaphysical in its essence, since it points to the dependency of the world on God. However, from the point of view of natural conceptualizations, not only the "vertical" dimension of metaphysics is relevant but also the "horizontal" dimension of causality within nature. On the horizontal plane of the interrelation among finite beings, contingency refers to a degree of indetermination, and a certain unpredictability in the connection between causes and effects. Moreover, whereas a theological perspective focuses on the radical contingency of that which exists as created being, natural philosophy addresses the *relationship* between contingency and necessity within nature, that is, between divine order and phenomenal imperfection. This relationship between that which is not necessary and that which is necessary *had to* be conceptualized and indeed was conceptualized as the relationship between the *absolutum* and the *conditionale* or *secundum quid*.

In Book 1 of the *Summa contra gentiles*, Thomas Aquinas defined contingency through its distinction from necessity. In the case of the contingent beings, as one reads in *Summa contra gentiles* I 67, a cause can produce its effect or not, whereas in the case of necessary beings, their cause cannot not produce them:

> The contingent differs from the necessary according to the way each of them is found in its cause. The contingent is in its cause in such a way that it can both not-be and be from it; but the necessary can only be from its cause. […] Just as from a necessary cause an effect follows with certitude, so it follows from a complete contingent cause if it be not impeded.[28]

A contingent cause, as one reads, will fulfill its tendency to produce a certain effect "si non impediatur," that is, if no impediment hinders its realization.

In Book 2 of the *Summa contra gentiles*, Thomas dealt extensively with the contingent being ("*omne quod est possibile esse et non esse*" and "[*id quod*] *ad utrumlibet se habet*").[29] According to him, the world is contingent insofar as it is created. In this general sense, "God is to all things the cause of being" (*Summa contra gentiles* II 15).[30]

[26] Vogt 2011, 50.

[27] Schepers 1965.

[28] Aquinas 1975, 221f: "Contingens a necessario differt secundum quod unumquodque in sua causa est: contingens enim sic in sua causa est ut non esse ex ea possit et esse; necessarium vero non potest ex sua causa nisi esse. […] Ex causa necessaria certitudinaliter sequitur effectus, ita ex causa contingenti completa si non impediatur."

[29] Thomas, *Summa contra gentiles* II,15. Cf. Aquinas 1975, 48: "everything that can be and not-be" and "it is indifferent to either."

[30] Aquinas 1975, 46: "Deus est omnibus causa essendi."

In particular, God's free will is the origin of this world. Nonetheless, Thomas does not exclude that natural reality is populated by both necessary and contingent beings. Absolute necessity (*necessitas absoluta*), he writes in *Summa contra gentiles* II 29, does not pertain to God, since His decision and action is independent from any constriction (*debitum*). Rather, absolute necessity pertains to the immaterial, or "separated" beings as well as to those bodies in which the form fulfills all potentialities of their matter, as is the case with the heavenly bodies transported in circles. As for terrestrial (sublunary) bodies, their forms are imperfectly realized. Matter, as the potentiality to take different forms, is at the origin of their contingency, that is, it is the source of the possibility to realize or not to realize a certain effect: "But in things whose form does not fulfill the total potentiality of the matter, there still remains in the matter potentiality to another form."[31] For the low realm of birth, corruption, and change, Thomas speaks of conditional necessity (*necessitas conditionalis*). In the sublunary sphere, contingency cohabits with absolute necessity (e.g., the inevitability of death for all animals and the hylemorphic composition of all bodies). Whereas necessity pertains to the formal determinations of natural phenomena, contingency is the partial fulfillment of necessary tendencies.[32]

According to Scholastic terminology, there is always a "quid" producing the deviation of material phenomena from their formal rule. We will call this outlook an "ontology and epistemology of contingency."[33] The Pythagoreanism of many Renaissance scholars such as Benedetti did not depart from a view stressing the contingent character of natural phenomena in general. As we will argue, one encounters in Benedetti's physics and mechanics a systematic use of theoretical tools implying natural contingency in the form of a distinction and interrelation between formal mathematical necessity and its material realization. In order to understand Benedetti's mathematical treatment of contingency it is useful to consider the medieval approaches to contingency, especially the science of weights (*scientia de ponderibus*) he relied upon.

The idea of contingency informing physics and mechanics was related to its use in other disciplines, even ethics. Whereas there can be no obstacle impeding the realization of God's will, which is therefore "absolute" (*voluntas absoluta*), human will, or *voluntas secundum quid*, is conditioned by circumstances. In other words, the realization of the highest aims of humankind is intrinsically contingent, as Dante expressed in the *Divine Comedy*:

> But utterance and feeling among mortals,
> For reasons which are evident to you,
> Have different feathers making up their wings.
> I, too, as man feel this disparity […].[34]

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[31] *Summa contra gentiles* II 30: "In quibus [rebus] vero forma non complet totam potentiam materiae, remanet adhuc in materia potentia ad aliam formam." Cf. Aquinas 1975, 87.

[32] *Summa contra gentiles* II 23: "Omnis enim agentis per necessitatem naturae virtus determinatur ad unum effectum. Et inde est quod omnia naturalia semper eveniunt in eodem modo, nisi per impedimentum: non autem voluntaria. Divina autem virtus non ordinatur ad unum effectum tantum […]. Deus non agit per necessitate naturae, sed per voluntatem." Cf. Aquinas 1975, 68: "For the power of every agent which acts by natural necessity is determined to one effect; that is why all natural things invariably happen in the same way, unless there be an obstacle; while voluntary things do not. God's power, however, is not ordered to one effect only […]. Therefore, God acts, not out of natural necessity, but by His will."

[33] Omodeo and Renn 2015.

[34] Alighieri 1984, 94; Dante Alighieri, *Paradiso* XV 79–83:

> "Ma voglia e argomento ne' mortali,
> per la cagion ch'a voi è manifesta,

Apart from ethical contingency, Scholastic authors also used *secundum quid* in logic. For instance, Petrus Hispanus explained the meaning of the so-called *secundum quid* fallacy in his *Tractatus sive summule logicales*, commenting on Aristotle's *On Sophistical Refutations* V (166b36–167a14).[35]

In logic, *secundum quid* meant either a "diminution" of a concept through restriction of its definition (*secundum quid et simpliciter*), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*). A *secundum quid* fallacy occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (or *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument "est animal pictum, ergo est animal" is not correct. In this case, there is a quid signalizing the gap between universal necessity and particular or concrete contingency.

4.3.2 Contingent Causation in the *scientia de ponderibus*

The *scientia de ponderibus* heavily drew on the idea of the conditional limitation of natural necessity depending on circumstances (*secundum situationem*, also literally meant as "depending on the position"). In particular, the concept of *gravitas secundum quid*, or positional heaviness, had a powerful explanatory function, most notably in the Aristotelian treatment of weights by Jordanus Nemorarius, and continued to be essential during the Renaissance in the reflections on mechanics by scholars such as Tartaglia, Cardano, and Benedetti himself.[36]

In mechanics the "limitation" or "determination" *secundum quid* implied that the dynamic tendency of a body was reduced or enhanced depending on intervening constraints or circumstances, in particular mechanical ones. The rotations of a lever around a pivot or of a balance around its fulcrum were conceptualized as constrained motions. In such displacements, the inherent ("necessary") vertical tendency of a weight resulted in a circular motion due to external constraints. Similarly, the heaviness (*gravitas*) of the bodies suspended at the extremities of a simple machine varied in relation to their changing positions within the system. In such cases, a "necessary" straightforward motion in accordance with natural order resulted contingently in a circular one. The implicit mental model for this kind of displacement was that "circular motion is constrained rectilinear motion." This means that, in the sublunary sphere of contingency, straightforwardness and rectilinear tendency had a higher onto-epistemological status than circularity since straightforwardness was necessarily rooted in natural order. By contrast, circularity, as the deviation from such order, had to be explained. As a consequence, circularity (in the elementary sphere) was allotted a derived and subordinated onto-epistemological status. In other words, circularity was an instance of nature departing from necessity owing to some rather elusive factor or *secundum quid*. From this viewpoint, it was seen as a deviant realization of given potentialities similar to moral deviation from the necessary laws of uprightness. In order to stress that the mechanical treatment of the *scientia de ponderibus* was embedded in the framework of contingency, we could also formulate the principle in this way: "circular motion is rectilinear motion modified by a contingency."

---

diversamente son pennuti in ali;
ond' io, che son mortal, mi sento in questa
disagguaglianza […]."

Also, see *Paradiso* IV, 87, IV, 109, IV, 113, and *Purgatorio* VII, 57.

[35] Hispanus 1972, 157–158.

[36] See Renn and Damerow 2012, especially the sections from 3.6 to 3.8.

Almost at the beginning of his small treatise "on the weights," Nemorarius stressed his Aristotelian commitment. In fact, his approach was based on the opposition between the *natural* vertical motion of the elements and the *violent* hindrances producing circular deviation. At the same time, he introduced the key concept of *gravitas secundum quid* (in some cases, also *levitas secundum quid*), which we will refer to as "positional heaviness."

> […] if equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter, to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously. I then say that since the longer arc of the circle is more contrary to a straight line than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the fall which the heavy body would have along the straight line than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc than over the shorter one; otherwise the motion would become heavier. Since something moves with more violence in the ascent [along the arc], it is apparent that there is more positional heaviness [*gravitas secundum situm*] and, as it is like that depending on position [*secundum situationem*], one can aptly call it 'positional heaviness' [*gravitas secundum situm*].[37]

In its circular descent along a circular path, a weight deviates from its natural tendency, or *intentio*, the more the arm of the balance departs from the horizontal position. Therefore, the "violence" is greater when the arc of displacement is longer, while the weight progressively loses its weight insofar as the vertical component in its motion is reduced.

According to Nemorarius, a weight that reaches the bottom of the circular arc described by the arm in its displacement is not "at rest" but only "lighter." In fact, a natural being is at rest only if it is fully accomplished, that is, once it has realized the aim, or act, toward which its power is directed teleologically. By contrast, a body is always in motion, or striving to move, until it has reached its end: "All motion strives toward its aim—indeed the whole nature strives towards actuality and is realized [in it]—hence the opposition occurs against [a displacement] contrary [to the natural tendency]."[38]

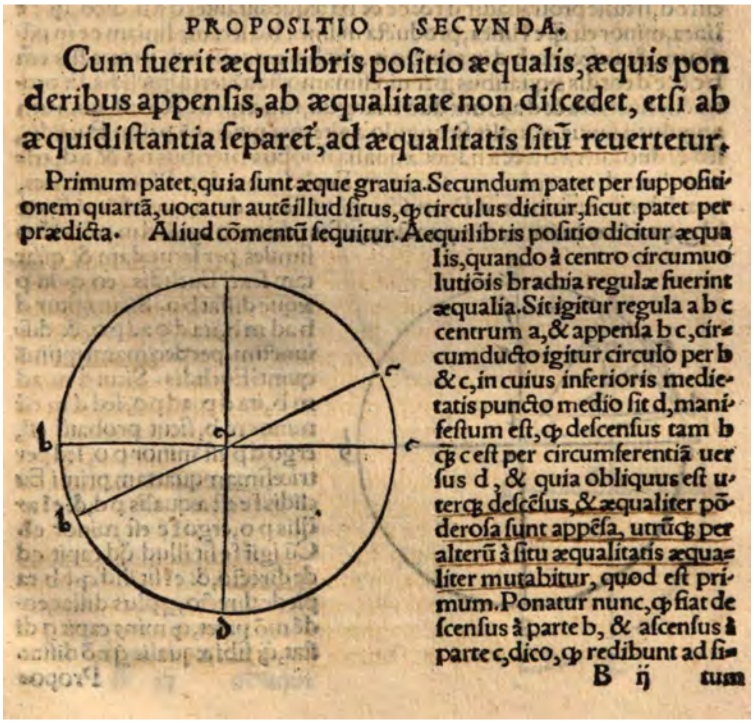
A body on one arm of the balance becomes lighter during its downward motion than an equal one located on the other extremity. Thus, as Nemorarius assumes, or tries to demonstrate, a balance removed from its state of equilibrium will tend to restore the original state. As one reads in the *propositio secunda* (with reference to the diagram in Figure 4.1), which is the second of a series of propositions developing the details of Nemorarius's doctrine of weights,

> Suppose now that the descent occurs on the side *B* and the ascent on the side *C*. I say that both will go back to the [horizontal] position of equality. In fact,

[37]Nemore 1533, f. A3*v* (emphasis added): "[…] si sumantur de circulo maiori et minori arcus aequales, corda arcum maioris circuli longior est. Propeterea posset ex hoc ostendi, quod pondus in libra tanto sit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod maior arcus circuli plus contrariatur rectae lineae quam minor, et casus gravis per arcum maiorem, plus contrariatur casui gravis, qui per rectam fieri debet, quam casus per arcum minorem. Patet ergo *maior est violentiam in motus secundum arcum maiorem*, quam secundum minorem. Aliter enim fieret motus magis gravis. Cum ergo plus in ascensu aliquod movetur violentiae, patet, quam maiore est *gravitas secundum situm, et quia secundum situationem* talium sic sit, dicatur gravitas secundum situm."

[38]Nemore 1533, ff. A3*v*–A3*r*: "In termino enim cuiscunque motus intenditur, intenditur et viget tota natura in actu, qui in motu sit quasi in potentia, secundum quem fiebat contrarietatis suae oppositio."

> *B* will not further descend, because its descent towards *D* is more oblique than the ascent of *C* towards the [horizontal position of] equality; in fact, *B* and *C* are equidistant from the place of equality.[39]



PROPOSITIO SECVNDA.

Cum fuerit æquilibris poſitio æqualis, æquis ponderibus appenſis, ab æqualitate non diſcedet, etſi ab æquidiſtantia ſeparet, ad æqualitatis ſitũ reuertetur.

Primum patet, quia ſunt æque grauia. Secundum patet per ſuppoſitionem quartã, uocatur autẽ illud ſitus, ꝙ circulus dicitur, ſicut patet per prædicta. Aliud cõmentũ ſequitur. Aequilibris poſitio dicitur æqualis, quando à centro circumuolutiõis brachia regulæ fuerint æqualia. Sit igitur regula a b c centrum a, & appenſa b c, circumducto igitur circulo per b & c, in cuius inferioris medietatis puncto medio ſit d, manifeſtum eſt, ꝙ deſcenſus tam b ꝗ̃ c eſt per circumferentiã uerſus d, & quia obliquus eſt uterq; deſcẽſus, & æqualiter põderoſa ſunt appẽſa, utrũq; per alterũ à ſitu æqualitatis æqualiter mutabitur, quod eſt primum. Ponatur nunc, ꝙ fiat deſcenſus à parte b, & aſcenſus à parte c, dico, ꝙ redibunt ad ſitum

B ij

Figure 4.1: Diagram accompanying preposition two in Apianus's 1533 edition of Nemorarius's *Liber de ponderibus* (1533, f. B2*r*). (Bayerische Staatsbibliothek)

Nemorarius's reasoning becomes clearer in light of propositions four and five:

> Fourth [proposition]: It is positionally heavier, insofar as its descent, in the same position, is less oblique.
>
> Fifth [proposition]: But a more oblique descent partakes less of the straight [descent], for the same quantity [of the path].[40]

In proposition five, it is suggested that the vertical components of the potential descents of the two beams could be identified and compared. This was the source of the idea that the variation of heaviness could also be determined by comparing the straightness of the descents. A similar procedure was later taken up and explained in detail in Niccolò Tartaglia's considerations in the *Questiti et inventioni diverse* (1546) about the manner of ascertaining the positional heaviness of two weights on the basis of the so-called angles of contact. These are the "curvilinear" or "mixed" angles between the circular path of the

[39] Nemore 1533, ff. B2*r*–*v*: "Ponatur nunc, quod fiat descensus a parte B, et ascensus a parte C, dico quod redibunt ad situm aequalitatis. Non enim ulterius descendet B, eo quod descensus eius versus D magis obliquus est, quam ascensus C ad aequalitatem; B enim et C iam aequaliter distant a situ aequalitatis."

[40] Nemore 1533, f. A3*r*: "Quarta [propositio]: Secundum situm gravius esse, quanto in eodem situ minus obliquus est descensus. Quinta [propositio]: Obliquiorem autem descensum minus capere de directo, in eadem quantitate." Translation from Renn and Damerow 2012, 63. For proposition four, see Nemore 1533, f. B3*v*–B4*r* and, for proposition five, Nemore 1533, f. B4*r*–C2*v*.

arms of a balance and the vertical lines connecting the weights to the cosmological center of gravity (see Figure 4.2). Tartaglia compared the angles of contact of two equal weights located on the extremes of a balance, and argued that the lifted one is always smaller than the lowered one. Thus, the lifted weight would face a descent that is more oblique. It would acquire a greater positional heaviness than its lowered counterweight and, as a further consequence, the inclined system would reestablish its horizontal balance, if not hindered to do so.

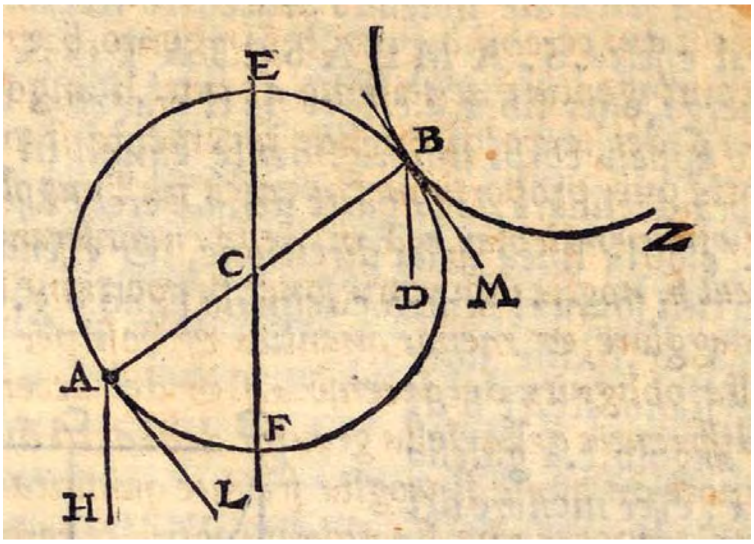


Figure 4.2: In the *Quesiti et invenzioni diverse*, Tartaglia argued that the relative positional heaviness of the weights A and B on a balance could be determined on the basis of the "mixed" angles of contact HAF and DBF. Since it is argued that DBF < HAF, the weight B will be heavier than A. Thus, the inclined system will strive toward the restoration of a horizontal equilibrium. (Max Planck Institute for the History of Science, Library)

In spite of his attempt to quantify the *quid* accounting for the alleged restorative motion of the inclined balance, Tartaglia's geometrical quantification maintained a margin of indeterminancy. As he stated, the ratio between the two mixed angles is less than that between any determined quantities. Therefore, it is impossible to stabilize the system in its inclined position by adding a small (no matter how small) weight on the lowered side of the balance. According to Tartaglia, it is impossible to counterbalance the positional heaviness of the lifted weight. Quite on the contrary, any additional weight added to the lowered side would make the balance rotate and reach the vertical position.[41]

4.4 The Epistemological Import of Benedetti's Generalization from Weights to Forces

As we have argued so far, in the medieval *scientia de ponderibus* circular motion is conceived of as constrained linear motion. Yet, within an Aristotelian cosmology, this mental model is restricted to the sublunary sphere, where motions cannot fulfill their nature. This is indeed the sphere of contingency, where a gap is to be witnessed between the necessary

[41] Tartaglia's approach was controversial, already in his time. See Renn and Omodeo 2013, sec. 3.6.

order of things (or “nature” as actuality) and the effective phenomena (subjected to “violence” or to external constraints). The four elements naturally tend toward their places through a straightforward descent or ascent. Heavy bodies, for instance, strive toward the center of gravity, which is, at the same time, the center of the cosmos. If their motion is hindered, as is the case with mechanical constraints, a certain factor or *quid* has to be taken into account, which explains the deviation from the rule. In this theoretical context, contingency is the concept expressing the relationship between the natural law and phenomenal reality, which follows a norm while deviating from it. The *secundum quid* is that which explains this deviation. Possibly, it has to be expressed through geometrical means, although it might prove unintelligible or infinitesimal, as was the case with Tartaglia’s ratio between mixed angles accounting for the *gravitas secundum quid* of the weights of a balance. In the treatment of weights, in particular of those on a balance, Nemorarius and his followers made a limited use of the mental model of curvilinear motion as constrained linear motion. In fact, they employed it to account for phenomena linked to gravity (i.e., the vertical fall of bodies explained in Aristotelian terms). It was Benedetti who made the decisive step toward the generalization of this model in the direction of inertial dynamics. Let us consider his application of it first to balances and then to centrifugal forces.

In the section on mechanics of the *Diversae speculationes*, Benedetti picked up and revised the Scholastic concept of *gravitas secundum quid*. Guidobaldo del Monte had already criticized Nemorarius’s and his followers’ conclusion that an inclined balance hinged on its fulcrum as its center of gravity would return to the horizontal position, but his criticism went so far as to renounce the concept of positional heaviness altogether.[42]

Relying on the Archimedean concept of the center of gravity of a body, Del Monte concluded that an equal-arms balance hinged on its fulcrum would remain stable in any position (a correct conclusion only if it is assumed, in modern terms, that the gravitational field is homogeneous): “*Propositio IV: Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus, ubicunque relicta manebit*.” (Fourth Proposition: Take a balance that is equidistant from the horizon and that has weights in its extremities which have the same weight and equally distant from the center (the latter being located in the balance itself). Whether it is displaced or not, it will remain in the same position in any position.)[43]

Benedetti shared the criticism of Nemorarius and Tartaglia with regard to their specific argumentation about the tendency of such an inclined balance to reach the horizontal position but based his judgement on a novel treatment of positional heaviness. The first chapter of Benedetti’s *De mechanicis* begins with the statement: “Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself.”[44]

Hence, he clearly committed himself to a mechanical theory of equilibrium based on positional heaviness. Benedetti’s technical terms are not always employed in a rigorous and consistent manner. He treats the *pondus* at times as the varying quantity to be taken into consideration, as is shown by expressions like “*proportio ponderis in C ad idem pondus in F*” and “*unde fit… pondus magis aut minus grave*,” in *De mechanicis* II (Benedetti 1585, 142). Given these semantic fluctuations, we will translate *pondus* as “body” or as

[42] Renn and Damerow 2012, 86–92. We will discuss the divergent interpretations of Benedetti and Del Monte later, in chapter 5.
[43] Damerow and Renn 2010, 65.
[44] Drake and Drabkin 1969, 166. Benedetti 1585, 141: “Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet.”

“weight” and *gravitas* as “heaviness” or as “weight,” depending on the context. At the beginning of chapter 1 of his book on mechanics, Benedetti talks of a varying quantity of heaviness, or gravity (*gravitas*), belonging to a weight (*pondus*) or a body placed on a balance beam. Hence, he makes a terminological distinction between *pondus*, as a kind of absolute weight or heavy thing, and *gravitas*, as a downward tendency that can act with more or less force on the body (depending on the inclination of the beam). In this case (as in most cases in the text), *pondus* has the essentialist meaning of a substance (a substratum or ὑποκείμενον). It is the body or weight on the balance, whose special property of being heavy, namely the *gravitas*, varies depending on a *quid*. This *quid* is the position, or *situm*.

Benedetti seeks to quantify it by means of a method he invented. He considers the line, which he calls *linea inclinationis* or *linea itineris*, connecting a weight on an inclined balance beam to the cosmological center of gravity. Note that Benedetti calls the elementary downward tendency an *iter* from a merely kinematic viewpoint, but also an *inclinatio* from a physical and more proper one. According to him, the major or minor heaviness of the weight can be assessed through the projection of the *linea inclinationis* on the horizontal line passing through the fulcrum (Figure 5.1). The more distant it is from the fulcrum, the heavier the positional heaviness becomes. Thus, the weight reaches a maximum of heaviness when the balance is horizontal, and its minimum when it is vertically resting (*nititur*) on the fulcrum or hanging (*pendet*) from it. Notably, this approach anticipates the one based on the determination of the torque in classical physics, and comes to the same conclusions.[45]

Additionally, Benedetti equates the heaviness to a *virtus*, *vis*, or *vigor*, i.e., a force, which might also act in different directions (in *De mechanicis*, Ch. 3) and is applied to the extremity of a constrained mechanical system, like a lever or a balance. This is a significant generalization from weights to forces, but for our present discussion the most important generalization relates to rectilinear tangential tendencies in systems set in circular motion.[46]

The relevant treatment is the epistle to Capra and is included in the *Diversae speculationes*. It deals with the rotation of a millstone and the question of whether its motion could be perpetual. Benedetti denies this by arguing that the rotation is impeded first by the friction of the air and, second and more importantly, by the resistance of the millstone’s parts. The latter have a straightforward tendency, an *inclinatio recte eundi*, along the tangential lines of their rotation (Figure 4.3). As one reads, this rectilinear inclination or impulse (*impetus*) can be bent only by violence. Moreover, the centrifugal tendency grows in proportion to the augmentation of the velocity, as witnessed by other cases, among them the rotation of a catapult or a sling (*machina missilis*). A centrifugal tendency is seen as a rectilinear natural inclination (*naturalis inclinatio recte eundi*).

> You ask me this question in your letter. Suppose a millstone rested on a virtually mathematical point and was set in circular motion, could that circular motion continue without end, assuming that the millstone is perfectly round and smooth?
>
> I answer that this kind of motion will certainly not be perpetual and will not even last long. For apart from the fact that the wheel is constrained by the air which surrounds it and offers resistance to it, there is also resistance from

[45] Renn and Damerow 2012, 138. We will deal with the details of Benedetti’s mechanics in the next section.
[46] Cf. Büttner 2008.

the parts of the moving body itself. When these parts are in motion, they have by nature a tendency [*impetus*] to move along a straight path. Hence, since all the parts are joined, and any one of them is continuous with another, they suffer constraint in moving circularly and they remain joined together in such motion only under compulsion. For the more they move, the more there grows in them the natural tendency to move in a straight line, and therefore the more contrary to their nature is their circular motion. And so they come to rest naturally: for, since it is natural to them, when they are in motion, to move in straight line, it follows that, the more they rotate under compulsion, the more does one part resist the next one and, so to speak, hold back the one in front of it.[47]

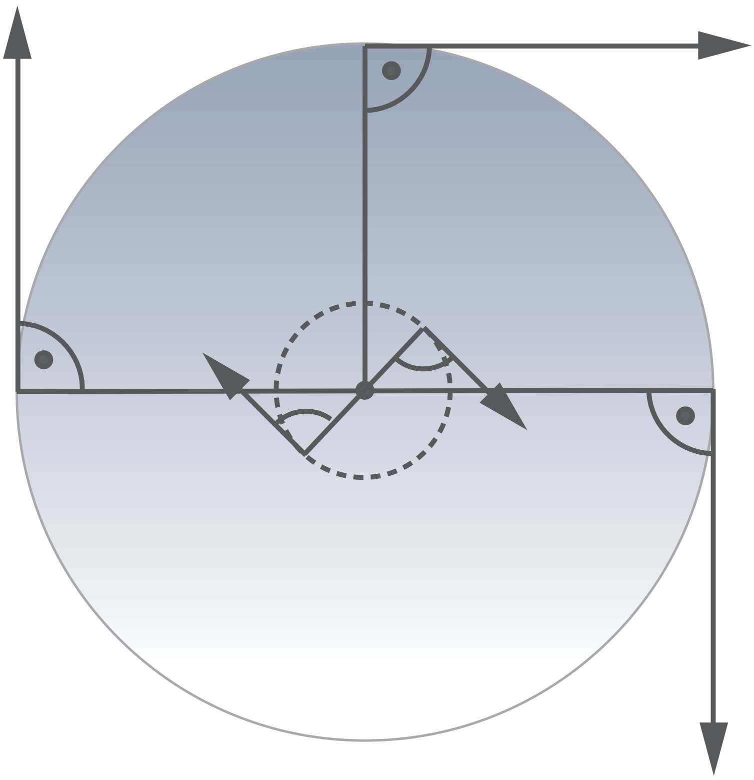


Figure 4.3: A diagram showing Benedetti's considerations on the rotating millstone stressing the centrifugal tendencies of its parts. (Drawing by Irina Tupikova)

The mental model of circular motion as constrained straight motion receives in Benedetti's treatment a higher degree of generalization. In this case, he argues that, since it contrasts

[47] Drake and Drabkin 1969, 229. Benedetti 1585, 285 (emphasis added): "Quaeris a me literis tuis, an motus circularis alicuius molae molendinariae, si super aliquod punctum, quasi mathematicum, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfecte rotundam, et laevigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia praeterquam quem ab aere qui ei circumcirca aliquam resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quae cum motae sunt, natura, impetum habent efficiendi iter directum, unde cum simul iunctae sint, et earum una continuata cum alia. *Dum circulariter moventur patiuntur violentiam, et in huiusmodi motu per vim unitae manent, quia quanto magis moventur, tanto magis in iis crescit naturalis inclinatio recta eundi, unde tanto magis contra suammet naturam volvuntur*, ita ut secundum naturam quiescant, quia cum eis proprium sit, quando sunt motae, eundi recta, quanto violentius volvuntur, tanto magis una resistit alteri, et quasi retro revocat eam, quae antea reperitur habere."

with a natural inclination, it cannot be eternal. Note that this assumption (violent motion cannot be eternal) is Aristotelian but emerges in a context in which this legacy is meant to be rejected.[48]

Another Aristotelian echo looms over Benedetti's statement that the linear tendency makes a body "lighter," since if it were freed from the constraint hindering its projection, it would not fall vertically but rather travel through a more or less rectilinear trajectory tangent to the circular motion of the constrained rotation. In the conclusion of his reflection on the natural rectilinear striving of the parts of a body set in circular motion, Benedetti stressed the originality of his treatment "without precedents" and its opposition to Aristotelian dynamics (according to which the projection of a body through a medium presupposes the support of the medium itself).

> But if you wish to see this truth more clearly, imagine that while the body, i.e., the top, is spinning around very rapidly, it is cut up or divided into many parts. You will observe not that those parts immediately fall toward the center of the universe, but that they move in a straight line, and, so to speak, horizontally. No one, so far as I know, has previously made this observation on the subject of the top.
>
> From such motion of the top or of a body of this kind it may be clearly seen how mistaken are the Peripatetics on the subject of the forced motion of a body. They hold that the body is driven forward by the air which enters [behind it] to occupy the space left by the body. But actually the opposite effect [that is to say, resistance] is produced by the air.[49]

We have so far observed two instances in Benedetti's work on mechanics in which a tension between mathematical laws of nature and their empirical realization emerges: his treatment of the rotation of a beam about its pole and that of a turning wheel. In both cases, natural straightforward tendencies are constrained and deviated into violent circular ones. The epistemological meaning of these concepts lies in the possibility of a geometrical treatment of natural contingency seen as the connection between the *necessity* of the rules and of the principles and their *necessitation*, that is, their deviation, as witnessed by the empirical reality of curvilinear motions.

4.5 From *inclinatio* to *inertia* and Beyond: Mechanistic Perspectives

René Descartes generalized the insights implicit in the idea that curvilinear motion is contingent rectilinearity at an epistemic level (through the expansion of their realm of application) as well as at an epistemological and ontological level (giving them a foundational meaning). In *Le Monde*, circular motion is treated as a deviation from rectilinear motion.

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[48] On Benedetti's anti-Aristotelianism, see Maccagni 1983.

[49] Drake and Drabkin 1969, 229–230. Benedetti 1585, 285: "Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, trochum scilicet, dum velocissime circunducitur secari, seu dividi in multas partes, unde videbis illas omnesque, non illico versus mundi centrum descendere, sed recta orizontaliter, ut ita dicam, moveri. Id quem a nemine adhuc (quem sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clare perspicitur, quam errent peripatetici circa motum violentum alicuius corporis, qui existimant aerem qui subintrat ab occupandum locum a corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur."

Descartes develops a general theory of the world in which circularity is the main characteristic of the motions of both the particles of matter as well as of planets revolving about the centers of their orbits.[50]

> […] when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is, the inclination they have to move, is different from their motion.[51]

This is the third of Descartes's three laws of nature (*loix or règles de la Nature*) as exposed in chapter 7 (" *Des loix de la nature de ce nouveau Monde*"). It follows the inertial law of conservation of the state of the bodies and that of the conservation of the quantity of motion. The third law is particularly relevant from the viewpoint of our epistemological inquiry into mathematics without necessity, since it clearly expresses the gap between law and effective reality, between the straightforward tendency of all bodies and their real circular motions, in a manner that is akin to medieval and Renaissance predecessors such as Benedetti. Note that Descartes calls the rectilinear tendency " *inclination*" just as Benedetti called it "*inclinatio recte eundi*." This terminological choice is apt to express its character as a natural inner tendency. The examples that Descartes choses to illustrate his claim are familiar to readers of Renaissance sources on mechanics: the wheel (*une roue*) and the sling (*fronde*) (Figure 4.4).

In the *Études galiléennes*, Koyré affirmed the complete independence of the law of inertia, which is only *in nuce* in Galileo's physics, from experience, since rectilinear motion is never observed in nature. "Contrairement à ce qu'on affirme bien souvent, la loi d'inertie n'a pas son origine dans l'expérience du sens commun et n'est ni une généralisation de cette expérience, ni même son idéalisation. Ce que l'on trouve dans l'expérience, c'est le mouvement circulaire ou, plus généralement, le mouvement curviligne. On n'est jamais—sauf le cas exceptionnel de la chute, qui n'est justement pas un mouvement inertial—en présence d'un mouvement rectiligne."[52]

In light of our reconstruction, this statement proves quite inaccurate. As we have seen, the vertical fall of a heavy body is not the only observable straight motion: the beginning of the trajectory of a projectile thrown with great speed also looks rectilinear. Slings and catapults are in fact the instruments with which turning wheels and rotating millstones were compared, and it was from these instruments that Benedetti, Descartes, and also Galileo in the Second Day of the *Dialogo sopra i massimi system del mondo*, derived the centrifugal tendencies of the parts of rotating objects. Is this not a generalization from experience? Such generalization went so far as to include the explanation of the behavior of bodies on a rotating Earth, in the case of Galileo, and the conceptualization of corpuscular and planetary motions, as was the case for Descartes. Moreover, before the classical law of inertia was defined, what took center stage was the observation of rectilinear motions—either the vertical fall or centrifugal tendencies—and of their circular deviations. A major physical problem faced by Scholastic and post-Scholastic mechanics

[50] On the Cartesian cosmos, see Aiton 1972, 30–64 and Gaukroger 2006, 304–317.

[51] Descartes 1998, 29. Descartes 1986, 43–44: " Lors qu' un corps se meut, encore que son mouvement se fasse plus souvent en ligne courbe, et qu' il ne s' en puisse jamais faire aucun, qui ne soit en quelque façon circulaire […], toutesfois chacune de ses parties en particulier tend toujours à continuer la sien en ligne droite. Et ainsi leur action, c' est à dire l' inclination qu' elles ont à se mouvoir, est differente de leur mouvement."

[52] Koyré 1986, 206.

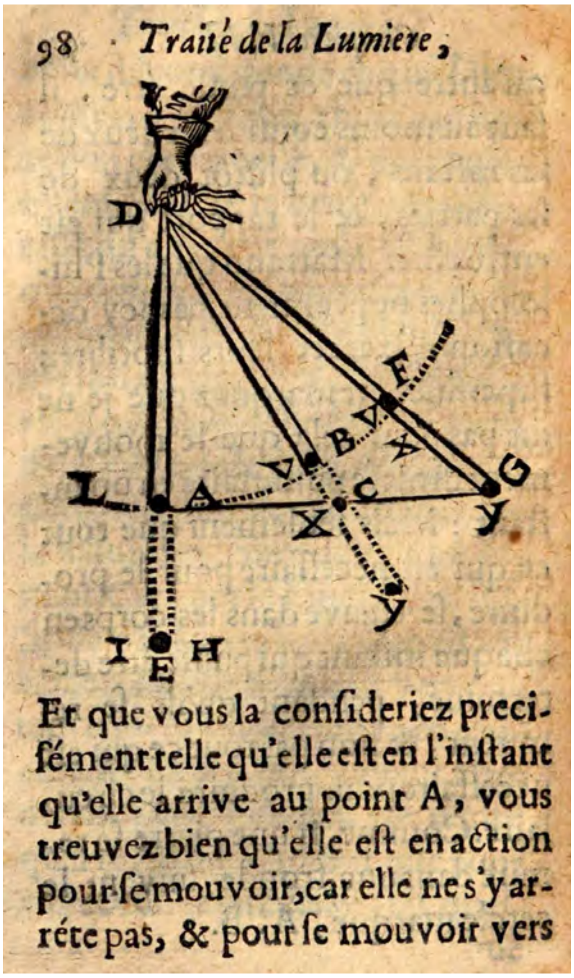

Figure 4.4: Descartes's visualization of the centrifugal tendency of bodies thrown by a sling, in *Le Monde*, Ch.7. (Bayerische Staatsbibliothek)

was precisely that of conceptualizing the relationship between curves and straight lines. In particular, against the backdrop of Aristotelian philosophy, curvilinear motion appeared as constrained. It was a derived displacement resulting from a *violent* external intervention bending the straightforward *natural* tendency of a moving body. In such an Aristotelian and post-Aristotelian context, circular motion was seen as contingent. That is to say, it was the deviation from natural order depending on an obstacle which was called the "secundum quid." As we have argued, the concept of "secundum quid" is embedded in the Scholastic reflections upon natural necessity, order, and contingency. It was referred to as a model of causality in which the observed phenomena represent a partial fulfillment of an underlying order, or of natural laws. Accordingly, elementary bodies express their necessary laws in a limited manner, that is, they have to be explained through the so-called *necessitas conditionata* or *necessitas secundum quid*. Contingency is the relation between necessary order and phenomenal reality. The gap has to be explained, and was explained with a quid, a factor, or a determination. Accordingly, a quid was introduced into mechanics to account for circular motions in terms of mechanical constraints.

In the medieval *scientia de ponderibus*, two determinations were considered for the equilibrium of a balance: first, the circle resulting from the inclusion of the vertical motions of the weights in a mechanical system, and second, the *situm* (location) of the weights in a mechanical system determining a variation in heaviness. The reflection on *gravitas secundum situm* (positional heaviness) from Nemorarius to Benedetti presupposes this twofold *quidditas* and focuses on the latter aspect (the variation of the heaviness).

The conviction that circular motion, as a violent motion, requires an explanation is based on the mental model that "circular motion is constrained (or *contingented*) straight motion." Although they were embedded in the medieval discourse on contingency, the several attempts to quantify the *quid* accounting for the deviation testify to the common effort to overcome the qualitative and indeterminable characterization of contingency as a form of causality. What was maintained, for instance in Descartes, was the idea of a gap between law and phenomenon. Yet, if the deviation from the law can be perfectly quantified, then the separation between the order of nature and its realization is virtually eliminated, that is, the fracture between absolute necessity and conditional necessity is recomposed. To be sure, this step toward the necessitation of nature, resulting from the abandonment of contingency in both senses (causal and epistemological), was accomplished only later, in the course of the seventeenth century.

The work of Benedetti and his onto-epistemology of contingency are representative of an age of transition from Scholastic and Renaissance natural philosophies to the various instantiations of the classical science of the next century. Benedetti's Pythagorean commitment to mathematics, seen as the most powerful logical means applied to all fields of knowledge and to nature in particular, is an illustrative case of the complex and non-linear history of scientific thought. His efforts to overcome Aristotelian conceptions could not really renounce the crucial assumption of the Aristotelian outlook under attack. This particularly concerns the ontology and epistemology underlying his scientific theories and practices. Mathematical determination, both in science and nature, did not imply necessity, neither at the level of material causation nor of explanation. The gap between the laws of nature and the effective processes reflected a Scholastic distinction between formal necessity and material imperfection. Such philosophical assumptions underpinned medieval treatments of phenomena, including statics, and Renaissance developments, especially in the line connecting Tartaglia and Cardano to Benedetti and Descartes. The distinction between formal necessity and phenomenal contingency offered them a horizon within which they could conceptualize general laws as well as their empirical instantiation. In particular, Benedetti could extend the area of application for the mental model that circular motion is a constrained (violent) deviation from the law of rectilinear motion. He did this by applying a model originating from statics to the area of dynamics, thus paving the way for the classical concept of inertia. However, we should not neglect the practical roots of his work in a Scholastic-embedded science of weights, which generalized observations of mechanical systems in order to make universal statements about nature.